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Abstract

Optimal monetary policy maximizes welfare, given frictions in the economic environment. Constructing a model with two sets of frictions – the Keynesian friction of costly price adjustment by imperfectly competitive firms and the Monetarist friction of costly exchange of wealth for goods – we find optimal monetary policy is governed by two familiar principles.

First, the average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions imply that the optimal nominal interest rate is positive.

Second, as various shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only small "base drift" for the price level path). Since expected inflation is roughly constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate – as there is expected growth or contraction of real economic activity.

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1 Introduction

Three distinct intellectual traditions are relevant to the analysis of how optimal monetary policy can and should regulate the behavior of the nominal interest rate, output and the price level.

The Fisherian view: Early in this century, Irving Fisher [1923,1911] argued that the business cycle was "largely a dance of the dollar" and called for stabilization of the price level, which he regarded as the central task of the monetary authority. Coupled with his analysis of the determination of the real interest rate [1930] and the nominal interest rate [1896], the Fisherian prescription implied that the nominal interest rate would fluctuate with those variations in real activity which occur when the price level is stable.

The Keynesian view: Stressing that the market-generated level of output could be inefficient, Keynes [1964 (1936)] called for stabilization of real economic activity by fiscal and monetary authorities. Within theoretical and quantitative models of macroeconomic activity constructed by his followers, stabilization policy typically mandated substantial variation in the nominal interest rate when shocks buffeted the economic system, particularly when there were shocks to aggregate demand. While most Keynesians viewed the price level as responding only gradually to these shocks, it typically changed over time as policy interventions focused on a real output target, with little importance attached to the path of the price level.

The Monetarist view: Evaluating monetary policy in a long-run context with fully flexible prices, Friedman [1969] found that an application of a standard microeconomic principle of policy analysis long used in public finance– that social and private cost should be equated – indicated that the nominal interest rate should be approximately zero. Using flexible price models of business fluctuations, later authors pointed out that the same reasoning also dictated that the nominal interest rate should not vary through time in response to real and nominal disturbances.

There are clear tensions between these three traditions if real forces produce expected changes in output growth that affect the real interest rate. If the price level is constant, then the nominal interest rate must mirror the real interest rate so that Friedman's rule must be violated. If the nominal interest rate is constant, as Friedman's rule suggests, then there must be expected inflation or deflation to accommodate the movement in the real rate so that Fisher's prescription cannot be maintained.

We construct a model economy that honors each of these intellectual traditions and study the nature of optimal monetary policy. There are Keynesian features to the economy: firms have market power, which means that output may be inefficiently low, and all prices cannot be frictionlessly adjusted. However, as in the New Keynesian research on price stickiness that begins with Taylor [1980], firms are forward-looking in their price setting, and this has dramatic implications for the design of optimal monetary policy. In our economy, there are also costs of converting wealth into consumption. These costs can be mitigated by the use of money, so that there are social benefits to low nominal interest rates as in Friedman's analysis. The behavior of real and nominal interest rates in our economy is governed by Fisherian principles.

Following Ramsey [1927], Lucas and Stokey [1983] and Ireland [1996], we determine the allocation of resources which maximizes welfare (technically, it maximizes the expected, present discounted value of the utility of a representative agent) given the resource constraints of the economy and additional constraints that capture the fact that the resource allocation must be implemented in a decentralized private economy. We assume that there is full commitment on the part of a social planner for the purpose of determining these allocations. We find that two familiar principles govern monetary policy in our economy:

The Friedman prescription for deflation: The average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions generally imply that there should be a positive nominal interest rate.

The Fisherian prescription for eliminating price-level surprises: As shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only a small "base drift" for the price level path). Since expected inflation is roughly constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate, i.e., as there is expected growth or contraction of real economic activity.

The organization of the paper is as follows. In section 2 we outline the main features of our economic model. In section 3, we identify four distortions present in our economic model, which are summary statistics for how its behavior can differ from a fully competitive, nonmonetary business cycle model. In section 4, we describe the nature of the general optimal policy problem that we solve. In section 5, we discuss optimal monetary policy in two special cases, for which analytical results can be derived. First, suppressing price stickiness, we discuss how Friedman's analysis carries over to an economy with imperfect competition. Second, we discuss how optimal policy works in an economy where the distortions associated with money demand are arbitrarily small. An exact case for price stabilization along Fisherian lines emerged in a previous application of this kind of setup by King and Wolman [1999], who studied an environment with monopolistically competitive firms and sticky prices, but without the "monetary distortions" emphasized by Friedman. Our analysis reviews this reference sticky price case and interprets the prior results along the lines of Adao, Correia and Teles [2000]. In section 6, we discuss how we choose the parameters of our model economy.

In section 7, we discuss the results which lead to the first principle for monetary policy. The nominal interest rate should be set at an average level that implies deflation, but it should be positive. We show how this steady-state rate of deflation depends on various structural features of the economy, including the costs of transacting with credit – which give rise to money demand – and the degree of price-stickiness. In our benchmark calibration, credit transactions costs are quite small, and the long run inflation ration rate under optimal policy is only slightly negative, approximately minus nine basis points of deflation per year. Hence, while the case for an average inflation rate of zero developed in King and Wolman [1999] does not apply here, we find only a small quantitative difference. A smaller degree of market power, less price stickiness, or a broader definition of money (lower velocity) all make for a larger deflation under optimal policy.

In section 8, we describe the near-steady state dynamics of the model under optimal policy. Looking across a battery of specifications, we find that these dynamics display only minuscule variation in the price level. Thus, we document that there is a robustness to the Fisherian conclusion in King and Wolman [1999], which is that the price level should not vary in response to a range of shocks under optimal policy. In fact, the greatest price level variation that we find involves less than a 1% long-run change in the price level in response to a productivity shock which brings about a temporary but large deviation of output from trend, in the sense that the cumulative output deviation is 20%. Across the range of experiments, output under optimal policy closely resembles output which would occur if all prices were flexible and monetary distortions were absent: real activity resembles that in a core "real business cycle" model which underlies our framework. At the same time, we find that the real interest rate under the optimal policy does not always closely mimic that in the underlying RBC framework. To help interpret these results, we contrast some of them to benchmarks, including a real business cycle model, our model with simple money growth or interest rate rules, and a version of our model with the money demand distortions eliminated. Section 9 concludes.

2 The model

The macroeconomic model we study is designed to be representative of two recent strands of macroeconomic research. First, we view money as a means of economizing on the use of costly credit.[1] Second, we use a new Keynesian approach to price dynamics, which views firms as imperfect competitors facing infrequent opportunities for price adjustment.[2] To facilitate the presentation of these mechanisms, we view the private sector as divided into three groups of agents. First, there are households which buy final consumption goods and supply factors of production. These households also trade in financial markets for assets, including a credit market, and acquire cash balances which can be exchanged for goods. Second, there are retailers, which sell final consumption goods to households and buy intermediate products from firms. Retailers can costlessly adjust prices.[3] Third, there are producers, who create the intermediate products that retailers use to produce final consumption goods. These firms have market power and face only infrequent opportunities to adjust prices.

2.1 Households

Households have preferences for consumption and leisure, which are represented by the time-separable expected utility function,

$$U_0 = \mathbf{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right\} \tag{1}$$

[1] As in Prescott [1986], Dotsey and Ireland [1996] and Lacker and Schreft [1996].

[2] Taylor [1980], Calvo [1983]

[3] It is possible to eliminate the retail sector, but fleshing it out makes the presentation of the model easier.

The momentary utility function $u(c, l)$ is assumed to be increasing in consumption and leisure, strictly concave and differentiable as needed. Households divide their time allocation – which we normalize to one unit – into leisure, market work n, and transactions time h_t:

$$n_t + l_t + h_t = 1 \tag{2}$$

Accumulation of wealth: Households begin each period with a portfolio of claims on the intermediate product firms, holding a previously determined share γ_{t-1} of the per capita value of these firms. This portfolio generates current nominal dividends of $\gamma_{t-1} Z_t$ and has nominal market value $\gamma_{t-1} V_t$. They also begin each period with a stock of nominal bonds (B_{t-1}) left over from last period which have matured and have market value $(1 + R_{t-1}) B_{t-1}$. Finally, they begin each period with nominal debt arising from consumption purchases last period, in the amount D_{t-1}. So, their nominal wealth is:

$$\gamma_{t-1} V_t + \gamma_{t-1} Z_t + (1 + R_{t-1}) B_{t-1} - D_{t-1} - T_t$$

where T_t is the amount of a lump sum transfer to or from the government.

With this nominal wealth and current nominal wage income $W_t n_t$, they may purchase money M_t, buy current period bonds in amount B_t, or buy more claims on the intermediate product firms. Thus, they face the constraint

$$M_t + B_t + \gamma_t V_t \geq \left(\gamma_{t-1} V_t + \gamma_{t-1} Z_t + (1 + R_{t-1}) B_{t-1} - D_{t-1} - T_t\right) + W_t n_t$$

We convert this nominal budget constraint into a real one, using a numeraire P_t. At present this is simply an abstract measure of nominal purchasing power but we are more specific later about its economic interpretation. The real flow budget constraint is

$$\begin{aligned} m_t + b_t + \gamma_t v_t \quad \geq \quad & \gamma_{t-1} v_t + \gamma_{t-1} z_t + (1 + R_{t-1}) \frac{P_{t-1}}{P_t} b_{t-1} \\ & - \frac{D_{t-1}}{P_t} - \frac{T_t}{P_t} + w_t n_t \end{aligned}$$

with lower case letters representing real quantities when this does not produce notational confusion.

Money and transactions: Although households have been described as purchasing a single aggregate consumption good, we now reinterpret this involving many individual products – technically, a continuum of products on the unit interval – as in many studies following Lucas [1980]. Each of these products is purchased from a separate retail outlet at a price $\overline{P}_t$. Each customer buys a fraction ξ_t of goods with credit and the remainder with cash. Hence, the households demand for nominal money satisfies

$$M_t = (1 - \xi_t)\overline{P}_t c_t \tag{3}$$

where $\overline{P}_t$ is the price which must be paid to a retailer for a unit of consumption. The customer's nominal debt is

$$D_t = \xi_t \overline{P}_t c_t.$$

which must be paid next period. If credit is used for a particular good, then there are time costs x_t. Total time costs are

$$h_t = \int_0^{F^{-1}(\xi_t)} x dF(x), \tag{4}$$

where $F(\cdot)$ is the cumulative distribution function for time costs. As in Prescott [1987], Dotsey and Ireland [1996] and Lacker and Schreft [1996], we think of each final consumption goods purchase as having a random fixed cost – perhaps, the extent to which small children are clamoring for candy in the checkout queue – that is known after the customer decides to purchase the product, but before the customer has decided on whether to use money or credit to finance the purchase. The household uses credit when the cost is below the critical level given by $F^{-1}(\xi_t)$ and uses money when the cost is higher.

Consumption demand and labor supply: Combining budget constraints, we can get a real flow budget constraint for the household,

$$\begin{aligned}(1-\xi_t)\frac{\overline{P}_t}{P_t}c_t + b_t + \gamma_t v_t \quad \geq \quad & \gamma_{t-1}v_t + \gamma_{t-1}z_t + (1+R_{t-1})\frac{P_{t-1}}{P_t}b_{t-1} \\ & -\xi_{t-1}c_{t-1}\frac{\overline{P}_{t-1}}{P_t} + w_t n_t - \frac{T_t}{P_t}\end{aligned} \tag{5}$$

To characterize the solution to the household's problem, we consolidate the three constraints, (2), (4) and (5) into one:

$$(1-\xi_t)\frac{\overline{P}_t}{P_t}c_t + b_t + \gamma_t v_t - \Big(\gamma_{t-1}v_t + \gamma_{t-1}z_t + (1+R_{t-1})\frac{P_{t-1}}{P_t}b_{t-1}$$
$$-\xi_{t-1}c_{t-1}\frac{\overline{P}_{t-1}}{P_t} + w_t\Big(1 - l_t - \int_0^{F^{-1}(\xi_t)} xdF\left(x\right)\Big) - \frac{T_t}{P_t}\Big) \geq 0$$

Let λ_t, which has the economic interpretation as the shadow value of wealth, represent the multiplier for this constraint at time t. The first-order conditions are given below.

$$c_t \ : \ \frac{\partial u(c_t, l_t)}{\partial c_t} = \lambda_t\left(1-\xi_t\right)\frac{\overline{P}_t}{P_t} + \beta E_t\left[\lambda_{t+1}\frac{\overline{P}_t}{P_{t+1}}\xi_t\right] \quad (6)$$
$$\xi_t \ : \ \lambda_t\frac{\overline{P}_t}{P_t}c_t = \lambda_t w_t F^{-1}\left(\xi_t\right) + \beta E_t\left[\lambda_{t+1}\frac{\overline{P}_t}{P_{t+1}}c_t\right] \quad (7)$$
$$l_t \ : \ \frac{\partial u(c_t, l_t)}{\partial l_t} = w_t\lambda_t \quad (8)$$
$$b_t \ : \ \lambda_t = \beta E_t\left[\lambda_{t+1}\left(1+R_t\right)\frac{P_t}{P_{t+1}}\right] \quad (9)$$
$$\gamma_t \ : \ v_t\lambda_t = \beta E_t\left[\lambda_{t+1}\left(v_{t+1}+z_{t+1}\right)\right] \quad (10)$$

The first efficiency condition states that the marginal utility of consumption must be equated to the full cost of consuming. The full cost of consuming involves a weighted average of the costs of purchasing goods with currency and credit. The second efficiency condition equates the marginal benefit of raising ξ – decreasing current expenditure on consumption – to its marginal cost – the sum of current time cost and future repayment.

2.2 Retailers

We assume that retailers create units of the final good according to a constant elasticity of substitution aggregator of a continuum of intermediate products on the unit interval. In general, this will imply that c units of final consumption are generated according to $c_t = [\int c_t(x)^{\frac{\varepsilon-1}{\varepsilon}}dx]^{\frac{\varepsilon}{\varepsilon-1}}$, where ε is a parameter which controls the degree of substitutability. In our setup, however, there will be groups of firms which will all charge the same price for their good within a period, so that they can be aggregated easily. Let the j-th group have fraction ω_j and charge price P_{jt}. Then the retailer allocates its demands for intermediates across the J categories, minimizing $[(1+R_t)\sum_{j=0}^{J-1}\omega_j P_{jt}c_{jt}]$ subject $c_t = [\sum_{j=0}^{J-1}\omega_j c_{jt}{}^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}$. Note that the nominal interest factor $(1+R_t)$ affects the retailer's expenditures because, as is further explained below, the retailer must borrow to finance current production. This cost minimization problem leads to intermediate input demands of the form

$$c_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} c_t \tag{11}$$

where the unit cost of production – an intermediate goods price level of sorts – is given by

$$P_t = [\sum_{j=0}^{J-1} \omega_j P_{jt}^{(1-\varepsilon)}]^{\frac{1}{1-\varepsilon}}. \tag{12}$$

This is the price index which we use as numeraire in the analysis above.

Since the retail sector is competitive and all goods are produced according to the same technology, it follows that the final goods price must satisfy:

$$\overline{P}_t = (1+R_t)P_t \tag{13}$$

For each unit of sales, the retail firm receives revenues in money or credit. Each of these are cash flows which are effectively in date t+1 dollars. If the firm receives money, then it must hold it "over night." If the firm takes credit, then it is paid only at date t+1 with no interest charges.

2.3 Producers

The producers of intermediate products are assumed to be monopolistic competitors and face irregularly timed opportunities for price adjustment. For this purpose, we use a generalized stochastic price adjustment model due to Levin [1991], as recently exposited in Dotsey, King and Wolman's [1999] analysis of state dependent pricing. In this setup, a firm which has held its price fixed for j periods will be permitted to adjust with probability α_j. The model is flexible in that it contains the Taylor [1980] staggered price adjustment model as one special case (a four quarter model would set

$\alpha_1 = \alpha_2 = \alpha_3 = 0$ and $\alpha_4 = 1$), the Calvo [1983] stochastic adjustment model as another (this setup makes $\alpha_j = \alpha$ for all j), and can be used to match microeconomic data on price adjustment. In a steady state situation, an economy with a continuum of firms will have a distribution with fractions ω_j which are determined by the recursions $\omega_j = (1-\alpha_j)\omega_{j-1}$ for $j = 1, 2, ...J-1$ and $\omega_0 = 1 - \sum_{j=1}^{J-1} \omega_j$.

Each intermediate product x on the unit interval is produced according to the production function

$$y_t(x) = a_t n_t(x)$$

with labor being paid a nominal wage rate of W_t and being flexibly reallocated across sectors. Nominal marginal cost for all firms is accordingly W_t/a_t.

Firms are assumed to maximize the present discounted value of their real profits (*current* profits are $\frac{Z_t(x)}{P_t} = \frac{P_t(x)}{P_t} y_t(x) - \frac{W_t}{P_t} n_t(x)$) given the intermediate product demand described by (11) above and the stochastic structure of nominal price adjustment.

The model economy is one in which all firms that are adjusting at date t will choose the same price, which we call P_t^*. This price is determined as part of the solution to the firm's dynamic programming problem

$$v_t^0 = \max_{P_t^*} \left\{ \frac{P_t^* y_{0t} - W_t n_{0t}}{P_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\alpha_1 v_{t+1}^0 + (1-\alpha_1) v_{t+1}^1 \right) \right] \right\}$$

where the maximization takes place subject to the demand curve and the production function

$$\begin{aligned} y_{0t} &= (\frac{P_t^*}{P_t})^{-\varepsilon} d_t, \\ y_{0t} &= a_t n_{0t}, \end{aligned}$$

where d_t is aggregate demand in period t. Aggregate demand will be made up of consumption (c_t) and exogenous, unproductive government spending (g_t):

$$d_t = c_t + g_t.$$

A few comments about the form of the dynamic program are in order. First, consistent with the discussion of the household, the dynamic program deflates the firm's nominal profits by P_t and establishes asset values using the multiplier λ_t, which is the household's shadow value of wealth. Second, the firm is constrained by its production function and by its demand curve, which depends on aggregate consumption and government demand. Third, the firm knows that there are two possible situations at date t+1. With probability α_1 it will adjust its price and the current pricing decision will be irrelevant to its market value (v^0_{t+1}). With probability $1-\alpha_1$ it will not adjust its price and the current price will be maintained, resulting in a market value (v^1_{t+1}), with the superscript j in v^j_t indicating the value of a firm which is maintaining its price fixed at the level set at date $t-j$, i.e., P^*_{t-j}. Thus, we have for $j = 1, \ldots, J-2$,

$$v^j_t = \frac{P^*_{t-j}y_{jt} - W_t n_{jt}}{P_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\alpha_{j+1} v^0_{t+1} + (1-\alpha_{j+1}) v^{j+1}_{t+1} \right) \right]$$

and

$$v^{J-1}_t = \frac{P^*_{t-(J-1)}y_{J-1,t} - W_t n_{J-1,t}}{P_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} v^0_{t+1} \right],$$

where

$$y_{jt} = \left(\frac{P^*_{t-j}}{P_t}\right)^{-\varepsilon} (c_t + g_t) \tag{14}$$

$$y_{jt} = a_t n_{jt}. \tag{15}$$

An optimal pricing decision therefore requires that

$$0 = \frac{1}{P_t} \frac{\partial Z_{0t}}{\partial P^*_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_1) \frac{\partial v_{1,t+1}}{\partial P^*_t} \right]$$

which is the requirement that, at the optimum, a small change in price have a zero effect on the present discounted value. It is straightforward to show that

$$\frac{\partial v^j_{t+j}}{\partial P^*_t} = \frac{1}{P_{t+j}} \frac{\partial Z_{jt+j}}{\partial P^*_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_{j+1}) \frac{\partial v_{j+1,t+j+1}}{\partial P^*_t} \right].$$

for $j = 1, \ldots, J-2$ and that

$$\frac{\partial v_{t+J-1}^{J-1}}{\partial P_t^*} = \frac{1}{P_{t+J-1}} \frac{\partial Z_{J-1,t+J-1}}{\partial P_t^*}.$$

Further,

$$\frac{1}{P_{t+j}} \frac{\partial Z_{jt+j}}{\partial P_t^*} = \frac{1}{P_{t+j}} (\frac{P_t^*}{P_{t+j}})^{-\varepsilon} (c_{t+j} + g_{t+j}) \left((1-\varepsilon) + \varepsilon \frac{w_{t+j}}{a_{t+j}} \left(\frac{P_t^*}{P_{t+j}} \right)^{-1} \right)$$

where $w_{t+j} \equiv \frac{W_{t+j}}{P_{t+j}}$ is the real wage at time $t+j$, $j = 0, \ldots, J-1$. Repeated substitution of these results into the optimal pricing decision implies that price setting requires

$$0 = E_t \left[\sum_{j=0}^{J-1} \beta^j \omega_j \lambda_{t+j} \left(\frac{1}{P_{t+j}} \frac{\partial Z_{j,t+j}}{\partial P_t^*} \right) \right] \tag{16}$$

i.e., that there is no expected, discounted reward from a slightly higher or lower price. In this expression, the weights ω_j serve as indicators of the probability of the price being held fixed for at least j periods, which is $\omega_j / \omega_0 = (1-\alpha_1)(1-\alpha_2)...(1-\alpha_j)$.

2.4 The government

The government plays two roles in our economy. First, the government is an actor in various markets of the economy. Second, the government is a social planner. We discuss each of these roles in turn.

2.4.1 The government as an actor in goods and other markets.

The government in our model economy takes limited fiscal actions and virtually unlimited monetary actions.

Fiscal actions: The government demands final goods g_t in an exogenous, stochastically-varying manner. It levies lump sum taxes to pay for these goods (the lump sum taxes are a determinant of the term T_t in the household's budget constraint above).[4] As in many macroeconomic analyses—including the optimal fiscal and monetary policy work of Lucas and Stokey [1983]—we assume that government purchases have no effect on either the utility of households or the productivity of firms, but simply involve a use of resources.

Monetary policy actions: We assume that the government can vary the money supply M_t in response to the underlying disturbances in the economy. The changes in M_t could be made through direct transfers to households so that $T_t = P_t g_t - (M_t - M_{t-1})$. Given the timing structure of our economy, in which asset markets are open prior to goods markets, a form of Ricardian equivalence should hold, so that such direct transfers should be equivalent to open market operations.

Notice that we do not permit our government to explicitly levy taxes on or make subsidies to the households or firms in our economy, despite the fact that there are good reasons that a government might wish to in our economy. For example, given the monopoly distortion in our economy, one use of fiscal policy might be to subsidize intermediate goods producers and levy lump-sum taxes on households to pay for these subsidies. A subsidy would be viewed as desirable by the residents of the economy because it could stimulate intermediate goods production, counteracting the effects of the monopoly power which intermediate goods firms have.

Notice also that we do not permit our government to pay interest on its money. Such a policy would typically be desirable as well.

2.4.2 The government as planner

The major focus of the paper is on the government as a planner, in particular on (i) how it would calculate the constrained optimal policy in our economy; and (ii) the characteristics of equilibria under the optimal policy. When we take this perspective, we assume that the government's objective is to maximize the expected utility of households, subject to the general equilibrium of the model. But we carry along the assumptions that the government has a limited set of instruments, ruling out the use of non-lump sum taxes or subsidies and the payment of interest on money. In this sense, our government is active as a traditional monetary policy decision-maker, varying the quan-

[4] By requiring the government to buy final, rather than intermediate goods, we are assuming that there is a common demand elasticity for private and public consumption of the sticky price goods, if we dropped the intermediate good interpretation.

We are also assuming that the government buys from retailers without using money.

tity of money to accomplish macroeconomic objectives. At the same time, it is passive as a fiscal decision-maker. We adopt this strategy because we are interested in the design of monetary policies which can be implemented without requiring simultaneous fiscal actions or changes in the nature of current monetary instruments.

3 Four distortions

Our macroeconomic model has the property that there are four, readily identifiable routes by which nominal factors can affect real economic activity. We discuss these four distortions in turn, using general ideas that carry over to a wider class of macroeconomic models.

Relative price distortions: In any model with asynchronized adjustment of nominal prices, there are distortions that arise when the price level is not constant. First, note that the definition of the perfect price index –which applies to both consumption and government spending–means that we can write nominal expenditure as

$$\sum_{j=0}^{J} \omega_j P_{jt}[c_{jt} + g_{jt}] = P_t[c_t + g_t].$$

Second, note that a simple sum aggregate output measure is given by

$$y_t = \int_0^1 y_t(x)dx = \sum_{j=0}^{J} \omega_j y_{jt}$$

and that this implicitly defines an implicit deflator $\mathcal{P}_t$ as

$$\mathcal{P}_t y_t = \sum_{j=0}^{J} \omega_j P_{jt} y_{jt}$$

Third, since the current model makes output linear in labor input, then we know that aggregate output y_t is simply related to aggregate labor input $n_t = \sum_{j=0}^{J} \omega_j n_{jt}$ so that the above may be combined to yield

$$c_t + g_t = \delta_t a_t n_t$$

The factor $\delta_t \equiv \mathcal{P}_t/P_t$ works like a productivity shock in an aggregate production function for our basic economy. In fact, this result carries over to the entire class of setups suggested by Yun [1996], in which firms have technologies that imply constant marginal cost and factors can be flexibly reallocated across sectors.[5] Variations in δ can be described in another complementary way. Using $y_{jt} = (P_{jt}/P_t)^{-\varepsilon}(c_t + g_t)$, we find that δ_t is related to relative prices:

$$\delta_t = [\sum_{j=0}^{J} \omega_j (P_{jt}/P_t)^{-\varepsilon}].$$

If all relative prices are unity, then δ takes on a value of one. If relative prices deviate from unity, which is the unconstrained efficient level given the technology, then δ_t measures the extent of lost aggregate output which arises for this reason.

The markup distortion: If all firms have the same marginal cost functions, then we can write

$$W_t = \Psi_t a_t$$

where W is the nominal wage, Ψ_t is nominal marginal cost and a_t is the common marginal product of labor. If we divide by the perfect (intermediate good) price index, then this expression can be stated in real terms as

$$w_t = \psi_t a_t = \frac{1}{\mu_t} a_t$$

so that real marginal cost ψ_t acts like a sales tax shifter in any such setup. In our model, the marginal product of labor is constant.

Some authors, such as Woodford [1995], King and Wolman [1996] and Goodfriend and King [1997], have described this second source of distortions in terms of the average markup $\mu_t \equiv P_t/\Psi_t$, which is the reciprocal of real marginal cost ψ_t. These authors have stressed that the monetary authority has temporary control over this markup tax because prices are sticky, enabling it to erode (or enhance) the markups of firms with preset prices in response to various disturbances. According to this convention, which we

[5] For example, if the production function had constant returns to scale labor and capital, then an analysis along the lines of Yun [1996] indicates that the text equation would be modified to $c_t + g_t = \delta_t a_t f(k_t, n_t)$.

follow here, a higher value of the markup lowers real marginal cost and works like a tax on productive activity.

Note that δ_t and μ_t (or ψ_t) are not necessarily related closely together, so thinking about these from the standpoint of fiscal analysis – in which there can be separate shocks to the level of the production function and its marginal products – is the relevant background to this analysis, rather than reasoning from the effects of productivity shocks which traditionally shift both in RBC analysis.

Inefficient shopping time: The next distortion is sometimes referred to as "shoe leather costs." But in our model, it is really "shopping time costs," as in McCallum and Goodfriend [1988], since it is in time rather than goods units. Using the notation above, it is

$$h_t = \int_0^{F^{-1}(\xi_t)} x dF(x) \tag{17}$$

From the standpoint of our economy, variations in h_t work like a shock to the economy's time endowment. Pursuing the fiscal analogy discussed above, this is similar to a conscription (lump sum labor tax).

The wedge of monetary inefficiency: In transactions-based monetary models, there is also an effect of monetary policy on the full cost of consumption. Beginning with the efficiency condition (6), using the bond efficiency condition (9) to eliminate the expectations term and substituting out the pricing of the final product using (13), we arrive at a simple version of the efficiency condition for consumption:

$$D_1 u(c_t, l_t) = \lambda_t \left[1 + R_t (1 - \xi_t)\right]. \tag{18}$$

This equation expresses the wedge of monetary inefficiency as the product of the nominal interest rate and the extent of monetization of exchange $(1-\xi_t)$. Pursuing the fiscal policy analogy discussed above, it is like a consumption tax relative to the non-monetary model.

4 Optimal policy

Our analysis of optimal policy is in the tradition of Ramsey [1928] and draws heavily on the modern literature on optimal policy in dynamic economies which follows from Lucas and Stokey [1983]. In general, the idea of optimal policy design is for the government to maximize expected utility subject to the conditions of dynamic equilibrium and the constraints on its instruments. Working in a dynamic competitive equilibrium setting, Lucas and Stokey showed the power of a multi-stage approach. First, one determines the conditions that circumscribe competitive equilibrium for arbitrary policies: in their initial analysis of a real economy subject to fiscal shocks, the relevant conditions which implicitly determined quantities and relative prices included the efficiency conditions of firms and households as well as the resource constraints of the economy plus private and public budget constraints. Second, these conditions were manipulated to eliminate all tax rates and relative prices, leaving only a group of constraints on real quantities. Third, the government maximized expected utility subject to the constraints on real quantities: this determined a unique path for real quantities. Fourth, relative prices and tax rates were determined which led these outcomes to be the result of a dynamic competitive equilibrium.

In this paper, as in King and Wolman [1999], we adopt a similar approach to an economy which has real and nominal frictions: monopolistic competition, price stickiness and the costly conversion of wealth into goods that can be altered by money holding. The outline of our multi-stage approach is as follows. First, we have already determined the efficiency conditions of households and firms that restrict dynamic equilibria in our economy, as well as the various budget and resource constraints. Second, we manipulate these equations to determine a smaller subset of restrictions that governs real quantities. However, for various reasons, we find it convenient to leave in one relative price – the multiplier on the households budget constraint– and two nominal variables–the inflation rate and the nominal interest rate– in the subset of equations. Third, we maximize expected utility subject to these constraints. Fourth, we find the remaining absolute prices and monetary policy actions which lead these outcomes to be the result of dynamic equilibrium.[6]

4.1 Organizing the restrictions on dynamic equilibrium

We begin by combining the household's first-order conditions with retailers' zero profit conditions. Using (9), (8) and (13) we eliminate expectations from (6) and (7).

[6] We do not consider the possibility that optimal policy might involve randomization, as suggested by Dupor [1999].

$$\begin{aligned} \frac{\partial u(c_t, l_t)}{\partial c_t} &= \lambda_t \left(1 + R_t \left(1 - \xi_t\right)\right) & (19) \\ \lambda_t R_t c_t &= \frac{\partial u(c_t, l_t)}{\partial l_t} F^{-1}\left(\xi_t\right). & (20) \end{aligned}$$

We rewrite the Euler equation (9) as

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{1 + R_t}{\Pi_{t+1}} \right], \tag{21}$$

where Π_t is the gross inflation rate:

$$\Pi_t = P_t / P_{t-1}.$$

Next, combining equations (2) and (4) we have a consolidated time constraint for the household in (22).

$$n_t + l_t + \int_0^{F^{-1}(\xi_t)} x dF\left(x\right) = 1. \tag{22}$$

The constraints upon the policymaker that arise directly from the household or retailers are (19) - (22). Equation (10) is an asset-pricing equation that determines the real share price, v_t, to ensure that the household holds the market portfolio thereby ensuring that it receives all profits. As such, we may ignore it in the analysis of optimal policy. Equation (8) will be used to eliminate the real wage in the firm's asset-pricing equation to which we turn next.

We eliminate nominal price and wage terms from the optimal firm price-setting equation. Multiplying (16) by P_t^* and manipulating the result using (8), (15) and (14) we have (23).

$$0 = E_t \sum_{j=0}^{J-1} \beta^j \omega_j X\left(\lambda_{t+j}, y_{j,t+j}, c_{t+j}, l_{t+j}, g_{t+j}, a_{t+j}\right) \tag{23}$$

where

$$X\left(\lambda, y_j, c, l, g, a\right) = \left[(1-\varepsilon)\left(\frac{y_j}{c+g}\right)^{\frac{\varepsilon-1}{\varepsilon}} \lambda + \varepsilon \frac{\partial u\left(c,l\right)}{\partial l}\frac{1}{a}\left(\frac{y_j}{c+g}\right)\right](c+g).$$

As discussed above, in any period t, $(1-\alpha_j)$ fraction of firms that set their prices j periods ago, $j = 1, \ldots, J-2$, will be unable to reset their prices. As a result, their price in period t, P^*_{t-j}, is unchanged from their price in the previous period, $P^*_{t-(j-1)}$. Equation (14) then implies that the demand for the output of such firms will evolve over time according to

$$y_{j,t}\Pi_t^{\varepsilon} = y_{j-1,t-1}\frac{(c_t + g_t)}{c_{t-1} + g_{t-1}}. \tag{24}$$

That is: the past behavior of various real quantities is one way of summarizing the past nominal prices which are the natural state variables of sticky price models.

Next, noting $n_t = \sum_{j=0}^{J-1} \omega_j n_{jt}$ and using (15), we find that

$$a_t n_t = \sum_{j=0}^{J-1} \omega_j y_{jt}. \tag{25}$$

Finally, the final goods production function is our last constraint.

$$(c_t + g_t) = [\sum_{j=0}^{J-1} \omega_j y_{jt}{}^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}} \tag{26}$$

4.2 Posing the optimal policy problem

Two of the implementation conditions, the household's Euler equation (21) and the firm's price-setting condition (23) introduce expectations of future variables into the time t constraint set. Thus the set of feasible policies for the monetary authority is constrained by the expectations of the private sector. The unusual nature of these constraints requires us to reformulate them prior to solving the optimal monetary policy problem. We begin by introducing Ω_t and Φ_t as the multipliers for (21) and (23), respectively. The optimal policy problem then solves

$$\min_{\{\Phi_t,\Omega_t\}_{t=0}^{\infty}} \max_{\{\mathbf{S}_t\}_{t=0}^{\infty}} \mathbf{E}_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \Bigg(u\left(c_t, l_t\right) \tag{27}$$

$$+ \Phi_t \mathbf{E}_t \left[\sum_{j=0}^{J-1} \beta^j \omega_j \mathbf{X}\left(\lambda_{t+j}, y_{j,t+j}, c_{t+j}, l_{t+j}, g_{t+j}, a_{t+j}\right) \right]$$

$$+ \Omega_t \mathbf{E}_t \left[\lambda_t - \left(1 + R_t\right) \beta \frac{\lambda_{t+1}}{\Pi_{t+1}} \right] \Bigg) \Bigg\}$$

where $\mathbf{S}_t = \left\{ c_t, \xi_t, l_t, n_t, \lambda_t, \Pi_t, R_t, \left(y_{j,t}\right)_{j=0}^{J-1}, \right\}$, subject to the additional constraints (19) - (20), (22), (24) - (26) in each period $t = 0, 1, \ldots,$ with $\left(y_{0,-1}, \ldots, y_{J-2,-1}, c_{-1}, g_{-1}, R_{-1}, a_0, g_0, z_0\right)$ given.

This problem is inherently nonstationary. Kydland and Prescott [1980] began the analysis of how to describe such problems using recursive methods. Important recent work by Marcet and Marimon [1999] formally develops a recursive approach to such problems. Following their method, we convert our dynamic optimization problem into a recursive saddlepoint problem. We reorganize the terms in (27) involving expectations of future variables at time 0. Grouping expectations of variables sharing the same date, we apply the law of iterated conditional expectation. Moreover, in re-organizing the constraints, we add terms involving variables dated before period 0 to ensure that the constraints in the first $J-1$ periods are identical to those appearing in subsequent periods. This requires the introduction of lagged multipliers into the problem. As is well known, these lagged multipliers effectively convert a nonstationary problem into a stationary or recursive one. The nonstationarity that would otherwise arise originates through initial conditions – here predetermined nominal prices – that the monetary authority may exploit. Augmenting the optimal policy problem with lagged multipliers does not necessarily eliminate this initial period issue; this depends on the lagged multipliers' starting values. However, the introduction of lagged multipliers as state variables does allow us to analyze a recursive problem.

The appendix describes how we rearrange (27) and introduce additional terms that allow us to reformulate the optimal policy problem under commitment to that in (28).

$$\min_{\{\Phi_t,\Omega_t\}_{t=0}^{\infty}} \max_{\{\mathbf{S}_t\}_{t=0}^{\infty}} E_t\Bigg\{\sum_{t=0}^{\infty}\beta^t\Bigg(u\left(c_t,l_t\right) \tag{28}$$

$$+\sum_{j=0}^{J-1}\beta^j\Phi_{t-j}\omega_j\mathbf{X}\left(\lambda_t,y_{j,t},c_t,l_t,g_t,a_t\right)$$

$$+\left(\Omega_t-\Omega_{t-1}\frac{1+R_{t-1}}{\Pi_t}\right)\lambda_t\Bigg)\Bigg\}$$

Here $\mathbf{S}_t$ is unchanged from (27) and the constraints are (19) - (20), (22), (24) - (26) as before. However, we now assume that $\left(y_{0,-1},\dots,y_{J-2,-1},c_{-1},g_{-1},R_{-1},a_0,g_0,z_0,\Phi_{-1},\dots,\Phi_{-(J-1)},\Omega_{-1}\right)$ is given. However, if we set $\Phi_j=0$, $j=-(J-1),\dots,-1$, and $\Omega_{-1}=0$, then we have not altered the original optimization problem.[7] Note that (28) does not contain constraints on the choice of current variables that involve the expectations of future quantities or prices.

4.3 Computation

We determine the efficiency conditions for the policy maker, given the problem posed in the previous subsection. We use these conditions in two ways. First, assuming certainty, we solve for a steady state. Second, we linearize around the steady state to study the responses to various disturbances.

After we calculate the solution to the optimal policy problem, it is also direct to calculate the values of various nominal variables that are relevant to monetary policy. For example, we can study the behavior of the price level by exploiting two aspects of the economy. First, given that the optimal policy problem determines quantities, we can determine the relative price of an adjusting firm, p_{0t}, using the demand behavior of retailers ($p_{0t}=\left(y_{0t}/y_t\right)^{-1/\varepsilon}$). Second, we can determine the behavior of the price level using the predetermined nominal prices, the current relative price p_{0t}, and the price level (12) determined in our analysis of the retailer's problem. Given that (28) determines ξ_t and R_t, we can also calculate the retail price level, the real

[7] In the numerical results we report below, the lagged multipliers that now augment the state vector $\left(\Phi_{-1},\dots,\Phi_{-(J-1)},\Omega_{-1}\right)$ are initially set to the long run values for Φ_t and Ω_t. This allows us to examine optimal monetary policy under commitment while abstracting from the transitory dynamics that arise during initial periods.

quantity of money, and the nominal quantity of money that are associated with the constrained optimal policy.

5 Optimal policy in two special cases

In two special cases of the model, one can characterize optimal policy analytically. One of those cases is well known: if prices are flexible it is optimal to equate the private and social costs of money holding, which means keeping the net nominal interest rate equal to zero – the Friedman rule. In the second special case, the distortions associated with money demand are assumed to be arbitrarily small. There we can show that under a familiar elasticity condition on preferences it is optimal for the price level to be constant in response to productivity shocks.

5.1 Flexible Prices

To make prices flexible, set $\omega_0 = 1$. This immediately eliminates the relative price distortions, since every firm charges the same price. The markup distortion is still present, but it cannot be affected by the monetary authority: the markup is constant across time and across states at $\varepsilon/(\varepsilon - 1)$. The only distortions that the monetary authority can affect are shopping time and the wedge of monetary inefficiency. Zero nominal interest rates eliminate both of these distortions, hence zero nominal interest rates represent optimal policy.[8] The only novel feature here is that the presence of monopolistic competition makes the Friedman rule outcome second-best. In a sense, the monetary authority would like to make the nominal interest rate negative, to offset the monopoly inefficiency. Of course the nominal interest rate cannot be negative. However, this incentive implies that in the full model with sticky prices, it *may* still be optimal to pursue the Friedman rule.

5.2 Absence of Monetary Distortions

If the time costs of credit are such that the shopping time and monetary wedge distortions vanish regardless of the level of interest rates, the conditions describing optimal policy simplify dramatically. King and Wolman [1999] showed that price stability is optimal in the long run for a particular

[8] Cole and Kocherlakota [1998] discuss policies that *implement* the Friedman rule.

specification of preferences. In fact one can derive a sharper result, for the case where government spending is absent.

To derive the result, we borrow the analytical approach of Adao, Correia and Teles [2000]. That is, we impose a constant price level on the equations describing optimal policy, and examine the conditions under which these equations are satisfied. The key implication of a constant price level across states is that the quantities produced by firms that set their prices at different times are identical. That is, $c_{j,t} = c_t$ and $n_{j,t} = n_t$. This implication allows us to derive the following condition, under which a constant price level is optimal in response to random variation in a_t:

D_t is constant across time and states,

where

$$D_t \equiv \left(\frac{c \cdot u_{cc}}{u_c} - \frac{c \cdot u_{cl}}{u_l} \right) - \frac{n}{l} \left(\frac{l \cdot u_{cl}}{u_c} - \frac{l \cdot u_{ll}}{u_l} \right).$$

This condition is clearly satisfied if preferences are constant elasticity and separable between consumption and leisure. Note that this is the same condition derived by Adao, Correia and Teles [2000] for a model with all prices set one period in advance. We have thus shown that the condition extends to a richer environment, specifically one with multi-period price-setting.

6 Choice of parameters

Given the limited amount of existing research on optimal monetary policy using the approach of this paper, and given the starkness of our model economy, we have chosen the parameters with two objectives in mind. First, we want our economy to be as realistic as possible, so we calibrate certain parameters to match certain features of the U.S. economy as discussed below. Second, we want our economy to be familiar to economists who have worked with related models of business cycles, fiscal policy, money demand, and sticky prices. Our benchmark parametric model is as follows, with the time unit taken to be one quarter of a year:

6.1 Preferences

We assume the utility function is logarithmic, with a share parameter chosen so that a real economy would have individuals working one-fifth of the time.

We assume also that the discount factor is such that the annual interest rate would be slightly less than four percent. This choice of the discount factor is governed by data on one year T-bill rates and the GDP deflator that we use in calibrating the distribution of credit costs.

$$\begin{aligned} u(c,l) &= \ln c + 3.3 \ln(l) \\ \beta &= 0.9928 \end{aligned}$$

We later explore some implications of a higher labor supply elasticity, assuming that $u(c,l) = \ln c + 3.3l$ which may be rationalized by indivisible labor as in Rogerson [1988] and Hansen [1985].

6.2 Monopoly power

We assume that the markup would be 10% over marginal cost if prices were flexible. Since the gross markup is $\mu = \frac{\varepsilon}{\varepsilon - 1}$, this implies that

$$\varepsilon = 10$$

We also explore some implications of a lower elasticity of demand.

6.3 Distribution of price-setters

A key aspect of our economy is the extent of exogenously imposed price stickiness. We use a distribution suggested by Wolman [2000], which has the following features. First, it implies that firms expected a newly set price to remain in effect for five quarters. That is: the expected duration of a price chosen at t, which is $\alpha_1 1 + (1-\alpha_1)\alpha_2 2 + (1-\alpha_1)(1-\alpha_2)\alpha_3 3 + ...$ is equal to 5. Second, this estimate is consistent with the recent empirical work on aggregate price adjustment dynamics by Gali and Gertler [1999] and Sbordone [1998]. Third, rather than assuming a constant hazard $\alpha_i = \alpha$ as in the Calvo model, our weights involve an increasing hazard, which is consistent with available empirical evidence and recent work on calibrated models of state dependent pricing. The particular adjustment probabilities α_i and the associated distribution are:

Table 1:
Price adjustment probabilities
and the associated distribution weights

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
0.014	0.056	0.126	0.224	0.350	0.504	0.686	0.897	1
ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
0.198	0.195	0.184	0.161	0.125	0.081	0.040	0.012	0.001

For a given distribution of price adjustment, we can also calculate the average age of a price which is in place, $\sum_{j=0}^{J-1} j\omega_j$. This average is 2.3 for the benchmark parameterization.

We explore some implications of assuming much greater price flexibility below.

6.4 Credit costs and the demand for money

The behavior of the demand for money in our model depends on the distribution of credit costs. We assume that distribution is a modified member of the beta family; the modification is that there is a mass point at zero credit costs. The distribution has four parameters: the two parameters of the standard beta distribution, which we will call ϕ_1 and ϕ_2, the upper bound of the support, which we call ϕ_3 (we assume the lower bound of the support is zero), and the mass on zero costs, which we call ϕ_4. To select values for these parameters, we use data on the GDP-velocity of domestically held currency, and nominal interest rates on one-year treasury bills.[9] It is common in work on business cycle models with money to view M1 as the relevant monetary aggregate. In our model, the choice between using money or credit is akin to that between using currency and a credit card. In addition, M1 has the drawback of not being controllable by the Federal Reserve under current – or easily imaginable institutional arrangements.

To calibrate the distribution of credit costs we begin by imposing $\phi_1 = \phi_2 = 1$, which means assuming a uniform distribution of costs. We then divide our data sample in half, and compute average velocity and the average nominal interest rate in the two subsamples. Average velocity and nominal interest rates in the first half of the sample were 11.48 and 9.3%, and in

[9] Our data on domestically held currency was provided by Phillip Jefferson, and is discussed in Porter and Judson [1996]. The data spans the period 1977:1 to 1995:4.

the second half of the sample 10.93 and 5.74%. The remaining distributional parameters are chosen to match these nominal interest rate - velocity pairs; this procedure yields $\phi_3 = 0.0419$ and $\phi_4 = 0.9018$. As a robustness check, we also present results for a calibration that has $\phi_1 = \phi_2 = 2$, which is a symmetric unimodal distribution.

Some readers may object to our use of home currency data to calibrate the model. We therefore present results for a calibration that involves again setting $\phi_1 = \phi_2 = 1$, but uses the GDP-velocity of M1 to pin down the other two parameters of the credit cost distribution. In this case we have $\phi_3 = 0.0342$ and $\phi_4 = 0.3380$.

7 Optimal policy in the long run

The preceding discussion established two reference points for thinking about optimal policy in the long run. The first reference point is Friedman's [1969] celebrated conclusion that the nominal interest rate should be sufficiently close to zero so that the private and social costs of money-holding coincide. At this point, the economy minimizes the costs of decentralized exchange. The second reference point is an average rate of inflation of zero, which minimizes relative price distortions in steady state: this is therefore the optimal long-run rate in the absence of the distortions which Friedman highlighted. In this section, we document the intuitive conclusion that the long-run inflation rate should be negative – but not as negative as suggested by Friedman's analysis – when both sticky price and exchange frictions are present.

In particular, when we solve the optimal policy problem for the benchmark model, we find that the asymptotic rate of inflation – the steady state under the optimal policy – is negative nine basis points ($-.09\%$ at an annual rate). Given that we assume a steady state real interest rate of 2.93% percent (as determined by time preference), the long-run rate of nominal interest is 2.84%. Hence, the long run more closely resembles the zero inflation case than it does the Friedman rule under the benchmark parameter values.

This result raises two sets of questions. First, how do the four distortions isolated earlier in the paper contribute to this finding? Second, how do variations away from the benchmark parameter values affect the optimal long-run inflation rate? Each of these questions is addressed below.

7.1 Behind the benchmark long-run inflation rate

In order to look behind the benchmark zero inflation steady-state, we think it is useful to take three steps. First, we consider how the economy would work in the zero-inflation steady state, even if this is not optimal. Second, we consider reevaluating the optimal policy problem if one or more of our four distortions is eliminated as a consideration for the monetary authority. Third, having isolated relative price distortions as a key feature under this benchmark set of parameter values, we look further into how these distortions depend on the steady state rate of inflation.

7.1.1 The (suboptimal) zero inflation steady state

If there is zero inflation in the benchmark economy, then it is relatively easy to determine the levels of the four distortions.

Let us start by considering the effects of sticky prices and imperfect competition: The *markup* is equal to that which prevails in the static monopoly problem, $\mu = \frac{\varepsilon}{\varepsilon-1} = 1.10$ so that price is ten percent higher than real marginal cost in the steady-state. There are *no relative price distortions* – all firms are charging the same, unchanging price P^*– so that $\delta = 1$. In this situation, the nominal and real interest rates are each equal to 2.93% per annum. The parameters of the credit cost technology imply that 90.6% percent of transactions are financed with credit ($\xi = .906$) and that the ratio of real money to consumption is about 9.5 percent.

Let us next consider the effects of costly exchange of wealth for goods: *The wedge of monetary inefficiency* is positive, but relatively small in this steady state. It is calculated from the above discussion as

$$(1 + (1 - \xi) * R) = (1 + (1 - .906) * .007) = 1.0007$$

where the calculation of the wedge uses the quarterly nominal interest rate .007. From the discussion above, we know that the *time cost h* is an extremely small number. At zero inflation, time costs associated with use of credit are approximately two-thousandths of a percent of labor time.

Even though the distortions associated with money demand are small at zero inflation, a monetary authority maximizing steady-state welfare would nonetheless choose a lower the rate of inflation, for the reasons stressed by Friedman [1969].

7.1.2 Optimal inflation with fewer distortions

We now imagine altering the monetary authority's problem – relative to the benchmark case – by selectively eliminating one or more distortions. For some of these modifications, there is an easy economic interpretation of our modified problem. For example, if we assume – as in King and Wolman [1999] – that there is interest on money at just below the market rate then there are no money demand distortions (no wedge and no resource costs). But to track down the origins of the benchmark inflation rate, it is sometimes necessary to consider other more abstract, modifications. Table 2 shows the effect of various modifications of the mix of distortions.

[10]

Table 2: Effect of eliminating various distortions on the long-run optimal inflation rate

(distortion eliminated is marked with an **x**)[11]

	mkup	h	wedge	A bench	B $\varepsilon = 4$	C $M1$	D $\phi_1 = \phi_2 = 2$	E ∞ l.s.	F $J = 3$
1				-.094	-.599	-.650	-.088	-.118	-.491
2			**x**	-.002	-.006	-.014	-.001	-.003	-.011
3		**x**		-.092	-.594	-.639	-.087	-.115	-.482
4		**x**	**x**	0	0	0	0	0	0
5	**x**			-.102	-.754	-.707	-.096	-.131	-.533

Why is disinflation desirable? Starting with the zero inflation steady state rate of inflation, the Table shows that both the wedge of monetary

[10]The table also presents results of the sensitivity analysis to be discussed below.

[11]This footnote explains the rows and columns of Table 2. In row 1, all distortions are present. In row 2, the wedge of monetary inefficiency is eliminated. In row 3, shopping time costs are eliminated, and in row 4, both forms of monetary distortion are eliminated. In row 5, the markup is fixed at $\varepsilon / (\varepsilon - 1)$.

The columns are as follows.

A. Benchmark calibration discussed in section 6.

B. Demand elasticity for the differentiated products set to 4 instead of 10.

C. The parameters ϕ_3 and ϕ_4 are chosen to be consistent with U.S. data on M1 rather than currency (see section 6.4). We maintain $\phi_1 = \phi_2 = 1$.

D. We maintain the currency calibration for ϕ_3 and ϕ_4, but set $\phi_1 = \phi_2 = 2$.

E. Instead of preferences that are logarithmic in leisure, we make them linear in leisure, which implies an infinite labor supply elasticity.

F. We modify the distribution of firms (ω) from that given in table 1 to $\omega = 0.4, 0.35, 0.25$. In this case, no firm goes more than three periods with the same price.

inefficiency and time costs play a role in reducing the inflation rate from zero to the benchmark level of -.09%.

No variation in the wedge of monetary inefficiency: Our discussion of the wedge of monetary inefficiency stressed that it captured the full price of converting wealth into final goods consumption, so that it was the product of the intensity of monetary exchange $(1-\xi)$ and the opportunity cost of holding money R. We now explore the implications of eliminating this wedge for the optimal rate of inflation. Mechanically, we fix the wedge at zero and re-solve the monetary authority's optimal policy problem. One rationalization of this procedure is that there is a consumption subsidy, introduced into the household's problem and then varied in a manner that would neutralize the wedge of monetary inefficiency, i.e.,

$$(1 + (1 - \xi_t) * R_t)(1 + \tau_t^c) = 1$$

Table 2 shows that there is a significant influence of this distortion on the optimal long-run rate of inflation. If it is eliminated by itself, then the inflation rate rises from -.09% to -.002%, so that the wedge accounts for almost all of the deviation from zero inflation.

Resource costs of credit: We can similarly eliminate the resource costs of credit usage from the optimal policy problem. Above, we used the idea that the wedge of monetary inefficiency is like a tax, so that it could be neutralized by a countervailing tax. In this case, we must envision a perturbation of the economy's resource constraint so that as the inflation rate is varied there are no effects on the economy's opportunities for work and leisure. That is: we must view the right-hand side of

$$l_t + n_t = 1 - h_t$$

as invariant the policymaker's choices. One possible interpretation is that a fiscal authority is adjusting the extent of a lump-sum confiscation of time to accomplish this elimination of resource costs of credit usage.

If we eliminate the resource costs by themselves, then the inflation rate barely rises, from -.094% to -.092%, so that time costs account for almost none of the deviation from the zero inflation position.

Why is there less deflation than at the Friedman Rule? If prices are flexible, then the Friedman rule is optimal even though there is imperfect competition. In fact, Goodfriend [1997] notes that a positive markup makes the case stronger in a sense because the additional labor supply induced by declines in the wedge and time costs yield a social marginal product of labor which exceeds the real wage.

To evaluate why there is a benchmark rate of inflation of -.09% per annum – as opposed to a Friedman rule level of -2.93% per annum – it is necessary to eliminate either the relative price distortion or the markup distortion. We suppose that the policy maker cannot alter the average markup of firms, but can influence all of the other distortions. Why might this be the case? We have stressed that the markup acts like a sales tax, so one possibility is that an explicit sales tax is levied on intermediate goods producers and that it is varied so that

$$(1+\tau_t^i)\frac{1}{\mu_t} = \frac{\varepsilon}{\varepsilon - 1}$$

i.e., so that the markup always stays at its zero inflation level (the level also consistent with imperfect competition but no price stickiness).

With the markup distortion fixed, Table 2 shows that there is a slightly more negative rate of inflation. This finding is consistent with results of previous studies which documented that the average markup (i) is decreasing in the inflation rate near zero inflation; and (ii) does not respond importantly to variations in the inflation rate near zero inflation. The first finding of the previous studies explains why eliminating the distortion makes the optimal inflation rate more negative, since the monetary authority does not encounter an increasing markup in the modified problem as it lowers the inflation rate from a starting point of zero. The second finding explains why the effect is a small one quantitatively: since the price adjustment decisions of firms are forward-looking, the markup is not too affected by the trend rate of inflation.

7.1.3 Assessing Relative Price Distortions

Given that relative price distortions play a major role in the determination of the steady-state inflation rate, it is desirable to investigate more closely how these depend on the extent of price stickiness and other factors. There are three ingredients of the relationship between δ and the inflation rate. First, in an inflationary steady-state, relative prices are linked together by

$$\frac{P_{jt}}{P_t} = \frac{P_{t-j}^*}{P_{t-j}\Pi^j} = p_0 \Pi^{-j}$$

where Π is the gross inflation rate. Second, the definition of the price level implies that

$$1 = [\sum_{j=0}^{J-1} \omega_j (P_{jt}/P_t)^{1-\varepsilon} = p_0^{1-\varepsilon}[\sum_{j=0}^{J-1} \omega_j \Pi^{j(\varepsilon-1)}]$$

where the second line involves the use of the steady-state behavior of relative prices. This equation implicitly determines p_0 as a function of Π. Third, the δ measure may be written as

$$\delta_t = [\sum_{j=0}^{J-1} \omega_j (P_{jt}/P_t)^{-\varepsilon}]^{-1} = p_0^{\varepsilon}[\sum_{j=0}^{J-1} \omega_j \Pi^{j\varepsilon}]^{-1}$$

where the second equality again follows from using the steady-state relative prices.

These expressions can be used to approximate the steady-state measure of relative price distortions as

$$\log(\delta) \approx -\frac{\varepsilon v}{2}(\Pi - 1)^2,$$

where $v = \sum_{j=0}^{J-1} j^2 \omega_j - [\sum_{j=0}^{J-1} j\omega_j]^2$ is a measure of the variance of the "age" of prices. The quality of this approximation can be evaluated since it is possible to calculate δ exactly and we have found that it is quite accurate for inflation rates between -1% and 10%, which would correspond to annual inflation rates of -4% to 40%.

This simple expression has a number of intuitive features. First, for small changes in the inflation rate near zero, there is no effect on the measure of relative price distortions.[12] For this reason, it is natural to conjecture that there will always be deflation in a setting which combines sticky prices and monopolistic competition with the costly conversion of wealth into consumption, although a markup which decreases with inflation can provide a disincentive for deflation. Second, distortions are larger if there is greater disparity between firms. Third, distortions are larger if there is a larger demand elasticity, which means that the inflation-induced changes in relative prices have a larger effect on the distribution of output across firms.

The relative price distortion implications of some commonly employed models of price adjustment due to Taylor [1980] and Calvo [1983] are easily

[12] $\frac{\partial \log(\delta_t)}{\partial \Pi} \approx -\varepsilon v(\Pi - 1)$, which is zero at Π=1

evaluated using this formula. Each of these models has a single parameter which determines the distribution of prices by age. For the Taylor model, it is the length that every firm's price is held fixed, J, and the mean is $[\sum_{j=0}^{J-1} j\omega_j] = \frac{1}{J}[\sum_{j=0}^{J-1} j] = \frac{J-1}{2}$. For the Calvo model, it is the probability of price adjustment, α, and the mean is $[\sum_{j=0}^{J-1} j\omega_j] = \alpha[\sum_{j=0}^{J-1} j(1-\alpha)^j] = \frac{1-\alpha}{\alpha}$. Further, letting the mean be $\overline{j}$ it can be shown that the variance measures take on the values $v = \frac{1}{3}(1+\overline{j})\overline{j}$ for the Taylor model and $v = (1+\overline{j})\overline{j}$ for the Calvo model. Accordingly, if the average age of prices is about 4 quarters, as suggested by the estimates of Gali and Gertler [1999] using the Calvo model, then $v = 20$. By contrast, a five quarter Taylor model – which implies that $[\sum_{j=0}^{J-1} j\omega_j] = 2$ – would have $v = 2$. These two common models suggest quite different costs of disinflation at the Friedman rule rate, if this is assumed, as in the balance of our analysis, to be -4% per annum (-1% per quarter) so that $\Pi - 1 = .01$. With a demand elasticity of $\varepsilon = 10$ then the formula implies that

$$\log(\delta) \approx -\frac{\varepsilon v}{2}(\Pi - 1)^2 \approx -\frac{10 * 20}{2}(.01)^2 = .01$$

or 1% of steady-state consumption for the Calvo specification. By contrast, with the five quarter Taylor structure, the welfare cost is only .1%.

Our benchmark parameterization implies that $\sum_{j=0}^{J-1} j\omega_j = 2.3$ and $v = \sum_{j=0}^{J-1} j^2\omega_j - [\sum_{j=0}^{J-1} j\omega_j]^2 = 3.3$, so that the δ lies between the example values given above: it is .165%. The relative price distortion at the Friedman rule is thus large (measured in terms of output) compared to the money demand distortions at zero inflation (discussed above). It is not surprising then that the solution to the optimal policy problem puts inflation much closer to zero than to the Friedman rule (this reasoning is informal, as the monetary authority balances *marginal* distortions).

7.2 Sensitivity Analysis

We now explore the sensitivity of the steady-state rate of inflation to various structural features of the model. These results are presented in Table 2.

Monopoly power: decreasing the demand elasticity (ε) to 4 leads to a larger deflation, 0.60% per year. This is to be expected, given the expression derived above for the relative price distortion: decreasing ε by a factor of 0.4 generates a corresponding decrease in the relative price distortion at any inflation rate. The money demand distortions become relatively more important, pushing the optimum closer to the Friedman rule.

Labor supply elasticity: with infinite labor supply elasticity, policy moves just a bit closer to the Friedman rule; steady state has 12 basis points of deflation per year.

Share of government spending: in our benchmark calibration, there is no government spending in the steady state. If the share of government spending is thirty percent, the steady state is closer to zero inflation – it involves deflation of 6.7 basis points per year. We conjecture that this occurs because an increase in government spending reduces the labor supply elasticity in response to a change in the real wage (our specification implies that the leisure demand elasticity is equal to negative one and the labor supply elasticity is $-l/n$ times this value; an increase in g raises the level of n and lowers the level of l, thus lowering this elasticity on both counts).

Concentration of credit cost distribution: we change the parameters of the distribution function $F(x)$ to $\phi_1 = \phi_2 = 2$, and then calibrate ϕ_3 and ϕ_4 to match velocity in the two halves of our 1977-1995 sample. This implies $\phi_3 = 0.0224$ and $\phi_4 = 0.9086$. With $\phi_1 = \phi_2 = 2$, instead of being uniform, the density has a classic bell shape. There is little effect on the long run inflation rate under optimal policy: it is -8.8 basis points.

Broader monetary aggregate: we return to $\phi_1 = \phi_2 = 1$, but calibrate ϕ_3 and ϕ_4 using data on $M1$ instead of domestically held currency. This results in a significant change in the long run inflation rate under optimal policy; it is -65 basis points, compared to -9 basis points for our benchmark calibration using currency. To understand this difference, note first that for $M1$, velocity in the two halves of the sample is 1.748 and 1.650, a larger percentage difference than for domestically held currency. For the model to generate a larger percentage difference in velocity given the same pair of nominal interest rates, it must generate a larger difference in the fraction of goods purchased with credit between those two nominal interest rates. From (20), this implies that for the M1 calibration, the additional credit costs incurred at the higher nominal interest rate exceed those for the currency calibration. For distributions that do not behave highly nonlinearly outside the range relevant for our sample, it follows that the sensitivity of credit costs to the nominal interest rate is higher for the M1 calibration.

Price stickiness: we change the distribution of prices (ω) to $[0.4, 0.35, 0.25]$. With this distribution, the expected duration of a newly adjusted price is 2.5 quarters. The inflation rate in the long run under optimal policy is -0.49%.

Optimal policy comes closer to the Friedman rule in this case because the relative price distortions associated with deviations from zero inflation are smaller the more flexible are prices.

8 Dynamics under optimal policy

We now discuss the dynamics of the model under optimal policy, local to the benchmark steady state described above. Impulse response functions are presented for shocks to the level of productivity and to government spending (aggregate demand). We also compare the dynamics under optimal policy to what occurs under two simple policy rules, and investigate the robustness of the benchmark dynamics to a change in the distribution of credit costs.

8.1 Responses to two shocks

Figures 1, 2 and 3 illustrate how the economy behaves under optimal policy in response to persistent shocks (AR(1), with $\rho = 0.95$) to productivity and aggregate demand. As a reference point in these figures, we plot the behavior of real variables that would occur in a real version of the model – with flexible prices and no money demand distortions. For both shocks, the most important aspect of the figures is that they display negligible movement in the price level. With prices virtually stable, it follows that the real and nominal interest rates essentially move together. There is, however, an important qualitative difference between optimal policy in response to productivity shocks and that in response to aggregate demand shocks. While there is little price level variation in either case, productivity shocks make the real interest rate behave approximately as in a real business cycle model, whereas aggregate demand shocks make the real interest rate vary significantly more in the initial periods than it would in a real business cycle model. We will therefore discuss the two sets of impulse response functions in more detail.

8.1.1 Productivity Shocks

In a pure real business cycle model, the response to a productivity shock can be traced analytically, since the only distortion present is the markup, and it is constant over time. The real model behaves as

$$\begin{aligned} c_t &= a_t\left(1-l_t\right), \\ w_t &= \frac{\varepsilon-1}{\varepsilon}\cdot a_t, \\ w_t\frac{\partial u(c_t,l_t)}{\partial c_t} &= \frac{\partial u(c_t,l_t)}{\partial l_t}. \end{aligned}$$

For our benchmark parameterization, these equations imply

$$c_t = a_t\left(1+\theta\cdot\left(\frac{\varepsilon}{\varepsilon-1}\right)\right)^{-1}, \tag{29}$$

and the real interest rate follows

$$1+r_t = \beta^{-1}\left[E_t\left(\frac{a_t}{a_{t+1}}\right)\right]^{-1} \tag{30}$$

in the real business cycle model. The lines marked 'RBC' in Figure 1 correspond to (29) and (30). Under optimal policy, our model with four distortions behaves almost identically to the RBC model. Recall that in a model with sticky prices, no money demand distortions and only productivity shocks, for the preference specification we use it is optimal to keep the price level constant (section 5). Here the money demand distortions mean that complete price level stabilization is suboptimal, but the money demand distortions are small enough that it remains optimal to approximately stabilize the price level, and this makes the model behave much like an RBC model. In the impact period there is a slight deviation from the RBC model. This occurs because with prices approximately constant, the fall in the real interest rate (see (30)) implies a fall in the nominal interest rate, which stimulates consumption by driving down the interest rate wedge and hence the full price of consumption, as shown in (18).

8.1.2 Government Purchases Shocks

In response to a government spending shock, the model under optimal policy does not behave like the RBC model. Consumption falls significantly more – and labor input rises significantly less under optimal policy than in the RBC model. To understand this behavior, we will focus on the monetary authority's implementation constraint, which is the efficient pricing rule of monopolistically competitive firms in a sticky price environment. For the reader's convenience, we reproduce (23) in a slightly altered form, which arises if we assume away all monetary distortions and we therefore also assume that the economy has a zero inflation stationary state.[13]

$$\begin{aligned} 0 &= E_t \sum_{j=0}^{J-1} \beta^j \omega_j \left[(1-\varepsilon) \left(\frac{y_{j,t+j}}{c_{t+j}+g_{t+j}} \right)^{\frac{\varepsilon-1}{\varepsilon}} (c_{t+j}+g_{t+j}) \lambda(c_{t+j}, l_{t+j}) + \right. \\ & \left. \varepsilon \frac{\partial u\left(c_{t+j}, l_{t+j}\right)}{\partial l_{t+j}} \left(\frac{y_{j,t+j}}{c_{t+j}+g_{t+j}} \right) n_{t+j} \right] \end{aligned} \tag{31}$$

The implementation constraint is then shifted by government spending. Goodfriend and King [2000] consider a simple related model in which prices must be set in advance, but only for one period, which would result in the related constraint

$$0 = E_{t-1}[(1-\varepsilon)\left(c_t + g_t\right)\lambda(c_t, l_t) + \varepsilon \frac{\partial u\left(c_t, l_t\right)}{\partial l_t} n_t] \tag{32}$$

For their simpler model, Goodfriend and King argue that the key to understanding the effects of government purchases under optimal policy is to understand that the government will choose consumption, taking into account its influence on the contingent claims price $\lambda(c_t, l_t) = \frac{\partial u(c_t, l_t)}{\partial c_t}$. In particular, relative to the benchmark RBC solution, the government will want to have less consumption when government purchases are high because this makes the contingent claims value of g_t high, making it easier to satisfy monopoly producers. (As in our calibrated example, GK work with an additively separable utility function so that the state price depends only on consumption and not on leisure as well).

Our staggered pricing model's implementability constraint displays a similar incentive, but a dynamic one: the monetary authority wants to depress the consumption path somewhat while there are predetermined prices.. (There may also be subtler effects on the composition of demand $(y_{jt}/(c_t + g_t))$, but these turn out to quantitatively negligible). In line with this, Figures 2 and 3 show that the optimal plan involves consumption which is transitorily low relative to the RBC solution. Because consumption is expected to

[13] These assumptions let us use $\lambda = 1/c$ and $c + g = an$ to simplify the X functions used in our above analysis, so that they take the form of the bracked expressions in (31).

grow toward the RBC path in these periods, the real interest rate is high relative to the RBC level.

In our setting, then, it is not efficient for the government to stabilize consumption in the face of government purchase shocks, even though it is feasible for it to do so. Rather, the optimal policy is to somewhat reinforce the negative effects that g has on consumption, thus attenuating the effects on employment and output. But, since the implied movements in real marginal cost are temporary, they have little consequence for the path of the price level.

8.2 Optimal policy versus simple alternatives

It is of interest to know how the paths that our model economy takes under optimal policy compare to what would happen under some simple policy rules which are commonly employed by macroeconomists. Figures 4-7 illustrate how the model's dynamics under optimal policy deviate from what they would be under a constant money growth rule, suggested by Friedman [1959], and an interest rate rule of the form suggested by Taylor [1993]. As previously, we look at the effects of both productivity and government demand shocks. We choose the interest rate coefficients suggested by Taylor's analysis applied to our model economy, specifically our interest rate rule is $(R_t = R^* + 1.5 \cdot (\pi_t - \pi^*) + 0.5 \cdot ((\ln(c_t) - \ln(c))$.[14]

Aggregate demand shocks: Under a constant money growth rule, as Gali [1999] has stressed, sticky price models typically predict that some real quantities are predetermined in the short-run, with adjustment occurring only as prices respond. In our context, with a money demand function that is close to $\ln M_t - \ln P_t = \kappa + \ln c_t$, consumption is the real quantity that is relatively unresponsive if money stock is unchanged and the price level is largely predetermined, as shown in panel A of Figure 4: consumption takes about 4 quarters to fall to the optimal policy solution, which is that it should be low while government purchases are high. The constant money growth rule consequently requires that output and labor input respond more elastically

[14] There is a subtlety here because of the issue of modeling the "output gap." The conventional measurement of the output gap is a slowly varying measure, so that we use stationary consumption in our model economy. However, Goodfriend and King [1997] argue that the in models such as ours capacity output is the real business cycle solution in our earlier graphs, so that the output gap would be $c_t - c_t^*$. This change would affect our results quantitatively, but not qualitatively (see footnote 14).

in the short-run to government purchases than under optimal policy, as also shown in panel A. There are dramatic implication for the real interest rate shown in panel B: the real interest rate falls by 70 basis points with constant money growth rather than rising by 60 basis points as under the optimal policy. However, because the reduction in real consumption demand lowers the demand for money, there is a temporary rise in the price level under the constant money growth rule (panel D) and anticipations of these movements imply that the nominal interest rate is relatively unresponsive to the increase in government purchases (panel C).

Under the interest rate rule, by contrast, there is a smaller difference between real quantities: Figure 5 shows that consumption and labor approach the optimal policy solutions much faster (panel A). At the same time, there is an initial period in which the Taylor rule produces incomplete accommodation – just as did the constant money supply rule – so that consumption does fall toward the optimal policy path for several periods. Accordingly, the real interest rate again falls in response to the shock (about 20 basis points on impact) rather than rising 60 basis points as under optimal policy.[15] The nominal interest rate rises persistently under the Taylor rule, and this rise is associated with a similar rise in expected (and actual) inflation. The persistent rise in inflation translates into important cumulative effects on the price level (panel D).

Productivity shocks: The same considerations are relevant for understanding dynamic responses to productivity. In figure 6, panel A, we see that the dynamic response to the productivity shock under a constant money growth rule is similar to the responses suggested by Gali. Consumption can rise only to the extent that prices are flexible, so that it increases by about .2% (in contrast to 1% under the optimal policy). Labor input must fall, since the small output response (with higher productivity) mandates fewer units of work effort. Again, this difference in the response of real quantities carries over to effects on the real interest rate: it initially rises in the face of the productivity shock, rather than falling, because there is sustained growth in consumption over the first few periods of the response. But, as with the aggregate demand shock, there are nearly offsetting movements in the price level which make the nominal interest rate largely invariant to the shock.[16]

[15] If we replace steady state consumption with the RBC level of consumption the Taylor rule comes closer to replicating optimal policy, while still deviating substantially: the real interest rate rises ten basis points on impact.

[16] The path of the nominal interest rate is the path of the real rate plus the path of

Under the interest rate rule, the monetary authority partly accommodates the productivity shock so that the real effects discussed above are attenuated (see panel A of Figure 7). But the Taylor rule again sets in motion persistent variations in inflation – deflationary changes when the productivity shock is positive – that have important consequences for the nominal interest rate (Figure 7, panel C) and the price level (Figure 7, panel D).

Combining the results: Taking these responses to demand and productivity response together, one can say that both the fixed money and Taylor rules imply less than the optimal degree of accommodation, so that the optimal policy solution is not reached and there are important effects on the price level. Typically, the Taylor rule is closer in terms of real responses.

8.3 Sensitivity to distribution of credit costs

In Figures 8 and 9, we investigate how the productivity shock dynamics under optimal policy are affected by two variations in the procedure for calibrating the distribution of credit costs. First, for Figure 8 we assume the distribution of credit costs is concentrated about the mean rather than being uniform. Specifically, as in section 7 we switch from $\phi_1 = \phi_2 = 1$ to $\phi_1 = \phi_2 = 2$, and recalibrate ϕ_3 and ϕ_4 to match the two pairs of velocity and nominal interest rates. The dynamics under optimal policy are virtually unaffected by this change in parameters. Next, for Figure 9 we set $\phi_1 = \phi_2 = 1$, but calibrate ϕ_3 and ϕ_4 using M1 rather than currency. The behavior of real quantities does not vary significantly from the benchmark case. While the price level is much more variable than in the benchmark case, it still moves by less than one percent in the long run, in response to a shock that has a cumulative impact on output of roughly twenty percent. Recall from section 7.2 that this case is one in which the steady state involves deflation of 0.65%.

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the expected inflation rate. With with our assumption on utility, the path of the nominal rate R_t is approximately $-E_t[\log c_{t+1} - \log c_t] + E_t[\log P_{t+1} - \log P_t]$, but constant money implies that the right-hand side is constant (since $\log P_t + \log c_t = \log M_t$). This is only an approximation since there are terms in the marginal rate of substitution that involve the wedge of monetary inefficiency. But it is a satisfactory approximation for our model.

9 Conclusion

We have developed a model monetary economy which includes four distortions relative to a real, imperfectly competitive model without capital. These distortions are introduced as a basis for examining the two most widely advocated views on the appropriate role of monetary policy, targeting either a zero nominal interest rate or price stability. The first two distortions involve production and encourage the monetary authority toward zero or positive rates of inflation. In our economy, there is imperfect competition and the resultant markup of price over nominal marginal cost leads to production that is inefficiently low. Moreover, the presence of staggered price adjustment implies that firms producing similar goods nonetheless have distinct nominal prices in the presence of trend inflation. This introduces a relative price distortion that leads to an inefficient mix of goods produced. Into this environment we introduce money as an alternative means of exchange through which households may economize on the time costs of using credit to finance consumption. The two remaining distortions, both of which encourage the monetary authority to pursue low nominal rates of interest, arise through the process of exchange. When nominal interest rates are positive, the absence of interest paid on money balances leads to inefficiently high levels of credit use. Additionally, the standard wedge of monetary inefficiency appears: positive nominal interest rates increase the full price of consumption.

We examine the role of optimal monetary policy under commitment, using a parameterized version of our model economy. Many of our parameter values are familiar ones in the equilibrium business cycle literature. The remainder are determined by matching some elements of recent U.S. monetary history.

Optimal long run inflation rates are close to zero. While the exchange distortions lead to some trend deflation, the production distortions maintain relatively high nominal interest rates. Examining the response of the optimal policy economy to productivity shocks and shocks to government spending, we find that the role of optimal monetary policy, to a first approximation, is to stabilize the price level around trend. In this sense, optimal policy is distinct from some commonly advocated rules, such as constant money growth or Taylor's rule.

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Appendix: Derivation of (28)

This appendix illustrates the derivation of (28) in section 4.2. Beginning with equation (27), the first step toward this recursive reformulation involves application of the law of iterated conditional expectations. Abbreviating $\mathbf{X}(\lambda_l, y_{j,l}, c_l, l_l, g_l, a_l)$ using $\mathbf{X}_{i,l}$ and expanding the summations in (27), we have the following.

$$
\begin{aligned}
&\min_{\{\Phi_t,\Omega_t\}_{t=0}^{\infty}} \max_{\{\mathbf{S}_t\}_{t=0}^{\infty}} \mathbf{E}_0 \Bigg\{ \Bigg(u(c_0, l_0) + \Phi_0 \Big[\omega_0 \mathbf{X}_{0,0} + \ldots + \beta^{J-1} \omega_{J-1} \mathbf{X}_{J-1,J-1} \Big] \\
&+\Omega_0 \Big[\lambda_0 - \frac{\beta(1+R_0)}{\Pi_1} \lambda_1 \Big] \Bigg) \\
&+\beta \Bigg(u(c_1, l_1) + \Phi_1 \Big[\omega_0 \mathbf{X}_{0,1} + \ldots + \beta^{J-1} \omega_{J-1} \mathbf{X}_{J-1,J} \Big] \\
&+\Omega_1 \Big[\lambda_1 - \frac{\beta(1+R_1)}{\Pi_2} \lambda_2 \Big] \Bigg) \\
&+\ldots \\
&+\beta^t \Bigg(u(c_t, l_t) + \Phi_t \Big[\omega_0 \mathbf{X}_{0,t} + \ldots + \beta^{J-1} \omega_{J-1} \mathbf{X}_{J-1,t+J-1} \Big] \\
&+\Omega_t \Big[\lambda_t - \frac{\beta(1+R_t)}{\Pi_{t+1}} \lambda_{t+1} \Big] \Bigg) \\
&+\ldots \Bigg\}
\end{aligned}
$$

The second step groups terms involving quantity variables and the household's multiplier λ_t of the same period. This makes immediate the inherent nonstationarity of the problem which we illustrate by explicitly including several additional periods below. Specifically, let $2 < i < J-1$ and $t > J-1$, we have

$$
\begin{aligned}
&\min_{\{\Phi_t,\Omega_t\}_{t=0}^{\infty}} \max_{\{\mathbf{S}_t\}_{t=0}^{\infty}} \mathbf{E}_0 \Bigg\{ \bigg(u\left(c_0, l_0\right) + \Big[\Phi_0 \omega_0 \mathbf{X}_{0,0}\Big] + \Big[\Omega_0 \lambda_0\Big] \bigg) \\
&+\beta \bigg(u\left(c_1, l_1\right) + \Big[\Phi_1 \omega_0 \mathbf{X}_{0,1} + \Phi_0 \omega_1 \mathbf{X}_{1,1}\Big] \\
&+\Big[\Omega_1 - \frac{\Omega_0\left(1+R_0\right)}{\Pi_1}\Big] \lambda_1 \bigg) \\
&+\beta^2 \bigg(u\left(c_2, l_2\right) + \Big[\Phi_2 \omega_0 \mathbf{X}_{0,2} + \Phi_1 \omega_1 \mathbf{X}_{1,2} + \Phi_0 \omega_2 \mathbf{X}_{2,2}\Big] \\
&+\Big[\Omega_2 - \frac{\Omega_1\left(1+R_1\right)}{\Pi_2}\Big] \lambda_2 \bigg) \\
&+\ldots \\
&+\beta^i \bigg(u\left(c_i, l_i\right) + \Big[\Phi_i \omega_0 \mathbf{X}_{0,i} + \ldots + \Phi_0 \omega_i \mathbf{X}_{i,i}\Big] \\
&+\Big[\Omega_i - \frac{\Omega_{i-1}\left(1+R_{i-1}\right)}{\Pi_i}\Big] \lambda_i \bigg) \\
&+\ldots \\
&+\beta^{J-1} \bigg(u\left(c_{J-1}, l_{J-1}\right) + \Big[\Phi_{J-1} \omega_0 \mathbf{X}_{0,J-1} + \ldots + \Phi_0 \omega_{J-1} \mathbf{X}_{J-1,J-1}\Big] \\
&+\Big[\Omega_{J-1} - \frac{\Omega_{J-2}\left(1+R_{J-2}\right)}{\Pi_{J-1}}\Big] \lambda_{J-1} \bigg) \\
&+\ldots \\
&+\beta^t \bigg(u\left(c_t, l_t\right) + \Big[\Phi_t \omega_0 \mathbf{X}_{0,t} + \ldots + \Phi_{t-(J-1)} \omega_{J-1} \mathbf{X}_{J-1,t}\Big] \\
&+\Big[\Omega_t - \frac{\Omega_{t-1}\left(1+R_{t-1}\right)}{\Pi_t}\Big] \lambda_t \bigg) \\
&+\ldots \Bigg\}.
\end{aligned}
$$

Note that the first $J-2$ periods involve less terms than all subsequent time periods making the optimal policy problem non-recursive. However, we introduce the additional multipliers $\Phi_{-(J-1)}, \ldots, \Phi_{-1}$ and Ω_{-1} and add, to the above problem, the following terms.

$$
\begin{aligned}
&\Phi_{-1}\omega_1 X_{1,0} + \ldots + \Phi_{-(J-1)}\omega_{J-1}\mathbf{X}_{J-1,0} + \Omega_{-1}\frac{(1+R_{-1})\,\lambda_0}{\Pi_0} \\
&+\beta\Big[\Phi_{-1}\omega_2 X_{2,1} + \ldots + \Phi_{-(J-2)}\omega_{J-1}\mathbf{X}_{J-1,1}\Big] \\
&+\ldots \\
&+\beta^i\Big[\Phi_{-1}\omega_{i+1} X_{i+1,i} + \ldots + \Phi_{-(J-1-i)}\omega_{J-1}\mathbf{X}_{J-1,i}\Big] \\
&+\ldots \\
&+\beta^{J-2}\Big[\Phi_{-1}\omega_{J-1}\mathbf{X}_{J-1,J-2}\Big]
\end{aligned}
$$

where $\mathbf{X}_{j,t} = \mathbf{X}\left(\lambda_t, y_{j,t}, c_t, l_t, g_t, a_t\right)$.

Figure 1

Response to a productivity shock under optimal policy

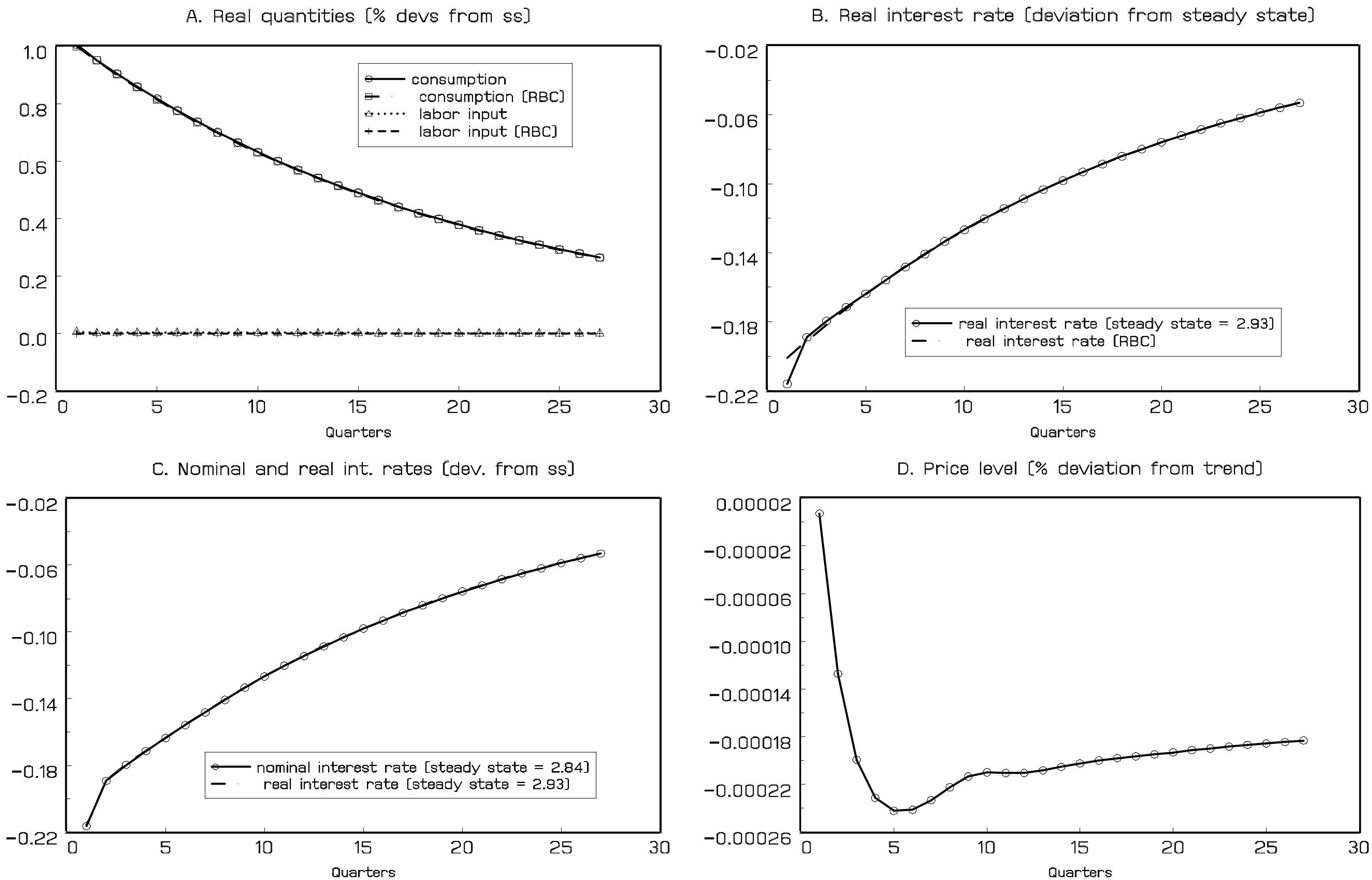

Figure 2

Response to a demand shock under optimal policy

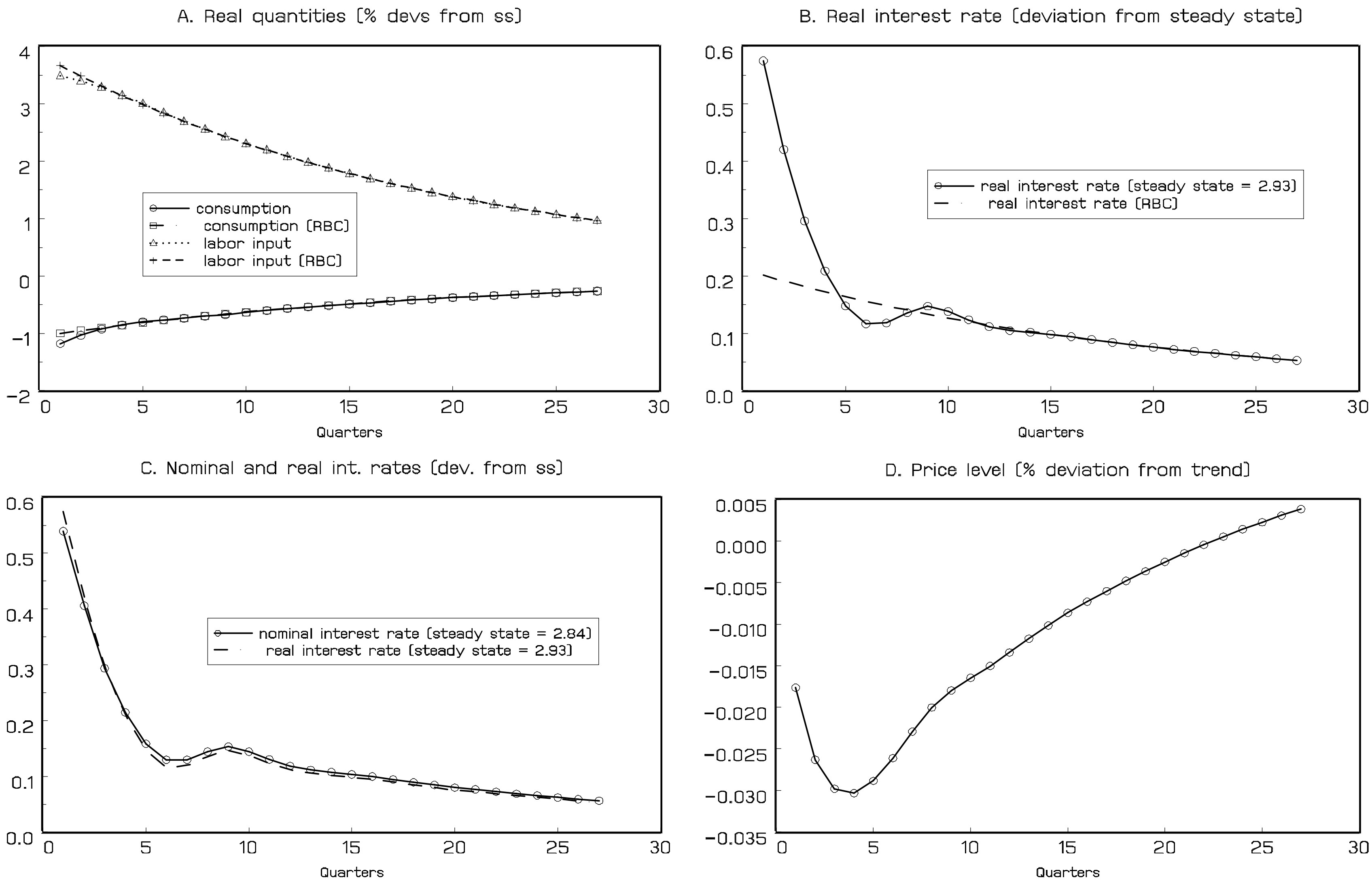

Figure 3
Productivity vs. aggregate demand shocks

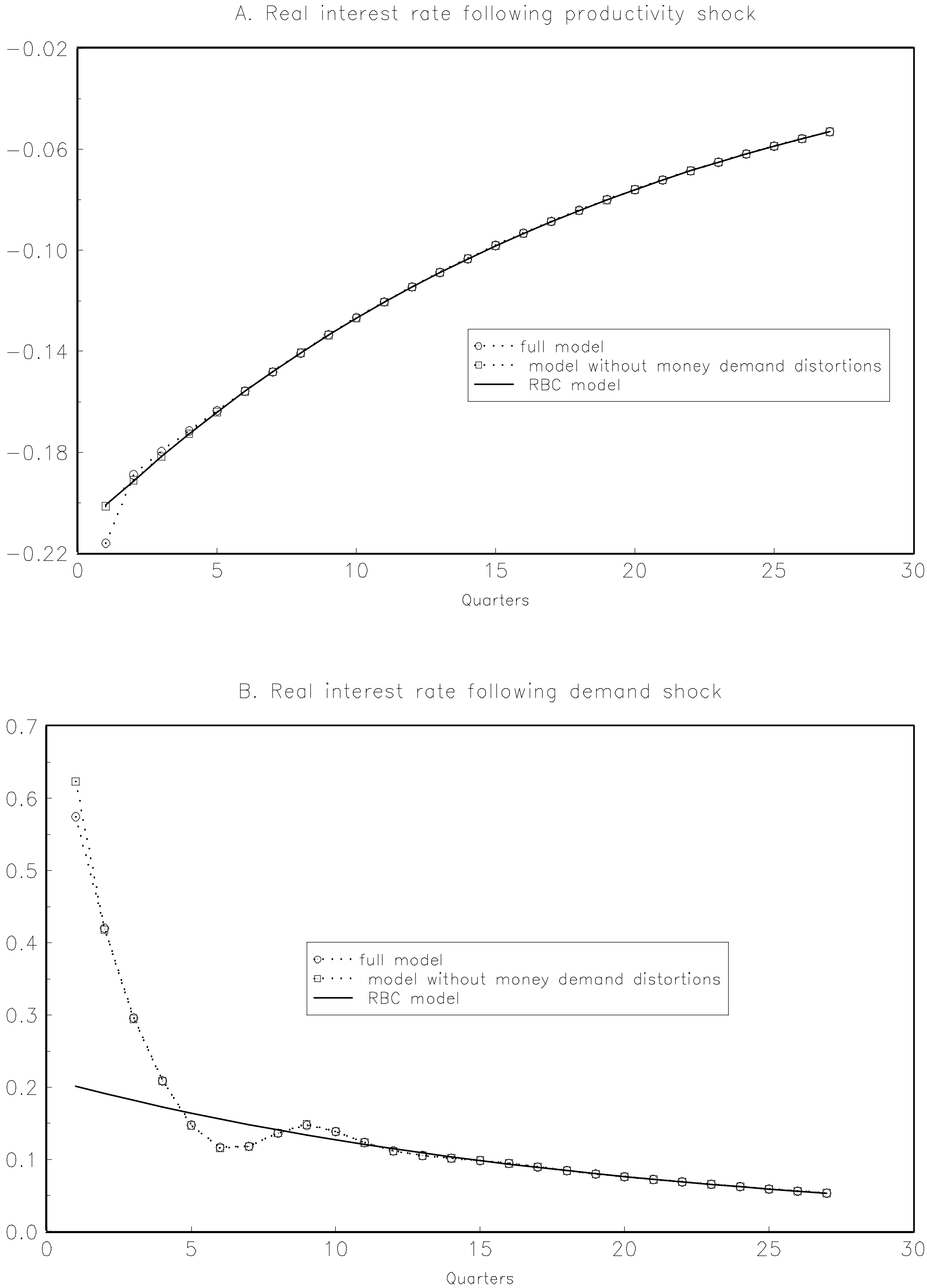

Figure 4: Aggregate Demand Shock
Comparison between constant money growth and optimal policy

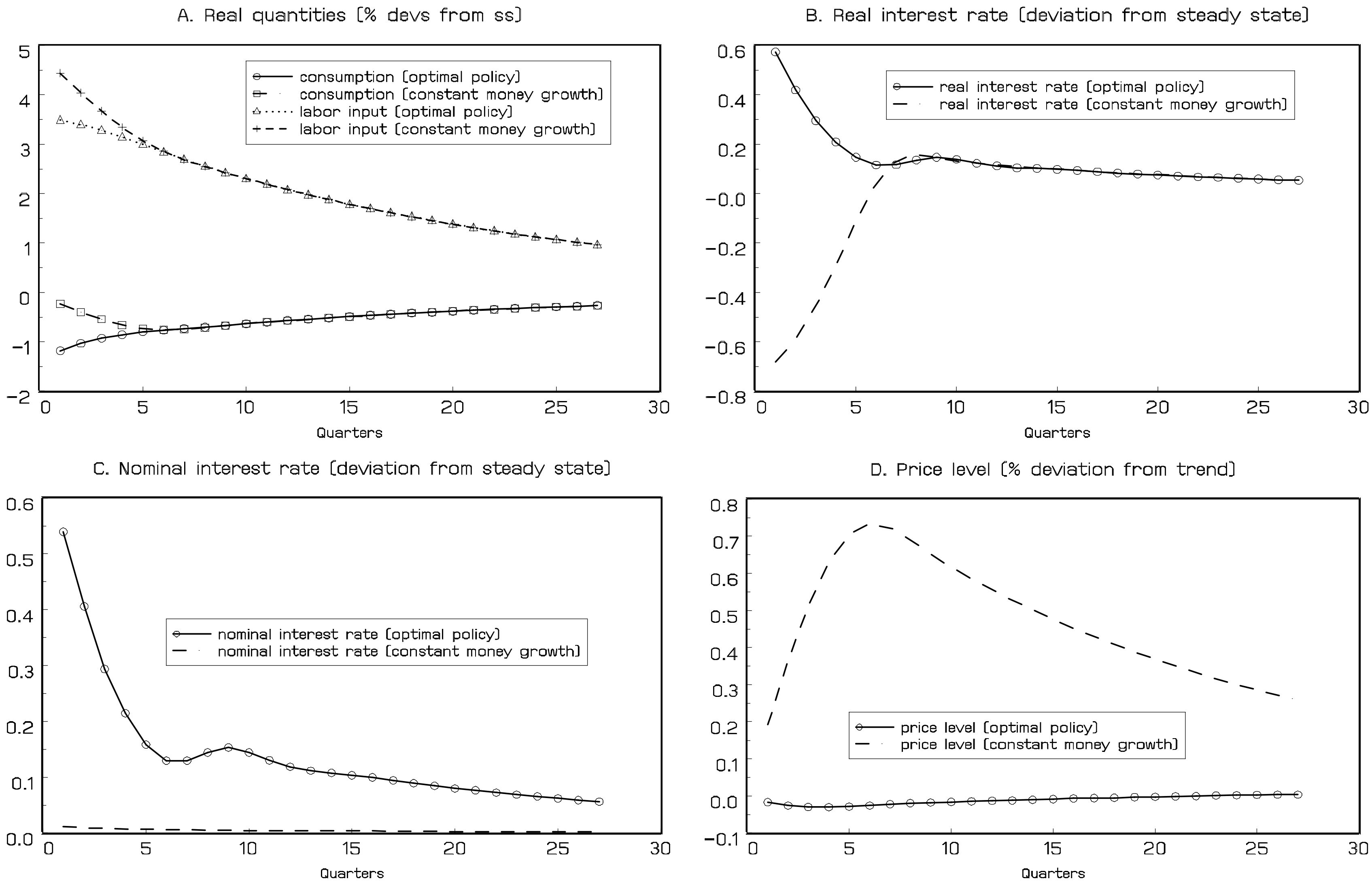

Figure 5: Aggregate Demand Shock
Comparison between Taylor rule and optimal policy

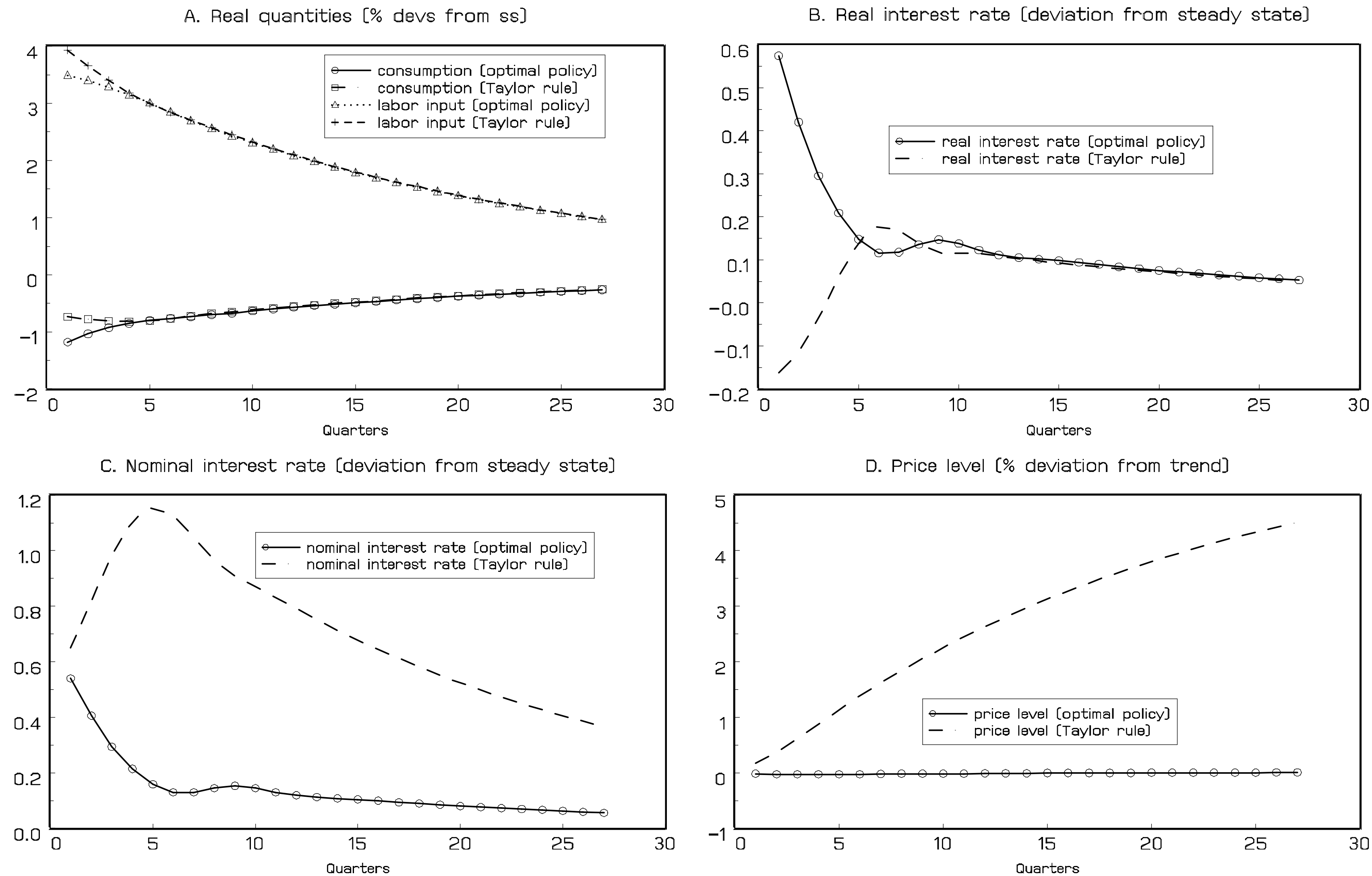

Figure 6: Productivity Shock

Comparison between constant money growth and optimal policy

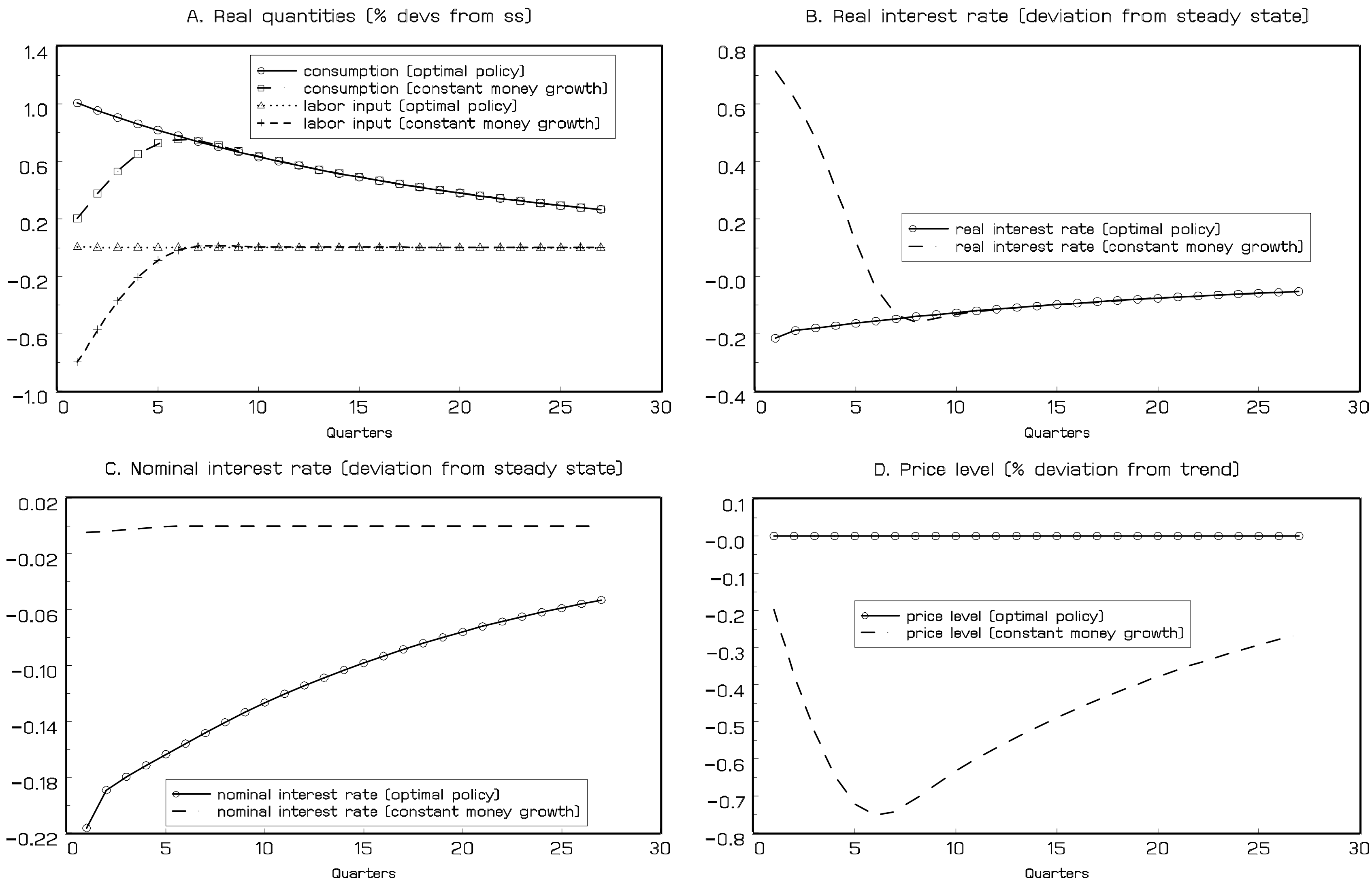

Figure 7: Productivity Shock
Comparison between Taylor rule and optimal policy

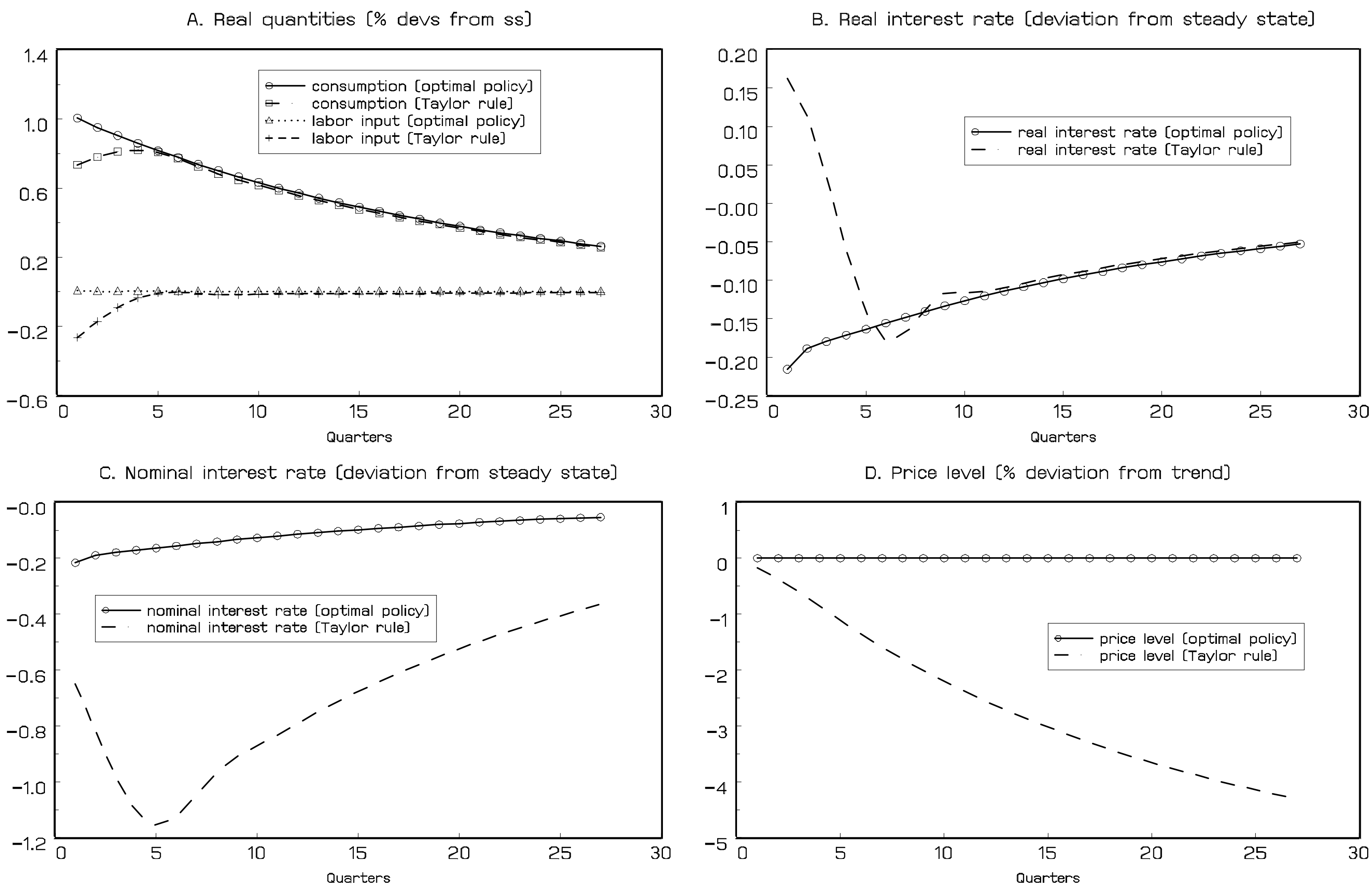

Figure 8: Productivity Shock

Comparison of benchmark and concentrated distrib. of credit costs

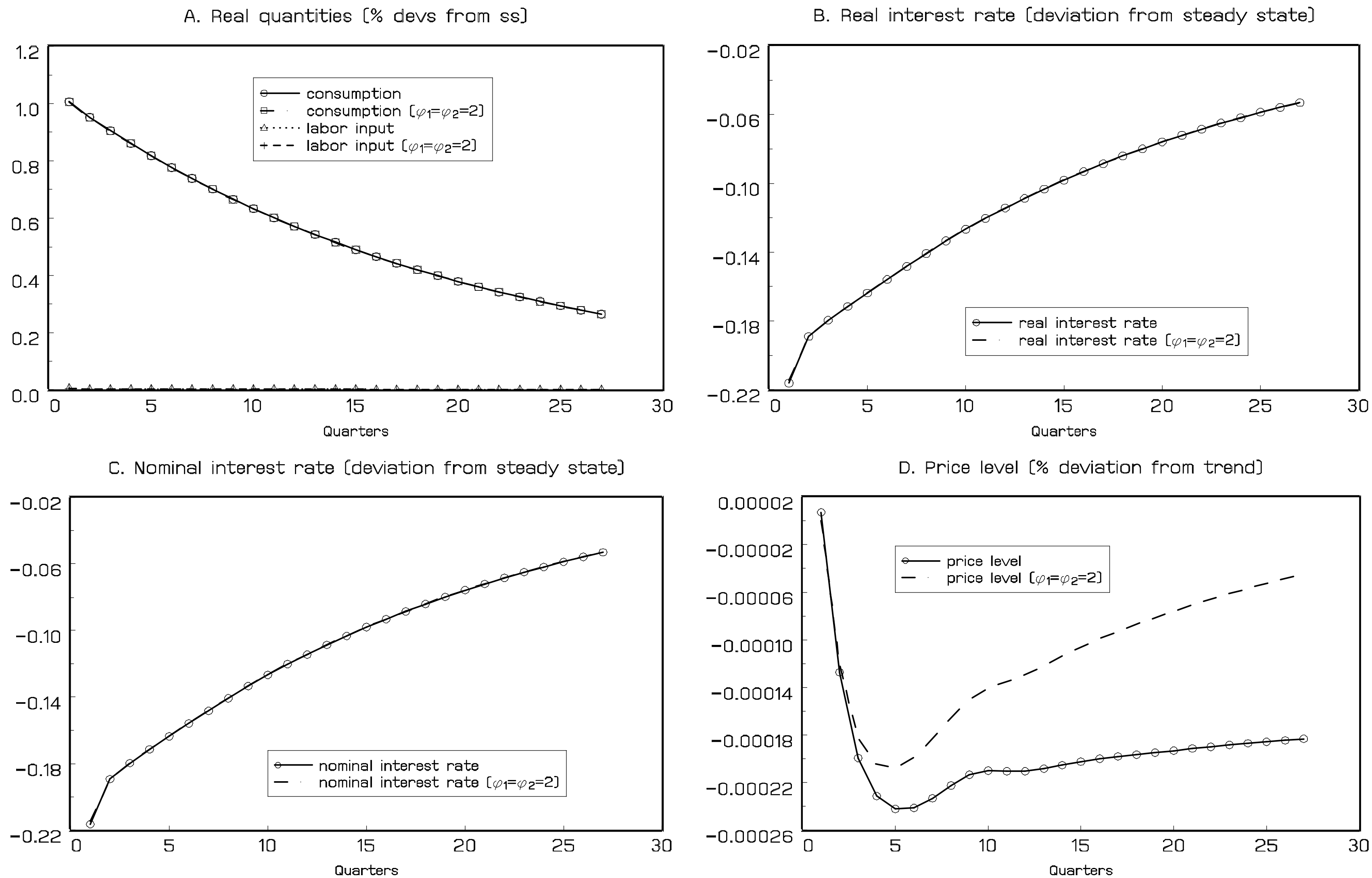

Figure 9: Productivity Shock
Comparison between benchmark and M1 calibration