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Abstract

In this paper we document real rate behavior. We do this by looking across a wide variety of constructed real rate series. These series are obtained by using a number of different methodologies for estimating expected inflation, using several different price series, and looking over different time periods. The evidence suggests that over the entire sample period, real rate increases follow output increases and the real rate is positively correlated with contemporaneous output. These results are generally contrary to those presented in the literature. The results, however, are sensitive to the price series used. That is, we find evidence of specification uncertainty. We also find that real rate behavior varies over different sample periods

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1 Introduction

With the recent advent of dynamic macroeconomic models that imbed nominal rigidities, there has been renewed interest in optimal monetary policy rules (King and Wolman (1998) and Woodford (1999)). One central characteristic of rules that use an interest rate instrument is that the nominal interest rate should respond to movements in the real interest rate. Unfortunately, the real rate of interest is unobservable implying that policy makers face severe implementation constraints. The central bank, however, does possess information on the state of the economy and, therefore, in making inferences about the real rate it would be helpful to know how movements in the real rate are correlated with the business cycle. In this paper we concentrate on the cyclical relationship between the ex-ante real rate of interest and real GDP.

Understanding this relationship is not only of significance to policymakers, but also to macroeconomic theorists. One challenging area of model development is matching modeled asset price behavior to data. In order to do so, one must have an accurate representation of autocorrelation and cross correlation functions in the data. Understanding the behavior of the real rate of interest has also been a topic of recent empirical interest. For example, Stock and Watson (1999) present data on the cross-correlations of the business cycle components of Chain-Weighted Gross Domestic Product with 70 macroeconomic time series over the 1947:1-1996:4 sample period, including a 90-day real treasury bill interest rate. Their findings suggest that the real interest rate is weakly to moderately negatively correlated with both the contemporaneous observation of output and with output up to four quarters leading and lagging the current quarter. King and Watson (1996) present similar contemporaneous and leading negatively correlated results, but find a positive correlation between the current real rate and prior output over the 1949:1-1992:4 sample period. As stated in King and Watson, the prevailing message from this recent work is that, "all prominent macroeconomic models...do a poor job at matching the interaction of real and nominal interest rates with real activity."

Before accepting these empirical results, it would be beneficial to analyze the empirical evidence in more detail. That is, we wish to more fully document real rate behavior, before accepting the evidence in these papers as a stylized fact. We do this by looking across a wide variety of constructed real rate series. These series are obtained by using a number of different methodologies for estimating expected inflation, using several different price series, and looking over different time periods. The evidence suggests that over the entire sample period, real rate increases follow output increases and that the real rate is positively correlated with contemporaneous output. These results are generally contrary to those presented in the previously cited empirical work. The results, however, are sensitive to the price series used. That is, we find evidence of specification uncertainty. We also find that real rate behavior varies over different sample periods.

To complete our study of the behavior of the ex-ante real rate of interest we also investigate its stationarity properties. We do this by looking at augmented Dickey-Fuller statistics and by using the Horvath and Watson (1995) procedure in the case where the cointegrating vector is known. Given the results in the literature it is not surprising that our conclusions regarding the stationarity of the ex-ante real rate depend on the sample period. These conclusions also depend on the price series and methodology used for calculating the real rate.

2 Construction of Real Rates

In this section we describe our various methods of constructing the real rate of interest. We use five basic methodologies and four different price series.

2.1 The Data

In order to ease comparison of our results with those presented by the benchmark model of Stock and Watson (1999), except where otherwise noted, data is taken from the Stock and Watson (SW) data set.[1] We selected four different price indices to use as measures of inflation: the monthly Consumer Price Index (CPI); the monthly CPI excluding food

[1] This data set was obtained from Mark Watson's web site at http://www.wws.princeton.edu:80/~mwatson/.

and energy for all urban consumers, sometimes referred to as the "Core CPI"; the quarterly GDP implicit price deflator (GDPD); and the quarterly personal consumption expenditures implicit price deflator (PCE).[2] In our forecasting equations, we also used the SW data set's nominal 90-day treasury bill rate (R3) and the unemployment rate for workers 16 years and older (U). The SW data was originally obtained from Citibase. Quarterly data for the Chain-Weighted Gross Domestic Product (GDP) is used as our output series in computing the cross-correlations with the real rate. [3]

2.2 Estimating Ex-Ante Real Rates

The ex-ante real rate is defined to be the nominal treasury bill rate, in period t, less agents expectations of inflation for period t+1, conditional on the information that becomes available by the end of period t. This is consistent with the notion that the ex-ante real rate should reflect the expected goods-valued return on an asset held over the maturity period, as well as the rental rate on capital used for future production of goods. Formally,

$$r_t = R3_t - E_t\left[\pi_{t+1}\right] \qquad (1)$$

For example, the month of December real interest rate is the December nominal rate, less the quarter one forecast of inflation made using information available through December 31.

In generating a series for $E_t\left[\pi_{t+1}\right]$,we use several different econometric forecasting methodologies as well as published forecasts produced by DRI. Compilation of the DRI real rate series did not require the use of an econometric forecasting model on our part, but rather the

[2] We also constructed real rates using the Investment deflator and an implicit price deflator for private GDP.

[3] The series mnumonics in Citibase and the Stock and Watson dataset are as follows:

CPI = punew, GDP Deflator = GDPD, PCE Deflator = GDC, 90-day t-bill = FYGM3, Unemployment Rate = LHUR, Chain-Weighted GDP = GDPQ, Other Data was obtained from DRI's, @uscen database, as follows: Core CPI = CPIU@f&E (12/1/98), Investment Deflator = PCWI (3/24/99), Private GDP Implicit Deflator = (GDP-G)/(GDP92C-G92C), that is the ratio of nominal GDP less government expenditures over chain-weighted GDP less government expenditures (3/34/99),and Private GDP = GDPD92C-G92C.

manual matching of data to form a quarterly real rate series.[4] A quarterly average of the nominal interest rate for the current quarter was matched to the subsequent quarter forecast made at the end of the current quarter (this forecast uses the first month of the subsequent quarter as the DRI control date). For example, the first quarter DRI real rate is the average of R3 for January, February and March, less the second quarter inflation forecast made at the end of March (DRI's April control date). The series for both the DRI GDP deflator and the DRI CPI real rates begin in the fourth quarter of 1973.[5]

2.2.1 Inflation Forecasting Methods

Four different econometric methods were used to compute a historical series of three-month inflation forecasts. Our benchmark model is an in-sample VAR. The in-sample VAR, however, is not wholly adequate for the purposes of constructing an ex-ante real rate. This is because the regression fits the model over the full sample of available data when forming parameter estimates. Thus the forecaster implicitly uses information not available at the time of the forecast. This, in turn, reduces the possibility of persistent one-sided errors that could occur when the economic environment is changing as it would, for instance, with a drastic change in monetary policy. To counter the deficiencies in the in-sample model, we also construct inflation forecasts using: an out-of-sample VAR forecast with a fixed start date; an out-of-sample VAR forecast with a rolling start date; and a time-varying parameter model using the Kalman Filter. We shall now describe each of the four procedures in turn.

In-Sample Forecasts - The Benchmark Models - In-sample forecasts using the four price indices were made using equations identical to those specified by Stock and Watson (1996). The model fits a VAR over the full sample of available data for each series. Therefore, the forecast in a given period uses information not available at the time of the forecast. The basic VAR forecasting equation is:

[4] Data used in the construction of the real rate using DRI historical forecast data for the CPI and GDP deflator was generously provided by Robert Hetzel. Further detail on this series is provided in Darin and Hetzel (1995).

[5] The GDP deflator series used the Implicit Price Deflator (up to 7/92), the Fixed-Weight Deflator (8/92 through 1/96) and the Chain-Weighted GDP Price Index (2/96 onward).

$$\pi_{t+1} = \alpha_0 + \sum_{i=0}^{3}(\beta_i \cdot \pi_{t-i}) + \sum_{i=0}^{3}(\gamma_i \cdot U_{t-i}) + \sum_{i=0}^{3}(\delta_i \cdot R3_{t-i}) + \varepsilon_t \tag{2}$$

where U and $R3$ are the unemployment rates and nominal interest rate series described above. Inflation, π_{t+1} is given by $\pi_{t+1} = 400*\ln(\frac{P_{t+3}}{P_t})$, when the price index, P,is a monthly series and by $400*\ln(\frac{P_{t+1}}{P_t})$, when the price index is a quarterly series. Expected inflation is just the in sample fitted values of equation (2). [6]

Out-of-Sample VAR Forecasts With Fixed and Rolling Start Dates - The first alternative set of forecasts we construct use out-of-sample VAR models, where the forecast is provided by selecting the order of the lag operators that exhibits the best out-of-sample forecasting performance up to the time that the forecast is made. Forecasting equations are based on (3) where we vary the number of lags used for each exogenous variable by using a data-dependant lag selection procedure. This procedure allows for the selection of between zero and five lags of the exogenous variables in making each period's forecast (a single constant term, representing the mean of the dependant variable, is also included in the set of possible forecasting models). The ex-ante inflation forecasts may be formally described by:

$$E_t[\pi_{t+1}] = \mu_t + \alpha_t(L) \cdot \pi_t + \beta_t(L) \cdot U_t + \gamma_t(L) \cdot R3_t \tag{3}$$

The degree of the lag polynomials, $\alpha(L)$, $\beta(L)$, and $\gamma(L)$ are chosen each period to minimize the Bayes Information Criterion (BIC), T*ln(RSS)+K*ln(T), where T is the number of observations, RSS is the residual sum of squares, and K is the number of regressors. Therefore, in each period, the forecaster is given the opportunity to select the best performing equation out of a pool of 6^3 VAR lag specifications.

Fixed start date versions of this model make use of all data up to the time of the forecast in computing the regression and rolling start date models use a regression estimate based on

[6] Regression results are available on request.

the prior ten years of data.[7] The rolling start date method is used to accommodate changes in the economic environment that could make basing forecasts on data in the far past a poor procedure. We develop both a fixed and rolling series of inflation forecasts for each price index, the fixed start date models are referred to as the 'recursive' models and the rolling start date models are termed the 'rolling' models.

Time-Varying Parameter Forecasts - Our time varying parameter (TVP) forecasting method is the univariate time-varying coefficients procedure described in Stock and Watson (1996). This class of time varying parameter models is found to have relatively good forecasting properties for the real interest rate by Bekdache (1998). The method allows the forecaster to continuously update his forecasting model. As with the out-of-sample VAR models, the forecasting exercise allows for the selection of the lag specification that provides the best performing forecast model in each period. Letting $\theta_t = (\mu_t, \alpha_{1t}, ...\alpha_{mt})'$, where m is the lag length and $z_t = (1, \pi_{t-1}, ..., \pi_{t-m})'$, then the TVP model is given by

$$\pi_t = \theta_t' z_t + v_t, \tag{4}$$

where the parameter θ_t evolves according to

$$\theta_t = \theta_{t-1} + u_t. \tag{5}$$

The TVP method forms an expectation of $\theta_{t+1,}$ $\theta_{t+1|t}$, and then uses this estimated parameter in forming the conditional forecast at time t. Forecast optimization occurs over six different lag specifications, m = 0,..,5. The variance-covariance matrix of u_t is given by $\lambda^2(\frac{1}{T}\sum_{t=1}^{T} z_t' z_t)$ and θ_t is estimated by a Kalman Filter for each value of λ. The lag length and λ values are chosen to minimize the conditional predictive least squares criterion, PLS $= \sum_{t_o}^{T}(y_t - y_{t|t-1})^2$. To reduce dependance on initial conditions, t_o was set at 120 for monthly data and 40 for quarterly data series.

[7] For both types of out-of-sample VAR's, a buffer set equal to the maximum number of lags was used to make the BIC statistics comparable. That is, for each of the 6^3 equations, the first regression begins in the sixth period after the data becomes available, regardless of the lag operator that would be chosen for the first regression by the BIC statistic.

2.2.2 Inflation Forecasting Model Performance

Table 1 compares the forecasting performance of the four models for each price series. It is clear that the TVP model is the best out of sample predictor for each of the price series, and even out performs the in-sample predictions for both the CPI and the Core CPI. These findings are consistent with both the parameter instability findings of Stock and Watson (1996) who investigate the CPI and Bekdache's (1999) assertion that the random TVP model is a good predictor of the real interest rate.[8]

Table 1 - Inflation Forecasting Performance of TVP Models					
	In-Sample VARs		Out-of-Sample VARs		TVP Model
	Period	S.E.E.	RMSE		RMSE
			Recursive	Rolling	
CPI	58:07-96:12	1.94	2.09	2.14	0.97
GDP defl	59:03-96:04	1.01	1.31	1.28	1.16
PCE defl	59:03-96:04	1.09	1.31	1.34	1.23
Core CPI	67:07-96:12	1.71	1.82	2.02	0.89

3 Examination of the real rate series

In this section we describe the behavior of our constructed real rate series. The main message we hope to convey is that although the various real rate measures are highly correlated their behavior is far from identical.

The correlation among expected real interest rates is given in Tables 2a and Tables 2b. Table 2a compares correlations using the different price indices holding the estimation methodology fixed. The ranges for the correlations, by econometric forecasting methodology are: 0.84-0.94 for the in-sample VAR methods; 0.760-0.93 for the recursive VAR; 0.63-0.86

[8] Bekdache's finding about the forecasting performance of the real rate is equivalent to statements about the forecasting performance of respective models for inflation. If $r_t = R_t - E_t[\pi_{t+1}]$, where r_t is the real interest rate and R_t is the nominal rate, then a forecaster predicting the real rate is actually predicting inflation, since R_t is known at the time of forecast.

for the rolling VAR; and 0.73-0.90 for the time-varying parameters models. Also, 0.85 is the correlation between the real rates based on the DRI CPI and GDP inflation forecasts.

Table 2b holds the price index fixed and varies the methodology. Overall, these series tend to be more highly correlated than those in Table 2a. For example, the frequency of correlations greater than 0.90 is approximately three times greater. The correlation between the series range from as low as 0.71 to as high as 0.97. For each given price index, there is a reasonably high correlation between the series constructed by the different econometric methods. Correlations between series constructed by the CPI range from 0.75 to 0.87. The GDP deflator produces series that are correlated between 0.90 and 0.95, while the PCE deflator series displays correlations between 0.90 and 0.97. The correlation across methods for real rates based on the Core CPI range from 0.71 to .91. Because of this higher correlation between real rates of the same price index, it is reasonable to assert that real rates are more sensitive to the price index used than to the construction method.

Table 2a

Correlations holding method fixed

	Var-in			Var-recursive				Var-rolling				tvp			
	core	pce	gdpd	core	pce	gdpd		core	pce	gpd		core	pce	gdpd	
cpi	.86	.92	.85	.87	.82	.76		.81	.78	.72		.85	.79	.73	
core		.85	.84		.80	.81			.64	.63			.81	.76	
pce			.94			.93				.86				.90	

Table 2b

Correlations holding price series fixed

	CPI			Core				PCE				GDPD			
	rec	roll	tvp	rec	roll	tvp		rec	roll	tvp		rec	roll	tvp	
var-in	.82	.75	.87	.91	.75	.88		.93	.90	.94		.91	.90	.95	
recursive		.84	.84		.85	.83			.91	.97			.94	.95	
rolling			.77			.71			.	.91				.93	

The general patterns of the series are observed by examining the real rates plotted in Figure 1. Figure 1a-1e plots estimates of the real interest rate based on each price index holding the estimation method constant, while Figured 1f-1i depicts the real rate of interest holding fixed the price index and allowing the estimation methodology to vary. There is a certain broad consistency between the real rates plotted in these figures. Each series tends to be low and positive during the 1960's and exhibits an upward trend toward the end of that decade. Also, the series tend to become both negative and more volatile, during the onset of the inflations of the 1970's. The series rise dramatically beginning with the disinflationary policy introduced by the Fed in October of 1979. Several of the series surge to more than eight percent during this period. The series then begin to gradually decline for the remainder of the sample period. The recession of the early 1990s is uniformly characterized by zero or slightly negative real rates.

Although such broad similarities exist between the series, it is evident that the various real rate measures are not identical. For example, the Core-CPI series plotted in Figure 1i clearly differ by as much as four percentage points during the early 1980s. Perhaps more striking, in 1980 figures 1a-d depict real rates that are both positive and negative in the same quarter depending on the methodology used.

4 Results of Business Cycle Decomposition

In this section we analyze the cyclical behavior of our various real rate series. To decompose each series into its cyclical component we use the approximate band pass filter developed by Baxter-King (1994). Following Stock and Watson (1999), we preserve cycles in the 6-32 quarter frequency range.[9] We display the cyclical behavior of our various real rate measures in figure 2.

Several general trends are observed in the filtered real rate series. The real rates tend to decline during a recession and bottom out with or shortly after the cyclical trough. Typically, the real rate peaks shortly prior to the onset of a recession, although there are examples of real rate peaks that do not precede an NBER business cycle peak. The real rates tend to exhibit their largest increase when the Federal Reserve raised interest rates during the early 1980s and for most series the largest decrease occurs immediately prior to that period.

[9] Further details on this method are in Stock and Watson (1998).

Again, in noting the differences in the series, the series plotted in Figure 2a-i show many large disparities in the real rates. There are several instances in which the filtered real rates move in different directions. For example, the VAR-recursive method in the second panel shows examples of such patterns in 1967, 1977 and again around 1979. In the third and fourth panels, similar patterns may be observed in 1977 and 1979, for the VAR-Roll and TVP methods, respectively.

Turning to the cross-correlations of our series with output, we first establish the benchmark findings of Stock and Watson (1999), reproduced in table 3.[10] This series is represented as our CPI in-sample VAR real rate.

Table 3 - Results From Stock & Watson (1999)									
Cross Correlation of Real Rates with Output (r_t,y_{t+k})									
Series	–4	-3	-2	-1	0	1	2	3	4
SW 1997	-0.05	-0.07	-0.12	-0.19	-0.28	-0.35	-0.38	-0.36	-0.29

Their results indicate that the real interest rate is negatively countercyclical leading and contemporaneously negatively correlated with output. Furthermore, the evidence in Stock in Watson indicates that the real rate is negatively countercyclical lagging. As pointed out

[10] The King and Watson (1996) results are as follows:

-4	-3	-2	-1	0	1	2	3	4
0.19	0.19	0.10	-0.07	-0.27	-0.43	-0.52	-0.50	-0.41

Note, however, that the King and Watson results are not striclty comparable to others presented in this study. First, their output series, instead of GDP, is Net National Product, less housing and farming and their price index is a deflator for this series. Second, our deflator series for both Gross Domestic Product and Personal Consumption Expenditures, and Gross Domestic Product are chain-weighted series, which only became available recently. Finally, the King-Watson methodology is quite different in that they use a VAR model to extract inflation and nominal rate functions and then compute the real intrest rates. Using this method, they then inverse Fourrier transform the data to exactly Band-Pass Filter the series (our BP Filter is an approximation as detailed in Stock and Watson (1997)). For comparison, we constructed an in-sample and ex-post real rate using their data. However, our results suggest that the approximate BP filtered data has a reasonably different cross correlation with output than the exactly filtered series. We also looked at a measure of private GDP, which excludes government spending. Omitting government spending did not meaningfully change our results.

in King and Watson (1996), existing macroeconomic models do not display this behavior.

But as noted in the previous section, many of the alternative series we have constructed differ substantially from this benchmark real rate. Indeed, it is conceivable that the incompatibility between the predictions of various theoretical models and the business cycle correlations presented in Table 3 is attributable to either the methodology used in constructing real rates or to the price series employed. The Stock and Watson real rate series is constructed using the CPI and an in-sample VAR forecast of inflation over the period 1947:1-1996:4. Such in-sample forecasting produces forecast errors with a mean of zero and does not simulate the conditions observed by forecasters who may make one-sided errors during times in which the state of the world is unknown. Hence, the Stock and Watson method may not produce an accurate measure of the ex-ante real interest rate. Furthermore, the reliance on inflation forecasts based on a single price measurement, namely the CPI, is problematic. Not only is the CPI time-series plagued by numerous unadjusted redefinitions over time, but also it is unclear what price measure is used by market agents when forming rational inflationary expectations.

The Figure 3 graphs of corr(r_t, y_{t+k}) represent a robustness check on the Stock and Watson findings. The results are substantially different from those presented in Table 3. Most notably, a positive contemporaneous correlation between output and the real rate is observed for thirteen of the eighteen series examined. Although several different inflation forecast methods were attempted, in general, differing forecast methods do not dramatically alter the cross-correlation properties of the real rate series. The major exceptions to this observation are the Core CPI rolling regression and the DRI CPI forecast in Figures 3.15 and 3.17 respectively.[11]

Results, however, were sensitive to the price index used. Series constructed using CPI inflation forecasts tend to preserve the negative current correlation of Stock and Watson,

[11] Note, however, that the DRI CPI real rate was constructed as a quarterly series and does not use a comparable information set to the other CPI series. For example, the DRI series for the first quarter is simply the fourth quarter average of the nominal treasury bill series, less the DRI January control date forecast for the first quarter. Other CPI series construct monthly real rates in a similar fasion, which are then averaged to form the quarterly series. Hence; the information available to the DRI forecaster is much greater since all fourth quarter information is known at the time of the forecast.

while other price indexes tended to produce positive contemporaneous cross correlations. Further many of the series, most notably those using the GDP deflator, produced positive correlations between lagged output and the real rate, suggesting that real rate increases follow output increases (making the real rate a positive lagging series). With the GDP deflator, high real rates are also a reasonably strong signal of an increase in output at least one quarter into the future. So using a broad measure of price behavior gives a very different perspective of the cyclical properties of the real rate from that using the CPI.[12]

If one were to use a narrower view of inflation, the PCE deflator could arguably be the preferred series. Although it measures a similar basket of goods, it is a chain weighted index and suffers less from measurement problems than the CPI because it is more consistently defined over the sample period. The cyclical behavior of the real rate constructed using the PCE deflator is much more similar to that of the real rate using the GDP deflator than it is to the real rate using the CPI. Results for the Core CPI are largely similar to those obtained with the PCE deflator. We shall examine these differences in more detail momentarily.

Instability Over The Business Cycle - Although the correlations presented thus far in our analysis generally support the hypothesis that the real rate is positively correlated with output, the correlations are not stable over time. The typical nature of this instability, which occurs across all methods and price series, is shown in Figures 4.1-4.12, using the TVP PCE deflator real rate series. These figures present the correlations of the filtered real rates series, divided into three subsamples: first quarter 1958 through fourth quarter 1973; 1974:1-1982:4; and 1983:1-1996:4.[13] The breakpoint dates were chosen to isolate a period

[12] We also performed this analysis on real rates constructed using the implicit price deflator for investment expenditure and private GDP. Results for the latter were largely similar to those for the GDP deflator series. The investment deflator produced mixed results, with the recursive and TVP series roughly similar to the PCE deflator or the Core CPI, but the in-sample and the rolling series producing negative correlations (although large standard errors for these correlations make the results fairly insignificant).

[13] Note: for all results presented in this subsection, we have used the same real rate series as in the previous section; that is, we have not reestimated the forecasting regressions based on the sample period data, but have simply isolated the sample periods.

that includes several oil shocks induced recessions, and low credibility monetary policy.[14]

The noticeable difference to be observed in these figures is that during the 1974:1-1982:4 period, real rates are negatively correlated with contemporaneous output and that real rate decreases lead and follow output increases.[15] This represents almost a total inversion of the relation observed outside of this period. This striking observation, can be partially accounted for by the dominance of temporary oil price shocks that were hitting the economy during this period. Such shocks typically generate a negative correlation between output and the real interest rate.

Explaining the Differences in Consumer Prices - A natural question that arises from the above analysis is: why are the results for the CPI real rate series so much different from those of the PCE deflator? After all, the PCE deflator series is constructed using much of the same raw data as the CPI and many of the disaggregated indices have similar inflation rates.

The Boskin Commission Report has recently gained attention for summarizing the problems associated with the measurement of the consumer price index, any of which could potentially contribute to the observed differences (see Boskin et. al 1996 and 1998). In addition, various nuances of the construction methodology are another potential source of difference (See BEA, 1982, Table 2, p12 for a prior reconciliation of the price indices). Some of these potential problems include:

1. The weighting of component indices - The CPI is constructed as a modified Laspeyres index, with a consumption bundle fixed at base year expenditures as recorded in the Consumer Expenditure Survey. As this method does not account for substitution between goods, it typically overstates the rate of inflation. The PCE deflator is a so-called "Fischer Ideal Index" and allows for some substitution between different goods.[16] Because different

[14] For conciseness, we have chosen to present these results for the kalman filter real rates. Rolling regression results were fairly similar.

[15] This behavior was exhibited by the PCE deflator series, the Core CPI series and the CPI series. Real rates constructed using the GDP and the TVP method produced an insignificantly positive contemporaneous correlation, but rolling regression results for the GDP series produced a negative correlation.

[16] See Moulton and Stewart, 1999 and Reinsdorf, 1999 for a precise formulation.

types of goods, durables versus non-durables for instance, have differing cyclical properties, the lack of substitution could imply different cyclical behavior in these two series.

2. Non-comparable items - Although, for the most part, both the PCE and the CPI use the same raw data, a number of items are not considered comparable between the two indices. Unpublished statistics compiled by Clint McCully of the National Income and Wealth Division of the Bureau of Economic Analysis estimate that about 28.7 percent of goods included in the PCE deflator have no identifiable counterpart in the CPI. Further, items in the CPI that have no comparable measure in the PCE deflator account for about twelve percent of the relative importance of the overall CPI. Items are deemed non-comparable for any number of reasons, including the method of construction (e.g. for certain medical expenses input costs of the producer are used), definition incompatibilities (e.g. the PCE deflator uses net premiums on auto and health insurance, whereas the CPI uses gross premiums), new items (which are more expeditiously incorporated into the PCE), or items in one index that are not defined in the other (e.g. foreign travel by U.S. residents abroad is included in the PCE).

3. Unrevised definitional changes - When an accounting or definitional change is made to the series, the BEA revises the PCE deflator back in time as far as possible. The CPI, however, is left unrevised prior to the introduction of the change.

While all of the above issues certainly contribute to the differences, we posit that it is this last measure that is of prime importance in reconciling the findings of the CPI with those of other price measures. Specifically, we suggest that the BLS's January 1983 introduction of a rental equivalence measure of home ownership value has a substantial impact on the results. We make this conjecture because it is primarily in the pre-1983 sample period that the CPI series differs from the other price series as illustrated in Figures 4.1-4.12.

To further illustrate the impact of this inconsistency in the CPI time series, we construct a real rate of interest using a modified version of the consumer price index. The series uses the conventional CPI described from January 1983 onward and a modified CPI from January 1967-December 1982, which reflects the current treatment of rental equivalence in the historical series. [17] Because the modified series is only available from 1967 onward,

[17] This series, termed the CPI X-1, was provided to us by Ken Stuart of the BLS.

we preserve the number of observations available for analysis by constructing the inflation forecasts using the in-sample VAR method detailed in Section 2.[18] As our previous analysis seems to indicate, the choice of methodology has only a limited impact on the business cycle correlations.

Figures 5.1, 5.3 and 5.4 plot the filtered correlations for the real rates constructed by the four consumer price series: the PCE deflator; the Core CPI; the CPI; and the CPI-X1 series. For ease of comparison, the sample for each of these quarterly series is restricted to the 1967:2-1996:4 period. Immediately, one notices the drastic improvement in matching the PCE deflator real rate that is gained from using the X-1 series instead of the CPI series. Whereas the CPI series essentially exhibits a moderately negative correlation with output for the contemporaneous, leading and lagging quarters, the X-1 real rate, by comparison, is significantly negatively correlated with future output and insignificantly correlated with contemporaneous and prior period output. These results seem to accord more closely with those for the PCE deflator real rate series than those of the CPI series.

A second, and more puzzling observation, arises in the comparison between the real rates constructed by the PCE deflator, the Core CPI and the CPI (Figures 5.1-5.3). Here it is observed that the Core CPI real rate produces results more similar to the real rate constructed using the PCE deflator than to the CPI. This appears to indicate that the treatment of housing, while important, is not the only factor influencing the odd behavior of the CPI and that the food and energy sectors also contribute to its peculiarity.

Figures 6.1-6.4 compare the filtered correlations with output of the *inflation rates* for the food and energy sectors of the PCE and CPI. The correlations for the energy sectors do not differ substantially. However, the contemporaneous correlation for the CPI food sector of .58 is greater than the PCE's 0.35. Presumably differences in the weights on these sectors also exert influence on the CPI's results, especially during the 1970s when the fixed weighting of the CPI would not allow for substitution away from energy goods as the price of oil exploded.

Therefore, it appears that some combination of the treatment of housing, food, and

[18] The in-sample regression was run on the monthly data for the period 67:05-97:03. The R^2-bar of the regression is 0.487506 and the standard error of the estimate is 1.974.

energy account for the differences between the CPI and the PCE, but clearly they are not the entire story. In light of our findings, one should probably view the CPI results with some degree of skepticism.

5 Stationarity of the Real Rate Series

In this section we investigate the stationarity of our various constructed expected real rate series. A number of author's have examined this issue and arrived at varying conclusions. The studies do have one element in common in that all the stationarity tests were conducted on ex-post real rates using CPI inflation. For example, Mishkin (1992) finds evidence that the real rate is stationary over the sample period 1953-1990 and the period 1953-1979. However, Gali (1992) over the sample period 1955:1 to 1987:3 cannot reject a unit root in the real rate, but appeals to the low power of the tests in maintaining the assumption of stationarity in the body of his empirical work. Similarly, King, Plosser, Stock, and Watson (1991) do not reject non-stationarity of the real rate over the period 1954:1-1988:4.

In the empirical work presented in table 4, we employ the augmented Dickey-Fuller test where the lag length was chosen by the step-down procedure suggested in Ng and Perron (1993). We also use this step-down procedure for determining the number of lagged first differences in the VECM that is estimated in the Horvath and Watson (1995) procedure for the case where the cointegrating vector is assumed to be known. Their results indicate that imposing a known cointegrating vector, which for us is (1,-1), greatly improves the power of the tests.

As one observes in the table, there is no clear cut evidence regarding the stationarity properties of the ex-ante real rate. For the shorter sample periods, which are chosen so that all the methodologies could be included, the results are not precise. The adf tests seems to indicate that the ex-ante real rate constructed using expected cpi inflation is stationary, while the Horvath Watson (hw) tests imply that the nominal interest rate and expected inflation as measured by the GDP deflator are cointegrated. For the expected real rate using pce inflation, only the recursive methodology indicates stationarity, while use of the Core-CPI yields stationarity for the rolling methodlogy. For longer sample periods that examine the expected real rate using the Var-in method, there is a strong indication that all four measures of the real rate are stationary. Thus, in analyzing the longer run stochastic properties of the real rate the relevant measure of inflation does not seem overly crucial. Of interest, however, is the fact that the TVP methodology, which generally produces the best forecasts of inflation, appears to indicate that all measures of the real rate are non-stationary over the shorter sample period.

Table 4						
Stationarity tests						
price measure used in real rate	sample period	test	var-in	recursive	rolling	tvp
cpi	1958:2-1996:4	hw	6.35	7.69	9.07+	5.98
		adf	-3.02*	-2.88+	-4.29**	-1.88
pce	1958:2-1996:4	hw	8.09	8.65+	7.42	7.36
		adf	-2.08	-2.97*	-3.62**	-1.94
gdpd	1958:2-1996:4	hw	9.65+	10.28*	8.38+	7.22
		adf	-2.26	-2.36	-2.27	-1.95
core	1967:2-1996:4	hw	7.90	8.13	13.77**	7.30
		adf	-2.03	-2.01	-3.90**	-2.40
cpi	1948:4-1996:4	hw	10.36*			
		adf	-2.99*			
pce	1948:4-1996:4	hw	8.74+			
		adf	-2.64+			
gdpd	1948:4-1996:4	hw	8.52+			
		adf	-3.58**			
core	1957:2-1996:4	hw	9.70+			
		adf	-3.09*			
+,*,** indicate rejection of the null hypothesis of no cointegration for the Horvath-Watson						
test and rejection of a unit root for the adf test at the critical values of 10%, 5%, and 1%,						
respectively.						

6 Conclusion

This paper has been motivated by a desire to better understand the time series properties of the real interest rate. This desire has been spurred by previous empirical findings that are inconsistent with theoretical model predictions and the relevance of real rate behavior for monetary policy. To examine real rates, we have presented an analysis of a multitude of different real rate measures and compared them with the results of previous studies.

Perhaps the strongest observation to be made is that real rate peaks and troughs appear to have an established relationship with business cycle peaks and troughs. Every post-war business cycle peak follows a peak in the real rate series and a real rate trough occurs either with or shortly after a business cycle trough. This observation is consistent across the eighteen series we constructed.

Far more important are our findings regarding the business cycle correlations of real rates. Contrary to the findings of earlier analyses, we believe that real rates are better characterized as positively correlated with contemporaneous output. We also find evidence that real rate increases tend to follow output increases by up to two quarters. The correlations are sensitive to the sample period used and may be related to the nature of shocks hitting the macroeconomy. These findings are generally consistent with the predictions of a standard Real Business Cycle model.

As a by-product of performing this analysis, we have presented a number of ancillary results. First, the random time varying parameter model produces the best forecasts of inflation over the given sample periods. TVP model results sometimes even outperform in-sample VAR models that implicitly take account of future information in making a forecast. Second, econometric forecasting methods do not drastically impact the business cycle correlation properties of the series. With only a few exceptions, results for the correlations of real rates generated using the TVP model of inflation forecasts were essentially the same as those for the in-sample, recursive and rolling VARs. Finally, regarding the unusual real rate behavior observed with series constructed with the Consumer Price Index, which differ substantially from those using the other three price series, we believe that a significant portion of this behavior is due to the way housing rent was incorporated into the CPI prior to 1983. Moreover, we suggest that definitional inconsistency in the CPI may render it an inappropriate series for constructing real rates.

With regard to the stationarity of the real rate our results are mixed. Using in-sample forecasts of inflation over our longer sample period produces the best evidence for the stationarity of the real rate, but to fully resolve this issue may require a longer data set or more powerful tests.

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Figure 1a. Real rates method: VAR-in

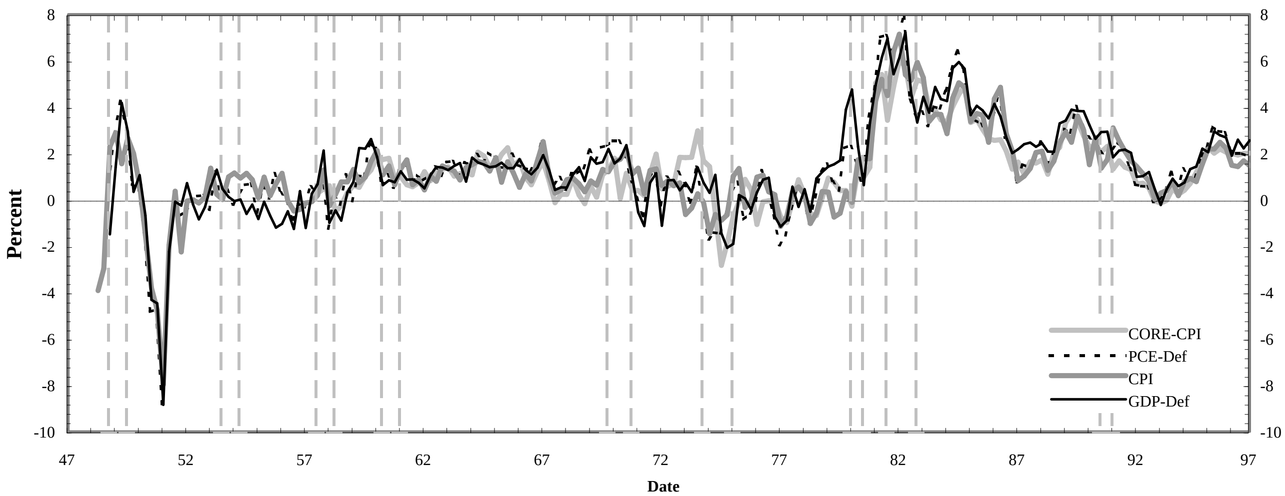


Figure 1b. Real rates method: VAR-recursive

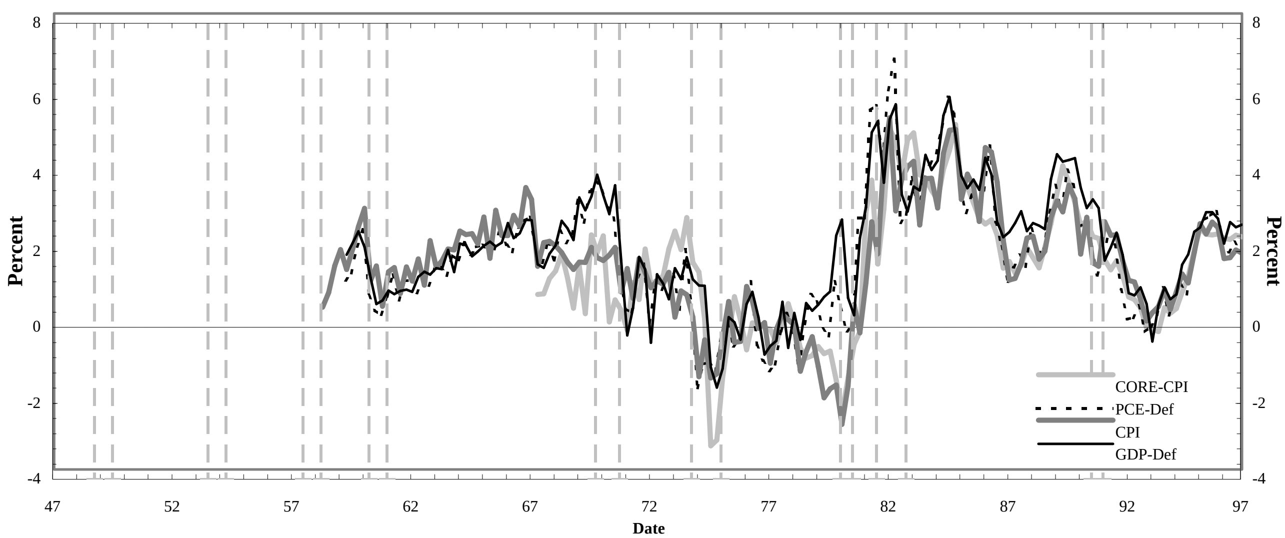


Figure 1c. Real rates method: VAR-roll

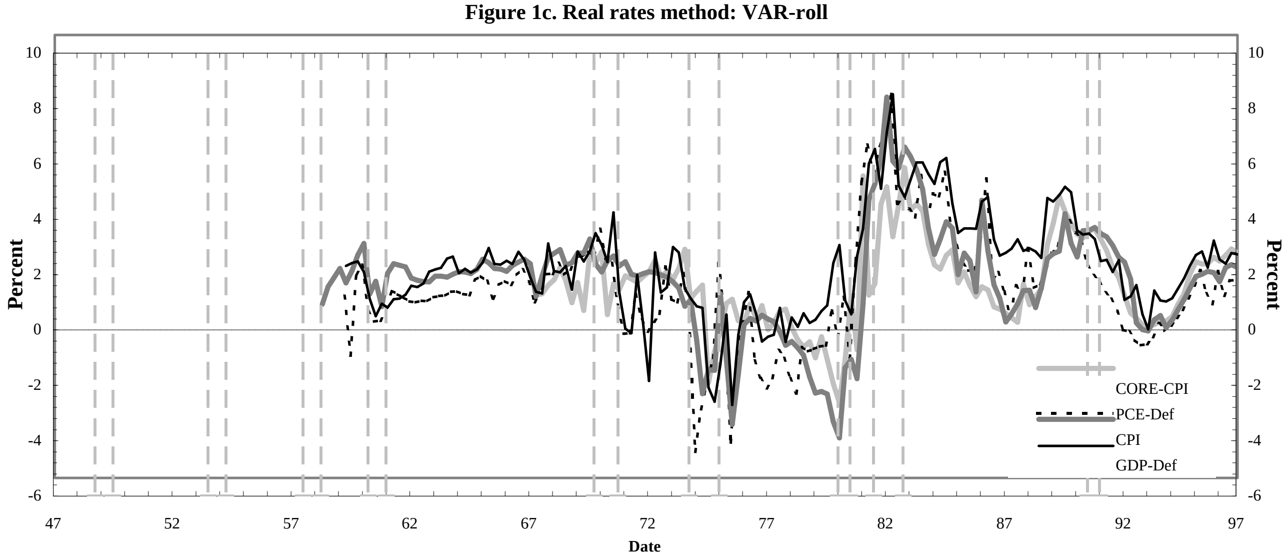


Figure 1d. Real rates method: TVP

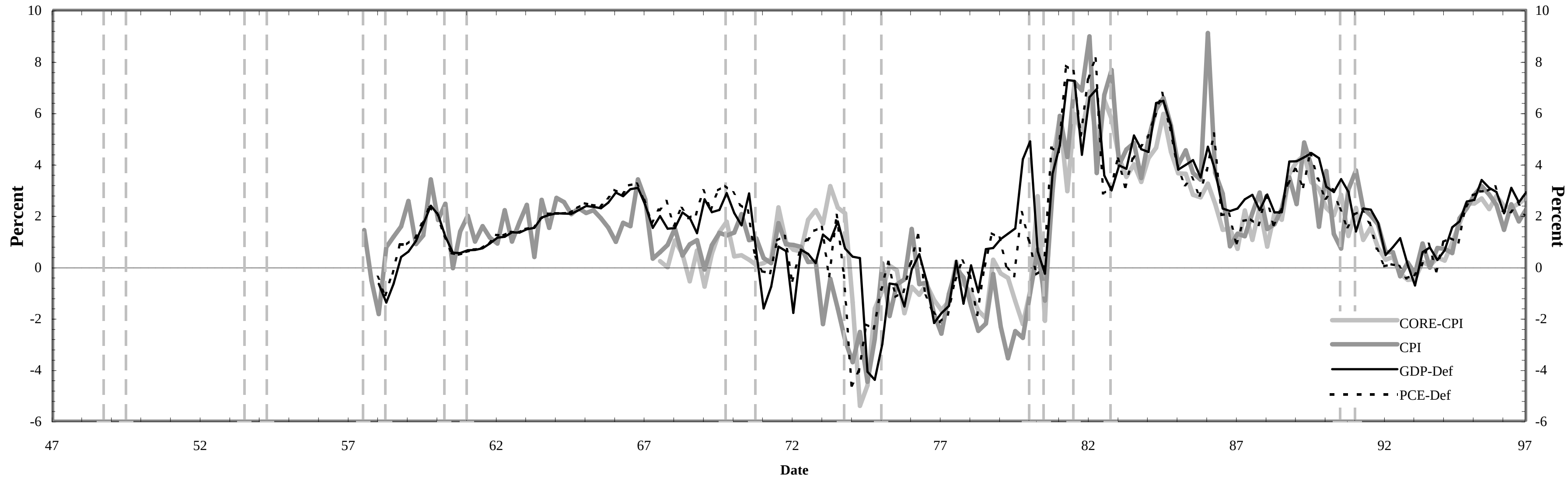


Figure 1e. Real rates method: DRI-publ

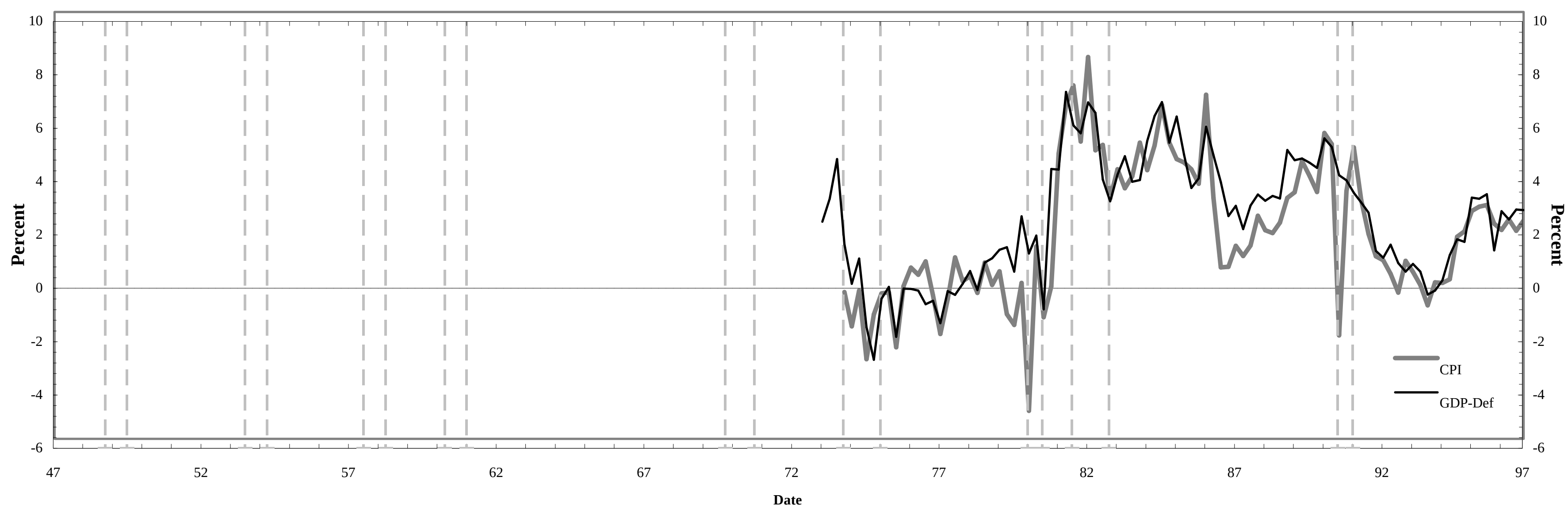


Figure 1f. Real rates method: CPI

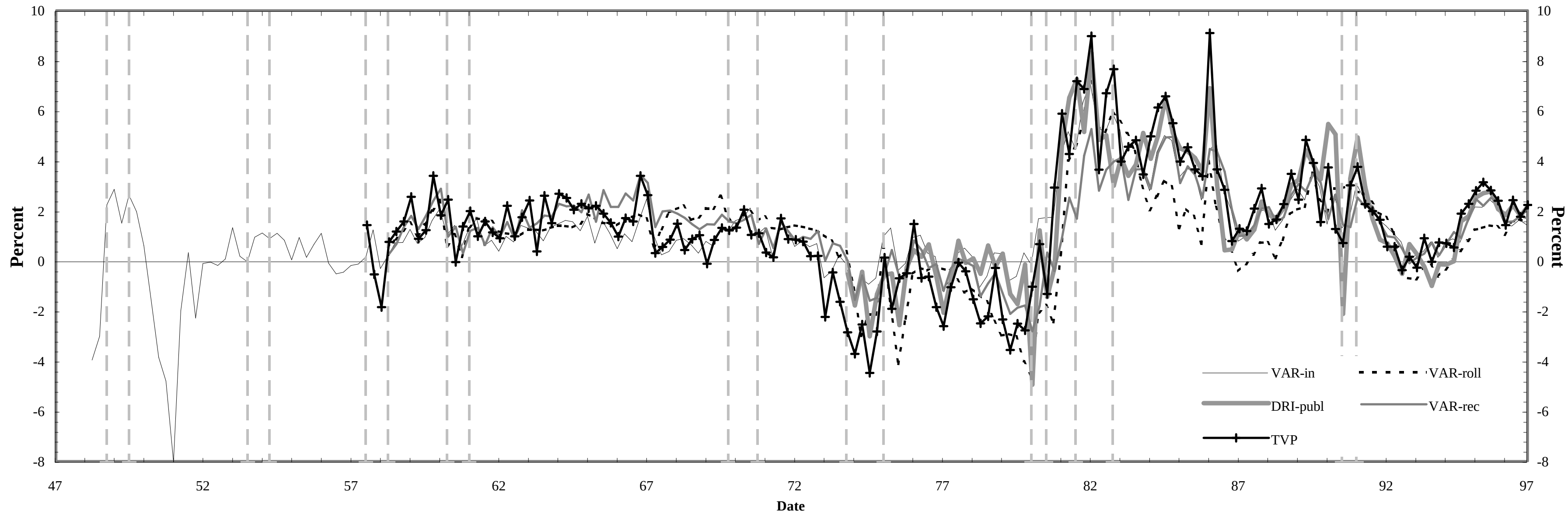


Figure 1g. Real rates method: GDP Deflator

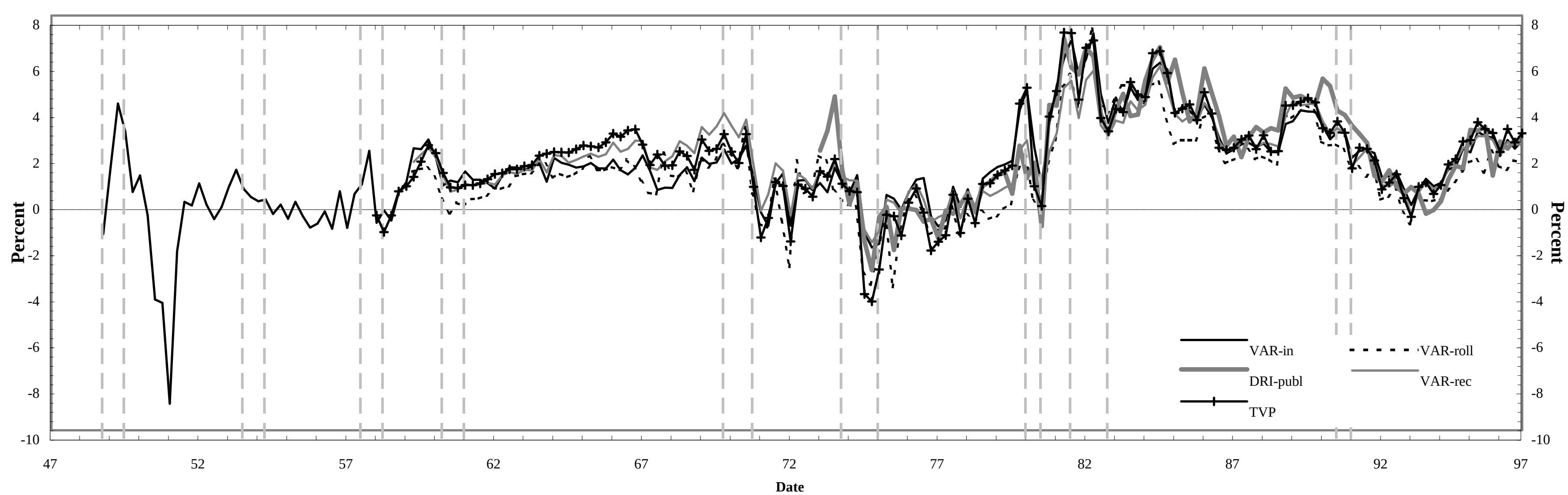


Figure 1h. Real rates method: PCE-Deflator

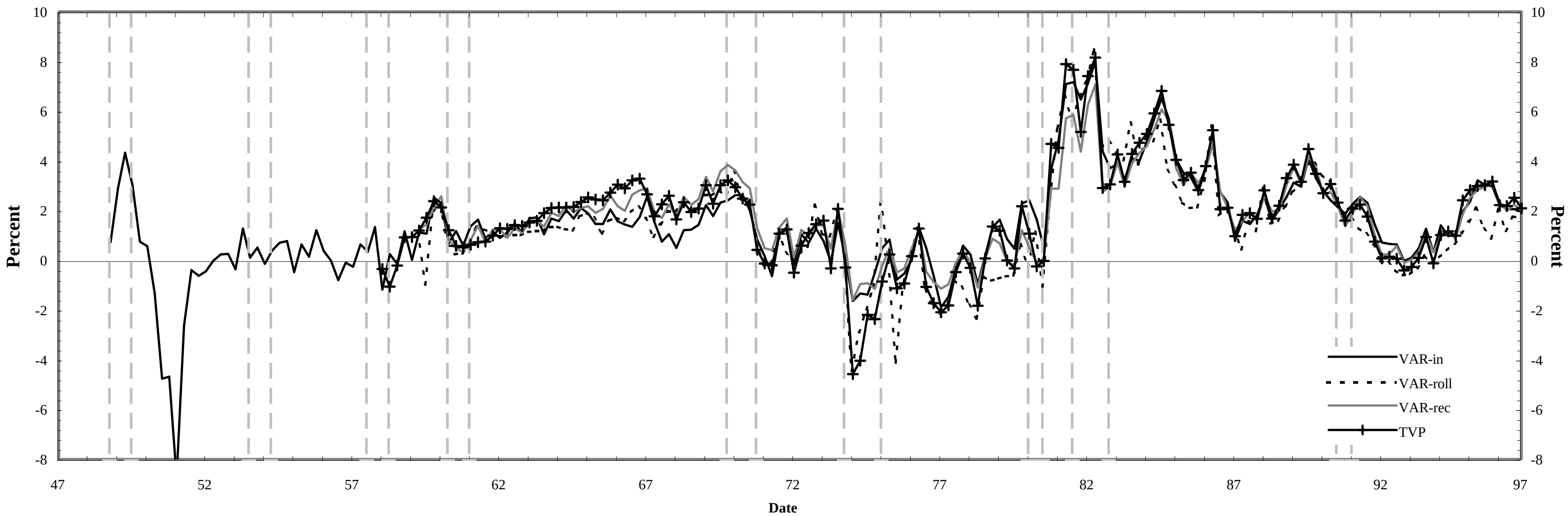


Figure 1i. Real rates method: Core CPI

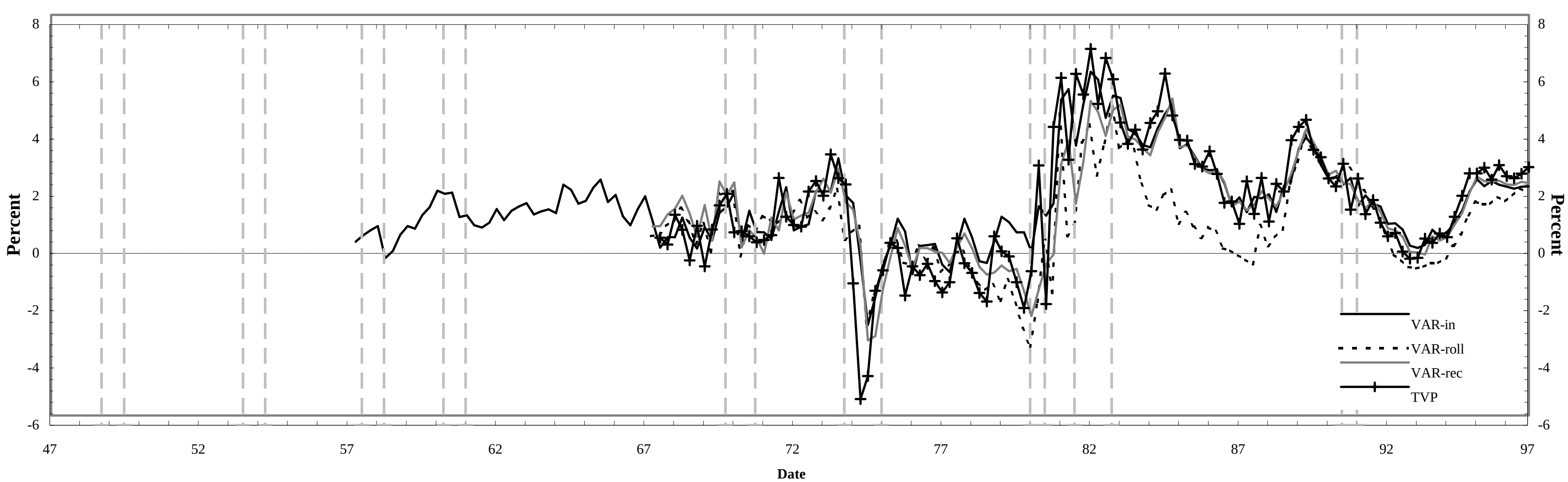


Figure 1j. Real rates method: INV-Deflator

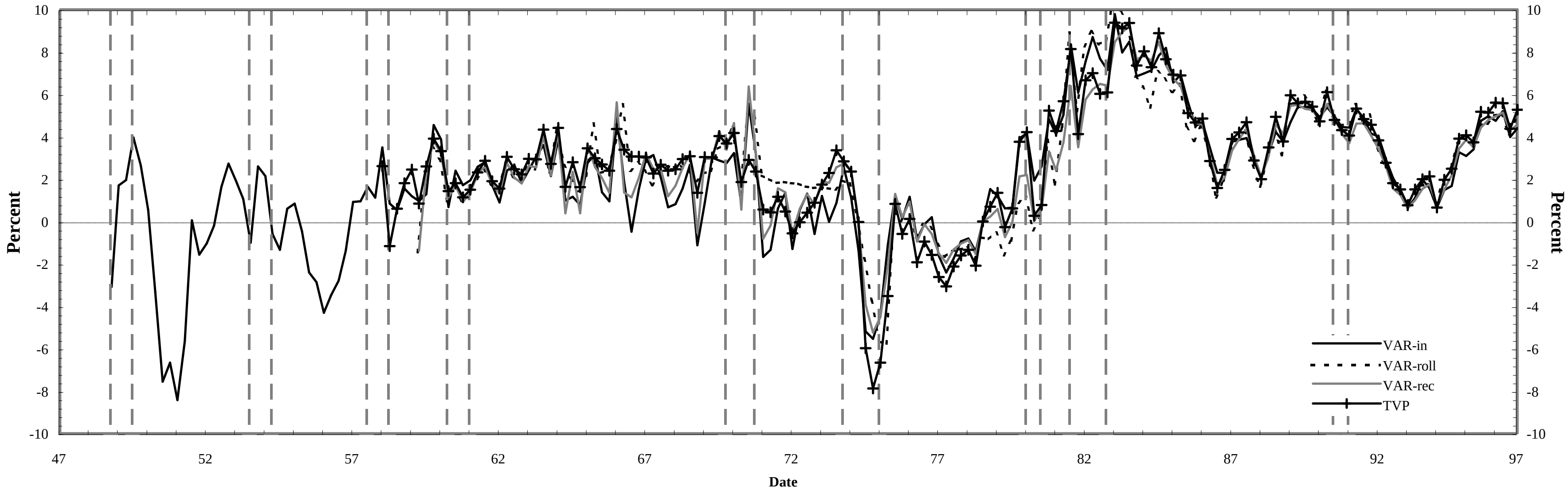


Figure 1k. Real rates method: GDP-Private

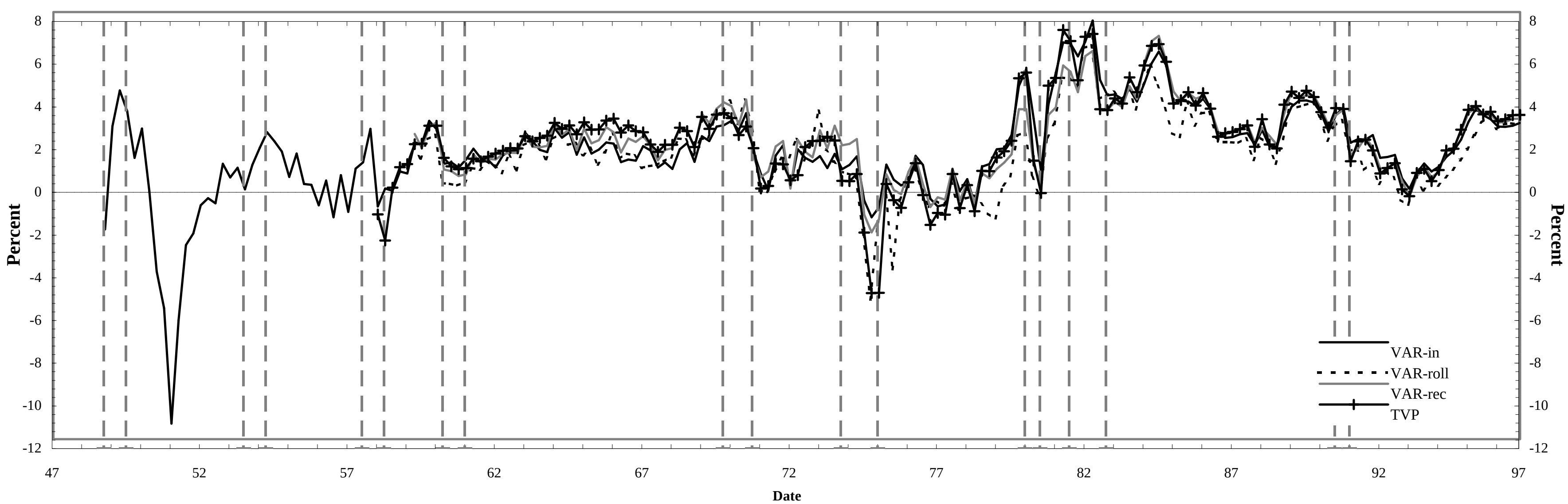


Figure 2a. Real rates method VAR-in

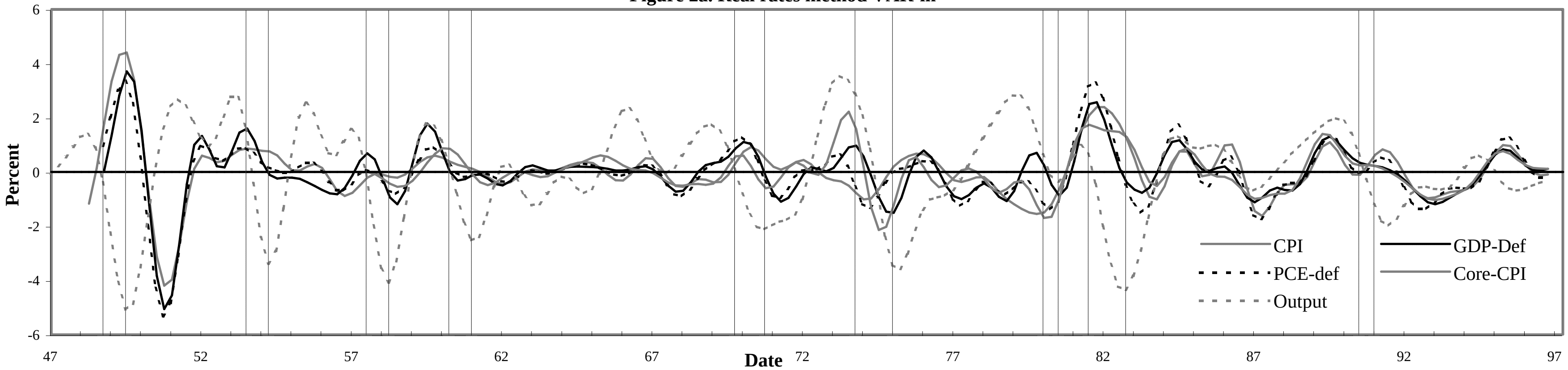


Figure 2b. Real rates method VAR-Recursive

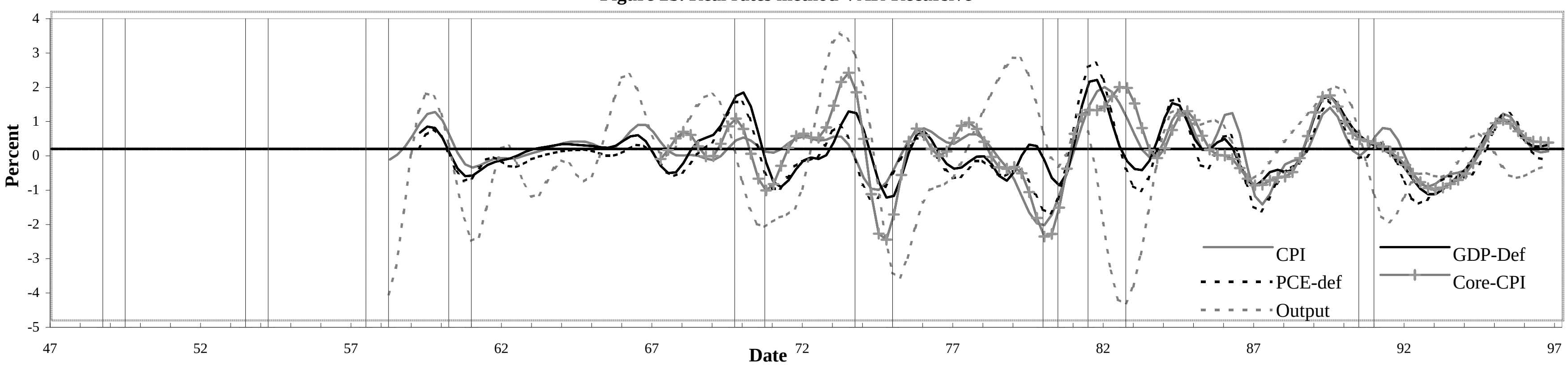


Figure 2c. Real rates method VAR-roll

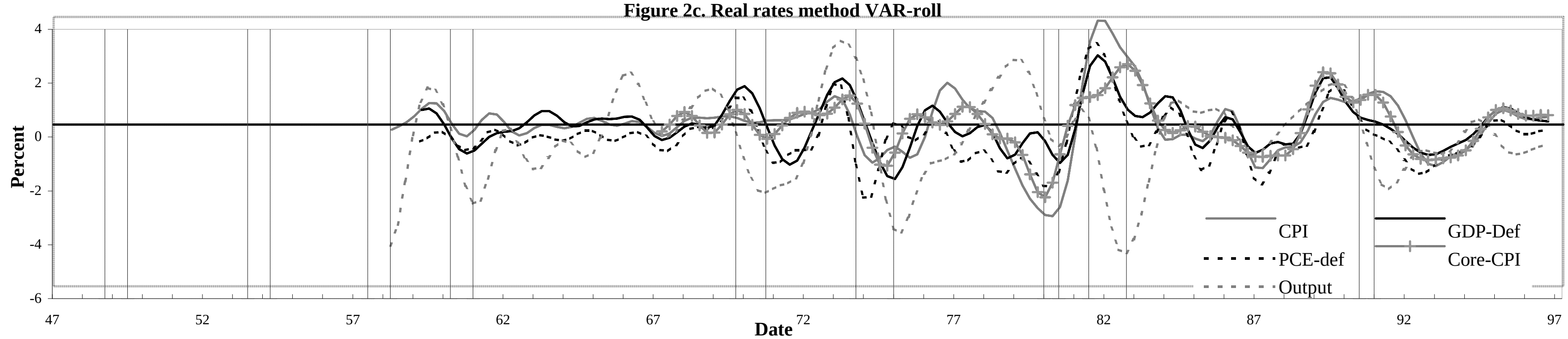


Figure 2d. Real rates method TVP

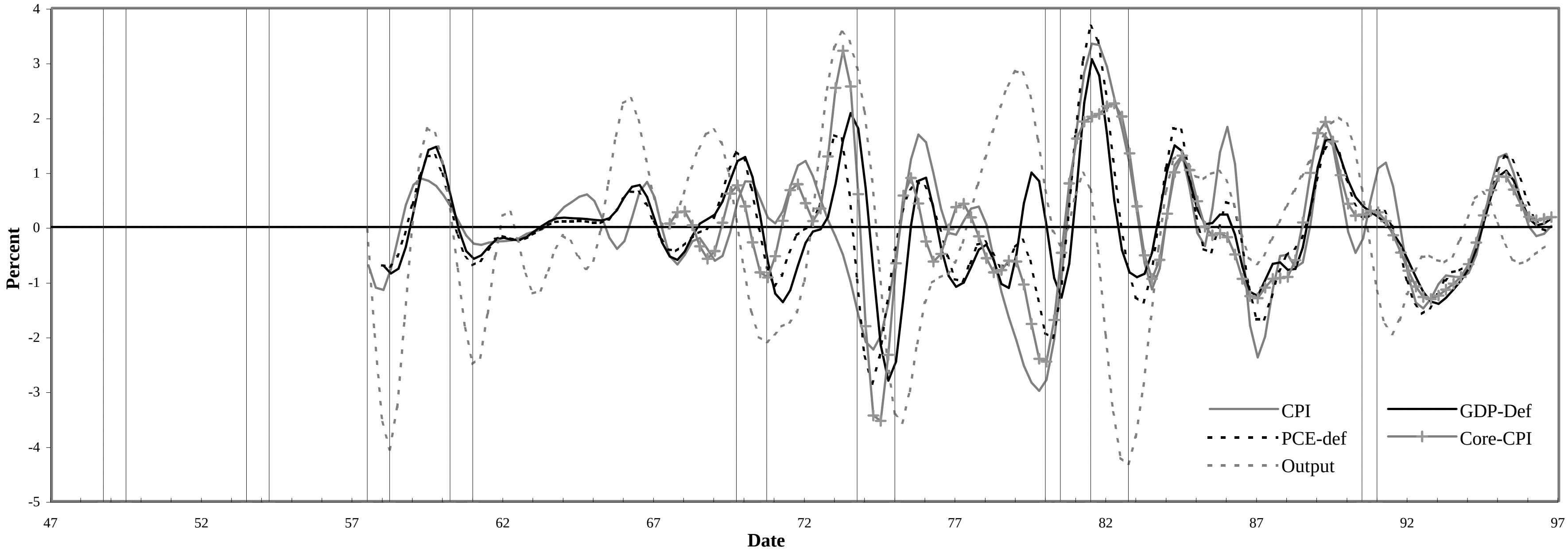


Figure 2e. Real rates method DRI-publ

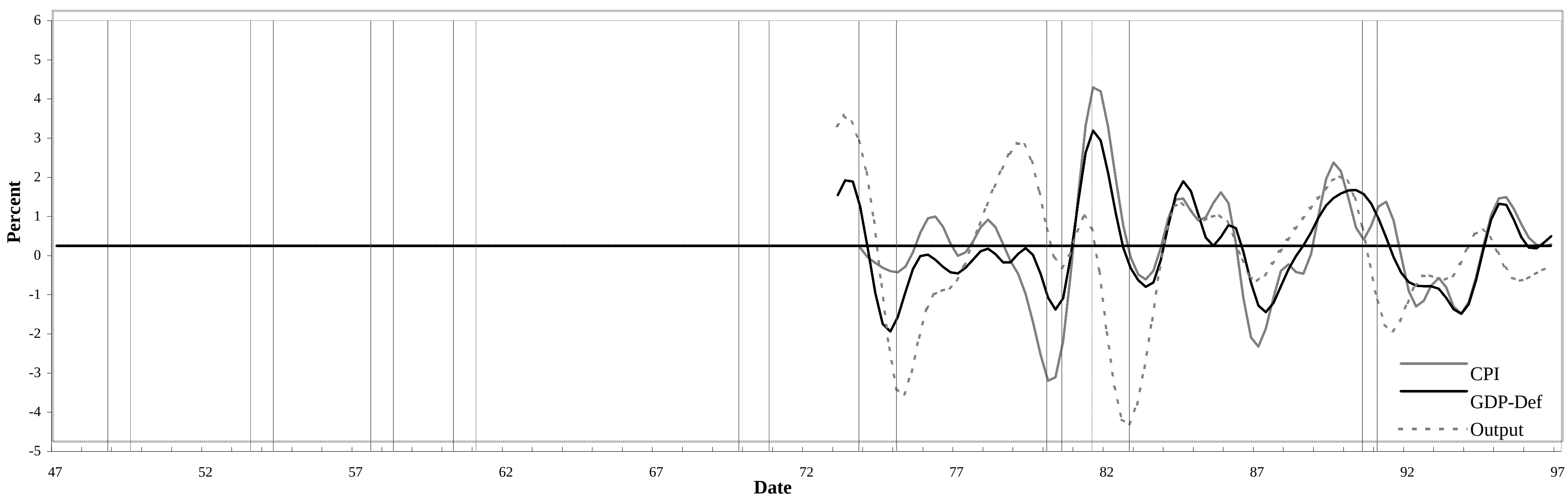


Figure 2f. Real rates method CPI

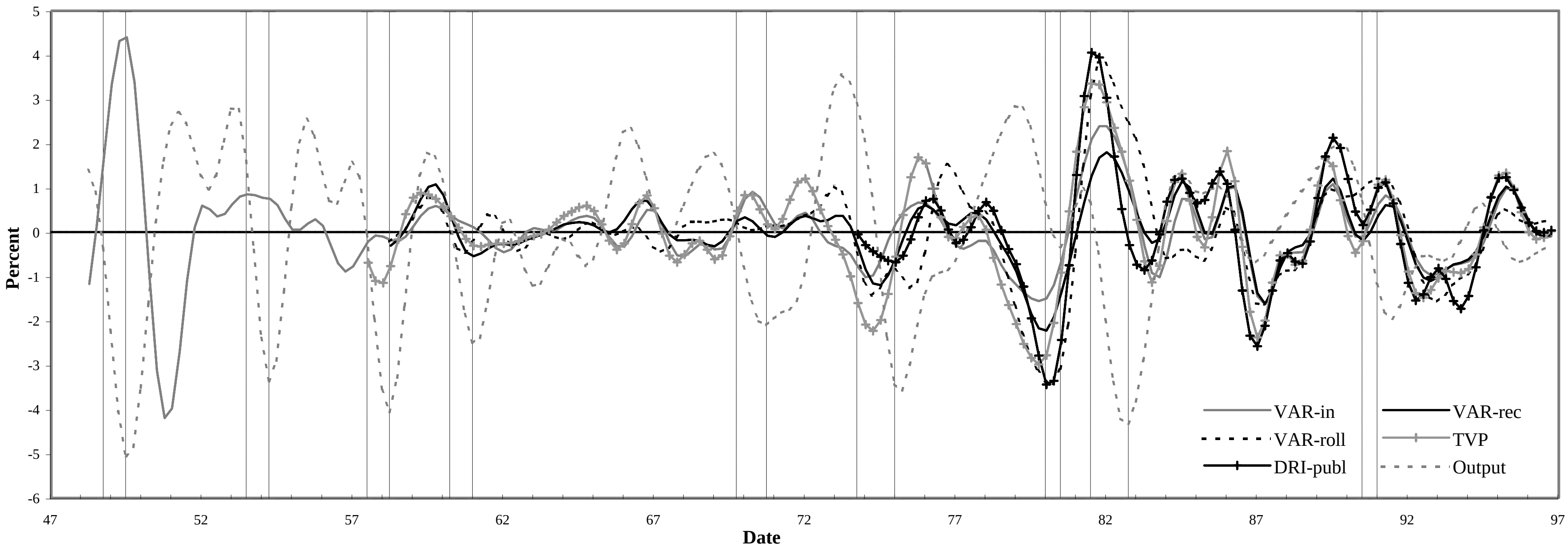


Figure 2g. Real rates method GDP-def

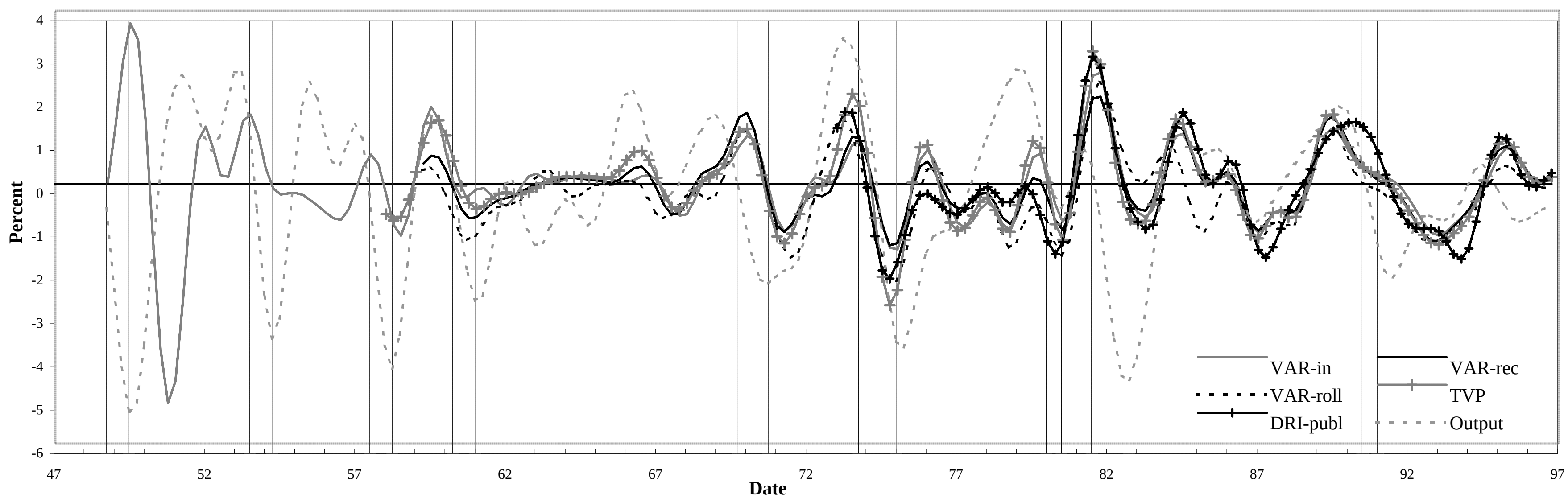


Figure 2h. Real rates method PCE-def

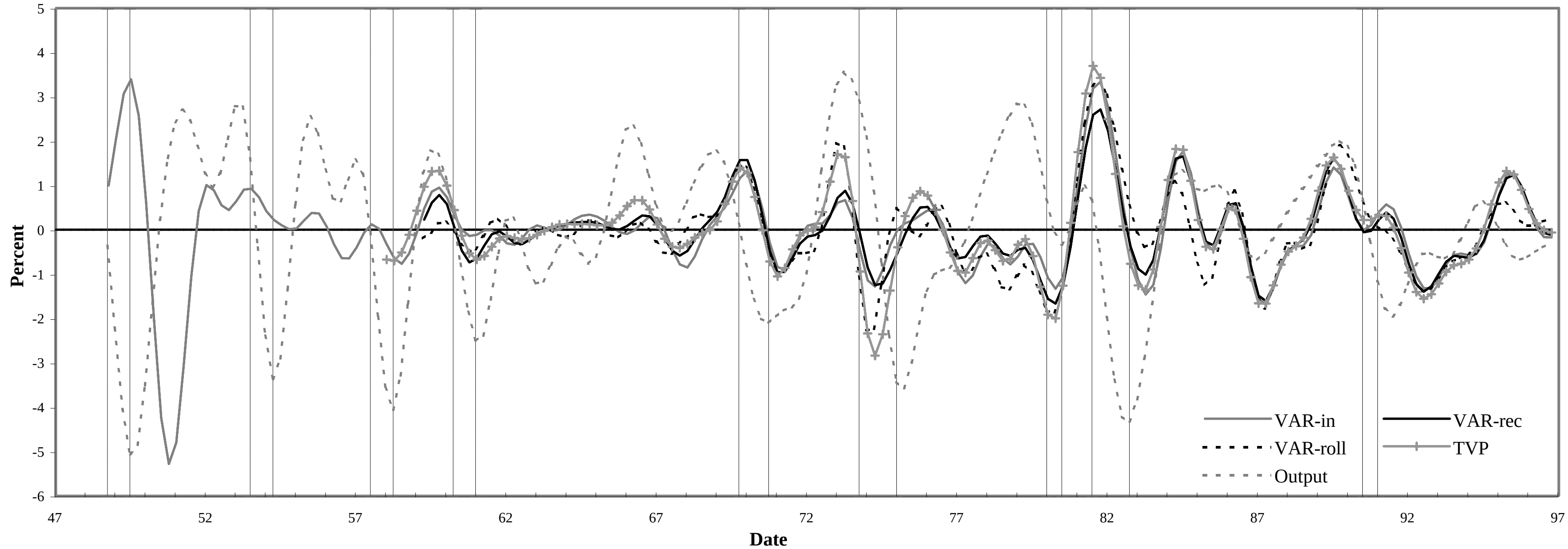


Figure 2i. Real rates method CORE-CPI

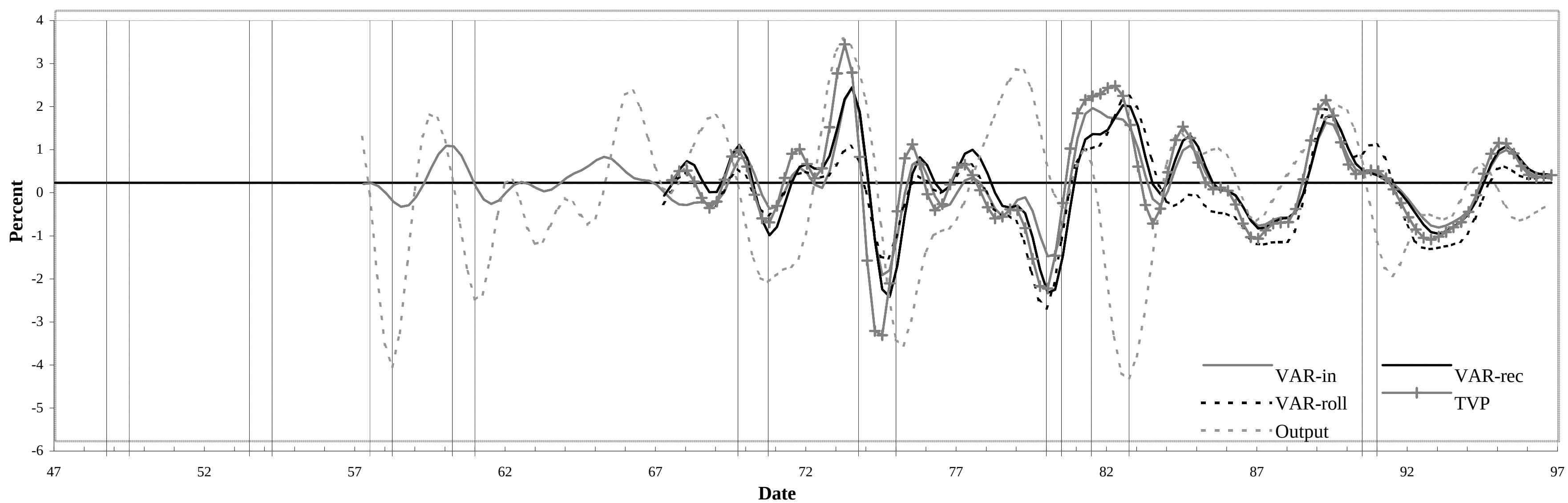


Figure 3.1 CPI VAR-IN

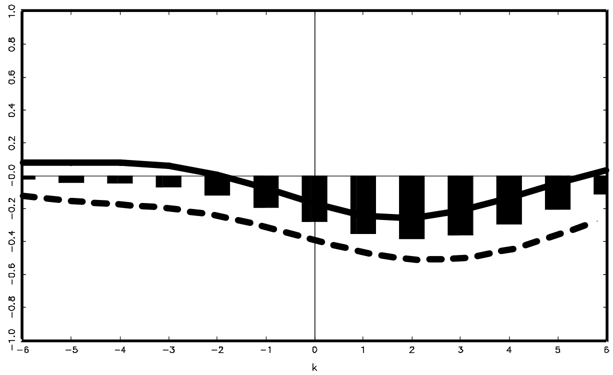
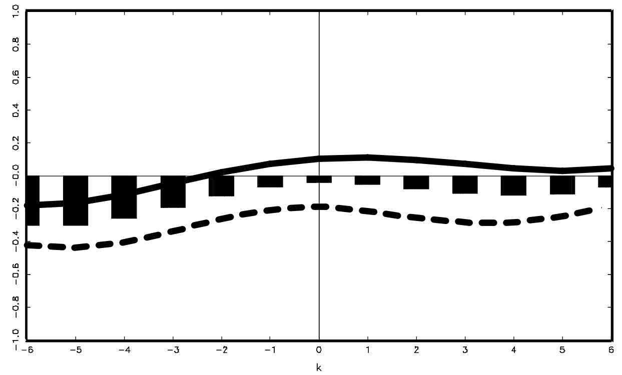


Figure 3.2 CPI VAR-REC



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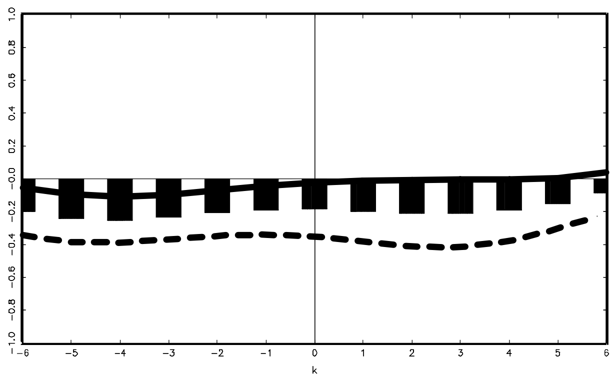


Figure 3.4 CPI TVP

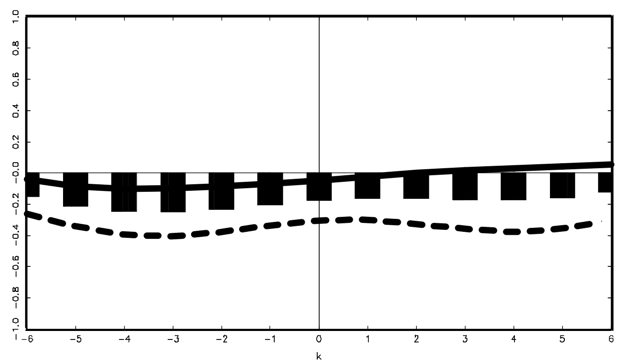


Figure 3.5 GDPD VAR-IN

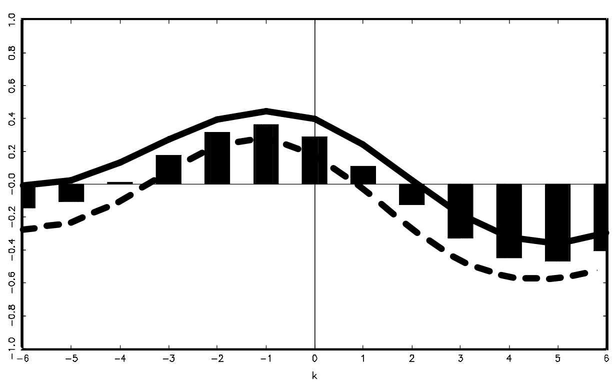


Figure 3.6 GDPD VAR-REC

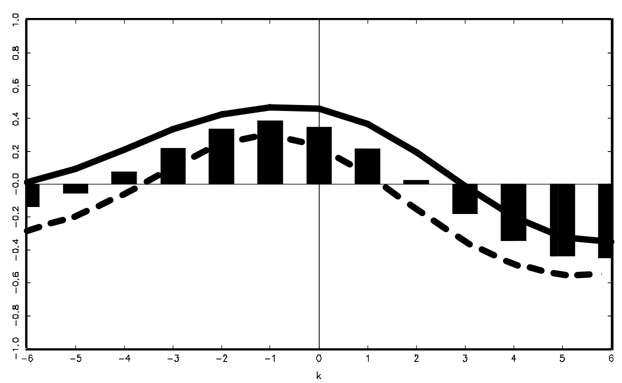


Figure 3.7 GDPD VAR-ROLL

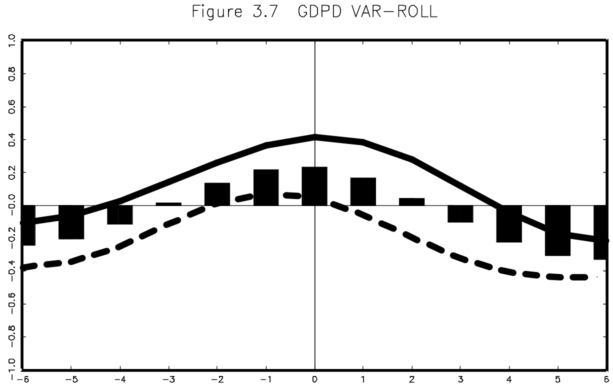


Figure 3.8 GDPD TVP

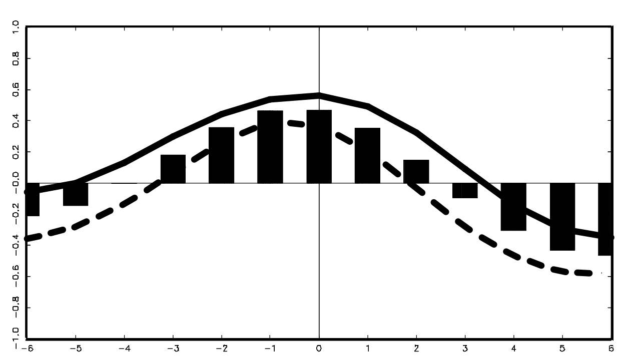


Figure 3.9 PCED VAR–IN

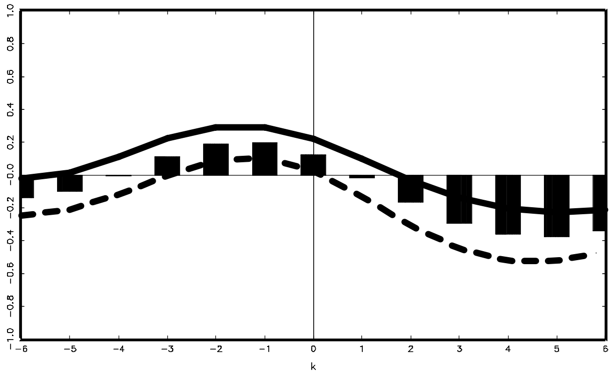


Figure 3.10 PCED VAR–REC

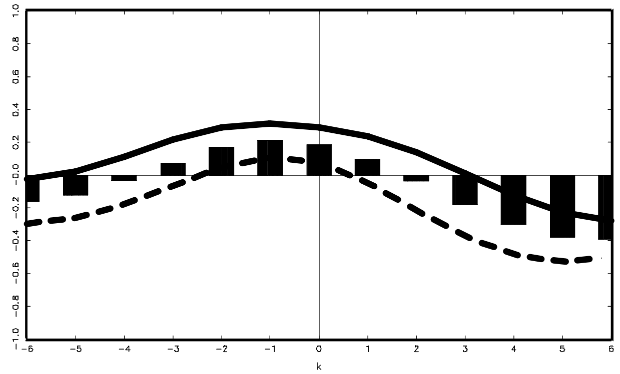


Figure 3.11 PCED VAR–ROLL

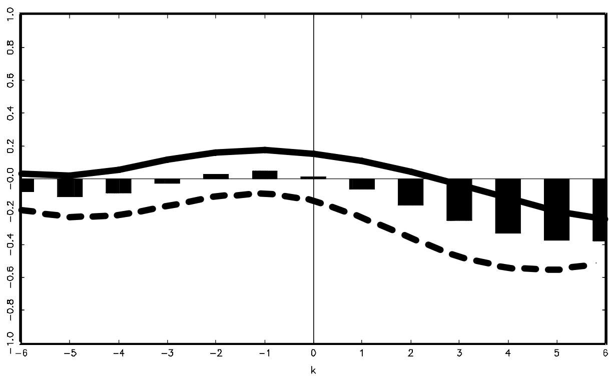


Figure 3.12 PCED TVP

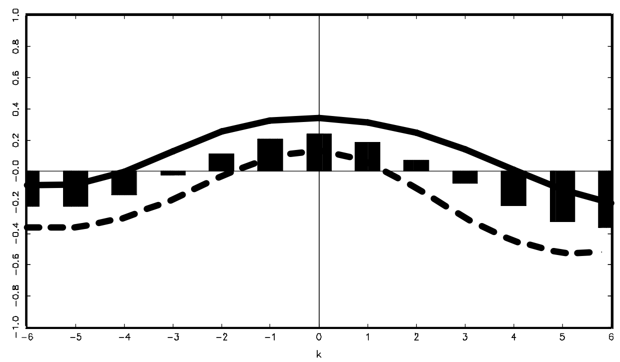
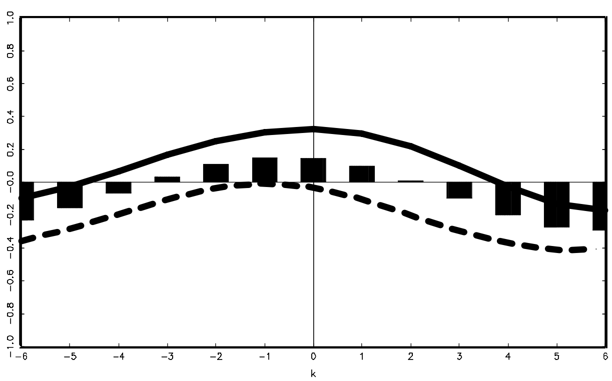


Figure 3.13 CORE CPI VAR–IN



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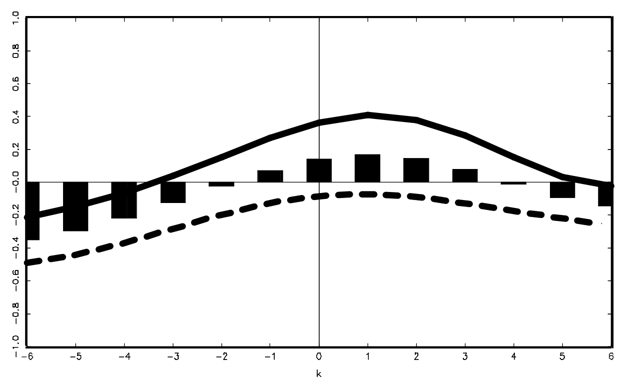

Figure 3.15 CORE CPI VAR–ROLL

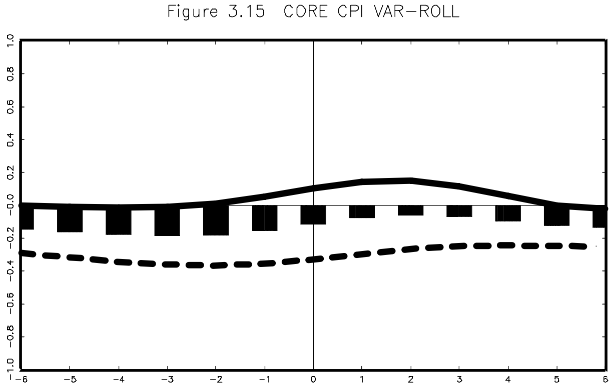

Figure 3.16 CORE CPI TVP

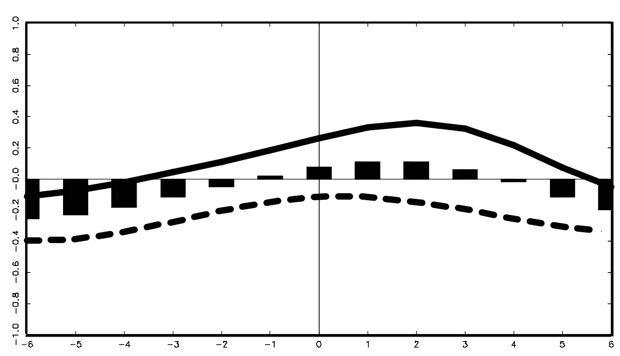

Figure 3.17 CPI DRI price forecast

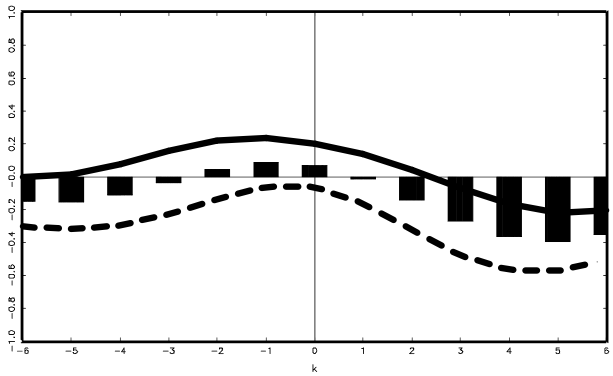

Figure 3.18 GDPD DRI price forecast

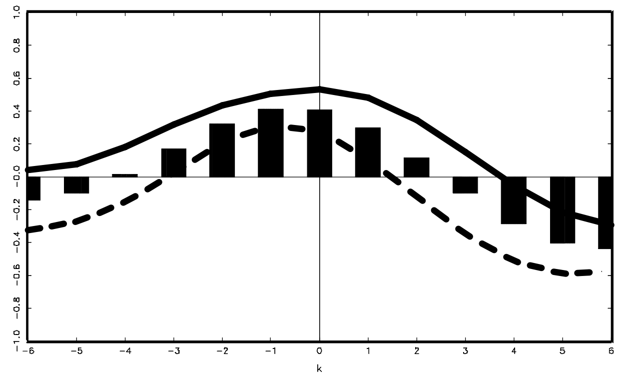

Figure 4.1 CPI TVP 1957:3 – 1973:4

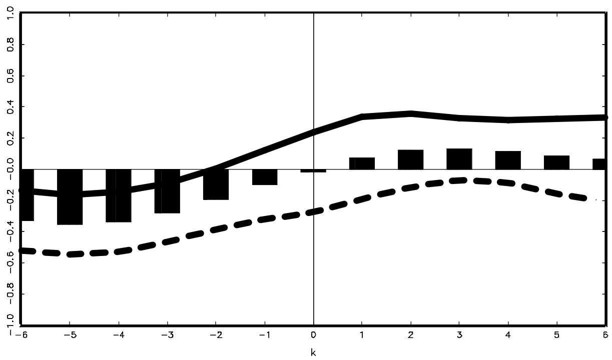

Figure 4.2 CORE CPI TVP 1967:3 – 1973:4

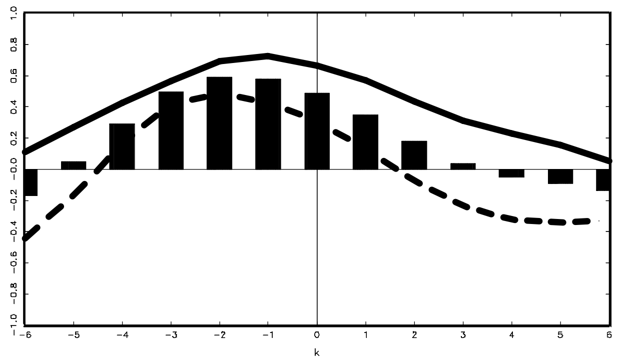

Figure 4.3 GDPD TVP 1958:1 – 1973:4

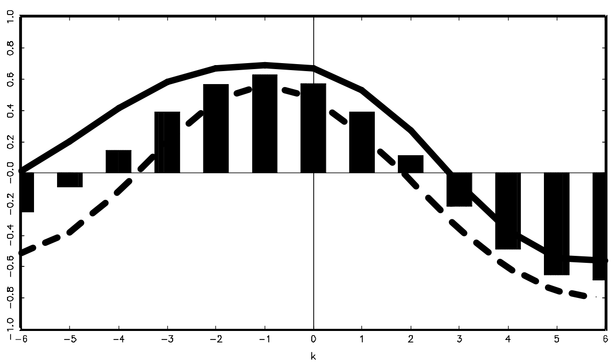

Figure 4.4 PCED TVP 1958:1 – 1973:4

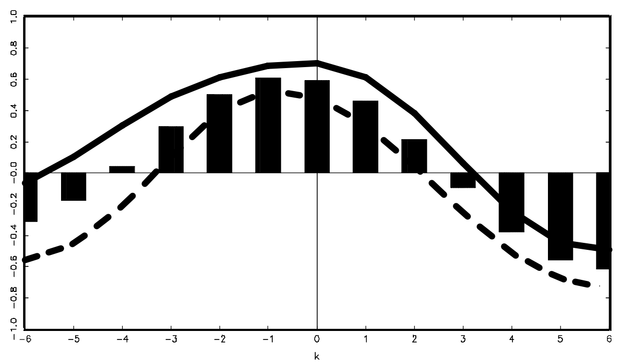

Figure 4.5 CPI TVP 1974:1 – 1982:4

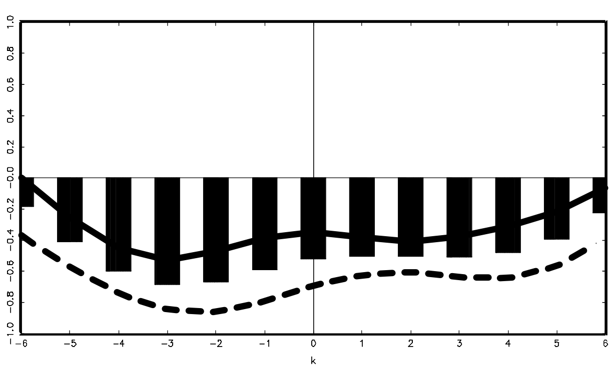

Figure 4.6 CORE CPI TVP 1974:1 – 1982:4

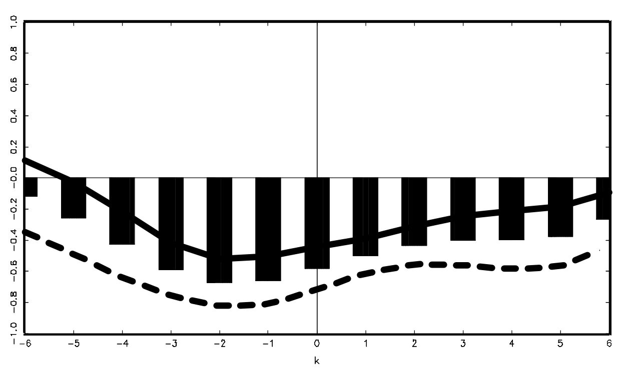

Figure 4.7 GDPD TVP 1974:1 – 1982:4

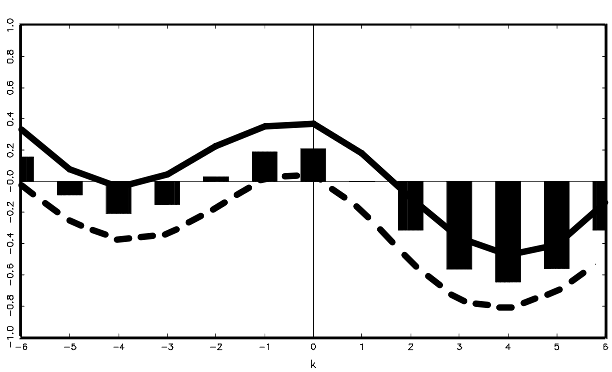

Figure 4.8 PCED TVP 1974:1 – 1982:4

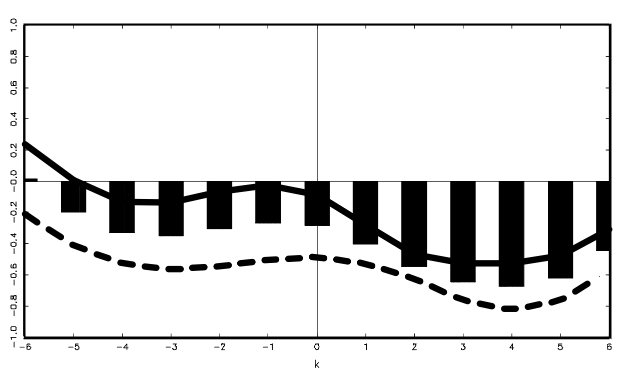

Figure 4.9 CPI TVP 1983:1 – 1996:4

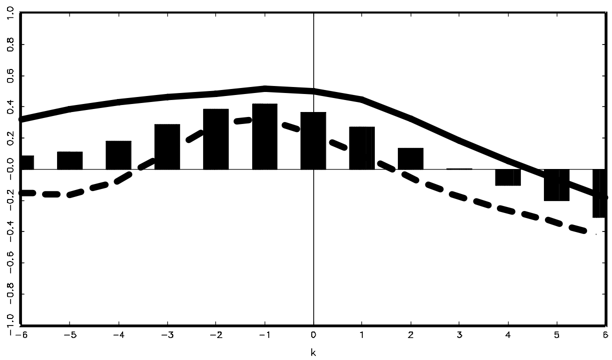

Figure 4.10 CORE CPI TVP 1983:1 – 1996:4

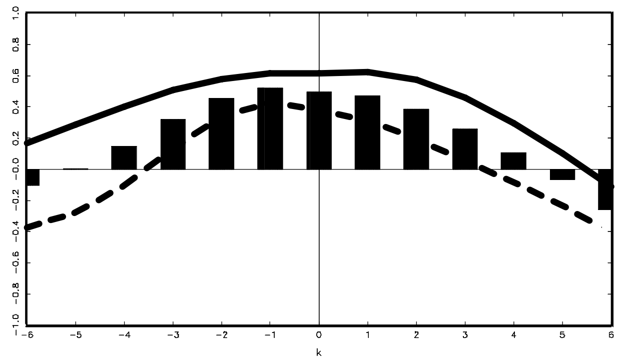

Figure 4.11 GDPD TVP 1983:1 – 1996:4

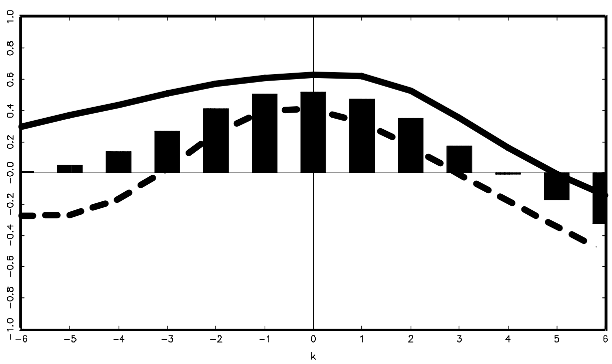

Figure 4.12 PCED TVP 1983:1 – 1996:4

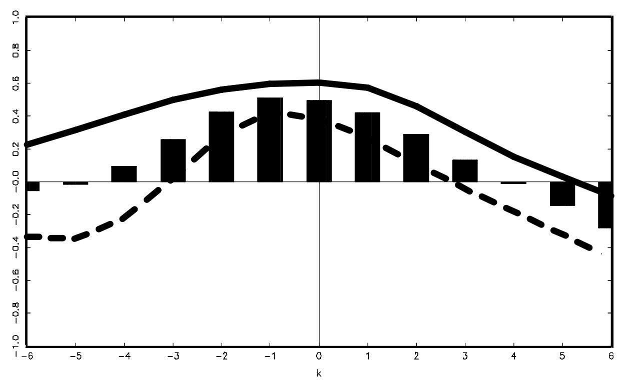

Figure 5.1 PCED VAR-IN (Start 1967:2)

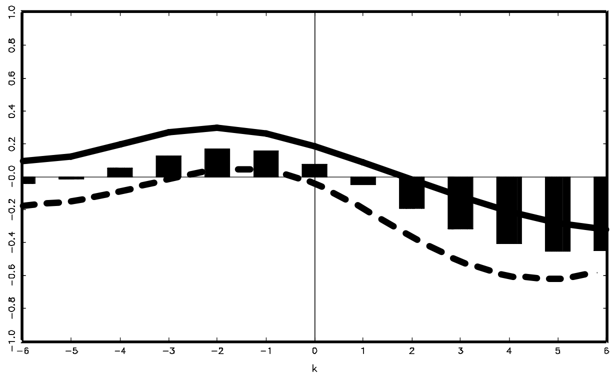

Figure 5.2 CORE CPI VAR-IN (Start 1967:2)

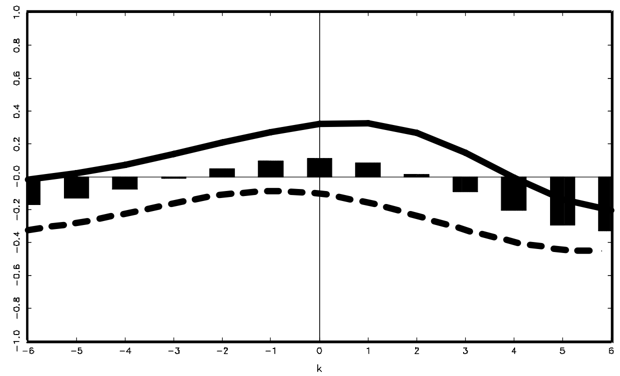

Figure 5.3 CPI VAR-IN (Start 1967:2)

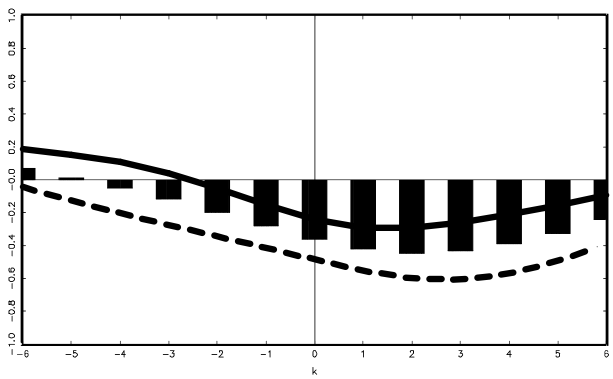

Figure 5.4 CPI-X1 VAR-IN (Start 1967:2)

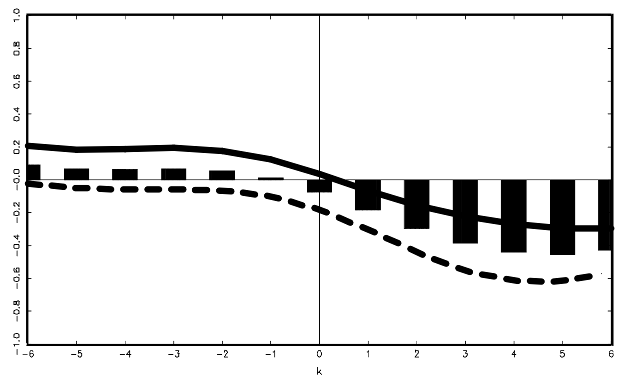

Figure 6.1 Inflation, CPI Energy

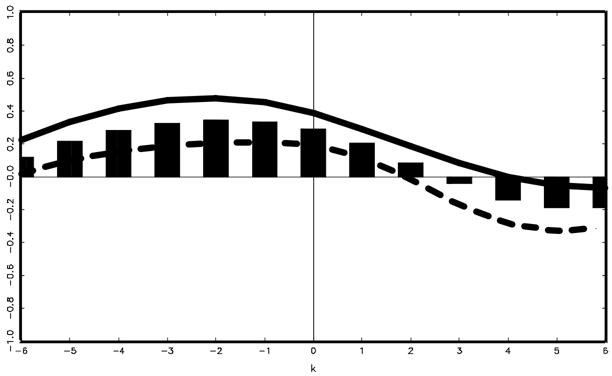

Figure 6.2 Inflation, PCED Energy

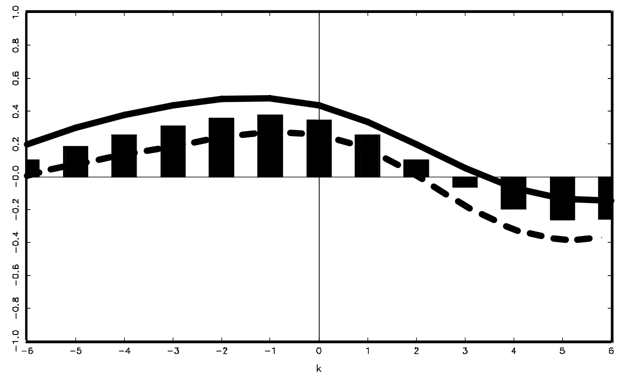

Figure 6.3 Inflation, CPI Food

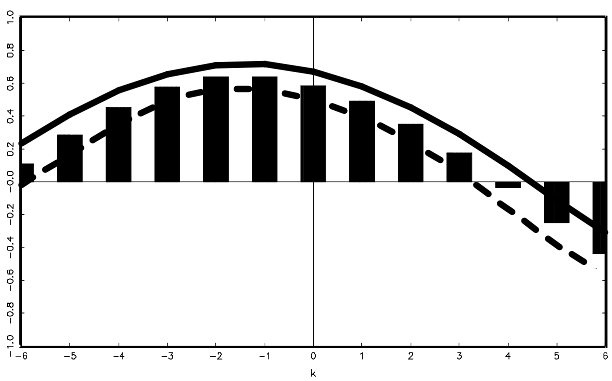

Figure 6.4 Inflation, PCED Food