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Abstract

If monetary policy succeeds in keeping average inflation very low, nominal interest rates may occasionally be constrained by the zero lower bound. The degree to which this constraint has real implications depends on the monetary policy feedback rule and the structure of price-setting. Policy rules that make the price level stationary lead to small real distortions from the zero bound. If policy imparts persistence into the inflation rate, the real implications of the zero bound are large in the presence of backward looking price-setting, and small if prices are set to maximize profits.

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1 Introduction

At least since the writings of Irving Fisher, economists have understood that nominal interest rates are bounded below by some number close to zero. No one would willingly hold a security, payable in money, which yielded a return below the storage cost of money. As long as storing money is nearly costless, a lower bound near zero applies. At least since the writings of John Maynard Keynes, some economists have also been concerned that very low nominal interest rates present a danger for a modern economy. Keynesian concerns – about a liquidity trap – involved the behavior of money demand at low nominal interest rates. While the behavior of money demand is one factor in understanding macroeconomic equilibrium, the simple fact recognized by Fisher potentially makes it undesirable for monetary policy to keep inflation very low. Whether or not this is so is equivalent to the question of whether or not the bound on *nominal* interest rates has *real* implications. We study this question in a dynamic equilibrium model with staggered price-setting.

A confluence of factors has led recently to much attention being focused on the effects of the nominal interest rate bound. First, since the inflation of the 1970s and early 1980s, the industrialized countries have begun to approach price stability. Assuming that the average level of real interest rates is roughly invariant to the average level of inflation, the Fisher equation implies that low inflation should correspond to low nominal interest rates. The second factor has been more acute: Japan has had short term nominal interest rates effectively equal to zero since 1998, and in the United States, the Fed Funds target currently stands at 1.00%. Finally, the evolution of macroeconomics has also stimulated work on the zero bound. A consensus has been reached that policy analysis must be conducted explicitly within the context of dynamic models, and the bound on nominal interest rates is a concrete object that can be analyzed in the context of modern models.[1]

Summers (1991) takes the position that the lower bound on nominal rates does make very low inflation undesirable, as it would occasionally prevent needed declines in real interest rates. We refer to this view as emphasizing the direct implications of the zero bound, and discuss it further in the next section. Fuhrer and Madigan (1997), Orphanides and Wieland (1998) and Reifschneider and Williams (1999) analyze the implications of the zero bound in dynamic rational expectations models that use variants of the Fuhrer-Moore (1995) specification of price-setting behavior. The first two of these papers find that the zero bound generates a significant real distortion at very low inflation rates, while Reifschneider and Williams find that certain specifications of the monetary policy reaction function can go a long way towards eliminating the real distortion.

Rotemberg and Woodford (1997,1998) study the zero bound's implications indirectly in a dynamic model that involves explicit optimizing behavior by consumers and firms. While they find that the zero bound has strong implications for the type of policy rules that are feasible, the effect on the optimal average inflation rate is small. Benhabib, Schmitt-Grohe and Uribe (2001) also analyze the zero bound's implications in an optimizing model; they argue that in the presence of the zero bound, certain types of policy rules lead to equilibria with "deflationary spirals." Eggertsson and Woodford (2003) study optimal policy in the presence of the zero bound, in an otherwise-linear dynamic New-Keynesian model. A linearized version of our model would look quite similar to the model used by Eggertsson and Woodford. Instead of studying optimal policy, however, we compare two simple policy rules. One is an inflation-targeting Taylor rule that is often described as a close approximation to actual central bank behavior. The other rule differs only by targeting the price level instead of the inflation rate.

Krugman (1998) blames Japan's recent economic problems partly on inappropriate monetary policy in the presence of the zero bound. Buiter and Panigirtzoglou (1999) and Goodfriend (2000) pursue the idea that by imposing a carry tax on money, the lower bound on nominal interest rates can be driven below zero. Goodfriend also analyzes quantitative monetary policy actions when the nominal interest rate is at its lower bound. McCallum (2000) studies two factors which mitigate the zero bound's real effects; first, the steady state real interest rate may be decreasing in the target inflation rate, and second, there is an exchange rate channel for monetary policy.

Our interest here is in understanding the conditions under which the zero bound has significant real implications, and thus could constitute an argument against very low inflation. We study a closed economy in which the steady state real interest rate is invariant to inflation, and we assume that the monetary authority follows an interest rate feedback rule. The paper is closest in spirit to the work of Reifschneider and Williams (2000). However, the framework here is a dynamic equilibrium model in which prices are set in the staggered fashion of Taylor (1980), and there is a demand for money motivated by a shopping time requirement, as in McCallum and Goodfriend (1987). The model is solved with a nonlinear method, so that the zero bound's implications can be analyzed directly. Wolman (1998), showed that for one parameterization of this model the zero bound did not constitute a significant real distortion. Here we show that the crucial determinants of the zero bound's real implications are the manner in which firms set their prices and the policy rule followed by the monetary authority. If prices are set to maximize expected present discounted profits, then a policy rule which keeps the price level stationary can keep the zero bound from significantly interfering with the behavior of the real economy. This result does not appear to be sensitive to the source of shocks hitting the economy.

The paper proceeds as follows. Section 2 heuristically discusses both the zero bound's direct implications and the reason that price level stationarity might be important for the equilibrium implications. Section 3 describes the model to be used for the formal analysis. Section 4 presents the main results, and section 5 presents additional results on uncertainty. Section 6 concludes.

2 A Heuristic Discussion

Partly because of the nonlinearity associated with the bound on nominal interest rates, the model laid out below is too complicated to solve analytically. As such, the results we will present are based on computation. To give the reader some background for interpreting those results, it will be useful to consider in isolation two components of the model – indeed components of all the models used to study this issue. Fisher's equation, relating the nominal interest rate to the real interest rate and expected inflation, is the source of the zero bound's direct implications. A general feature of the monetary policy rule – whether it keeps the price level trend-stationary – helps us to move from the direct implications to thinking about equilibrium outcomes.

2.1 Direct Implications of the Zero Bound

The Fisher equation states that the gross nominal interest rate $(1 + R_t)$ is approximately equal to the product of the gross real interest rate $(1 + r_t)$ and expected inflation $(E_t P_{t+1}/P_t)$:[2]

$$1 + R_t \approx (1 + r_t)\left(E_t P_{t+1}/P_t\right).$$

Suppose we treat inflation and expected inflation as fixed. Then the lower bound on the nominal interest rate translates directly into a lower bound on the real interest rate. Taking as given that there is some underlying "natural" real rate behavior which would be optimal, the zero bound could inhibit that behavior. At very low levels of inflation the zero bound would almost surely inhibit that behavior. This direct implication is the basis for Summers' (1991) argument that monetary policy should not aim for a very low (zero) inflation rate. Implicit in the argument is the idea that inflation would be stabilized in a narrow band. In practice, however, it is difficult to imagine that policy would stabilize inflation in a narrow band; this has not occurred in countries with explicit inflation targeting policies. One is then led to ask whether certain policy rules might generate temporary variation in expected inflation that would allow the desired variation in real interest rates to occur even in the presence of low average inflation.

2.2 The Potential Importance of Price-Level Trend-Stationarity[3]

Anticipating results to be presented below, we consider here one general feature of policy rules: their implications for whether the price level is trend stationary. Among rules that promise to achieve zero inflation on average, one class keeps the price level stationary around a constant level, and the other class allows base drift in the price level but always brings inflation back toward zero. To simplify the discussion, consider feedback rules that set the nominal interest rate as an increasing linear function of either the inflation rate or the log price level:

$$R_t = \max\left\{r^* + f_p \cdot \pi_t, 0\right\}, \tag{1}$$

where $\pi_t = P_t/P_{t-1}$, or

$$R_t = \max\left\{r^* + f_p \cdot \left(\ln\left(P_t\right) - \ln\left(P^*\right)\right), 0\right\}. \tag{2}$$

Now consider a situation where inflation is negative in the current period, the nominal interest rate is at zero, and the natural real rate is negative. This is a situation where the direct implications of the zero bound would suggest that the natural real rate cannot be achieved. Figure 1.A plots the long run path of inflation for rules that allow base drift in the price level, and the point marked with an asterisk is the current period being described. Because there is deflation in the current period, if the rule successfully stabilizes inflation it will cause next period's expected deflation rate to be lower. But with the current nominal rate at zero, there needs to be positive expected inflation in order to achieve a negative real rate.

Figure 1.B plots the long run path of the *price level* under rule (2), and the point marked with an asterisk indicates a current period, where the nominal rate is zero, the price level

is below its average, and the natural real rate is negative. Because the price level is below average in the current period, if the rule successfully stabilizes the price level it will cause an expected increase in the price level next period, meaning positive expected inflation. Positive expected inflation is precisely what is needed to accomplish a negative real rate when the nominal rate is zero.

This simple analysis suggests rules that keep the price level stationary may make it possible for low average inflation to coexist with unimpeded behavior of the real interest rate. Alternatively, more complicated policies which allowed base drift in the price level could be consistent with natural real rate behavior if they generated "overshooting" of expected inflation under certain circumstances.

Before moving to the formal model, note that the discussion above was not framed in terms of particular types of shocks; what matters is the shocks' implications for the appropriate real rate of interest. Note also that for a policy rule to be successful in allowing appropriate real rate variability, there must be some flexibility in expected inflation. If the structure of the economy is such that expected inflation is insufficiently flexible, it will be impossible to prevent real implications of the zero bound.

3 The Model

As in other recent work on the zero bound, the model used here is one in which there is assumed to be sluggish price adjustment. The model is in the tradition of Taylor (1980), in that price-setting is staggered: each firm sets its price for 2 periods, with 1/2 of the firms adjusting each period. There are a continuum of firms, and they produce differentiated consumption goods using, as the sole input, labor provided by consumers at a competitive wage. It has been more common in recent work with sticky-price models to use the Calvo framework, in which firms face an exogenous, constant probability of being able to adjust their price. We use the two-period Taylor framework for two reasons. First, it seems to be a closer approximation to reality at a microeconomic level: in the Calvo environment, a positive fraction of firms charge prices that were set arbitrarily far in the past. Second, although the Calvo model is known for its analytical tractability, the Taylor model with two-period pricing has fewer state variables than the Calvo model.

Consumers are infinitely lived, and purchase consumption goods using income from their labor, which is supplied elastically. Consumers hold money in order to economize on transactions time, as in McCallum and Goodfriend (1987). Monetary policy is given by an exogenous rule which is known to all agents. Aside from the internal dynamics associated with price stickiness, the model is driven by exogenous shocks to labor supply or consumption demand. The analysis will concentrate on the internal dynamics.

3.1 Consumers

Consumers have preferences over a consumption aggregate (c_t) and leisure (l_t) given by

$$E_t \sum_{t=0}^{\infty} \beta^t \cdot \left(\ln\left(c_t + \vartheta_t\right) + \chi_t \cdot l_t\right), \tag{3}$$

where ϑ_t and χ_t are random variables which follow persistent two-state Markov processes. The discount factor β is set to 0.995, and the means of ϑ_t and χ_t are set to 0 and 3, respectively.[4] The consumer's budget constraint is

$$c_t + \tau_t + M_t/P_t + \left(1 + R_t\right)^{-1}\left(B_t/P_t\right) = M_{t-1}/P_t + B_{t-1}/P_t + w_t n_t + d_t,$$

and the time constraint is

$$n_t + l_t + h(M_t/(P_t c_t)) = E, \tag{4}$$

where τ_t is a real lump-sum tax (or transfer if negative), P_t is the price level, M_t is nominal money balances chosen in period t, to carry over to $t+1$, B_t is holdings of one-period nominal zero-coupon bonds, issued by the government, maturing at $t+1$, R_t is the interest rate on nominal bonds, w_t is the real wage, n_t is time spent working, d_t is real dividend payments from firms, $h(M_t/(P_t c_t))$ is time spent transacting, and E is the time endowment. Defining real balances to be $m_t \equiv M_t/P_t$, the function $h(\cdot)$ is parameterized as in Wolman (1997):

$$h(m_t/c_t) = \phi \cdot (m_t/c_t) - \tfrac{\nu}{1+\nu} A^{-1/\nu} (m_t/c_t)^{\frac{1+\nu}{\nu}} + \Omega, \text{ for } m_t/c_t < A \cdot \phi^{\nu},$$
$$h(m_t/c_t) = \tfrac{1}{1+\nu} A \phi^{1+\nu} + \Omega, \text{ for } m_t/c_t \geq A \cdot \phi^{\nu}, \tag{5}$$

with $\phi = 1.4 \times 10^{-3}$, $A = 1.7 \times 10^{-2}$, and $\nu = -0.7695$.[5] Transactions time is thus decreasing in real balances and increasing in consumption, up to a satiation level of the ratio of real balances to consumption.

Goods market structure. Following Blanchard and Kiyotaki (1987), we use the Dixit and Stiglitz (1977) monopolistic competition framework. The consumption aggregate is an integral of differentiated products, $c_t = [\int c(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega]^{\frac{\varepsilon}{\varepsilon-1}}$. We set $\varepsilon = 10$, which implies that every producer faces a downward sloping demand curve with constant elasticity 10.

Since all producers that adjust their prices in a given period choose the same price, it is easier to write the consumption aggregate as

$$c_t = c(c_{0,t}, c_{1t}) = (0.5 \cdot c_{0,t}^{\frac{\varepsilon-1}{\varepsilon}} + 0.5 \cdot c_{1,t}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $c_{j,t}$ is the quantity consumed in period t of a good whose price was set in period $t-j$. The constant elasticity demands for each of the goods take the form:

$$c_{j,t} = \left(P^*_{t-j}/P_t\right)^{-\varepsilon} \cdot c_t, \tag{7}$$

where P^*_{t-j} is the nominal price at time t of any good whose price was set j periods ago, and P_t is the price index at time t, given by

$$P_t = \left[0.5 \cdot (P^*_t)^{1-\varepsilon} + 0.5 \cdot \left(P^*_{t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{8}$$

Optimization. If we attach a Lagrange multiplier λ_t to the budget constraint, so that λ_t is the marginal value of real wealth, the first order conditions for the individual's maximization problem, with respect to c_t, l_t, B_t and M_t, are

$$(c_t + \vartheta_t)^{-1} = \lambda_t - \chi_t \cdot h'(\cdot)(\frac{m_t}{c_t^2}) \ , \tag{9}$$

$$\chi_t = w_t \cdot \lambda_t, \tag{10}$$

$$\frac{\lambda_t}{P_t} = \beta (1 + R_t) \cdot E_t (\lambda_{t+1}/P_{t+1}) \,, \tag{11}$$

and

$$(\lambda_t/P_t) \cdot (1 + w_t \cdot h'(\cdot)(1/c_t)) = \beta E_t (\lambda_{t+1}/P_{t+1}) \,. \tag{12}$$

In choosing consumption optimally (9), the individual weighs the benefit of consuming a marginal unit, which is the left hand side of (9), against the cost, which consists of both forfeited real wealth (the first term on the right hand side) and time spent transacting (the second term on the right hand side). In choosing leisure optimally (10), the individual balances the marginal utility of leisure against the wage earnings that the time would yield. The choice of bond holdings (11) balances the marginal value of nominal wealth today against $(1 + R_t)$ times the marginal value of nominal wealth tomorrow. And finally, optimal money holdings (12) imply that the individual balances the transactions-facilitating benefit against the foregone interest cost of holding money.[6]

3.2 Firms

Each firm produces with an identical technology:

$$c_{j,t} = n_{j,t}, \quad j = 0, 1, \tag{13}$$

where $n_{j,t}$ is the labor input employed in period t by a firm whose price was set in period $t - j$. Given the price that a firm is charging, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their price in a given period can thus be thought of as passive. Given that it has set a relative price P^*_{t-j}/P_t, real profits for a firm of type j are

$$\left(P^*_{t-j}/P_t\right) \cdot c_{jt} - w_t \cdot n_{jt}, \tag{14}$$

that is, revenue minus cost.

We will analyze two specifications for the determination of P_t^*. Firms either choose P_t^* to maximize the present discounted value of profits over the two periods that the price will be charged, or they choose P_t^* according to the partially backward-looking specification of Fuhrer and Moore (1995). The former specification is the natural one suggested by the rest of the model. The latter has been found useful in generating inflation persistence similar to that seen in the data. Note that for both specifications, we assume that the firm's nominal price is fixed for two periods. An alternative assumption – made for example by Yun (1996) – is that in between adjustments, a firm's nominal price automatically adjusts at the steady state inflation rate. Our results for the case of a zero inflation target would be unchanged by this assumption. For the case of a positive inflation target analyzed below, the overall message would not change. We will return to this point in section 4.

Profit maximizing pricing: Maximization of present value implies that a firm chooses its current relative price taking into account the effect on current and expected future profits. Substituting into (14) the demand curve (7) and the technology (13), the present discounted value of expected profits is given by

$$\begin{gathered} c_t \cdot \left[\left(P_t^*/P_t\right)^{1-\varepsilon} - w_t \cdot \left(P_t^*/P_t\right)^{-\varepsilon} \right] + \\ \beta E_t \tfrac{\lambda_{t+1}}{\lambda_t} \cdot c_{t+1} \cdot \left[\left(P_t^*/P_{t+1}\right)^{1-\varepsilon} - w_{t+1} \cdot \left(P_t^*/P_{t+1}\right)^{-\varepsilon} \right], \end{gathered} \tag{15}$$

for the two periods over which a price will be in effect. Differentiating (15) with respect to P_t^* and setting the resulting expression equal to zero, the optimal relative price satisfies

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \left\{ \frac{\lambda_t c_t w_t + \beta E_t \lambda_{t+1} c_{t+1} (P_{t+1}/P_t)^{\varepsilon} w_{t+1}}{\lambda_t c_t + \beta E_t \lambda_{t+1} c_{t+1} (P_{t+1}/P_t)^{\varepsilon - 1}} \right\}. \tag{16}$$

It is easiest to understand this expression under perfect foresight, in which case the optimal relative price is a constant markup $(\varepsilon / (\varepsilon - 1))$ over a weighted average of current and future real marginal cost (here real marginal cost is equal to the real wage). The weights represent marginal revenue shares associated with the current and future periods. In a non-inflationary steady state, the firm would choose a markup over current marginal cost of $(\varepsilon / (\varepsilon - 1))$. In an inflationary or deflationary steady state, an adjusting firm's markup over current marginal cost differs from $(\varepsilon / (\varepsilon - 1))$, as adjusting firms take into account the future erosion (or increase) of their relative price. Out of steady state, the markup over current marginal cost becomes time varying: it depends on the current and future marginal utility of wealth, price level, and aggregate demand. With uncertainty, the weighted average representation of (16) would need to be augmented by a covariance term.

Backward-looking pricing: The second specification we will consider has firms choosing their relative price as a function of the relative price chosen by firms in the previous period, and that expected to be chosen in the next period, as well as "output gaps" in the current and immediate future periods:

$$\ln \left(P_t^* / P_t\right) = 0.5 \cdot \left(q_t + \gamma \ln \left(c_t / c\right)\right) + 0.5 \cdot E_t \left(q_{t+1} + \gamma \ln \left(c_{t+1} / c\right)\right) + z_t, \tag{17}$$

where

$$q_t = 0.5 \cdot \ln \left(P_t^* / P_t\right) + 0.5 \cdot \ln \left(P_{t-1}^* / P_{t-1}\right), \tag{18}$$

and c is the steady state level of consumption. This specification is close to that in Fuhrer and Moore (1995), although the term z_t – an increasing function of the real wage – is absent from that paper and others using the same general specification. This term is necessary in the context of the optimizing model used here, as without it the model would not be completely specified.[7] The important feature of (17) is that the relative price chosen by adjusting firms in the current period depends directly on the relative price chosen by adjusting firms in the previous period. This feature has been useful in generating aggregate price dynamics similar to those in U.S. data. However, there has thus far been little success in deriving an equation like (17) from optimal firm behavior.

3.3 Monetary and Fiscal Policy

The government issues non-interest bearing currency and interest bearing discount bonds, imposes lump sum taxes or distributes lump sum transfers, and purchases some amount of the same basket of differentiated products that individuals purchase. It is also natural to consider the nominal interest rate as one of the policy variables. As Woodford (1995) explains, it is not feasible for the government to choose each of these variables independently: individuals' optimal choice of money and bond holdings together with the government's budget constraint limits to three the number of policy variables the government can independently determine. We assume that the government fixes the real quantity of interest-bearing debt and the real quantity of government purchases at constant levels. The quantity of money is endogenously determined by money demand, given the nominal interest rate implied by the government's feedback rule. Lump sum taxes or transfers are determined as a residual, to satisfy the government's flow budget constraint:

$$M_t + B_t/\left(1 + R_t\right) = M_{t-1} + B_{t-1} + p_t g_t - p_t \tau_t. \tag{19}$$

The nominal interest rate feedback rules are similar to the ones already discussed, but allow for a response to real activity as well as prices. They are in the spirit of Taylor (1993). We consider the following rule that makes the price level trend-stationary,

$$R_t = \max\left\{R^* + f_p \cdot \left(\ln P_t - \ln \overline{P_t}\right) + f_c \cdot \left(\ln c_t - \ln \overline{c}\right), 0\right\}, \tag{20}$$

and a rule that allows for base drift in the price level:

$$R_t = \max\left\{R^* + f_p \cdot \left(\left(P_t/P_{t-1}\right) - \pi^*\right) + f_c \cdot \left(\ln c_t - \ln \overline{c}\right), 0\right\}. \tag{21}$$

In these rules, R^* is a "target" nominal interest rate, which is equal to the steady state real interest rate plus the targeted inflation rate. In the targeted steady state, the net nominal interest rate will equal R^*, and the gross inflation rate will equal π^*.[8] Rules similar to the inflation-based rule (21) have been widely argued to approximate actual central bank behavior. The price-level based rule is a simple alternative, and the reasoning given earlier suggests that the two rules may lead to quite different implications of the zero bound.

Because money is assumed costless to hold, there is a potential indeterminacy at a zero nominal interest rate. Individuals demand real balances at least equal to $cA\phi^v$, but they are indifferent among any levels of real balances at least that large. We resolve this indeterminacy by assuming that in states of the world where the nominal interest rate is zero, the government accommodates nominal money demand at exactly $PcA\phi^v$. An alternative assumption that would lead to virtually identical results would be for the monetary rule to impose a bound on the nominal interest rate of $\epsilon > 0$, where ϵ is arbitrarily small.

3.4 Solving the Model

We solve the model using the finite element method (see McGrattan (1996)). This involves picking a grid of points for the model's endogenous state variables, and then finding values of the "control" variables (nonpredetermined variables) numerically for each grid point and each value of the discrete random forcing variable such that the model's equations are satisfied. The method uses interpolation to calculate expectations of future variables, which in general will not fall on the original grid. The solution consists of functions mapping from the states to the controls. Those functions can be used in conjunction with the Markov process for the forcing variable to simulate the model.

One feature of this solution method is that it restricts consideration to equilibria in which policies are functions of only the economy's natural state variables. That is, the equilibria we find are Markovian. This description makes it clear that the method cannot be used to investigate certain types of multiplicity. For example, Benhabib, Schmitt-Grohe and Uribe (2001) describe equilibria where initial conditions are not uniquely pinned down; our solutions always involve unique initial conditions. In Section 6 we will return to the work of Benhabib, Schmitt-Grohe and Uribe.

Depending on the combination of policy regime and pricing specification, and depending on whether we are considering variation in the preference parameters, the size of the system of equations needed to solve the model varies between $3 \times N$ and $6 \times N_1 \times N_2$, where N is the number of grid points for a case with one endogenous state variable, and N_1 and N_2 are the natural analogues for cases with two endogenous state variables. The other element of the expression (3 or 6) comes from the fact that we can reduce the model to three fundamentally nonlinear equations in three nonpredetermined variables, plus a law of motion for the state variable(s). In the case of shocks, the number of equations is doubled to account for the two-state Markov process.

4 The Zero Bound in a Perfect Foresight Equilibrium (Transitional Dynamics)

Concern about the zero bound typically involves unusual economic conditions that one would not think of modelling using perfect foresight. However, it turns out that our main findings can in fact be conveyed in an environment without uncertainty, and without variation in exogenous state variables. Studying the model's transitional dynamics – its behavior starting from arbitrary values of the *endogenous* state variable(s) – reveals the same flavor of results as studying the response to shocks to *exogenous* state variables. This is the framework we will use to illustrate the importance of the policy rule and price-setting in determining whether the zero bound significantly interferes with the real economy. Later we will provide an example illustrating how similar results are obtained in a stochastic environment. In each of the cases that follow, the parameters of the policy rule are $f_p = 1.5$ and $f_c = 0.125$. These correspond to the standard Taylor-rule values, because the real activity argument of the policy rule is a quarterly measure. We will compare the model's transitional dynamics at 5% average annual inflation and 0% average annual inflation.

4.1 A Case Where the Direct Implications Dominate

The direct implications of the zero bound are that if inflation and expected inflation are taken as given, then the zero bound prevents negative real rates from occurring. Our earlier discussion suggests that this scenario is most likely to occur in equilibrium if the monetary authority allows drift in the price level and firms set their prices in a backward-looking manner. Both of these features impart inertia into the inflation rate.

For the policy rule that makes the price level nonstationary after linear detrending (21), and the backward-looking price-setting rule (17), Figure 2.A displays equilibrium behavior – or "policy functions" – for key variables, when the average inflation rate is 5% annually.[9] The x-axis in each panel is the relative price chosen by firms in the previous period (P^*_{t-1}/P_{t-1}). This is the single state variable needed to solve the model. While consumption, real and nominal interest rates and expected inflation are the variables of primary interest, the policy function for the current relative price (P^*_t/P_t) and the 45^o line allow one to trace out dynamics for an arbitrary initial condition. The fact that current relative price is increasing with respect to the lagged relative price, but with a slope less than unity, means that the relative price of adjusting firms converges monotonically to its steady state level. The policy function for expected inflation (panel iii) then implies similar persistence: expected inflation converges monotonically to its steady state level.

Monotonic convergence of inflation suggests that if steady state inflation is zero, then the zero bound may affect the behavior of real rates (this follows from our discussion of trend-stationarity). Our suspicion is confirmed by Figure 2.B, which displays the same policy functions as Figure 2.A, for an average inflation rate of zero. In states where the nominal interest rate is zero, the real rate "should be" negative (that is, it would be in the absence of the zero bound), but in fact the more extreme the initial condition is in demanding a low real interest rate, the *higher* the equilibrium real interest rate is. The combination of backward-looking price-setting and a policy rule that imparts some inertia into inflation creates an environment in which the direct implications of the zero bound are a reasonable approximation to the equilibrium implications.

The results illustrated in Figure 2 are consistent with previous work by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998): Fuhrer-Moore type price-setting combined with a policy rule that stabilizes inflation (not the price level) leads to significant real distortions associated with the zero bound if the average inflation rate is kept near zero. Unlike the models used by those authors however, our model has a nontrivial demand for money. That feature does not significantly exacerbate or mitigate the real effects of the zero bound.

4.2 The Role of Policy

Maintaining backward-looking price setting, we now ask whether the zero bound's direct implications are counteracted significantly by a policy rule that keeps the price level stationary. Setting the stage, Figure 3.A displays the equilibrium for 5% average inflation, under the stationary price level rule (20) and backward-looking price setting (17). Both the previous period's detrended price level $\left(P_{t-1}/\left(\pi^*\right)^{t-1}\right)$ and the previous period's relative price set by adjusting firms are state variables in this case. Rather than plotting three-dimensional policy functions, the figure plots variables as functions of P^*_{t-1}/P_{t-1}, for a value of $P_{t-1}/\left(\pi^*\right)^{t-1}$

near the steady state. Plotting the three-dimensional policy functions would not change the basic message. From panels iii and iv, note that inflation does not converge monotonically to steady state. Correspondingly, expected inflation is higher when the nominal interest rate is lower. These features suggest that under zero average inflation, policy rule (20) may be more effective than (21) in allowing necessary variation in real interest rates. And indeed Figure 3.B shows this to be the case. There is a slight distortion apparent in the behavior of consumption and real interest rates under zero average inflation: real rates cannot fall as low as they would at a higher average inflation rate. However, the distortion is much less extreme than under the rule allowing base drift, where real rates are prevented from falling below zero at all.

The intuition for how the stationary price level rule helps to mitigate real effects of the zero bound is essentially that given in section 2 above, and by Duguay (1994) and Coulombe (1998): by promising to return the price level to a fixed path, policy automatically generates expected inflation in situations where the nominal rate hits the zero bound and the real rate needs to be negative. One could question the relevance of this case, given that central banks in the industrialized countries generally focus on inflation rather than the price level. However, this analysis is aimed at generating a better understanding of what factors affect the zero bound's real consequences. Our finding about the importance of the policy feedback rule suggests one rationale for why central banks should focus on the price level rather than the inflation rate.[10]

4.3 The Role of Price Setting

Figures 2 and 3 are both based on an ad-hoc, partially backward-looking pricing specification. The advantage of this specification is that it has been successful in explaining aggregate U.S. data (Fuhrer and Moore (1995)). However, the natural assumption about price setting in our model is that adjusting firms set a price which maximizes the expected value of their present discounted profits. Profit maximizing price-setting in this class of model was first described by Yun (1996). Unlike the alternative, profit maximizing pricing is derived from optimizing behavior, and as such we can more confidently conduct policy analysis with it.[11]

Figure 4.A shows the usual set of policy functions for 5% average annual inflation, when policy keeps the price level stationary (rule (20)) and firms set their prices to maximize expected present discounted profits. For this specification, the single state variable is the detrended nominal price set by adjusting firms in the previous period $((P_{t-1}/(\pi^*)^{t-1})$. The policy rule makes expected inflation high when the nominal interest rate is low. Not surprisingly then, we see in Figure 4.B that at zero average inflation, the bound on nominal interest rates causes only a small distortion in real variables. In contrast, a rule that allows base drift in the price level prevents the real rate from going negative under profit maximizing pricing. Figure 5 shows that in this case, states where the nominal interest rate is zero all involve the real interest rate being equal to some barely negative number.

Note that it would be incorrect to describe any of the examples here as involving nominal and real rates being "stuck." Tracing out the transitional dynamics in Figure 5, for example, we find that if the nominal interest rate starts at zero it only stays there for one period. The closest thing to a trap that we observe is in Figure 2, Fuhrer-Moore pricing with base drift in the price level. There, if the initial condition is significantly below the steady state, then the economy can experience several consecutive periods of zero nominal interest rates and high but declining real interest rates.

Comparing the two pricing specifications, conditional on the policy rule, the real effects of the zero bound are greater under the backward looking pricing scheme. This accords with the intuition developed in section 2: any factor that contributes to the inflation rate being persistent will exacerbate the zero bound's distortionary effect. Recall from above that all of the results for a zero inflation target would carry over to a world where the price charged by "nonadjusting" firms automatically increased at the steady state inflation rate, as in Yun (1996). Results for the five percent inflation target would not be identical, and thus the quantitative real implications of the zero bound would differ under this alternative assumption. However, to a great extent the real implications are visible from looking at the zero inflation results in isolation; for example, comparing figures 2.B and 3.B reveals the importance of a stationary price level in mitigating the zero bound's real implications. This sort of comparison would be unchanged with automatic price changes for nonadjusting firms.

5 Incorporating Uncertainty

We claimed earlier that the same flavor of results would be conveyed by analyzing an environment with uncertainty as by studying the model's pure transitional dynamics. To support that claim, we present examples in which there is random variation in first ϑ_t and then χ_t; ϑ_t is a consumption demand shifter, and χ_t is a labor supply shifter. The examples involve profit maximizing pricing with a stationary price level. This case yielded small real effects of the zero bound under certainty, so it is an interesting one to pursue in more detail.

Figure 6 shows policy functions for the two realizations of ϑ_t, under the assumption that ϑ_t follows a symmetric, persistent process with probability 0.2 of exiting the current state. The distortion in real rate and consumption behavior that occurs in the region where the nominal rate is zero is not qualitatively different than for the transitional dynamics alone. Figure 7 repeats this information for random variation in χ_t, with similar results. In short, the nature of the shocks does not seem to affect conclusions about the zero bound's real implications.

We have not considered a "price shock," something that affects firms' pricing decisions in a way that the monetary authority cannot respond to within the period. One might suspect that the zero bound would be a more intractable problem in the presence of price shocks. An unusual string of such shocks might keep the economy stuck at the zero bound. Recall however that whether an economy can "withstand" the zero bound depends crucially on whether policy can create *expected* inflation, not whether it can affect the price level instantaneously. Thus it is not clear that price shocks would affect our results qualitatively.

6 The Zero Bound and Multiple Steady States

Benhabib, Schmitt-Grohe and Uribe (2001) have argued that the zero bound on nominal interest rates interacts with policy rules similar to (21) to generate both multiple steady state equilibria and the likelihood that equilibrium paths will converge to a steady state different from the one targeted by the policy rule. In the models they analyze, there are no state variables (i.e. predetermined variables), there are only two dynamic variables, and it is feasible – though nontrivial– to describe the models' global dynamics. Here there is a state variable (or two), and there are three dynamic variables. Thus, while we will determine whether the zero bound leads to the presence of a second steady state, we will not undertake global analysis of the model's dynamics.

To determine whether a second steady state equilibrium exists at zero nominal interest rates, we begin by imposing such a steady state on all of the equations in the model apart from the monetary policy rule. From (11), this steady state involves an inflation rate of $P_t/P_{t-1} = \beta - 1$; from (16) and (8) we can calculate the real wage (w_{zb}), and then (9), (10) and (12) yield λ_{zb}, real balances and, most importantly c_{zb}, where a variable with subscript zb indicates the value that variable would take on in a steady state with a zero net nominal interest rate. The final step is to evaluate the policy rule at this candidate steady state; if the rule returns a nominal interest rate of zero, then there is a second steady state at the zero bound.

For the stationary price level rule, a second steady state at the zero bound exists if

$$R^* + f_p \cdot \left(\ln\left(P_0 \cdot \beta^t\right) - \ln\left(P_0 \cdot (\pi^*)^t\right)\right) + f_c \cdot (\ln c_{zb} - \ln \overline{c}) < 0. \tag{22}$$

In this expression, P_0 is the initial price level, and $\overline{c}$ is the steady state level of consumption in the targeted steady state, where the gross inflation rate $\pi = \pi^*$. Note that $R^* = (\pi^*/\beta) - 1$. Condition (22) holds if

$$\ln c_{zb} < \ln \overline{c} - (1/f_c) \cdot ((\pi^*/\beta) - 1) + t \cdot [(f_p/f_c) \cdot (\ln \pi^* - \ln \beta)] .$$

If $(f_p/f_c) > 0$, then for large enough t this condition will hold, because by assumption $\beta < \pi^*$. Thus, for the stationary price level rule, there exists a second steady state with zero nominal interest rates for the parameterization we considered.

For the rule that allows for base drift in the price level, a steady state at the zero bound exists if

$$R^* + f_p \cdot (\beta - \pi^*) + f_c \cdot (\ln c_{zb} - \ln \overline{c}) < 0. \tag{23}$$

Note that c_{zb} and $\overline{c}$ are independent of the policy rule parameters f_p and f_c . For f_p large enough then, condition (23) holds because $\beta < \pi^*$. If $f_c = 0$, then a second steady state exists if $f_p > 1/\beta$. If $f_c > 0$, then f_p needs to be higher in order for there to be a steady state equilibrium, because c_{zb} is generally higher than $\overline{c}$. For the values of f_p and f_c that we chose, there does exist a steady state equilibrium with zero nominal interest rates.

Knowing that multiple steady states exist is only a first step toward understanding their implications for the model's dynamics. It is beyond the scope of the current paper to analyze global dynamics in the models used here. However, as explained by Eggertsson and Woodford (2003), a generalization of the monetary policy rules we use would eliminate the zero nominal interest rate steady state as an equilibrium. The generalization involves a commitment by the monetary authority not to let the nominal stock of money decline when the nominal interest rate is zero. This is a modification of our assumption at the end of section 3.3; instead of putting agents exactly at their satiation point for real balances when $R = 0$, the monetary authority supplies additional money. With the real quantity of interest-bearing government debt fixed, in a zero nominal interest rate steady state the ever-increasing quantity of real balances would violate the agent's transversality condition for nominal assets.

Our results on nonlinear dynamics presented above show that, even when a nontargeted steady state exists at the zero bound, it is not inevitable that the dynamics will lead to that steady state. McCallum (2001) makes a stronger claim, that the deflationary spiral equilibria of Benhabib, Schmitt-Grohe and Uribe (2001) are implausible, because they are not least-squares learnable. Thus, even without a commitment not to let the money stock decline, there is reason to believe that zero nominal interest rate steady states may be unlikely. We should note however, that the model here differs somewhat from that studied by McCallum, and the nonlinearity makes it less straightforward to study learnability.

7 Concluding Remarks

The economy's underlying dynamics (the nature of price-setting) interact with the policy rule to determine whether the zero bound has significant real effects in equilibrium. With nominal rigidities, the policy rule is an important factor in determining equilibrium dynamics (see Dotsey (1999)). In choosing a policy rule, policymakers should be well-informed about what kinds of rules enable the economy to function normally when inflation is low enough that the zero bound might be encountered. Intuitively, the crucial feature is that a rule should generate positive expected inflation in situations where the nominal interest rate is zero and the real interest rate needs to be negative.[12] While we have found that price level stabilization rules such as (20) have this feature, there are undoubtedly other rules which do not result in a stationary price level but nonetheless generate the appropriate temporary expected inflation. Reifschneider and Williams (1999) show that in the Federal Reserve

Board's FRB/US model, rules which are close to (21), but deviate downward in periods when rates have been zero in the recent past, help to mitigate real implications of the zero bound.

Any discussion of "good" policy naturally leads one to think about optimal policy. King and Wolman (1998) show that in the model discussed here, if there is no distortion associated with money demand, then optimal monetary policy involves keeping the price level constant, so that the nominal interest rate moves identically with the real interest rate. Their analysis does not take into account the zero bound, and this omission matters if the real interest rate ever needs to fall below zero. In the model used here, which includes a money demand distortion, optimal policy will almost certainly be affected by the zero bound: abstracting from price stickiness, optimal policy would involve a zero nominal interest rate *in every period*. This is the Bailey (1956) and Friedman (1969) welfare cost of inflation argument. With both price stickiness and money demand, there is a tug of war between zero inflation, which eliminates the sticky price distortion, and a zero nominal interest rate, which eliminates the money demand distortion.[13] The steady state welfare costs of small inflation or deflation associated with sticky prices tend to be small, so the optimal policy might well occasionally involve zero nominal interest rates. Khan, King and Wolman (forthcoming) describe optimal policy in a model close to the one in this paper – the money demand specification is different – but their analysis does not address the zero bound issue because they approximate the dynamics under optimal policy with a linear system.

Wolman (1998) uses the model in the current paper to conduct a limited welfare analysis that explicitly deals with the zero bound. He compares moderate inflation and moderate deflation under a rule that keeps the price level trend-stationary, and finds that the latter generates higher welfare even though the zero bound is encountered regularly. We have conducted similar analyses for the other cases in this paper (policy rule and price-setting specification) and found the same qualitative results. That is, even though low inflation targets exacerbate the frictions associated with hitting the zero bound, these costs are outweighed by the benefits of zero nominal interest rates associated with eliminating money demand frictions. Clearly, the specification of money demand is important for these results.

Eggertsson and Woodford (2003) take into account the zero bound in their analysis of optimal monetary policy in a Calvo staggered pricing model. Their findings reinforce the message that price-level targeting has benefits from the standpoint of the zero bound on nominal interest rates. Optimal policy is not exactly price-level targeting, but they show that a simple price-level targeting rule approximates optimal policy much more closely than a simple inflation targeting rule that would achieve identical outcomes absent the zero bound.

It would be worthwhile to study optimal policy in our model, which contains nonlinearities and money demand distortions absent from Eggertsson and Woodford's analysis. This would be feasible using the same nonlinear method we applied. Policy would be derived from an optimization problem rather than an exogenous rule, but this does not present significant computational obstacles. We chose to focus on a comparison of two simple rules rather than to compute optimal policy. These simple rules represent popular prescriptions for monetary policy, and it is important that policymakers understand how such rules behave in the face of the zero bound on nominal rates.

8 Footnotes

1. The fact that the zero bound involves a nonlinearity makes computation more difficult, and may have delayed work in this area.

2. In general, the equality is not exact because of covariance between inflation and the growth rate of the marginal utility of consumption. See for example Sarte [1998].

3. The basic idea in this section is already present in Duguay [1994] and Coulombe [1998].

4. We interpret the model as describing quarterly data. This value of β thus implies a steady state real interest rate of roughly 2% per annum.

5. These parameters were estimated using U.S. data on M1.

6. The transactions facilitating benefit is given by $\frac{w_t \lambda_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t})$, and the foregone interest cost is $\frac{\lambda_t}{P_t} - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$ (see (11)). A conventional money demand equation can be derived by combining (11)-(12): $m_t/c_t = A \cdot (\phi + (R_t/(1+R_t)) \cdot (c_t/w_t))^{\nu}$.

7. To see this, note that in steady state, (17) collapses to $0 = 0$ without z_t. We specify z_t so that in steady state (17) is equivalent to (16):

$$z_t = k_f \cdot (w_t - w),$$

where w is the steady state real wage implied by profit maximizing price-setting, and k_f is a positive constant.

8. Note that in contrast to the earlier discussion, we now allow the average inflation rate to be non-zero, for both the trend-stationary and nonstationary cases. That is, π^* may not be equal to one, and $\overline{P}_t$ may be growing at a constant rate π^*.

9. The figures show net quarterly interest rates and gross quarterly inflation rates.

10. For a different argument in favor of price level targeting, see Svensson [1999].

11. This criticism is especially relevant with respect to the zero bound on nominal interest rates, as the U.S. economy has not approached the zero bound during the sample in which Fuhrer-Moore price-setting specifications have been estimated.

12. Mishkin [1996] makes this point in arguing that monetary policy can work through expanding the money supply when the nominal interest rate is zero.

13. Uhlig [2000] highlights the conflicting nature of conventional views about zero nominal interest rates. On one hand there is the optimality of the Friedman rule, while on the other there is the danger of a deflationary trap.

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Figure 1.A
Stabilizing Inflation

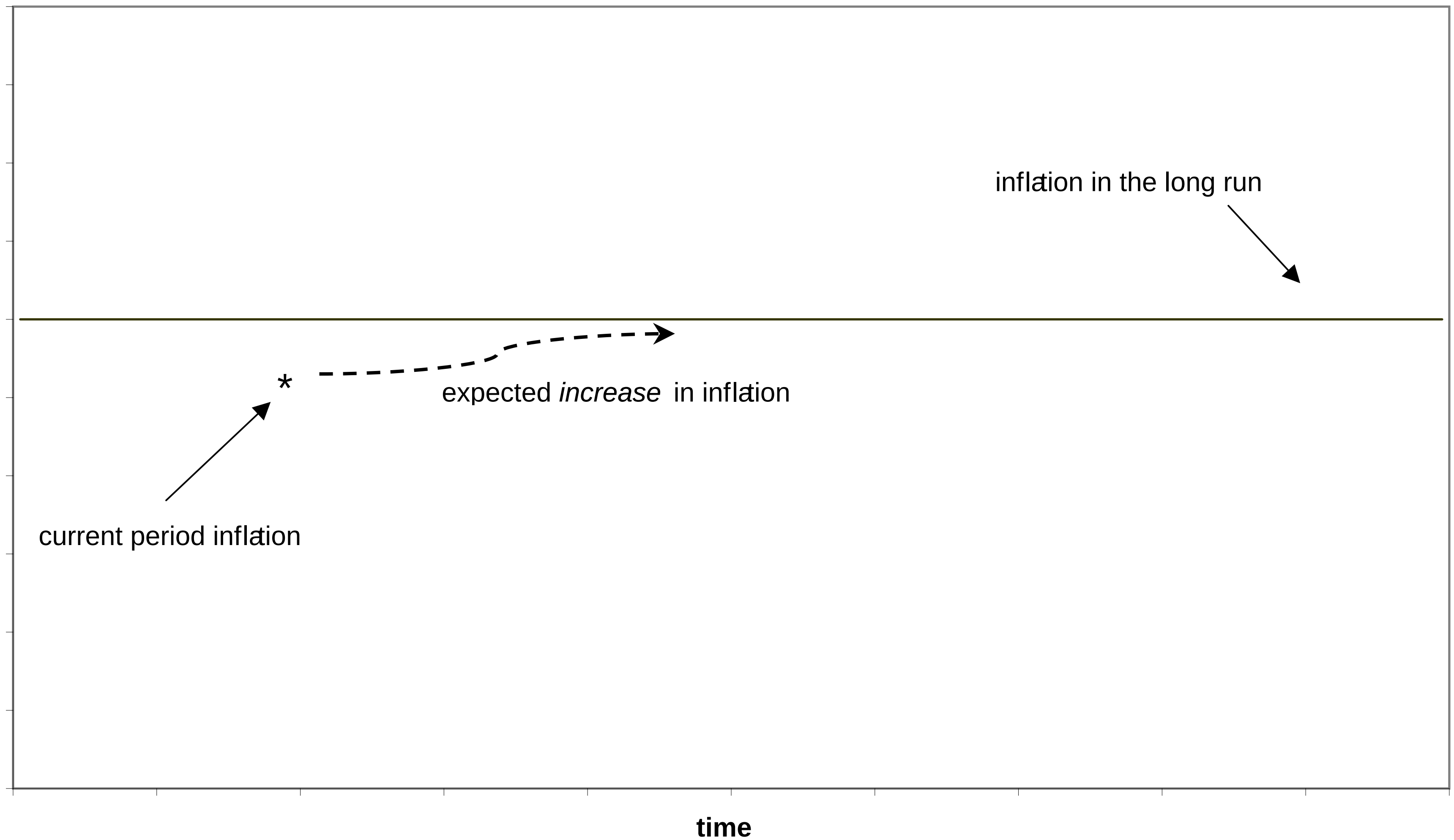

Figure 1.B
Stabilizing the price level

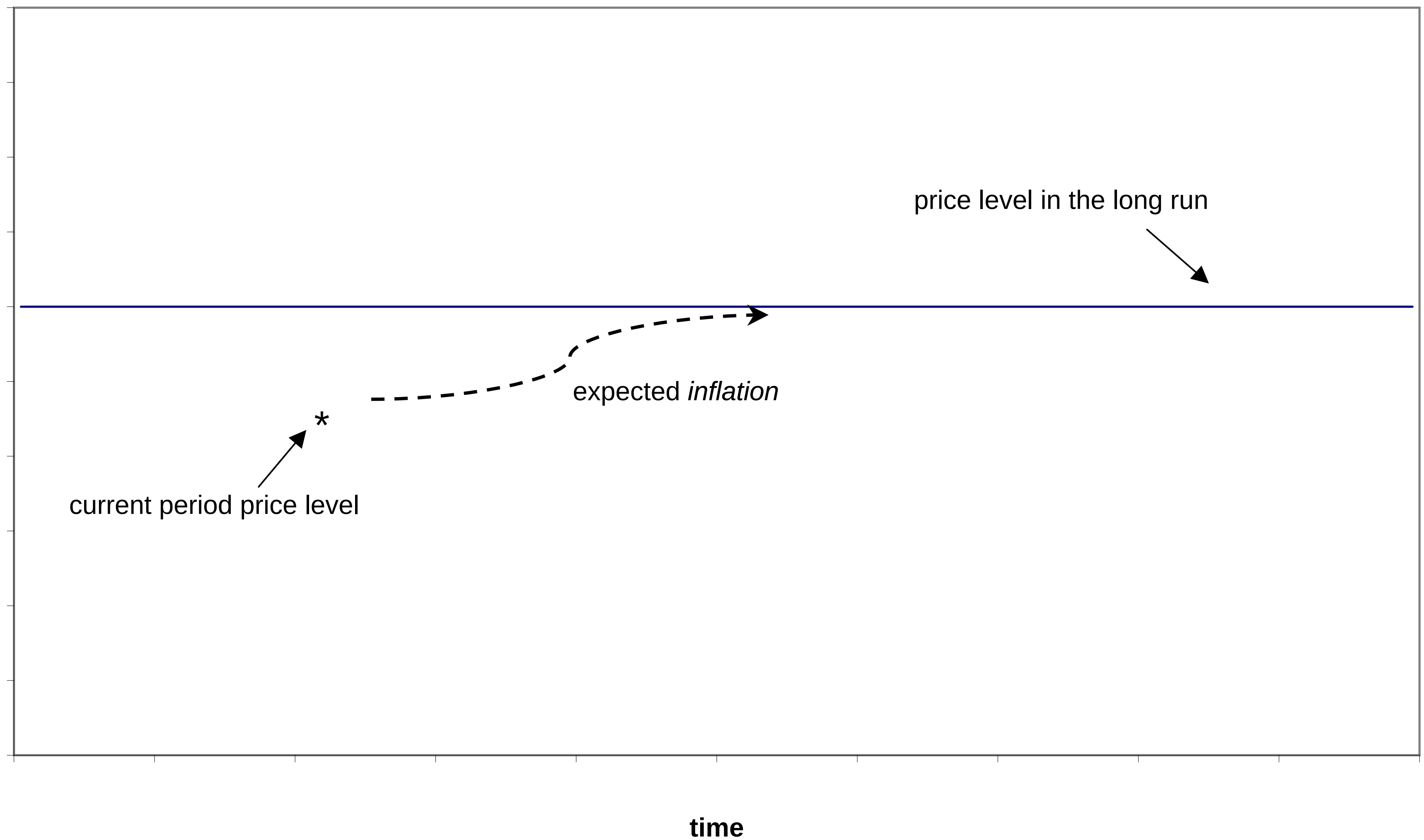

Figure 2.A. 5% inflation, backward pricing, nonstationary price level

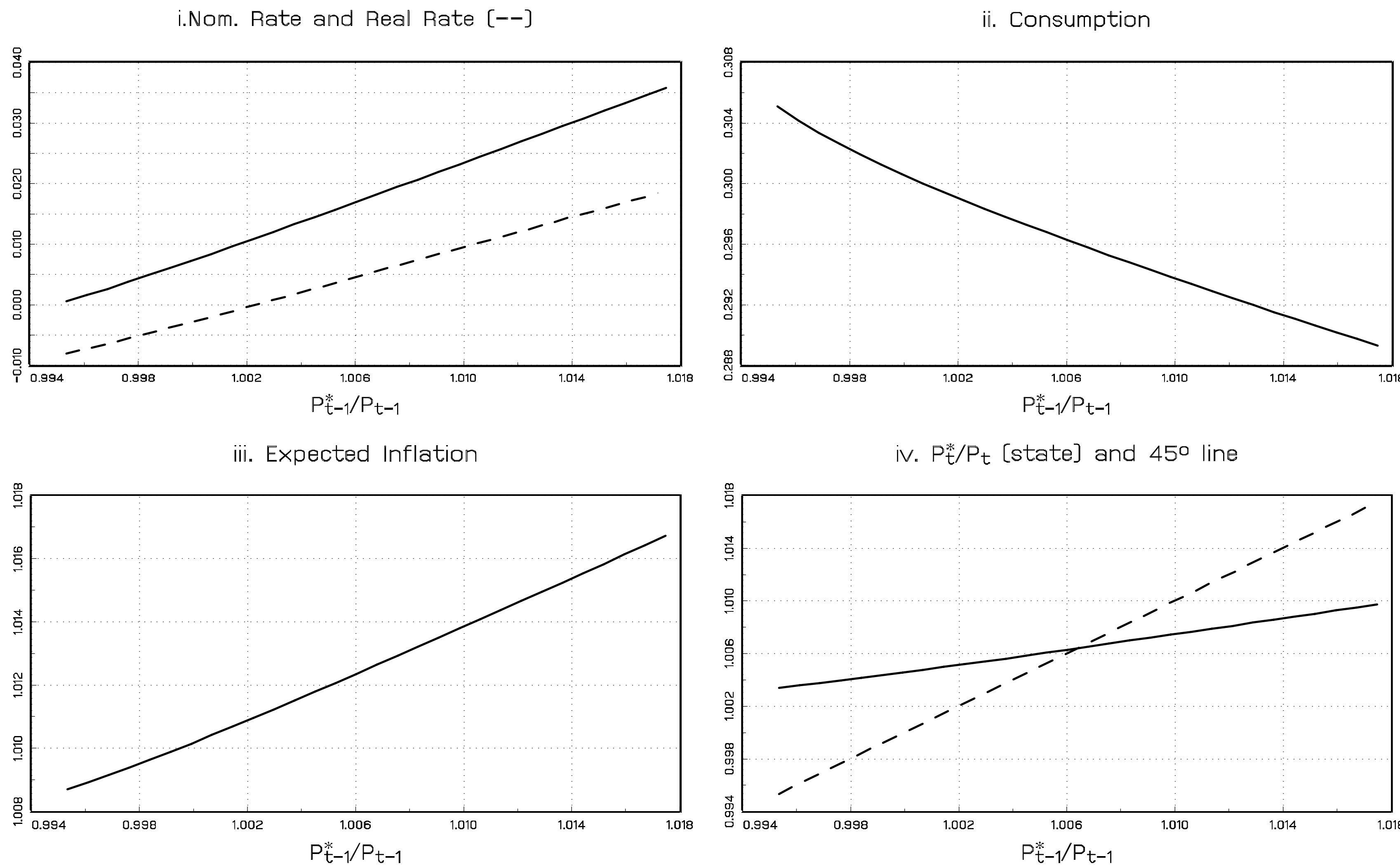

Figure 2.B. Zero inflation, backward pricing, nonstationary price level

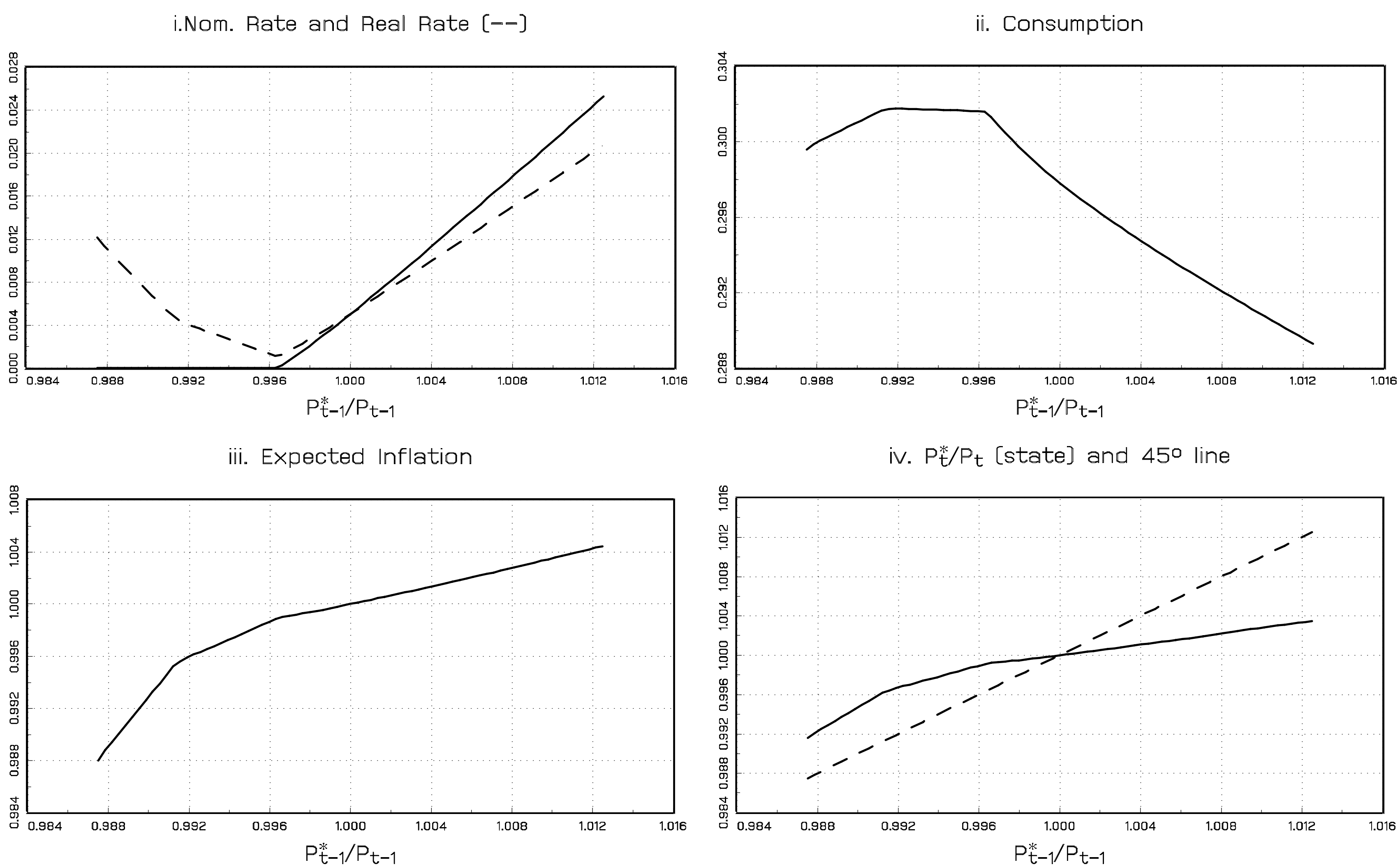

Figure 3.A. 5% inflation, backward pricing, stationary price level

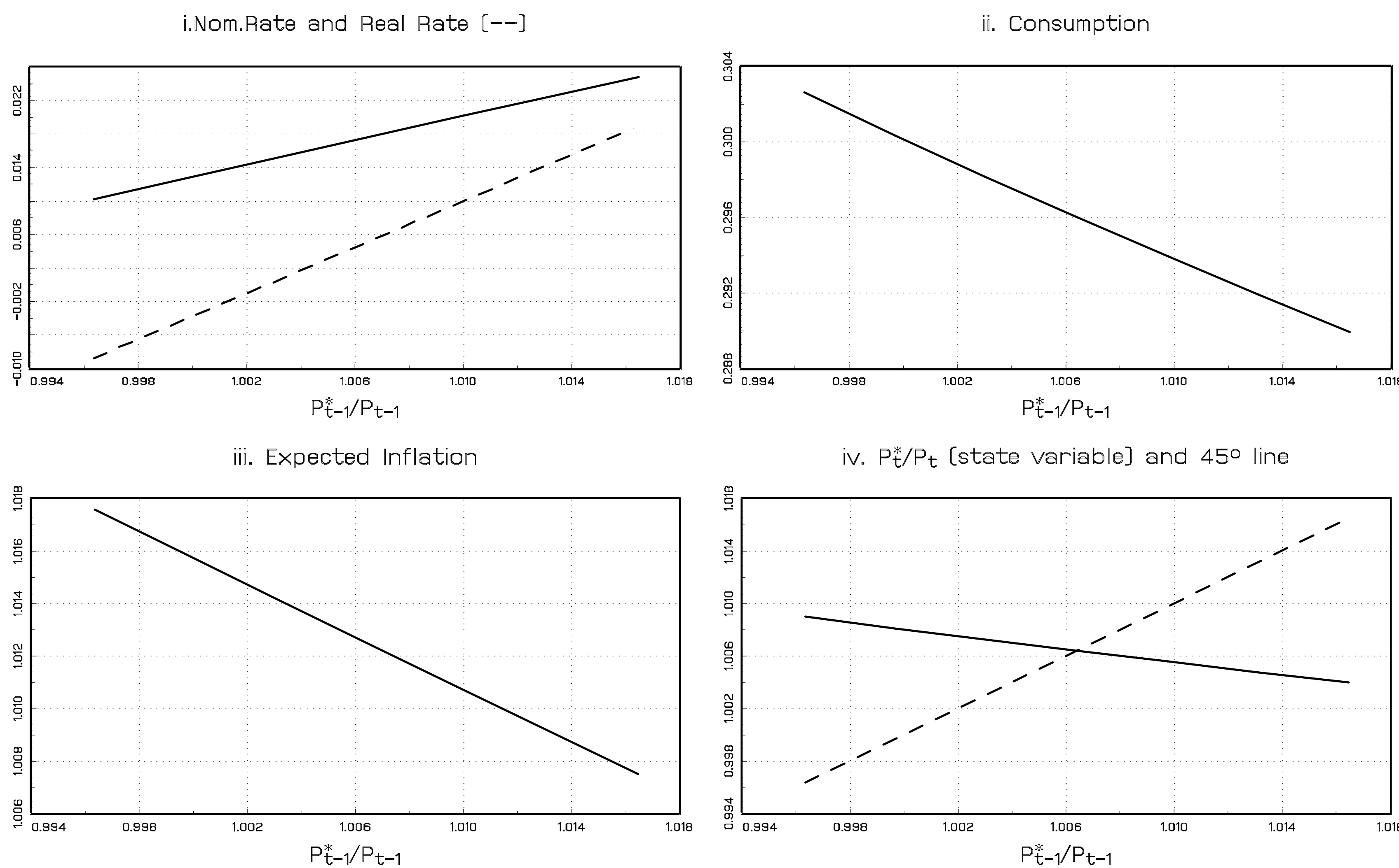

Figure 3.B. Zero inflation, backward pricing, stationary price level

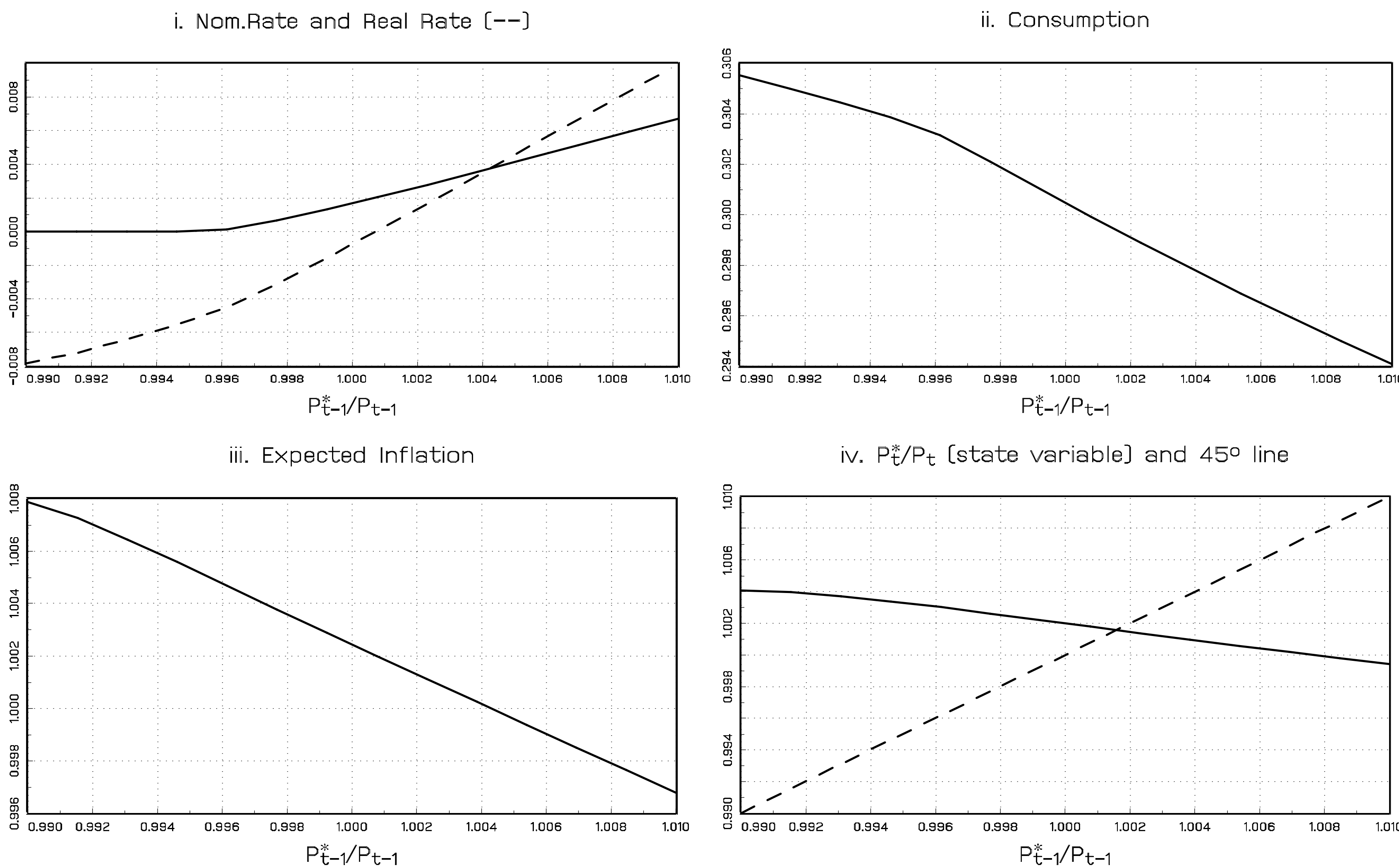

Figure 4.A. 5% inflation, optimal pricing, stationary price level

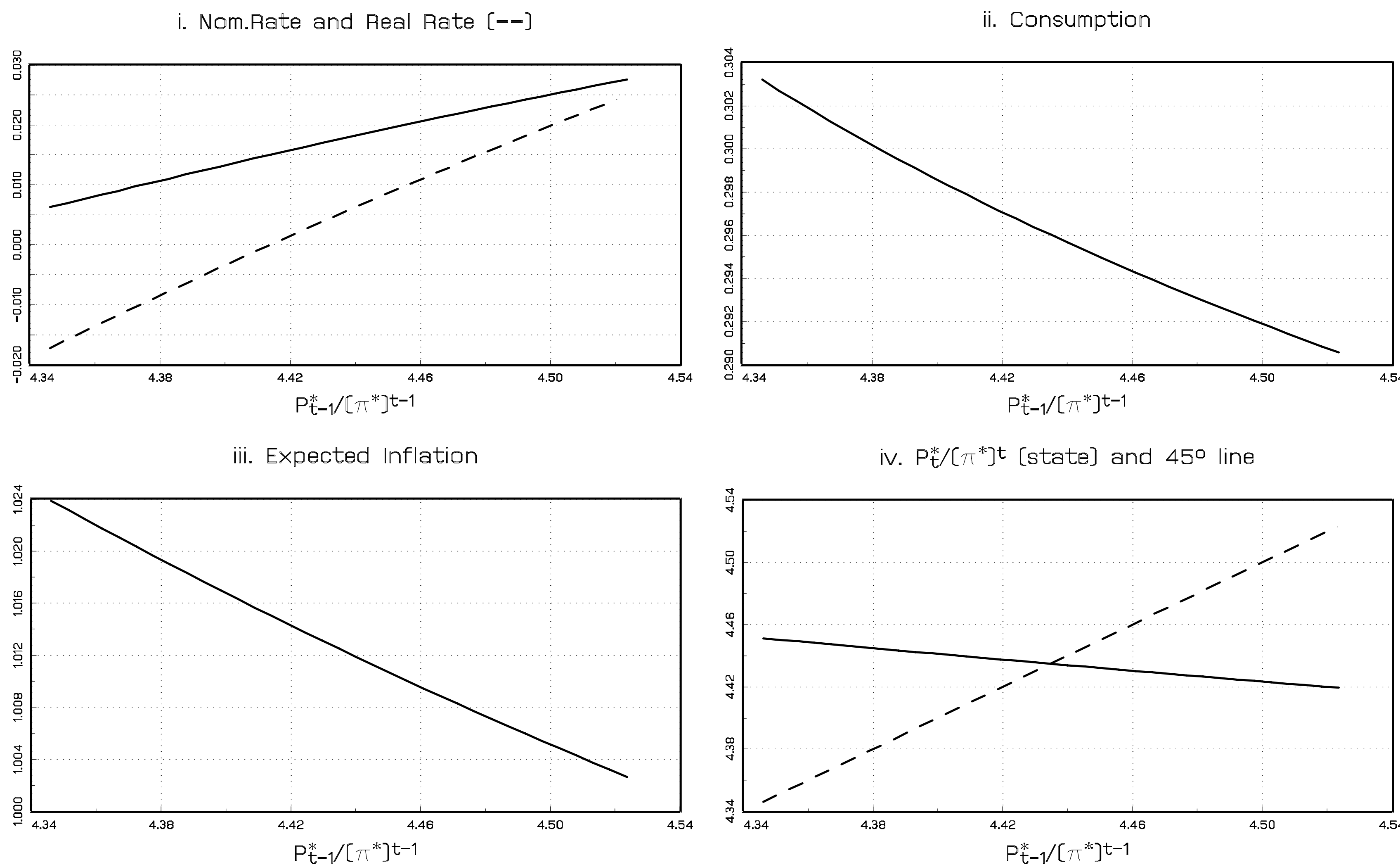

Figure 4.B. Zero inflation, optimal pricing, stationary price level

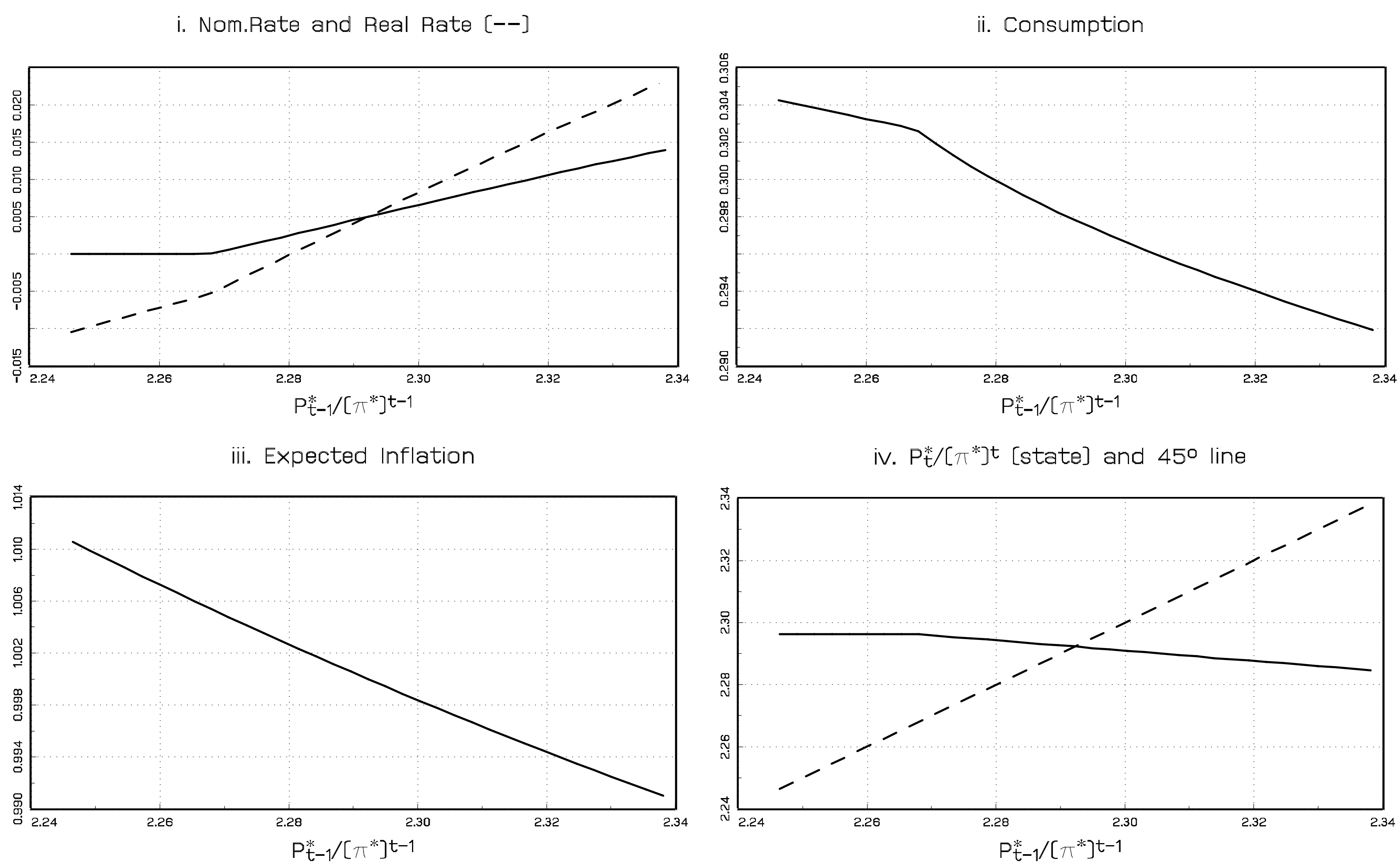

Figure 5. Zero inflation, optimal pricing, nonstationary price level

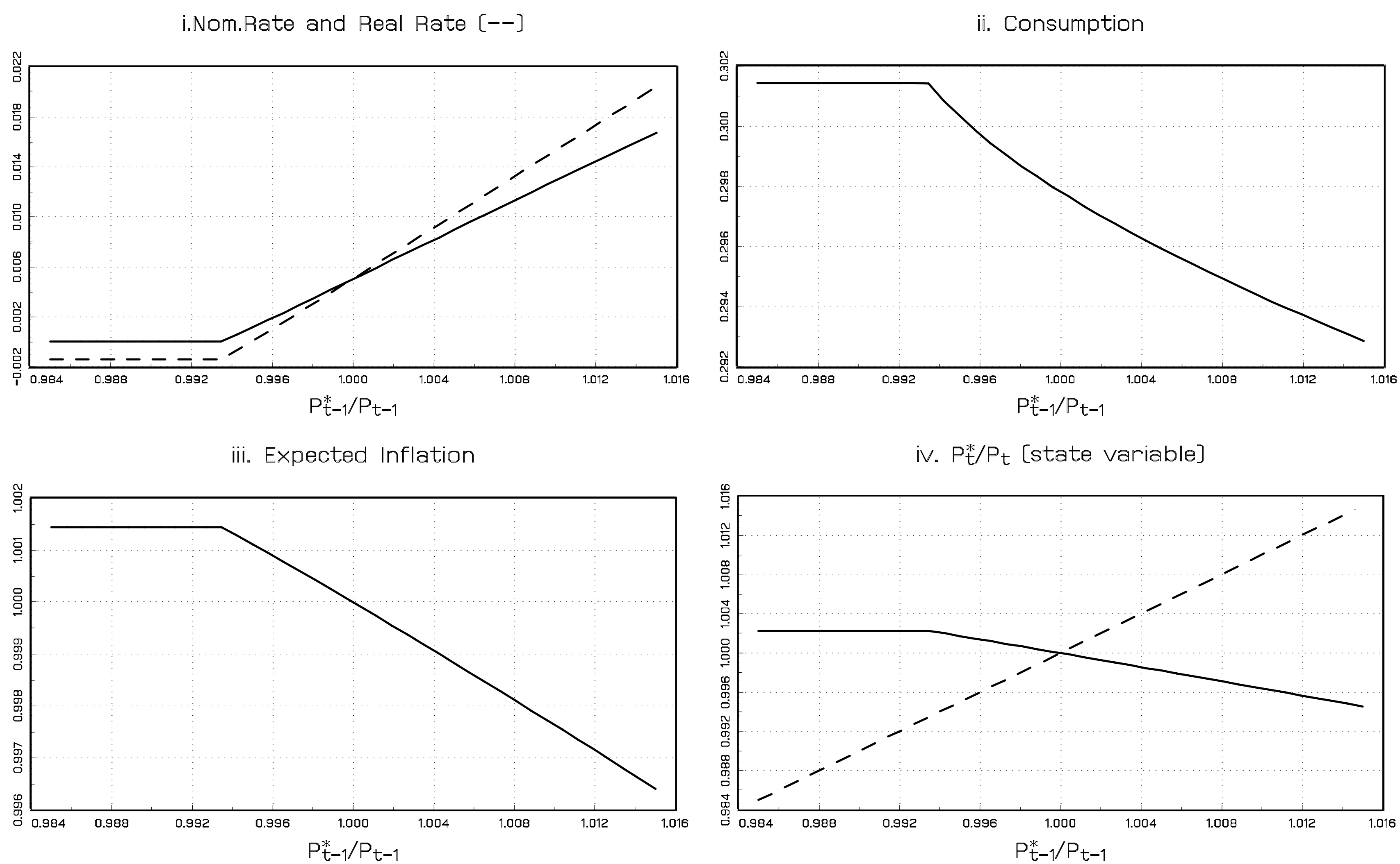

Figure 6. Random theta, zero inflation, optimal pricing, stationary price level

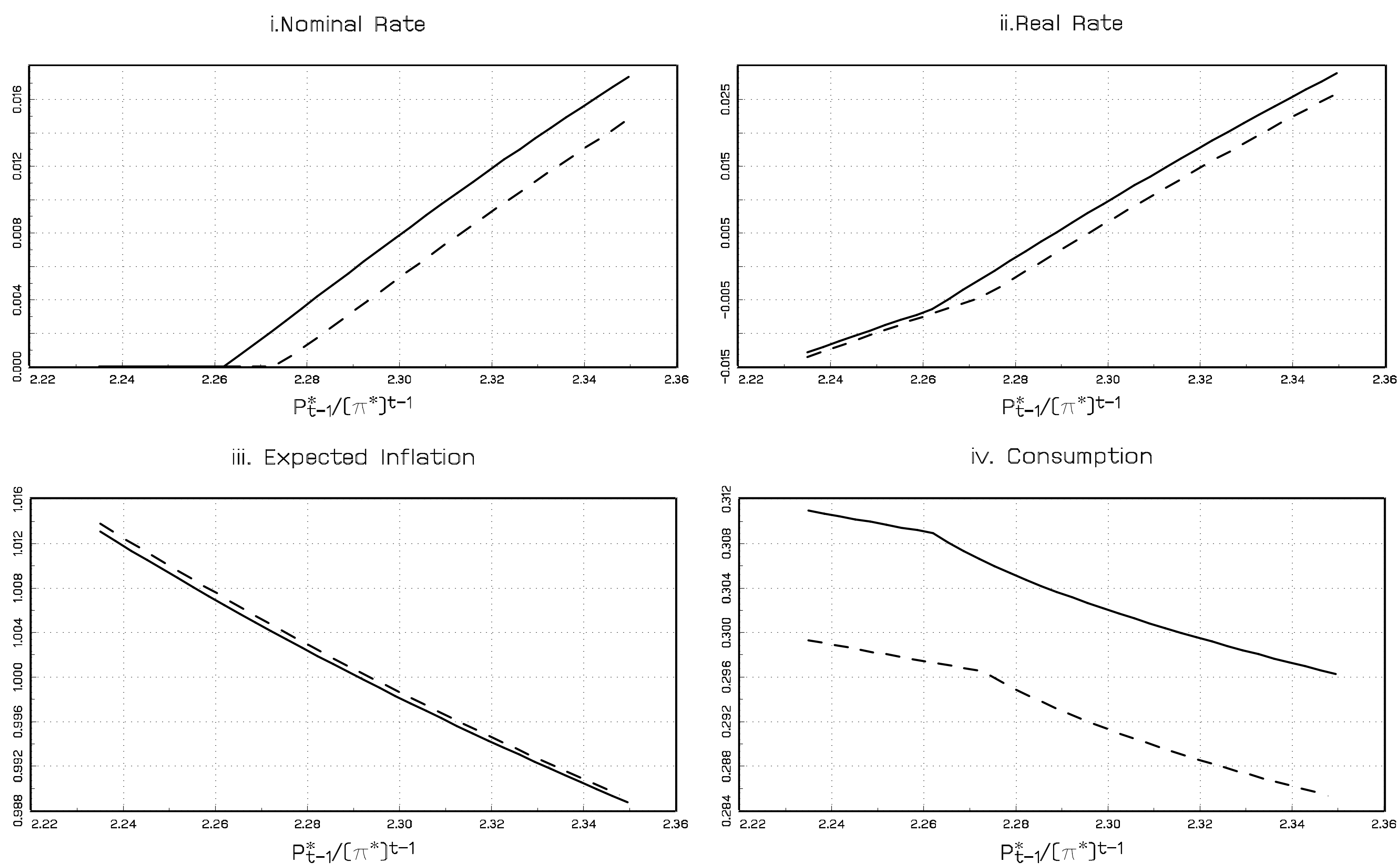

Figure 7. Random chi, zero inflation, optimal pricing, stationary price level