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Mechanism Design and Assignment Models

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Abstract

This mechanism design paper studies the assignment of people to projects over time. Inability to communicate interim shocks is a force for long-term assignments, though exceptions exist for high risk aversion. In contrast, costless reporting of interim shocks makes switching powerful for virtually all environments. Switching elicits honest reports and mitigates incentive constraints allowing, in particular, beneficial concealment of project quality. Properties of the production technology are also shown to matter. Substitutability of intertemporal effort is a force for long-term assignments while complementarity with Nash equilibrium strategies is a force for job rotation.

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1 Introduction

This paper examines a variant on the classical assignment problem of Koopmans and Beckmann (1957). There is a mechanism designer who must use people to evaluate and operate a set of projects. Each project operates over more than one stage, but in any single stage the designer may assign only one person to it. The designer's information about each project is limited; he observes a project's output, but not its interim shock nor its labor inputs. Only the agent assigned to a project at a particular stage observes that stage's relevant variable, be it the interim shock or his own labor input. In addition to setting standard contractual terms such as output-dependent consumption, the designer has the ability to reassign agents among the projects. Providing conditions under which these reassignments occur is the goal of this paper. The conditions we examine include communication possibilities, agent heterogeneity, conditions on preferences, and technological complementarities.

Organizations regularly face assignment problems. Conglomerates must decide how to allocate executives across divisions. Firms must decide how to allocate managers across departments. Managers must decide how to allocate employees across jobs. Frequently, these decisions have time and contingent components. How long should a manager be assigned to a project? Under what conditions, should he be reassigned? Regular periodic job rotation is one strategy undertaken by many organizations. Executives are rotated across divisions, and managers are often rotated across functional areas. Even within a function employees may be rotated. For example, many large banks rotate their loan officers among lending offices.[1] This solution to the assignment problem is costly. Job-specific knowledge is lost and time is spent learning details specific to the new assignment. Yet, despite these costs organizations still regularly reassign people.

There are several theories of job reassignment. In Meyer (1994), reassignments help an organization learn about the ability of workers. Reassignments can also provide training for managers who are later promoted. In Ickes and Samuelson (1987), rotation can solve "a ratchet effect." For incentive reasons long-term contracts are beneficial, but rotation is the only way the organization can commit to the long-term contracts.

[1] Rotation may occur at the economy level as well. Germany requires that its firm periodically hire new auditors, presumably to prevent collusion between management and auditors.

Our goal in this paper is to identify additional forces – complementary to those identified by the literature – that lead to reassignments. The forces we identify include limited information revelation, information scrambling, as well as properties of production technologies. Unlike Ickes and Samuelson (1987), we do not rely on limited commitment by the mechanism designer.

Section 2 lays out the general environment and compares it with the Koopmans and Beckmann (1957) assignment problem. The remaining sections study various simplifications to it. Sections 3 and 4 contain two-stage models with an interim shock in the first stage followed by a labor effort in the second. The interim shock is observed only by the agent initially assigned to the project, while the labor effort is observed only by the agent assigned to the project in the second stage. Between the two stages the agents can be reassigned to different projects.

In Section 3, the agent cannot report interim shocks to the mechanism designer. This lack of information greatly impedes reassignments. Yet, despite this impediment, we still find conditions such as high risk aversion under which reassignment is optimal. The problem studied in this section is related to literatures that examine the value of allowing the agent to know information that is private to him and not observed by the designer.

In Section 4, agents send reports to the designer, who uses these reports to make second-stage assignments. We find that reassignment is always optimal with this communication and often strictly dominates allocations without reassignment. Also, we find that the designer may *not* want to inform the agent about the interim state of his newly assigned project. Instead, the designer may want to make agent reassignment random, in effect scrambling the information communicated to the agent. Under some conditions this scrambling is valuable because it relaxes second-stage moral hazard constraints.

Section 5 studies a model in which the interim shock stage is replaced by one in which the agent takes a hidden effort. In this extension, we find that optimality of reassignment depends on the substitution and complementarity properties of production over the two project stages. Substitution is a force for staying put, as the agent takes full responsibility for all stages of production effort. Complementarity is a force for reassignment; the Nash equilibrium in efforts alleviates incentive constraints. Finally, Section 6 discusses extensions.

2 The Environment

There is a continuum of agents and a continuum of projects, both of measure one. The continuum assumption should be viewed as an approximation to the large number of people and projects that make up a firm or an economy. This abstraction avoids the need to worry about aggregate uncertainty that may arise when there is a finite number of agents and shocks are identically and independently distributed.

Production on a project takes two stages. In the first stage there is an interim shock θ to each project. Each project's shock is drawn from the same probability distribution $h(\theta)$ and there is a finite number of possible realizations of θ. Shocks are independent across projects, but with the continuum assumption $h(\theta)$ can also be viewed as the fraction of projects experiencing shock θ. In the second stage of production, each project requires a labor input $b \in B$. In some sections of the text the set B is finite, while it is a continuum in others. This set typically contains the zero effort point. On each project, the shock and the labor input determine the conditional probability distribution of the project's output $q \in Q$; the set Q is finite. We write the conditional distribution as $p(q|b, \theta)$. Shocks θ and labor inputs b applied to one project do not affect production on other projects. Later, in Section 5 below, we modify this technology by replacing the interim shock with an initial effort.

In each stage only one agent may be assigned to a particular project. The agent assigned to a project in the first stage observes the initial shock θ. The agent assigned to a project in the second stage supplies the labor input b. Both the initial shock and the labor input are private information, while the output is publicly observed. The designer may reassign agents between stages using whatever information he has available. We assume, however, that an agent assigned to a new project in the second stage does not observe its interim shock.

There is a finite number of agent types i. Each type is a positive fraction of the population $\alpha_i > 0$. An agent's preferences are

$$U_i(c) - V_i(b),$$

where c is his consumption, with $c \in \Re_+$. We assume that the functions U_i and V_i are increasing, that U_i is weakly concave, and that V_i is weakly convex. Since we will be solving for Pareto optima, we assign Pareto weights $\lambda_i > 0$ to type-i agents.

Because all projects are ex ante identical, the initial assignment of agents does not matter. Any agent can be assigned to any project. The interesting assignment problem occurs later, after the interim shock θ has been realized. At that point, the designer may reassign agents using whatever information he has available.

2.1 Full Information and the Classical Assignment Problem

If shocks θ and effort levels b were publicly observed, then this problem would be a version of the classical assignment problem. Let $\delta_i(\theta)$ be the probability a type-i agent is assigned to a project of quality θ. Equivalently, $\delta_i(\theta)$ is the fraction of type-i agents assigned to a project of quality θ. Let $c_i(\theta)$ and $b_i(\theta)$ be the consumption and effort, respectively, of a type-i agent who is assigned to a project of quality θ. The assignment problem is

Program 1:

$$\max_{\delta_i(\theta), c_i(\theta), b_i(\theta)} \sum_i \alpha_i \lambda_i \sum_\theta \delta_i(\theta)(U_i(c_i(\theta)) - V_i(b_i(\theta)))$$

$$\text{s.t. } \sum_i \alpha_i \sum_\theta \delta_i(\theta) \left(c_i(\theta) - \sum_q p(q|b_i(\theta), \theta) q \right) \leq 0, \tag{1}$$

$$\forall \theta, \ \sum_i \alpha_i \delta_i(\theta) = h(\theta), \tag{2}$$

$$\forall i, \ \sum_\theta \delta_i(\theta) = 1. \tag{3}$$

Equation (1) is a standard resource constraint; total consumption is less than total output. Equations (2) are the assignment constraints. They guarantee that the number of agents assigned to project type θ equals the number of such projects in the population. The last constraint, equation (3), ensures that the δ_i are probability measures, or equivalently, that all agents of type i are assigned to some project type.

Solutions to Program 1 are easily characterized. Because of the public information, agents work as hard as they are told. Agents assigned to more productive projects work harder than agents assigned to less productive projects. Agents are fully insured over output and with separable preferences consumption does not depend on the assignment θ. Any differences in consumption levels among agents in the cross section will be the result of differences in preferences and the Pareto weights.

Program 1 is a version of the classical assignment problem of Koopmans and Beckmann (1957).[2] In the simplest application of that problem, there are n heterogeneous jobs and n heterogeneous agents, with differing quality of matches. The problem is to match each worker with a job in a way that maximizes the objective function. In our problem, the quality or productivity of the job corresponds to the interim shock θ, while the heterogeneity in agents correspond to the agent type i. Heterogeneity in preferences or Pareto weights, provides reasons to reassign agents. For example, low Pareto weight agents or agents with less aversion to work are assigned to high productivity projects. If agents are homogeneous, however, then the assignment problem is of little interest. With separability in preferences, consumption is not tied to project assignment θ. Furthermore, even though effort may be higher for productive projects, projects are randomly assigned so as to maximize ex ante expected utility and maintain equal utility treatment.

Private information alters the analysis. The designer's assignment rule affects his ability to elicit information about the projects, information that he knows a priori in the classical assignment problem. These factors affect the quality of matches. They even make the assignment problem important when the agents are homogenous, that is, even when the driving forces for matches in the classical model are not operating.

3 No Communication

In this section, we assume that shocks θ in the first stage are private to the agent initially assigned to the project, and that effort b is private to the agent assigned to the project in the second stage. Furthermore, we assume that an agent *cannot* report the shock on his initial project. Despite his ignorance of the interim state of a project, the designer may reassign agents across projects after the first stage. If he does this, the reassignment must, by necessity, be random across project shocks θ.

[2] Also, see the survey in Sattinger (1993).

This environment is the most difficult one for generating reassignments of agents. In the classical problem, the designer knows the quality of a project; his problem is simply to match heterogenous agents to heterogenous projects. But in this problem, not only is the designer ignorant of this information, he does not receive a report on it.

As we will see in the next section, reporting is so valuable that preventing it by fiat here requires some justification. The applications we have in mind for this section have production processes in which it is difficult to completely describe the interim state of production. For example, agricultural shocks not only include whether it rained or not, but also information on how much it rained and where the runoff was. This kind of information, particularly if not formally measured, is difficult to communicate. Reporting a state is not simply a matter of reporting a value from the real line; it requires that a sender and receiver make a substantial investment in time and expertise to establish a communication channel between them (Arrow (1971)). As an approximation, we make the extreme assumption that an agent cannot communicate any information on the interim stage of the project to the designer. It is too extreme a description for many applications, but it illustrates the point nicely and makes the analysis manageable.[3]

An inability to communicate interim shocks should be particularly important when some sort of coordination is needed across stages in production. Consider a production technology on each project, $p(q|b, \theta)$, that requires a different effort b for each realization of θ; if the wrong effort is chosen then output is low. An example of such a technology is one in which the type of fertilizer to be applied to a crop depends on the moisture content of the soil, influenced by but not solely dependent on previous rainfall. If agents cannot report on the interim state then switching is ineffective. The agent could not tell the designer the moisture content of the soil, so whoever was assigned to that plot in the second stage would not know how much fertilizer to apply. In this section, we analyze a production technology

[3] The results in this section are related to literatures that examine the value of allowing an agent to gather information that the principal does not observe. Results are mixed in the accounting literature. Christensen (1981) and Penno (1989) contain examples where the agent knowing certain information is bad. Baiman and Sivaramakrishnan (1991) study a similar model with the addition of second-stage moral hazard. They find conditions on the production function in which it is always more desirable for the agent to know more information. Finally, in a monopolist problem with varying demand, Lewis and Sappington (1994) find conditions under which it is, and is not, desirable for the monopolist to help potential buyers observe a parameter indicating how much they like the monopolist's project.

where some coordination is desirable, in the sense that it is valuable to work harder on more productive projects, but the coordination is not as valuable as in the fertilizer example.

Specifically, we work with a simple deterministic second-stage production function $q = b\theta$, and unless otherwise specified all possible realizations of θ are positive. We also assume that agents are homogenous so we drop subscripts i and remove explicit references to fractions α_i and λ_i from the problem.

We start by analyzing no-switching contracts. If agents are not switched then $\delta(\theta) = h(\theta)$, that is, the probability that an agent is assigned to a project of type θ is simply the fraction $h(\theta)$ of those projects in the population. The problem is a standard hidden information problem. The assigned agent knows θ and decides on effort b, but the designer observes neither of them.

The designer chooses state-contingent effort, $b(\theta)$, and output-dependent consumption, $c(q)$. Because of the deterministic production function, however, consumption can be made explicitly a function of θ. Notice that $c(q) = c[b(\theta)\theta]$, so we write c as if it depended directly on θ, namely, $c(\theta) \equiv c[b(\theta)\theta]$.

Because of the private information, incentive constraints are needed to guarantee that an agent who receives shock θ takes effort $b(\theta)$. If the project type is θ and the agent contemplates deviating to another effort consistent with the output produced by a θ'-type agent, then to rationalize his output he must produce the output q that would be consistent with effort $b(\theta')$ at shock θ', even though the true shock is θ. The requisite deviating effort would be $b(\theta')\theta'/\theta$. The incentive constraints (5), below, prevent this deviation. One related concern is that an agent will decide to work a level of b that is inconsistent with the output of any type. In this case, the designer knows the agent has deviated. For simplicity, we abstract from the details of the punishments and assume that the designer can automatically prevent deviations that have a positive probability of revealing a deviation.[4]

[4] Alternatively, we could restrict the designer's punishments by setting consumption equal to zero. For preferences in which $U(0) = -\infty$, like log utility, the effect is the same. For other preferences, a constraint that limits agents' utility to be at least equal to receiving zero consumption and taking zero effort would be necessary. This constraint is easy to analyze. The analogous constraint for the switching case, however, is not so simple to analyze.

The programming problem is

Program 2:

$$\max_{c(\theta),b(\theta)} \sum_{\theta} h(\theta)(U(c(\theta)) - V(b(\theta)))$$

$$\text{s.t.} \quad \sum_{\theta} h(\theta)(c(\theta) - \theta b(\theta)) \leq 0, \tag{4}$$

$$U(c(\theta)) - V(b(\theta)) \geq U(c(\theta')) - V(\frac{b(\theta')\theta'}{\theta}), \quad \forall \theta, \theta'. \tag{5}$$

Equation (4) is the resource constraint. It guarantees that consumption does not exceed output. Equations (5) are the incentive constraints discussed earlier.

If agents are switched, neither the designer *nor the agent newly assigned to a project* know its interim shock. Consequently, all agents must be assigned the same effort level b. Furthermore, there is no incentive constraint. For any deviation from the recommended effort b there is a positive probability that the deviation will be revealed by an output impossible in equilibrium, causing the unmodeled non-pecuniary punishment to be imposed.[5] As in the no-switching model, the designer can infer θ from q. Here, the inference follows from $\theta = q/b$. Therefore, as in the no-switching model, we write $c(\theta)$.

The problem if agents are switched is

Program 3:

$$\max_{c(\theta),b} \sum_{\theta} h(\theta) U(c(\theta)) - V(b)$$

$$\text{s.t.} \quad \sum_{\theta} h(\theta)(c(\theta) - \theta b) \leq 0. \tag{6}$$

Equation (6) is the resource constraint.

[5] The random variable θ can only take on a finite number of values, none of which are zero. If an effort less than b is taken then the lowest value of θ will reveal the deviation. Similarly, for an effort greater than b the highest value of θ plays that role. Finally, depending on the possible values of θ, intermediate values of θ may be revealing as well.

All agents work the same effort. The common effort level is chosen so that the marginal disutility of effort equals expected marginal product, that is, $V'(b) = E(\theta)$. Finally, with risk aversion consumption is constant.

Each regime has an advantage and a disadvantage relative to the other. If agents are not switched, their efforts can be tailored to the relative productivities of each project but there are incentive constraints. If agents are switched, the same effort is applied to all projects regardless of relative productivities but there are no incentive constraints. As the next two propositions demonstrate, risk aversion is an important factor in determining which regime is better.

Proposition 1 *If agents are risk neutral with respect to consumption then no-switching dominates switching.*

Proof: With risk neutrality incentive constraints do not bind in the no-switching model. Let $b(\theta)$ satisfy $V'(b(\theta)) = \theta$ and give each agent his own product, $c(\theta) = \theta b(\theta)$. This contract is incentive compatible because it satisfies the first-order conditions to the agents' problems. It also is a solution to the full-information problem. Consequently, it is better than the best switching contract. **Q.E.D.**

With risk neutrality there is no distortion from the incentive constraints. Without the incentive distortion, the ability to tailor effort levels to marginal productivities is unambiguously good. This is not true when agents are risk averse. The designer wants to reduce the variability of consumption but that limits the incentives of productive agents to work hard. If agents are risk averse enough, it is more important to remove the disincentive to work than it is to tailor efforts to marginal productivities.

We prove the theorem for preferences where any variation in consumption is undesirable. Preferences are *Leontief* if $U(\{c(\theta)\}) = \min_\theta U(c(\theta))$. Under Leontief preferences there will not be any state-dependent variation in consumption, that is, $c(\theta)$ will be a constant. If it is not, any consumption paid out in excess of the minimal received consumption level is wasted. These preferences punish variation in consumption and in this sense are analogous to an extreme form of risk aversion. After the proof, we provide an example using a standard utility function in which switching still dominates.

Under Leontief preferences, low-productivity types work relatively hard and high-productivity types work relatively little to ensure that there is no variation in output-dependent consumption. With switching, output can be made higher by having both agents apply identical effort. The loss of output from low-productivity types who were working hard is more than compensated by the increased output from high-productivity types.

Proposition 2 *If preferences are Leontief and if $\theta_j > 0$ for all j, then switching dominates.*

Proof: See Appendix.

Figure 1 illustrates Proposition 2 for the case where there are two shocks that occur with equal probabilities. Under the no-switching contract each agent knows the quality of his project. With Leontief preferences agents are so risk averse that they dislike *any* variation in their consumption. This assumption forces solutions to the no-switching model to satisfy $c(\theta) = c$. Because consumption is not state-contingent, the solution to the optimal no-switching problem is characterized by $b(\theta)$ satisfying $\theta b(\theta) = \theta' b(\theta')$ for all θ, θ', that is, all agents must produce the same output regardless of their shock θ. Consequently, the less productive agents work the hardest. In Figure 1 both agents produce q_{ns}.

A switching contract that does better is for both agents to work $b_s = 0.5b(\theta_l) + 0.5b(\theta_h)$. Disutility of effort is less and total output is higher because the increase in the high-type's output is greater than the decrease in the low-type's output.

The result is not just applicable for the extreme preferences of the proposition. For example, if preferences are $U(c) = -(c + 0.01)^{-0.18} - 0.5b^{1.22}$, with two types, and $\theta_l = 1$, $\theta_h = 2$, then switching dominates.

There is a simple case where we can prove that not switching dominates switching, regardless of the preferences. Assume that there are two possible shocks, and unlike in Proposition 2 above, assume that $\theta_l = 0$ and $\theta_h > 0$. Production is still $q = b\theta$. Because the lowest value of θ is zero, an agent can always work zero and pretend he received the θ_l shock. This creates an incentive constraint under switching that is

$$h(\theta_l)U(c(\theta_l)) + h(\theta_h)U(c(\theta_h)) - V(b) \geq U(c(\theta_l)) - V(0), \tag{7}$$

where the summation over θ indicates the uncertainty the agent has over the interim state of his newly assigned project. Here, on the right-hand side, the agent contemplates taking

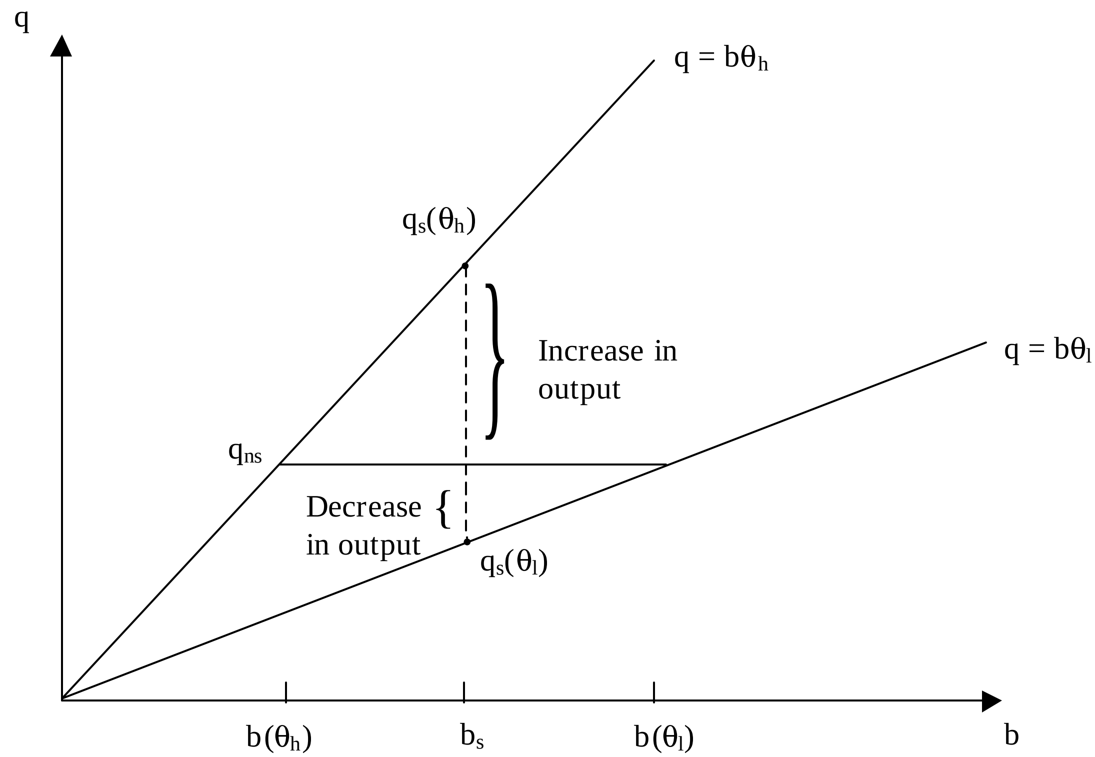

Figure 1: Illustration of proof of Proposition 2 for case with two types and $h(\theta_l) = h(\theta_h) = 0.5$: Under no-switching, agents work $b(\theta_l)$ and $b(\theta_h)$ and total output is q_{ns}. Under switching both agents work b_s and total output is $0.5q_s(\theta_l) + 0.5q_s(\theta_h)$. Total output under switching is greater because $q_s(\theta_h) - q_{ns} > q_{ns} - q_s(\theta_l)$.

the deviating effort, $b = 0$, ensuring that output will be zero. In equilibrium, zero output is produced by agents with the low shock, so $c = c(\theta_l)$. On the left-hand side, effort b is taken so that output is $q = b\theta$ and consumption depends on the type of project they are assigned to.

The resource constraint is

$$h(\theta_l)c(\theta_l) + h(\theta_h)c(\theta_h) \leq h(\theta_h)\theta_h b. \tag{8}$$

For $b > 0$ the optimal, incentive-compatible switching contract is characterized by higher consumption for higher output, that is, $c(\theta_h) > c(\theta_l)$.

If the agents are not switched, they know their type θ, and then we have the earlier private information case. The optimal no-switching contract is characterized by $b(\theta_l) = 0$

and $b(\theta_h) > 0$. For θ_l, the problem is trivial in the sense that there is no value to effort. The incentive constraints, (5), only apply to the high type θ_h so

$$U(c(\theta_h)) - V(b(\theta_h)) \geq U(c(\theta_l)) - V(0). \tag{9}$$

The resource constraint is

$$h(\theta_l)c(\theta_l) + h(\theta_h)c(\theta_h) \leq h(\theta_h)\theta_h b(\theta_h). \tag{10}$$

In this special case, no-switching dominates switching.

Proposition 3 *If there are two shocks, $\theta_l = 0$, and if the optimal switching contract is characterized by $c(\theta_h) > c(\theta_l)$, then no-switching strictly dominates switching.*

Proof: Take an optimal switching contract $(b, c(\theta_l), c(\theta_h))$. Now consider the no-switching contract $b(\theta_l) = 0$, $b(\theta_h) = b$, with $c(\theta)$ unchanged. Under this contract the resource constraint is unchanged; equations (8) and (10) are identical. Furthermore, $U(c(\theta_h)) > h(\theta_l)U(c(\theta_l)) + h(\theta_h)U(c(\theta_h))$. Substituting this inequality and $b = b(\theta_h)$ into the left-hand side of equation (7) delivers the no-switching incentive constraint (9). We thus have a feasible no-switching contract that is better than the optimal switching contract. **Q.E.D.**

4 The Roles of Information and Communication

Without communication, the optimality of reassignment depends heavily on risk aversion. In this section, we examine the importance of the previous no-communication assumption by allowing an agent to report to the designer on his initial project's shock. The combination of reporting and switching is useful for three reasons: it reveals information, it helps to make better matches, and it allows for the scrambling of information communicated to the agent.

To study the heterogeneity issue, we drop the representative consumer simplification and return to heterogenous agents. As in the environment of Section 2, let there be a finite number of observably different types, with each type i constituting a positive fraction, $\alpha_i > 0$, of the population. An agent's type is public information. Pareto weights are described by λ_i, with $\sum_i \lambda_i = 1$. All agents of a given type are treated identically ex ante.

4.1 Information revelation

In this section, we study the prototypical moral hazard problem by adding a stochastic element to second-stage production. The production function is $p(q|b,\theta) > 0$ for all q, b, θ, so p is a *non-degenerate* probability distribution. Second-stage effort b is private information. The set of efforts, B, is a finite set.

For reasons illustrated shortly, we allow for some randomization in contractual terms. As in the no-switching model, the contract contains a recommended effort level that depends on the interim state of the project. Now, however, this recommendation may be random. It is described by the conditional probability distribution $\pi_i(b|\theta)$ for type i. Because of the randomized effort, consumption needs to be a function not only of the interim state θ and the output realization q, but also the realized recommended effort b. We write the compensation schedule of type i as $c_i(q, b, \theta)$. We could have allowed randomization in the compensation schedule as well but with separable preferences consumption will be degenerate so we dropped explicit consideration of this source of randomization.

We first consider no-switching contracts. When a type-i agent stays on his project with probability one, he knows the state θ. By the Revelation Principle the contract needs to satisfy incentive constraints that induce truthful reporting of θ and then, given a truthful report, other constraints that ensure that the agent takes the recommended effort. The truth-telling constraints are

$$\forall i, \theta, \quad \sum_{q,b} p(q|b,\theta)\pi_i(b|\theta)[U_i(c_i(q,b,\theta)) - V_i(b)] \tag{11}$$

$$\geq \quad \sum_{q,b} p(q|\phi(b),\theta)\pi_i(b|\theta')[U_i(c_i(q,b,\theta')) - V_i(\phi(b))], \quad \forall \theta' \neq \theta, \ \forall \phi : B \to B.$$

Constraints (11) ensure that telling the truth, θ, and then taking the resulting recommended effort b, is preferable to lying, *i.e.,* sending a report $\theta' \neq \theta$, and then taking any deviation strategy, ϕ, which maps recommended effort b to alternative effort b'. For more details on these constraints, see Myerson (1982) for the original treatment or Prescott (2003) for an exposition in a similar model.

In addition to constraints (11), the Revelation Principle requires constraints that ensure that an agent who truthfully reports θ takes recommended effort b. For all i, θ, and b such that $\pi_i(b|\theta) > 0$,

$$\sum_q p(q|b,\theta)[U_i(c_i(q,b,\theta)) - V_i(b)] \geq \sum_q p(q|\hat{b},\theta)[U_i(c_i(q,b,\theta)) - V_i(\hat{b})], \quad \forall \hat{b}. \tag{12}$$

With communication a strikingly simple mechanism improves upon no-switching contracts.

Proposition 4 *Regardless of risk aversion, heterogeneity in preferences and welfare weights, and the specification of technology $p(q|b,\theta) > 0$, switching and telling the agent the state of his newly assigned project θ weakly dominates not switching him. Dominance is strict if incentive constraints (11) bind.*

Proof: Consider the following contract: After agents report on their interim shocks, the designer switches them and then makes their new assignment and compensation independent of the report they sent. Under this contract, an agent's utility does not depend on his report so he reports the true shock. Next, assume that the quality of each agent's assigned project is randomly drawn from the distribution $h(\theta)$ and the designer tells each agent the quality of his new project θ. The compensation schedule is still described by $c_i(q,b,\theta)$, but now θ is the quality of an agent's *newly* assigned project.

Because the designer and the agent know the state of the project, there are no truth-telling constraints as in equations (11). The only incentive constraints left are those on the agent's effort, which are identical to constraints (12) in the no-switching scheme. Thus, the set of no-switching contracts is a subset of the switching contracts. Consequently, switching weakly dominates no-switching. The dominance is strict if truth-telling constraints (11) bind in the no-switching optimum, as these are eliminated in the switching regime. **Q.E.D.**

In the first-stage of the switching scheme, agents are just information monitors. They report the true shock because they are indifferent to what they observe and what they report.[6] The arrangement is essentially a moral-hazard economy, with the added feature that there is a random, publicly observed shock to the production technology. It should be noted that this result does not depend on the second-stage moral hazard. The information revelation result still applies to the deterministic production technology analyzed in the previous section.

[6] A similar idea is used in Hirao (1993).

4.2 Information scrambling

In the contract described above the designer tells the agent the interim state of his newly assigned project. That property of the contract was imposed by fiat. While sufficient to illustrate the information revelation role of switching, it need not be optimal. Indeed, sometimes the designer would choose *not* to tell the agent the state of his newly assigned project. In this case, not only does switching remove truth-telling constraints but it also scrambles information. The agent now has to infer the quality of his newly assigned project. As we will see below, scrambling weakens second-stage incentive constraints. Ignorance is bliss here.

In the switching stage, the designer assigns agents to projects according to the assignment probabilities $\delta_i(\theta)$. As before, the designer recommends an effort level b, according to the possibly stochastic rule $\pi_i(b|\theta)$.

If the designer does not tell the agent the quality of his new project then the agent has to infer it. He has three pieces of information from which to form his inference: his type i, the assignment rule $\delta_i(\theta)$, and the recommended effort rule $\pi_i(b|\theta)$. A type-i agent who is recommended effort b forms a posterior over project quality of $pr_i(\theta|b)$. The posterior is related to the other objects by the relationship $pr_i(\theta|b) = \delta_i(\theta)\pi_i(b|\theta)/\pi_i(b)$, where $\pi_i(b)$ is the unconditional probability that a type-i agent is recommended effort b.

The incentive constraint can be written directly in terms of the posterior probabilities, $pr_i(\theta|b)$, but it is more convenient to substitute out for this term. Again, there are no truth-telling constraints. The moral hazard incentive constraints are for all b such that $\pi_i(b|\theta)\delta_i(\theta) > 0$,

$$\sum_{q,\theta} p(q|b,\theta)\pi_i(b|\theta)\delta_i(\theta)[U_i(c_i(q,b,\theta))-V_i(b)] \geq \sum_{q,\theta} p(q|\widehat{b},\theta)\pi_i(b|\theta)\delta_i(\theta)[U_i(c_i(q,b,\theta))-V_i(\hat{b})], \quad \forall \hat{b}, \tag{13}$$

where the $\pi_i(b)$ cancel out of both sides.

Compare these moral hazard constraints, (13), with the moral hazard constraints, (12), used by the other two schemes. For a given b, (13) is a convex combination of all the incentive constraints (12) corresponding to θ for which b was recommended. We can now prove the following theorem.

Proposition 5 *A switching contract where the designer does not tell the agent the shock θ of his newly assigned project weakly dominates a switching contract where the designer tells the agent the value of θ of his newly assigned project.*

Proof: Any contract satisfying (12) for each θ will satisfy (13), but not necessarily vice versa. **Q.E.D.**

If each agent assigned to a different quality project was recommended a different effort level then there would not be any value to scrambling. Each agent would be able to infer from his recommended effort level the quality of his new project. But, if agents cannot make that inference, because more than one project quality is assigned a particular effort level, then the dominance in Proposition 5 may be strict. The numerical example below demonstrates.

4.3 Private information and the assignment problem

We now formulate the economy-wide mechanism design problem and then present an example that illustrates not one but three reasons for switching: the classical role of matching heterogenous agents to heterogenous jobs, information revelation, and information scrambling.

We take as a starting point the observation that at no cost the designer can induce truthful revelation of project qualities, θ, by the schemes described above. The program is

Program 4:

$$\max_{\delta_i(\theta)\geq 0,\pi_i(b|\theta)\geq 0,c_i(q,b,\theta)} \sum_i \alpha_i\lambda_i \sum_{q,b,\theta} p(q|b,\theta)\pi_i(b|\theta)\delta_i(\theta)[U_i(c_i(q,b,\theta)) - V_i(b)]$$

$$\text{s.t.} \sum_i \alpha_i \sum_{q,b,\theta} p(q|b,\theta)\pi_i(b|\theta)\delta_i(\theta)(c_i(q,b,\theta) - q) \leq 0, \tag{14}$$

(13) (the incentive constraints),

$$\forall\theta, \sum_i \alpha_i\delta_i(\theta) = h(\theta), \tag{15}$$

$$\forall i,\theta \ni \delta_i(\theta) > 0, \sum_b \pi_i(b|\theta) = 1. \tag{16}$$

Constraint (14) is the resource constraint. Constraints (15) are the assignment constraints and constraints (16) guarantee that the $\pi_i(b|\theta)$ are probability measures.

4.4 A numerical example

Let there be two types of agents, $i = 1$ (type 1s) and $i = 2$ (type 2s). The Pareto weights on the types are $\lambda_1 = 0.2$ and $\lambda_2 = 0.8$. The low status agents are a much larger fraction of the population than the high status agents. Specifically, $\alpha_1 = 0.8$ and $\alpha_2 = 0.2$.

Preferences are separable in consumption and effort. Utility from consumption is $U(c) = c^{0.5}/0.5$. Agents may choose from only three possible efforts, b_1, b_2, or b_3. The effort portion of the utility function is described by $V(b_1) = V(b_2) = 0$, and $V(b_3) = 0.5$.

There are three different types of projects, indexed by θ_1, θ_2, and θ_3. Project types are random and drawn from the distribution $h(\theta_1) = h(\theta_2) = 0.4$, and $h(\theta_3) = 0.2$. Each type of project may produce either a low output, $q_l = 0$, or a high output, $q_h = 1$. The probability distribution of the outputs is a function of the project type θ and the effort b and is described by $p(q|b,\theta)$. Output on each project is independent of other projects. Table 1 describes the $p(q|b,\theta)$ production function used in the example.

	θ_1			θ_2			θ_3	
	q_l	q_h		q_l	q_h		q_l	q_h
b_1	0.99	0.01	b_1	0.70	0.30	b_1	0.99	0.01
b_2	0.70	0.30	b_2	0.99	0.01	b_2	0.99	0.01
b_3	0.40	0.60	b_3	0.40	0.60	b_3	0.99	0.01

Table 1: A production technology, $p(q|b,\theta)$, that generates switching.

The first two projects, indexed by θ_1 and θ_2, are the most productive as long as effort b_3 is taken. The two types of projects are identical except that b_i, $i = 1, 2$ has a different effect on each project. If b_i is worked on a θ_i, $i = 1, 2$, project then the project is extremely unproductive. The third project is so unproductive that the highest effort has no marginal effect at all.

Table 2 lists the solution to the example. The top half of the table lists the optimal allocation for type-1 agents and the bottom half lists it for type-2 agents. The second column in each half lists the probability of being assigned to each project type, and the third column contains the effort taken conditioned on the project assignment. Finally, the fourth and fifth columns list consumption given outputs q_l and q_h respectively.

	Type 1's allocation			
	$\delta_1(\theta)$	Action	$c(q=0)$	$c(q=1)$
θ_1	0.5	b_3	0	0.32
θ_2	0.5	b_3	0	0.32
θ_3	0	-	-	-
	Type 2's allocation			
	$\delta_2(\theta)$	Action	$c(q=0)$	$c(q=1)$
θ_1	0	-	-	-
θ_2	0	-	-	-
θ_3	1	b_1 or b_2	1.65	1.66

Table 2: Optimal Allocation as a Function of the Agent's Type

Type-1 agents are assigned randomly to the productive projects, θ_1 and θ_2, while the high-status type-2 agents are always assigned to the low quality projects, θ_3. On the productive projects the type-1 agents are induced to work hard, b_3, by a contract that gives them 0 consumption if output is low and 0.32 units of consumption if output is high. Type-2 agents work either b_1 or b_2, since both are just as unproductive and gives the same disutility. Consistent with their high status, they receive large consumption transfers. Also, because they is no incentive problem on their effort, they are virtually fully insured over outputs.[7] Type-2 agent's are truly the idle rich.

Randomizing over θ_i, $i = 1, 2$, in a type-1 agent's project assignment mitigates incentive constraints. Half of the time a type-1 agent is assigned to a θ_1 project and half of the time he is assigned to a θ_2 project. Since he does not know his assigned project type, he forms a posterior probability based on his information set. His knowledge at that point is based on his type and his recommended effort. He infers from the equilibrium distribution that since he is a type-1 agent and was recommended effort b_3 there is a 50% chance he has been assigned to a θ_1 project and a 50% chance he has been assigned to a θ_2 project. He knows for sure that he is not working a θ_3 project because only type-2 agents are assigned to θ_3

[7] The two consumption levels differ only because of the grid-based numerical methods we used to solve the example.

projects. Now consider the choice facing this agent after being recommended effort b_3. If he takes either effort b_1 or b_2 he gets less disutility but there is a $\frac{1}{2}(.99)+\frac{1}{2}(.70) = 0.845$ chance of producing the low output, as is evident from Table 1. The dependence of consumption on output is sufficient to preclude this strategy.

The 0.845 chance of the low output is directly due to the scrambling of project assignments. Consider the case where the designer tells a type-1 agent the state of his project. If he is assigned to a θ_1 project and is recommended effort b_3 then an incentive compatible contract must stop him from taking effort b_1 or b_2. The very unproductive effort, b_1, is easy to prevent, but the other effort, b_2, is much harder to detect since it produces the low output 70% rather than 99% of the time. Compare the lower number with the 0.845 chance of the low output if the agent deviates under the scrambling regime. The incentives necessary now to make b_3 incentive compatible are costlier than when there is scrambling. This result can be confirmed if the above program is solved when there are no θ_2 projects, 80% of the projects are type θ_1, and the rest are type θ_3. Now, implementing the b_3 effort is so costly that the program gives up on implementing it. Instead, effort b_2 is taken on all type θ_1 projects and output declines.

Finally, switching is also beneficial because it allows for the standard better match between agents and projects. Without switching some type-2 agents would work high quality projects, while some type-1 agents would work low quality projects. Such an assignment is clearly not optimal because given the distribution of Pareto weights it is desirable for type-1 agents to work hard. Switching ensures that workers who work hard are assigned to projects where their hard work is productive and it ensures that idle workers are assigned to projects where hard work is not productive.

5 The Role of Efforts

In the previous sections, the quality of a project was determined by an initial random shock. With communication that random shock could be elicited at no cost by the designer. An agent's role in the first stage of a production process was simply to gather information. There are many situations, however, where the quality of a project would be determined by the efforts taken by an agent. In this section, we study this question by replacing the interim shock θ in the first stage with an initial effort level a. This effort level is taken by the agent initially assigned to a project and is private information to him. As with the interim shock, an agent assigned to a *new* project in the second stage does not observe the effort a taken on it by the initial agent, and must take the recommended second stage effort b, regardless of effort a. We focus our analysis on complementarity in production between the two efforts.

To keep this problem tractable, we restrict our analysis to symmetric equilibria in which there are no heterogeneous types, all the agents are treated identically ex ante, and all face the same effort profile over the two stages. Output on a particular project is a function solely of efforts a and b taken on that project and a project-specific random shock. These latter shocks across projects are uncorrelated. Specifically, if agents are *not* switched, the production function on an agent's project is written simply as $p(q|a, b)$, where p denotes the probability of output q given efforts a and b in the two stages. If agents are switched, then it is necessary to keep track of output on both projects to which an agent was assigned. The production function from his perspective is $p(q_1|a, b^*)p(q_2|a^*, b)$, where a^* and b^* are the first and second stage efforts recommended to others, either after the agent leaves or before he arrives, respectively. We adopt the convention that from the agent's perspective, q_1 refers to his initial project whether or not he is switched and q_2 refers to his second-stage project if he is switched from his original project. Similarly, the consumption of an agent who is not switched is $c(q_1)$, while the consumption of a switched agent is $c\,(q_1, q_2)$.

Agents receive utility from consumption and disutility from efforts. Utility is separable and written $U(c) - V(a, b)$, where U is strictly concave and V is strictly convex.

For analytical reasons we make several simplifications. First, we model switching as a discrete decision made by the designer at the time of contracting. Thus, all participants in the economy know beforehand whether or not they will be switched. Our second simplification is to allow only the designer to send messages (recommending efforts) immediately after contracting and not at an interim stage. This assumption precludes the designer from recommending an interim-stage effort a and then, after the interim stage effort is taken, sending a random message which recommends a final-stage effort b.[8] Finally, as noted, we

[8]Examples can be generated where such a strategy is beneficial. Unfortunately, it greatly complicates the analysis of the switching problem.

also restrict our focus to symmetric contracts. Symmetry here means that agents face the same contract *and* are recommended the same sequence of efforts. Essentially, our framework reduces the incentive constraints to a symmetric, pure strategy, Nash equilibrium in the game played between the two agents. However, realizations of output may still differ across the two projects.

We start by considering the no-switching contract. The optimal no-switching contract solves

Program 5:

$$\max_{c(q_1),a,b} \sum_{q_1} p(q_1|a,b)U(c(q_1)) - V(a,b)$$

$$\text{s.t.} \sum_{q_1} p(q_1|a,b)(c(q_1)-q_1) \leq 0, \tag{17}$$

$$\sum_{q_1} p(q_1|a,b)U(c(q_1)) - V(a,b) \geq \sum_{q_1} p(q_1|\widehat{a},\widehat{b})U(c(q_1)) - V(\widehat{a},\widehat{b}), \forall \widehat{a},\widehat{b}. \tag{18}$$

Equation (17) is the resource constraint and equations (18) are the incentive constraints.

If agents are switched the problem changes. Now consumption can depend on output of the two projects where an agent has worked. Since the treatment of the agents is symmetric, we can keep the problem relatively simple.

Program 6:

$$\max_{c(q_1,q_2),a,b} \sum_{q_1,q_2} p(q_1|a,b)p(q_2|a,b)U(c(q_1,q_2)) - V(a,b)$$

$$\text{s.t.} \sum_{q_1,q_2} p(q_1|a,b)p(q_2|a,b)(c(q_1,q_2)-q_1) \leq 0, \tag{19}$$

$$\begin{aligned} & \sum_{q_1,q_2} p(q_1|a,b)p(q_2|a,b)U(c(q_1,q_2)) - V(a,b) \\ \geq \quad & \sum_{q_1,q_2} p(q_1|\hat{a},b)p(q_2|a,\hat{b})U(c_1(q_1,q_2)) - V(\hat{a},\hat{b}),\ \forall \hat{a},\hat{b}. \end{aligned} \tag{20}$$

Equation (19) is the resource constraint. Notice that the only output entering it is q_1. Some kind of formulation like this is needed because under switching, two different agents work on a given project but there is really only one project per agent and we need to avoid double counting. Put differently, since the projects are identical and all agents are assigned the same effort levels we could have just as easily replaced q_1 with q_2.

The incentive constraints (20) reflect the ability of the agent to affect output on his initial project through effort a and output on his second project through effort b. The other agent's efforts on both of these projects are taken as given, that is, b when the agent in question contemplates a or $\widehat{a}$, and a when the agent in question contemplates b or $\widehat{b}$.

5.1 Substitutes

We now consider an environment in which the initial and interim efforts a and b are perfect substitutes in the production function, that is, on each project the probability distribution of output is described by $p(q|a+b)$. It is sometimes useful to write total effort as $e \equiv a+b$. A function satisfies the *monotone likelihood ratio condition* (MLRC) if $p'(q|e)/p(q|e)$ is nondecreasing in q. Let $P(q|e)$ be the cumulative distribution function. Then, a function satisfies the *convexity of the distribution function condition* (CDFC) if $P''(q|e) \geq 0$ for all q and e. Finally we assume that $V(a,b) = V(a) + V(b)$. These assumptions are sufficient for the use of the first-order approach in single agent problems, which will be used in the proof.

Proposition 6 *If initial and interim stage efforts are perfect substitutes in production, the production function satisfies MLRC and CDFC, and $V(a,b) = V(a)+V(b)$, then the optimal no-switching symmetric equilibria strictly dominates all switching symmetric equilibria.*

Proof: If an agent is switched, then he faces the option of deviating on project one, project two, or both. The first-order conditions to the agent's problem are:

$$\sum_{q_1,q_2} p'(q_1|a+b)p(q_2|a+b)U(c(q_1,q_2)) - V'(a) = 0, \tag{21}$$

$$\sum_{q_1,q_2} p(q_1|a+b)p'(q_2|a+b)U(c(q_1,q_2)) - V'(b) = 0. \tag{22}$$

The first-order approach to incentive problems is not necessarily sufficient in the switching case. Nevertheless, these conditions are still necessary for a solution and that is all we need for our proof. Our strategy is to show that given a contract satisfying (21) and (22), a better, incentive compatible, no-switching contract can be designed. To do this, take any contract satisfying (21) and (22). Construct a no-switching contract $\widetilde{c}(q_1)$ by

$$U(\widetilde{c}(q_1)) = \sum_{q_2} p(q_2|a+b)U(c(q_1,q_2)),\ \forall q_1, \tag{23}$$

or, equivalently,

$$\widetilde{c}(q_1) = U^{-1}(\sum_{q_2} p(q_2|a+b)U(c(q_1,q_2))),\ \forall q_1, \tag{24}$$

where U^{-1} is the inverse of the utility function U. Substitution of (23) into (21) delivers

$$\sum_{q_1} p'(q_1|a+b)U(\widetilde{c}(q_1)) - V'(a) = 0. \tag{25}$$

Let $\widetilde{V}(e) = \min_{a,b} V(a) + V(b)$ subject to $a+b=e$. This object is the indirect disutility of effort received by the agent. By the Theorem of Maximum (actually minimum here) $\widetilde{V}$ is a convex function like V. Because of the symmetry, any solution to the agent's problem will be characterized by $a = b = e/2$, which implies that $\widetilde{V}'(e) = V'(e/2) = V'(a) = V'(b)$. Substituting into (25) delivers

$$\sum_{q_1} p'(q_1|e)U(\widetilde{c}(q_1)) - \widetilde{V}'(e) = 0. \tag{26}$$

This is the first-order condition to the agent's problem expressed in terms of total effort e. Furthermore, the first-order approach is sufficient for the no-switching problem because of the assumptions of MLRC and CDFC (see Rogerson (1985) or Hart and Holmstrom (1987)). Therefore, the contract $(\widetilde{c}(q_1), a, b)$ is incentive compatible in the no-switching regime. Furthermore, by concavity of U, $\widetilde{c}(q_1) < \sum_{q_2} p(q_2|a+b)c(q_1,q_2))$ which means that there are more resources available in the economy. By a continuity argument, there is a feasible no-switching contract that improves upon $(\widetilde{c}(q_1), a, b)$. Therefore, no-switching strictly dominates switching. **Q.E.D.**

No-switching contracts are powerful in this model because they allow incentives to be focused on one project rather than two and thereby reduce consumption variation.

5.2 Complements

In contrast, switching can be valuable for production functions with complementarity in the two inputs. For this result, we consider the production function $p(q|f(a,b))$, where complementarities are expressed through the functional form of f. A probability function p_1 exhibits *first-order stochastic dominance* over another probability function p_2 if $\sum_x p_1(x)g(x) \geq \sum_x p_2(x)g(x)$ for all weakly increasing functions g. We assume that p exhibits first-order stochastic dominance in the value of f, that is, for $f(a,b) > f(\widetilde{a},\widetilde{b})$, $p(q|f(a,b))$ first-order stochastically dominates $p(q|f(\widetilde{a},\widetilde{b}))$. The function f is symmetric in the two inputs. More importantly, it is characterized by expected output being highest along the diagonal of (a,b) space, increasing as both inputs are increased, but dropping rapidly as one moves off the diagonal. Specifically,

Assumption 1 *The function $f(a,b)$ is symmetric. Higher efforts a and b with $a = b$, that is, upward movement along the diagonal, increase $f(a,b)$. However, increasing one effort while maintaining the other - movement off the diagonal - reduces $f(a,b)$ the larger the difference between a and b. Formally, the value of f decreases as $|a-b|$ increases.*

One implication of Assumption 1 is that for $a \neq b$, $f(a,b) < f(a,a)$ and $f(a,b) < f(b,b)$. An example of a production function that satisfies Assumption 1 is $f(a,b) = \min(a,b) - |a-b|$.

Proposition 7 *If $p(q|f(a,b))$ satisfies first-order stochastic dominance, f satisfies Assumption 1, $V(a,b) = V(a) + V(b)$, and the consumption sharing rule in the solution to the no-switching program, $c(q_1)$, is weakly monotonically increasing then the best switching contract strictly dominates the best no-switching contract.*

Proof: See Appendix.

When an agent is switched, he plays a Nash-like game in efforts with the other agents assigned to his projects. As a consequence, all of his feasible deviations push him to off-diagonal effort pairs, which are extremely unproductive and relatively inexpensive to prevent. In contrast, an agent who is not switched can deviate in both stages so as to take on-diagonal effort pairs. These are more productive and thus more expensive to prevent.

6 Discussion

The previous sections isolated forces for reassignment in the assignment model. In many applications several of these forces, even opposing ones, may operate simultaneously. One interesting class of models builds upon the previous sections by having the initial effort a determine the interim state θ stochastically by the production function $h(\theta|a)$. As in Sections 2 and 3, θ is private information. Next, like the earlier models, the agents report on their shock, reassignments are made, and then the agents take their private second-stage efforts b that determine the publicly observed outputs q according to the production function $p(q|b,\theta)$; the shock θ is the interim state of the project an agent is assigned to in the second stage.

Analyzing this model is difficult. Even writing down the switching version of the model is not straightforward. One version that can be analyzed, however, is the case where $q = b\theta$. If agents are not switched, there are lots of incentive constraints. Working backwards there are no incentive constraints on the second-stage effort if there is a truthful report and the initial action was taken. The deterministic production function allows the designer to infer effort b from the output and θ. However, there are still a myriad of incentive constraints on the report on θ as well as on the initial action.

The following switching contract eliminates the truth-telling constraints on θ: Upon receipt of the agents' reports, the designer randomly assigns agents to projects of quality θ by the distribution $h(\theta|a)$ and then recommends each agent an effort b that may depend on the quality of their new project. In equilibrium the designer knows the state of each project so – just like in the the no-switching contract considered above – deviations in b can be detected and thus prevented. But now the same process can also be used to eliminate the truth-telling constraints. Unlike in the no-switching contract, if an agent lies about the state of his original project he cannot continue the cover-up by adjusting his effort from the recommended level. Instead, some other agent will work the recommended effort and then produce an output that reveals the initial deception. Consequently, θ is public information for all intents and purposes and the problem collapses to an almost standard moral hazard problem where the only incentive problem is on the initial effort a.

In this example, perfect inference is only possible because of the deterministic production function. For more general production functions $p(q|b, \theta)$, where inference is less than perfect, the analysis is not so clear. Forces like those studied in Section 5.1 would push towards no-switching assignments. Limits on communication could work in a similar direction. Still, which force would dominate seems hard to ascertain at this point.

A Proofs

Proposition 2 *If preferences are Leontief and if $\theta_j > 0$ for all j, then switching dominates.*

Proof: Leontief preferences forces solutions to the no-switching model to satisfy $c(\theta) = c$. Because consumption is not state-contingent, a solution to the no-switching problem is characterized by $b(\theta)$ satisfying $\theta b(\theta) = \theta' b(\theta')$, for all θ, θ', that is, all agents must produce the same output regardless of their shock θ. The less productive a worker is the harder he works. Formally, $b(\widetilde{\theta}) > b(\theta)$ for $\widetilde{\theta} < \theta$ and total output for the no-switching contract, q_{ns}, is

$$q_{ns} \equiv \sum_i h(\theta_i)\theta_i b(\theta_i).$$

Now consider a switching contract with $b = \sum_i h(\theta_i)b(\theta_i)$, the previous average effort of the no-switching contract. Then total output for switching, q_s is

$$
\begin{aligned}
q_s &= \sum_i h(\theta_i)\theta_i(\sum_{\text{all } j} h(\theta_j)b(\theta_j)) \\
&= \sum_i h(\theta_i)\theta_i h(\theta_i)b(\theta_i) + \sum_i h(\theta_i)\theta_i(\sum_{j\neq i} h(\theta_j)b(\theta_j)) \\
&\quad \text{(adding and subtracting a common term)} \\
&= \sum_i [h(\theta_i)\theta_i h(\theta_i)b(\theta_i) + \sum_{j\neq i} h(\theta_j)\theta_i h(\theta_i)b(\theta_i) - \sum_{j\neq i} h(\theta_j)\theta_i h(\theta_i)b(\theta_i)] \\
&\quad + \sum_i h(\theta_i)\theta_i(\sum_{j\neq i} h(\theta_j)b(\theta_j)) \\
&= \sum_i \theta_i h(\theta_i)b(\theta_i) - \sum_i\sum_{j\neq i} h(\theta_j)\theta_i h(\theta_i)b(\theta_i) + \sum_i h(\theta_i)\theta_i(\sum_{j\neq i} h(\theta_j)b(\theta_j)) \\
&= q_{ns} - \sum_i\sum_{j\neq i} h(\theta_j)\theta_i h(\theta_i)b(\theta_i) + \sum_i h(\theta_i)\theta_i(\sum_{j\neq i} h(\theta_j)b(\theta_j)) \\
&= q_{ns} - \sum_{i=2}^{n}\sum_{j=1}^{i} h(\theta_i)h(\theta_j)(\theta_i b(\theta_i) + \theta_j b(\theta_j)) + \sum_{i=2}^{n}\sum_{j=1}^{i} h(\theta_i)h(\theta_j)(\theta_i b(\theta_j) + \theta_j b(\theta_i)) \\
&= q_{ns} + \sum_{i=2}^{n}\sum_{j=1}^{i} h(\theta_i)h(\theta_j)(\theta_i - \theta_j)(b(\theta_j) - b(\theta_i)) \\
&> q_{ns}.
\end{aligned}
$$

The last inequality holds because, again, $\theta_i > \theta_j$ and $b(\theta_j) > b(\theta_i)$. As a consequence, the switching contract produces more output, which means (common) consumption c is higher than under the no-switching contract. Furthermore, since effort b is a convex combination of the no-switching effort levels and since V is convex, disutility from effort is less with switching. Therefore, the best switching contract is strictly better than the best no-switching contract. **Q.E.D.**

Proposition 7 *If $p(q|f(a,b))$ satisfies first-order stochastic dominance, f satisfies Assumption 1, $V(a,b) = V(a) + V(b)$, and the consumption sharing rule in the solution to the no-switching program, $c(q_1)$, is weakly monotonically increasing then the best switching contract strictly dominates the best no-switching contract.*

Proof: We start with an optimal solution to the no-switching problem $(a, b, c(q_1))$. It is characterized by $a = b$. Now consider the following switching contract. The agent is switched with probability one, effort on project one is a, effort on project two is b, and consumption is

$$c(q_1, q_2) = U^{-1}(0.5U(c(q_1)) + 0.5U(c(q_2))), \tag{27}$$

where $c(q_1)$ and $c(q_2)$ are the terms of the no-switch contract applied to both of the projects the agent works. When the two outputs differ, consumption is set to a level that gives the same utility as if the contract randomized over the two outputs in the no-switching contract. If $q_1 = q_2$, then $c(q_1, q_2) = c(q_1)$, that is, consumption is unchanged. This contract gives the same utility as the no-switching contract because

$$\begin{aligned}
& \sum_{q_1} p(q_1|f(a,b)) \sum_{q_2} p(q_2|f(a,b)) U(c(q_1,q_2)) \qquad (28) \\
= & \sum_{q_1} p(q_1|f(a,b)) \sum_{q_2} p(q_2|f(a,b)) \left(0.5U(c(q_1)) + 0.5U(c(q_2))\right) \\
= & \; 0.5 \sum_{q_1} p(q_1|f(a,b)) U(c(q_1)) + 0.5 \sum_{q_1} p(q_2|f(a,b)) U(c(q_2)) \\
= & \sum_{q_1} p(q_1|f(a,b)) U(c(q_1)).
\end{aligned}$$

The last line holds because of the symmetry in the problem. Furthermore, this contract also uses less resources than the no-switching contract. Our strategy is to show that it is incentive compatible under the switching regime. A continuity argument will then show that better switching contracts exist.

Consider the deviating strategy $\widehat{a}, \widehat{b}$. The consumption part of utility is

$$\begin{aligned}
& \sum_{q_1} p(q_1|f(\widehat{a},b)) \sum_{q_2} p(q_2|f(a,\widehat{b})) U(c(q_1,q_2)) \qquad (29) \\
\leq & \sum_{q_1} p(q_1|f(\widehat{a},\widehat{a})) \left(\sum_{q_2} p(q_2|f(a,\widehat{b})) U(c(q_1,q_2)) \right) \\
\leq & \sum_{q_2} p(q_1|f(\widehat{a},\widehat{a})) \sum_{q_2} p(q_2|f(\widehat{b},\widehat{b})) U(c(q_1,q_2)).
\end{aligned}$$

The first inequality holds because $f(\widehat{a},\widehat{a}) \geq f(\widehat{a},b)$, p satisfies first-order stochastic dominance, and the term $\sum_{q_2} p(q_2|f(a,\widehat{b}))U(c(q_1,q_2))$ is an increasing function in q_1. (To see this, substitute (27) for $c(q_1,q_2)$ and recall that $c(q_1)$ is weakly increasing by assumption.) The second inequality holds based on a similar argument with respect to q_2.

Now, consider the entire utility from taking deviating strategy $\widehat{a}$, $\widehat{b}$. Utility from this strategy is

$$
\begin{aligned}
& \sum_{q_1} p(q_1|f(\widehat{a},b)) \sum_{q_2} p(q_2|f(a,\widehat{b}))U(c(q_1,q_2)) - V(\widehat{a}) - V(\widehat{b}) \\
\leq \; & \sum_{q_1} p(q_1|f(\widehat{a},\widehat{a})) \sum_{q_2} p(q_2|f(\widehat{b},\widehat{b}))U(c(q_1,q_2)) - V(\widehat{a}) - V(\widehat{b}) && (30) \\
= \; & 1/2 \left(\sum_{q_1} p(q_1|f(\widehat{a},\widehat{a}))U(c(q_1)) - V(\widehat{a}) - V(\widehat{a}) \right) \\
& +1/2 \left(\sum_{q_2} p(q_2|f(\widehat{b},\widehat{b}))U(c(q_2)) - V(\widehat{b}) - V(\widehat{b}) \right) && (31) \\
\leq \; & 1/2 \left(\sum_{q_1} p(q_1|f(a,b))U(c(q_1)) - V(a) - V(b) \right) \\
& +1/2 \left(\sum_{q_2} p(q_2|f(a,b))U(c(q_2)) - V(a) - V(b) \right) && (32) \\
= \; & \sum_{q_1} \sum_{q_2} p(q_1|f(a,b))p(q_2|f(a,b))U(c(q_1,q_2)) - V(a) - V(b), && (33)
\end{aligned}
$$

which is the incentive constraint. Inequality (30) follows from (29). Equality (31) follows from substituting (27) for $c(q_1, q_2)$ and using (28). Inequality (32) follows from the no-switching incentive constraints. Finally, (33) is obtained by using (28).

The constructed switching contract is feasible, gives the agent the same utility, and uses less resources. By continuity, a feasible switching contract exists that increases the agent's utility. **Q.E.D.**

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