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The Value of Information and Heterogeneous Agents and Partially Revealing Prices

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Abstract

This paper studies how the arrival of information affects welfare in a general equilibrium exchange economy with incomplete and differential information. It considers a setup in which agents differ in their attitudes toward risk. This introduces gains from trade. In equilibrium, the information sets differ across agents, i.e., they hold heterogeneous beliefs. For certain structures of primitives, the latter introduces an adverse effect on welfare. In this case, the arrival of information has opposite effects: on the one hand it weakens the adverse effect on trade, and on the other hand it strengthens the Hirshleifer effect. The first effect fosters and the second one discourages risk-sharing trades. When the first effect dominates, welfare increases upon the arrival of more precise information.

Keywords: *Information and Welfare, Heterogeneous Agents, Partially Revealing Prices*

JEL Classification: *D8*

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1 Introduction

Thanks to the explosive development of the Internet and the World Wide Web, the last decade has witnessed a dramatic change in the technology used to access information. At the same time, advances in computer technology have made it possible to process significantly larger amounts of information than was possible a few years ago. The changes are particularly shocking when we look at financial markets. Nowadays, any individual can gain access to specialized reports, data series, or the latest news before trading. Thus, no one would question that market participants have become much more sophisticated relative to what they were twenty years ago. The question remains as to whether these changes have been accompanied by an increase in welfare. Intuition suggests that the arrival of more precise information should lead to better decisions and, hence, should improve welfare. The examples below, however, challenge this common view and illustrate cases where the receipt of more information may be harmful.

Lerman et al. [26] describe the results of a survey conducted among the family members of individuals with BRCA1-linked hereditary breast-ovarian cancer.[1] They find that 57 percent of the individuals interviewed declined to take a free BRCA1 test. One of the reasons listed against being tested was the possibility of losing health insurance. Quaid and Morris [31] report similar behavior in a sample of individuals who were offered a free test for Huntington's disease.

Since 1994 the Federal Reserve has routinely announced interest rate changes at pre-scheduled and publicly available dates. The decisions are made public after the meetings of the Federal Open Market Committee. The Fed deviated from this scheme in January and April of 2001, when it decided to cut interest rates well before the next scheduled meeting. Banerjee and Seccia [4] find evidence of abnormal volumes of trade in interest rate futures the day before scheduled meetings, but they find no evidence of excess trade before the two unscheduled announcements. They conclude that the two unexpected interest rate cuts might have had a negative impact on welfare if a fraction of the abnormal trade is motivated by hedging purposes. In those two opportunities, agents did not have the chance to insure against interest rate changes. This example illustrates a theoretical result present in Sulganik and Zilcha [35]. They argue that when futures markets are available, where agents can share risk, the value

[1] The isolation of the BRCA1 gene allows one to learn if the individual carries a cancer-predisposing mutation that increases the probability of developing breast or ovarian cancer.

of information may not always be positive. A similar conclusion is obtained by Drees and Eckwert [10]. They argue that transparency in the foreign exchange market can reduce welfare when agents can hedge against currency risk.

The previous papers illustrate a general principle: the arrival of more accurate information may be harmful if it precludes risk sharing trades. Agents cannot insure against events that are no longer uncertain. Drèze [11] was the first to identify the possibility that information may have a negative value, but the result is commonly known in the literature as the "Hirshleifer effect." Hirshleifer [21] formalized Drèze's argument using a general equilibrium framework. Consider an Edgeworth box economy with one good, two agents, and two states of the world. The state probabilities are common knowledge. In such a framework, if markets open before the state realization is known, agents will trade to a point on the contract curve. If markets open after the state of the world has been publicly disclosed, no trade takes place. The example shows how better information may lead to worse consumption allocations.

The present paper analyzes how the receipt of more precise information affects welfare in a pure exchange economy with incomplete information and one round of trading. The model has two main features: there is uncertainty about the state that will be realized in the future, and agents are affected asymmetrically in different states. The latter allows them to share risk and partially insure against the states in which they are unlucky. The structure of the model captures an important feature of financial markets: agents trade not only to share risk, but also because they have different expectations about the assets being traded.

We consider a pure exchange economy where agents can invest in two assets: a risk-free bond and a risky asset, namely a tree. The tree pays either high or low dividends. The ex-ante probabilities of these events are not known, but each agent receives private signals about the tree. Each signal can be either good or bad. The probability of receiving a good signal depends on the probability that the tree pays high dividends. The last feature is what makes the signals informative. Agents also receive a riskless endowment that may take either a high or low value. The fraction of agents receiving a high riskless endowment –hereafter referred to as the rich ones– is also unknown. We assume agents share the same preferences, which are represented by a concave utility function with a decreasing coefficient of absolute risk aversion. Thus, richer individuals are "de-facto" less risk-averse than poor individuals, so the former tend to insure the latter. Markets open before the tree pays off. The equilibrium price of the risky asset depends on two variables: the probability of high dividends (which determines the distribution of signals across agents), and the fraction of rich agents. Agents face a nontrivial signal extraction problem: they see a price but cannot infer its two underlying determinants. Their posterior beliefs are based on three pieces of information: the price, the private signals about the tree, and the individual realization of the riskless endowment. If agents could observe the fraction of rich agents before they trade, they could use the equilibrium price to infer the actual probability with which the tree pays high dividends. In our framework however, agents are not able to infer that probability because the distribution of endowments is also unknown.

In the extreme case where everyone is fully informed, the model predicts that rich agents insure poor agents by purchasing the risky assets and selling risk-free bonds. This prediction may not hold in the partial information economy considered in the present paper. The reason is the following: If an agent receives a low endowment, he infers that the fraction of poor agents in the economy is relatively high. In other words, agents who are hit with shocks that induce them to sell stocks (i.e., a low endowment), believe that there are many others in the same situation. So a low endowment serves as a signal that the excess demand of stocks is low. Conversely, rich agents believe that the excess demand of stocks is high. This leads to different interpretations of the market price. For a given price, poor (rich) agents perceive that there are more (less) agents who dislike risk, so they infer that the reason why agents demand stocks is because they offer a "high" (low) expected return. In summary, agents who are hit with a low endowment shock tend to be more "optimistic" than agents with high endowments. This effect dampens the incentives to share risk. As agents become more sophisticated and acquire more precise information, i.e., more signals, the dispersion of beliefs shrinks, which tends to offset the previously described negative effect.

We find that more precise information about the underlying source of risk can enhance or reduce the possibilities to share risk among agents, leading to an increase or decrease in welfare, respectively. The Hirshleifer effect is still present in our model, but there is another channel through which information affects the equilibrium allocation: the dispersion of beliefs. This paper describes a case where the heterogeneity of beliefs discourages trading. In this case, better information reduces the dispersion of beliefs and, hence, has a positive welfare effect.

If the previous information structure was abandoned and agents received common signals instead of private signals, the model would belong to the set of economies studied by Schlee [34]. In this case, we would observe a decrease in individual welfare after the arrival of better information. The present paper, however, offers a different conclusion.

1.1 Related Literature

Several authors have analyzed the robustness of Hirshleifer's result. Marshall [28] analyzes the value of public information when individuals can trade before and after the arrival of information:

> If the impact of information is insured before its arrival, that insurance precludes further trade based on the news. ... In all, the information has no impact on distribution, no impact on satisfaction, and hence, no value.
>
> In the contrary case when the news must arrive before its impact is insured in a preliminary market, the information is harmful. People can afford to pay something to suppress the news or to delay its arrival. ... Public information in this situation is harmful; at best its impact can be counteracted by prior insurance.[2]

A similar conclusion is obtained in Ng [30] and Green [15]. Hakansson et al. [18] provide sufficient and necessary conditions for public information to have positive social value.[3] Their results, though, rely on the fact that agents can trade prior to the arrival of information. If that market is missing, they cannot rule out the possibility that better information may decrease social welfare. Schlee [34] focuses on the last case, where individuals trade after the receipt of information. He concludes that better information typically reduces everyone's welfare in the following situations: there is no aggregate risk, some agents are risk-neutral, or the economy behaves as if there were a representative agent.

It must be noted that with the exception of Green [15], the papers described above assume a common signal structure. Agents update their beliefs after the signal is announced and then they trade. The common signal assumption simplifies the analysis considerably, as agents do not need to learn from prices. In effect, prices do not convey any information that is not already contained in the signal

[2] Marshall [28], p 380.

[3] They show that if the initial endowments are an equilibrium allocation without information, then the arrival of public information leads to a Pareto-superior consumption allocation in either of the following cases: the marginal rates of substitution are not equalized across agents, or there is sufficient asymmetry in the posterior beliefs.

itself. This makes the problem tractable, but as the present paper shows, it may bias the conclusions. Moreover, the assumption that individuals hold homogeneous beliefs lacks empirical support.

The assumption that prices reveal valuable information has been extensively studied in the literature. The first generation of models developed by Grossman [16], Grossman and Stiglitz [17], Hellwig [20], Diamond and Verrecchia [9], Verrecchia [36], and Admati [1] prevents prices from fully revealing the fundamentals of the economy by assuming the existence of noise traders. The latter display a random behavior and distort the information conveyed by the price. Even though these models have given us valuable insight into many areas, the assumption of noise traders makes them unsuitable for welfare analysis. Among the papers that have been able to obtain partially revealing prices without resorting to noise traders, we should mention Ausubel [2]. Since the structure of our model shares some similarities with his work, we defer a more detailed discussion until the end of Section 2.

There is another family of models, considered in Bhattacharya and Matthew [6], Rahi [33], and Marín and Rahi [27], that also allows for the existence of partially revealing equilibria without assuming the presence of noise traders. These models introduce information asymmetries across agents, i.e., there is a group of agents that is more informed than the rest. This generates an adverse selection effect, since uninformed agents reduce their participation in the market because of their informational disadvantage. In that framework, the receipt of more precise information not only strengthens the Hirshleifer effect, but also dampens the adverse selection effect. Thus, more information does not necessarily lead to a decrease in welfare. Even though the conclusion is similar to the one described in the present paper, the mechanisms explaining the result are different. The disparities in the level of information appear to be a sensible assumption when we consider trades on assets issued by private companies, in which case there may be leakages of privileged information. However, it is a harder assumption to justify when we look at trades on assets linked to aggregate variables like interest rate futures or exchange rate futures. Besides, the modelling strategy followed in the present paper allows for a straightforward comparison with the existing literature. In fact, our model could be mapped into the set of economies considered by Schlee [34], in the case where agents receive public signals instead of private signals.

Berk [5] also studies the relationship between welfare and information in economies with partially revealing equilibria. He analyzes a simple dynamic game and concludes that it is possible for an equilibrium to exist where agents choose to purchase information, even if all agents, including those who purchased the information, are made strictly worse off from an ex ante perspective.

Citanna and Villanacci [8] study a class of models with partially revealing prices, multiple goods, asymmetric information, and heterogeneous wealth. They find that welfare may increase after the arrival of more precise information. The reason is that the wealth effects due to price changes outweigh the Hirshleifer effect. Gottardi and Rahi [14] also analyze a model with asymmetric information and reach a similar conclusion. In addition, they are able to disentangle the different channels through which the arrival of better information affects welfare.[4] Morris and Shin [29] study the value of public information in a game-theoretic context with private information. In their setup, the receipt of a more precise public signal plays an additional role: it helps to coordinate individual beliefs.

The present paper also utilizes a model with partially revealing prices and asymmetric information to assess the value of public information. Unlike Citanna and Villanacci [8], the beliefs' updating scheme is endogenous and is interlinked across agents. Also, their results cannot be extended to the present paper since they rely on the assumption of a countable number of states and more than two goods. Unlike Morris and Shin [29], we focus on a competitive environment without strategic behavior.

Krebs [24] studies an economy with incomplete markets and private signals. He finds that it is possible to find cases where information has a positive effect on welfare. This is regardless of whether equilibrium prices fully reveal the signals received by other agents, or whether prices are non-revealing and therefore, agents hold heterogeneous beliefs. The reason why information can have a positive effect on welfare is because it may help to correct for the inefficiencies due to the lack of complete markets.

Finally, we should mention that a positive relationship between the precision of public information and welfare can also be observed once the endowment economy framework is abandoned. In a model with production, the early arrival of public information has an additional effect –it may allow for better investment decisions. In order to gauge the negative impact of information, Hirshleifer [21] provides an example where this effect dominates and better information enhances welfare. In a recent paper, Eckwert and Zilcha [13] study more formally the value of information in an economy with production. They assume that the production function is subject to a productivity shock that consists of the sum of two random variables. Agents have partial information about the realization of one of these variables

[4] The structure of the model considered in their paper allows only for a fully revealing equilibrium. Thus, in order for the asymmetric information assumption to play any role, they have to use a definition of equilibrium that is less restrictive than the standard rational expectations equilibrium concept used in the literature.

and trade on that information. The authors show that information about non-tradable risks always has a positive value. Krebs [25] studies a two-country framework with a linear production technology and concludes that there are cases where at least one country benefits from the arrival of more precise information.

The paper is organized as follows. Section 2 introduces a simple model that enables us to provide an analytical characterization of the main result in the paper. In Section 3 we set out a more general model and define the equilibrium concept used in the rest of the paper. Section 4 describes the baseline parameterization. Section 5 presents the results, and Section 6 concludes.

2 A model with heterogeneous priors

This section introduces a simple model that enables us to provide an analytical characterization of the result described in the paper. The simplicity derives from ad hoc assumptions that are abandoned in later sections.

We consider a pure exchange economy with heterogeneous agents. There is a single risky asset in the economy: a tree. The tree pays high dividends with probability ν and low dividends with probability $1-\nu$. The tree pays off once and then dies. There is a measure 1 of agents in the economy, and everybody is initially entitled to a share of the tree. Agents also receive a riskless endowment. Half of the population receives a high riskless endowment, while the other half receives a low riskless endowment.

The value of ν is drawn from a discrete distribution with support $\{\nu_l, \nu_h\}$, where

$$\nu_h = 0.5 + \Delta,$$

$$\nu_l = 0.5 - \Delta.$$

Agents do not "observe" the actual realization of ν. Instead, they learn from a public signal. The signal (denoted by s) has the following probability distribution:

$$Pr\left(s=1\right) = \begin{cases} \frac{1+\rho}{2} & if \quad \nu = \nu_h \\ \frac{1-\rho}{2} & if \quad \nu = \nu_l \end{cases} .$$

The previous formulations says that it is more likely that agents observe a good signal (s =1) when ν takes a high value, while it is more likely that they observe a bad signal (s=0) when ν takes a low value. Thus, the parameter ρ captures how informative the signal realization is. When ρ equals zero, the signal is completely uninformative. When ρ equals one, the signal reveals the actual value of ν. It is easy to verify that the signal structure becomes more informative as ρ increases.[5]

The key assumption in this section is that agents have different priors about the probability distribution from which ν is drawn. This explains why agents hold heterogeneous beliefs in spite of the fact that they learn from a public signal. It is assumed that agents with high endowments are "born" pessimistic, i.e., they believe that ν_h is drawn with probability $0.5 - b$. On the other hand, agents with low endowments are "born"optimistic, they believe that ν_h is drawn with probability $0.5 + b$. The value of b determines how large is the bias. In the next sections we consider an environment in which agents share a common prior and receive private signals. The equilibrium price is partially revealing and the beliefs are endogenously determined. In this scenario, we show that the negative correlation between beliefs and endowments arises naturally as long as the forces that determine the desire to buy or sell stocks (in this model, the endowment realization) are not highly correlated with the dividend payoffs.

Agents use Bayes' rule to update their beliefs, i.e., the conditional expectation of ν. Let us denote by $\tilde{\nu}^{\bar{s}}$ the belief about ν of an agent with high endowment and when a public signal s is observed. Similarly, let $\tilde{\nu}^{\underline{s}}$ denote the belief about ν of an agent with low endowment and signal s. Formally,

$$\tilde{\nu}^{s} = E\left[\nu \mid s\right] = \frac{Pr\left(\nu_h\right) Pr\left(s \mid \nu_h\right) \nu_h + Pr\left(\nu_l\right) Pr\left(s \mid \nu_l\right) \nu_l}{Pr\left(\nu_h\right) Pr\left(s \mid \nu_h\right) + Pr\left(\nu_l\right) Pr\left(s \mid \nu_l\right)}.$$

Agents can transfer resources across the two states of nature that can be realized: the high and low dividends states. This means that consumers can trade in two Arrow-Debreu securities. One of them pays 1 unit of consumption goods if the high dividends state is realized. Otherwise, it pays zero. The other security only pays (1 unit) in the low dividends state. There is only one price to be determined: the relative price between these two securities.

The equilibrium price depends on the signal realization. If the signal is good, more weight is assigned to the high dividends state, i.e., agents believe that it is more likely that $\nu = \nu_h$. In this case, the relative

[5]We use Blackwell's definition to compare two information structures. A formal definition of Blackwell's criterion is provided in Appendix B.

price of the contingent claim paying in the high state is higher. On the contrary, when the signal is bad, the relative price is low.

Agents maximize their expected utility of consumption. For example, an individual with high endowment solves the following optimization problem:

$$\begin{aligned} &\underset{c_h, c_l}{Max} \left\{ \tilde{\nu}^{\bar{s}} u\left(c_h\right) + \left(1 - \tilde{\nu}^{\bar{s}}\right) u\left(c_l\right) \right\} \\ &\text{subject to} \quad (1 - p^s) c_l + p^s c_h = W = \bar{a} + (1 - p^s) d_l + p^s d_h \\ &c_l, c_h \geq 0, \end{aligned} \tag{1}$$

where: $u(\cdot)$ represents the utility function; c_j denotes planned consumption in state j; $\bar{a}$ denotes the riskless endowment; d_j denotes the dividends paid by the tree in state j; and W denotes individual wealth. The sum of the prices of the Arrow-Debreu securities is normalized to 1. The probability distribution used to compute the expected utility varies across agents since they share different priors.

In order to provide a further analytical characterization, we focus on the case in which agents display a logarithmic utility function. Equation (2) describes the equilibrium price of a contingent claim paying in the high dividends state, for a public signal realization of s.

$$p^s = \frac{\bar{a}\tilde{\nu}^{\bar{s}} + \underline{a}\tilde{\nu}^{\underline{s}} + d_l \left(\tilde{\nu}^{\bar{s}} + \tilde{\nu}^{\underline{s}}\right)}{\bar{a} + \underline{a} + 2d_h - \left(d_h - d_l\right)\left(\tilde{\nu}^{\bar{s}} + \tilde{\nu}^{\underline{s}}\right)} \tag{2}$$

In the present model there are two forces that affect the equilibrium trading volume. First, agents display different attitudes toward risk. The assumption that agents share a utility function with constant coefficient of relative risk aversion implies that individuals with high endowment are "de facto" less risk averse than individuals with low endowment. Thus, in equilibrium we would expect the former to be net buyers of the risky asset (the tree), and the latter to be net sellers of the risky asset. Second, agents disagree about the dividend probability distribution. Conditional on the wealth level, optimistic agents demand more risky assets than pessimistic agents. This section focuses on the case where both motives for trade are perfectly negatively correlated: agents who in principle are willing to buy shares of the tree are pessimistic, while the agents who in principle are willing to sell shares are optimistic.

2.1 The value of information

We are interested in analyzing how individual welfare is affected when agents receive more precise information about the aggregate state probability distribution. In a partial information setup, more information is always welfare enhancing. This is because the receipt of more precise information enables agents to make better decisions. However, it is a well known result that in a general equilibrium framework, the arrival of information may be detrimental to individual welfare. In the current setup, more information is equivalent to a more precise signal structure (higher ρ). As the signal becomes more informative, there are two effects that work in opposite directions. On the one hand, the dispersion of beliefs shrinks: the agents with high endowments become more optimistic and demand more trees, while the agents with low endowments become more pessimistic and demand less trees. This increases the equilibrium volume of trade. On the other hand, as the public signal becomes more informative, agents are able to identify better the aggregate state that is more likely to occur. This reduces the level of uncertainty in the economy, and compresses the possibilities to share risk. The latter is known as the Hirshleifer effect. From an ex ante perspective, i.e., before the signal is realized, the first effect increases welfare, and the second effect reduces it.

Schlee shows that when agents hold homogeneous beliefs, and the economy behaves as if it is inhabited by a representative agent, the Hirshleifer effect is dominant. In this case, every agent experiences a decrease in welfare as everyone is more informed. The following proposition shows that when the representative agent setup is abandoned and agents hold heterogeneous beliefs, the opposite result can be observed.

Proposition 1 *The ex ante expected utility of both types of agents increases with ρ for values of ρ sufficiently close to 1, when Δ equals 0.5 and* $b > \frac{\Delta(d_h - d_l)(\bar{a} - \underline{a})}{\underline{a}(2a+d_l+d_h)+d_h(a+d_l)+d_l(a+d_h)}$.

Proof. Let $\bar{\mathcal{U}}$ denote the ex ante expected utility of an agent with high endowment, and $\underline{\mathcal{U}}$ denote the ex ante expected utility of an agent with low endowment. Formally,

$$\bar{\mathcal{U}} = Pr^{\bar{a}}(s=1, d_h)\, u\left(c_h^{\bar{1}}\right) + Pr^{\bar{a}}(s=1, d_l)\, u\left(c_l^{\bar{1}}\right) + Pr^{\bar{a}}(s=0, d_h)\, u\left(c_h^{\bar{0}}\right) + Pr^{\bar{a}}(s=0, d_l)\, u\left(c_l^{\bar{0}}\right), \quad (3)$$

and

$$\underline{\mathcal{U}} = Pr^{\underline{a}}(s=1,d_h)\,u\left(c_h^{\underline{1}}\right) + Pr^{\underline{a}}(s=1,d_l)\,u\left(c_l^{\underline{1}}\right) + Pr^{\underline{a}}(s=0,d_h)\,u\left(c_h^{\underline{0}}\right) + Pr^{\underline{a}}(s=0,d_l)\,u\left(c_l^{\underline{0}}\right), \quad (4)$$

where $Pr^j(s,d_i)$ denotes the probability assigned by agent j to the joint event that signal s is observed and the tree pays dividends d_i. Type j is determined by the riskless endowment realization.

An increase in the precision of the signal affects welfare through various channels. From a partial equilibrium perspective, a more precise signal structure changes the probability distribution over the four possible realizations of signals and dividend payoffs. For example, a more precise signal increases the probability that a good public signal is observed before the tree pays high dividends and decreases the probability that a bad signal is observed before the tree pays high dividends. Second, more information enables agents to make better decisions and to allocate more resources in the states that are more likely to occur. Finally, in a general equilibrium framework there is a third channel: equilibrium prices are affected by the precision of the signal. The three terms in the equation below describe the different channels that affect welfare in the case of a highly endowed agent. The case of an agent with low endowment is entirely analogous.

$$\begin{aligned}\frac{\partial\bar{\mathcal{U}}}{\partial\rho} = &\left[\frac{\partial Pr^{\bar a}(s=1,d_h)}{\partial\rho}u\left(c_h^{\bar 1}\right) + \frac{\partial Pr^{\bar a}(s=1,d_l)}{\partial\rho}u\left(c_l^{\bar 1}\right) + \frac{\partial Pr^{\bar a}(s=0,d_h)}{\partial\rho}u\left(c_h^{\bar 0}\right) + \frac{\partial Pr^{\bar a}(s=0,d_l)}{\partial\rho}u\left(c_l^{\bar 0}\right)\right] + \\ &\left[\begin{array}{l}\left(Pr^{\bar a}(s=1,d_h)\,u'\left(c_h^{\bar 1}\right)\frac{\partial c_h^{\bar 1}}{\partial\tilde\nu} + Pr^{\bar a}(s=1,d_l)\,u'\left(c_l^{\bar 1}\right)\frac{\partial c_l^{\bar 1}}{\partial\tilde\nu}\right)\frac{\partial\tilde\nu^{\bar 1}}{\partial\rho} + \\ \left(Pr^{\bar a}(s=0,d_h)\,u'\left(c_h^{\bar 0}\right)\frac{\partial c_h^{\bar 0}}{\partial\tilde\nu} + Pr^{\bar a}(s=0,d_l)\,u\left(c_l^{\bar 0}\right)\frac{\partial c_l^{\bar 0}}{\partial\tilde\nu}\right)\frac{\partial\tilde\nu^{\bar 0}}{\partial\rho}\end{array}\right] + \\ &\left[\begin{array}{l}\left(Pr^{\bar a}(s=1,d_h)\,u'\left(c_h^{\bar 1}\right)\frac{\partial c_h^{\bar 1}}{\partial p} + Pr^{\bar a}(s=1,d_l)\,u'\left(c_l^{\bar 1}\right)\frac{\partial c_l^{\bar 1}}{\partial p}\right)\frac{\partial p^1}{\partial\rho} + \\ \left(Pr^{\bar a}(s=0,d_h)\,u'\left(c_h^{\bar 0}\right)\frac{\partial c_h^{\bar 0}}{\partial p} + Pr^{\bar a}(s=0,d_l)\,u\left(c_l^{\bar 0}\right)\frac{\partial c_l^{\bar 0}}{\partial p}\right)\frac{\partial p^0}{\partial\rho}\end{array}\right]\end{aligned}$$

When the utility function is logarithmic, the second term vanishes. Equations (5) and (6) show how the ex ante expected utility of rich and poor agents reacts to changes in the precision of the signal when $\rho = 1$, i.e., when agents have full information.

$$\left.\frac{\partial\bar{\mathcal{U}}}{\partial\rho}\right|_{\rho=1}=\frac{1}{2}\left[\begin{array}{l}\left[(0.5-b)(0.5+\Delta)-(0.5+b)(0.5-\Delta)\right]\left[u\left(c_h^{\bar{1}}\right)-u\left(c_h^{\bar{0}}\right)\right]+\\ \left[(0.5+b)(0.5+\Delta)-(0.5-b)(0.5-\Delta)\right]\left[u\left(c_l^{\bar{0}}\right)-u\left(c_l^{\bar{1}}\right)\right]\end{array}\right]+$$

$$A\times\left(\frac{\bar{a}+d_l}{a+d_l}-\frac{\bar{a}+d_h}{a+d_h}\right)\frac{2b\left[\bar{a}\left(2a+d_l+d_h\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]-2\Delta\left(d_h-d_l\right)\left(\bar{a}-\underline{a}\right)}{\left[\begin{array}{l}\left[\bar{a}\left(2a+d_l+d_h\right)+\left(d_h-d_l\right)\Delta\left(\bar{a}-\underline{a}\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]\times\\ \left[\bar{a}\left(2a+d_l+d_h\right)-\left(d_h-d_l\right)\Delta\left(\bar{a}-\underline{a}\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]\end{array}\right]} \tag{5}$$

$$\left.\frac{\partial\underline{\mathcal{U}}}{\partial\rho}\right|_{\rho=1}=\frac{1}{2}\left[\begin{array}{l}\left[(0.5+b)(0.5+\Delta)-(0.5-b)(0.5-\Delta)\right]\left[u\left(c_{\underline{h}}^{1}\right)-u\left(c_{\underline{h}}^{0}\right)\right]+\\ \left[(0.5-b)(0.5+\Delta)-(0.5+b)(0.5-\Delta)\right]\left[u\left(c_{\underline{l}}^{0}\right)-u\left(c_{\underline{l}}^{1}\right)\right]\end{array}\right]+$$

$$A\times\left(\frac{\underline{a}+d_h}{a+d_h}-\frac{\underline{a}+d_l}{a+d_l}\right)\frac{2b\left[\underline{a}\left(2a+d_l+d_h\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]-2\Delta\left(d_h-d_l\right)\left(\bar{a}-\underline{a}\right)}{\left[\begin{array}{l}\left[\underline{a}\left(2a+d_l+d_h\right)+\left(d_h-d_l\right)\Delta\left(\bar{a}-\underline{a}\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]\times\\ \left[\underline{a}\left(2a+d_l+d_h\right)-\left(d_h-d_l\right)\Delta\left(\bar{a}-\underline{a}\right)+d_h\left(a+d_l\right)+d_l\left(a+d_h\right)\right]\end{array}\right]} \tag{6}$$

where,

$$a=\frac{\bar{a}+\underline{a}}{2},$$

and

$$\begin{aligned}A&=\Delta\left(1-4b^2\right)\frac{\left(1+4b^2\right)\left(e+d_l\right)\left(e+d_h\right)+b\left(\bar{a}-\underline{a}\right)\left(2a+d_l+d_h-2\Delta\left(d_h-d_l\right)\right)}{\left(1-2b\right)^2\left(1+2b\right)^2}\\ &\geq\Delta\left(1-4b^2\right)\frac{\left(1+4b^2\right)\left(e+d_l\right)\left(e+d_h\right)+2b\left(\bar{a}-\underline{a}\right)\left(a+d_l\right)}{\left(1-2b\right)^2\left(1+2b\right)^2}>0.\end{aligned}$$

The first term in equations (5) and (6) equals zero when $\Delta=0.5$, i.e., when there is no further uncertainty once the value of ν is revealed. In this case, knowing ν is equivalent to knowing the future dividend payoff with certainty. Thus, in equilibrium there is no trade, and agents consume their endowments.

$d_h = 1$	$d_l = 0.1$
$\bar{a} = 1$	$\underline{a} = 0.5$
$\Delta = 0.3$	$b = 0.05$

Table 1: **Parameter values**

The second term captures how information affects welfare through changes in the equilibrium price. The signs of these terms are determined by the signs of the numerators,

$$2b\left[\bar{a}\left(2a + d_l + d_h\right) + d_h\left(a + d_l\right) + d_l\left(a + d_h\right)\right] - 2\Delta\left(d_h - d_l\right)\left(\bar{a} - \underline{a}\right)$$

and

$$2b\left[\underline{a}\left(2a + d_l + d_h\right) + d_h\left(a + d_l\right) + d_l\left(a + d_h\right)\right] - 2\Delta\left(d_h - d_l\right)\left(\bar{a} - \underline{a}\right).$$

Note that both terms are negative when $b = 0$. The reason is that in this case only the Hirshleifer effect is present. However, when $b > 0$, there is an additional effect due to the decrease in the dispersion of beliefs. If the bias in the priors is significant, the latter offsets the Hirshleifer effect, inducing a positive relationship between welfare and information. It is trivial to verify that the set of parameters that satisfy $b > \frac{\Delta(d_h - d_l)(\bar{a} - \underline{a})}{\underline{a}(2a + d_l + d_h) + d_h(a + d_l) + d_l(a + d_h)}$ is nonempty.

■

The previous proposition illustrates that it is possible that every agent in the economy benefits from the arrival of more precise information. Even though the proposition considers only the case where the signal is fully revealing, it can be easily extended to cases where ρ is sufficiently close to 1. An analytical characterization cannot be provided for other values of ρ. However, the figures below show that the positive relationship between welfare and the precision of the signal is not only observed when the signal is close to being fully revealing. The parameter values used to draw the graphs below are specified in Table 1.

Figures 1 and 2 show that it is possible to observe a positive monotonic relationship between welfare and information. The result is observed without relying on a wide dispersion of beliefs (the bias in the priors is small). The welfare measure described in the graphs corresponds to equations (3) and

(4). They capture the expected utility of an agent before the signal is observed, and markets open. In order to be consistent with the belief updating scheme, the probability distribution used to compute the expected utility is based on the biased distribution perceived by the agents.

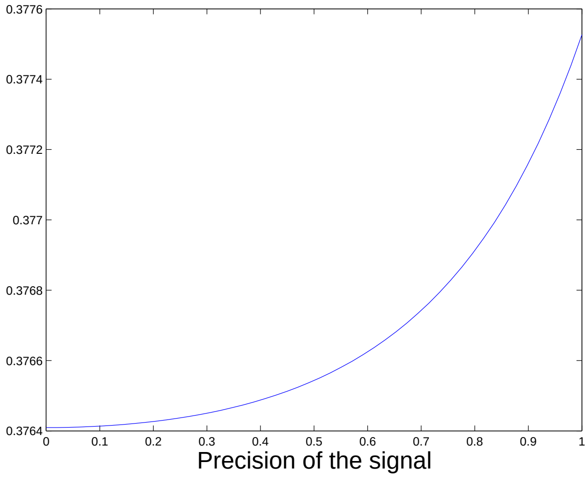

Figure 1: **Welfare of an agent with high endowment when the ex ante expected utility is computed using the agent's prior.**

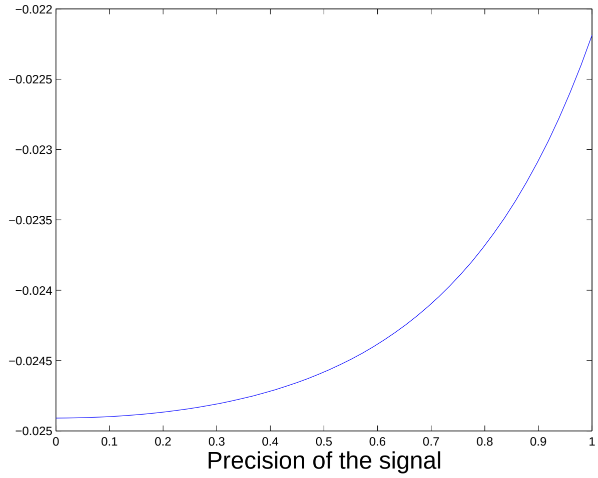

Figure 2: **Welfare of an agent with low endowment when the ex ante expected utility is computed using the agent's prior.**

Figures 3 and 4 show that a positive relationship between welfare and the precision of the signal is also observed when the agents' welfare is evaluated using the correct probability distribution. Figure 5 describes the ex ante expected utility before the endowment realization. This can also be interpreted as the expected utility of a utilitarian planner that assigns equal weight to every agent. The figure shows that for certain parameter values the "optimal" amount of information conveyed by the signal is away from the boundaries.

This section introduced a simple model that shows how information can have positive value in an exchange economy with complete markets. Some comments about the robustness of the result are in order. For tractability reasons, the section focused on a specific parametric framework. We expect the welfare effects described before to be present in a more general setting, with a concave utility function that displays a decreasing degree of absolute risk aversion. Second, the heterogeneity of beliefs was a direct consequence of the assumption that agents are endowed with heterogeneous priors. However, we were silent about what could originate the discrepancy in the priors. This can be viewed as an important

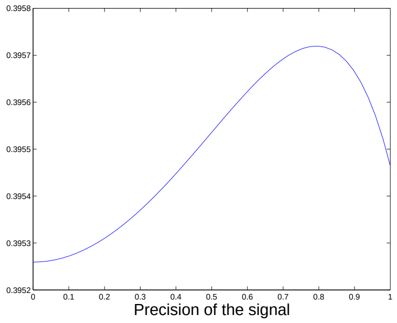

Figure 3: **Welfare of an agent with high endowment when the ex ante expected utility is computed using the correct distribution.**

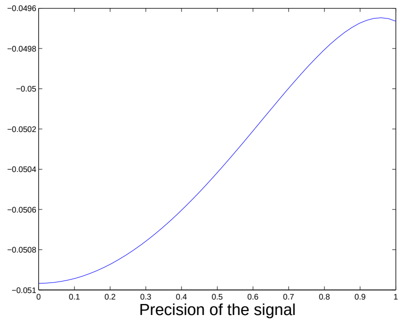

Figure 4: **Welfare of an agent with low endowment when the ex ante expected utility is computed using the correct distribution.**

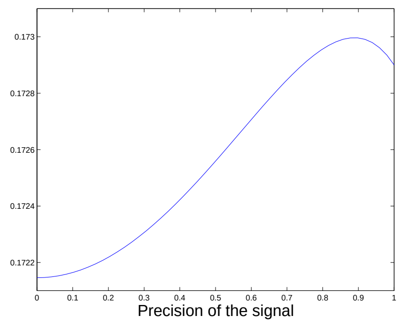

Figure 5: **Welfare of a utilitarian planner.**

limitation given that this feature plays a key role to explain the result. The model considered in the following section corrects this problem. In the rest of the paper we focus on a framework that follows the Harsanyi doctrine, where agents share a common prior. The heterogeneity of beliefs is thus endogenously generated by the model. This will help us to understand what structure of primitives may induce the class of heterogeneity that leads to a positive welfare impact.

3 A model with private signals and common prior

The model analyzed in this section shares many features with the one introduced before. The main point of departure is that the model below features agents that learn from private signals and the equilibrium price. We borrow the basic structure from the model studied in Hatchondo et al. [19]. Even though the model is simple, it does not allow for a tractable analytical solution, so we must rely on numerical techniques to characterize the equilibrium.

We consider a pure exchange economy with differential information and heterogeneous agents. There is a single risky asset in the economy: a tree. The tree pays high dividends with probability ν and low dividends with probability $1-\nu$. The tree pays off only once and then dies. There is a measure 1 of agents in the economy, and everybody is initially entitled to a share of the tree. Agents also receive a riskless endowment, though some of them are luckier than others, i.e., a fraction ϕ of the population receives a high endowment, while a fraction $1-\phi$ receives a low endowment.

The parameters ν and ϕ are drawn from a joint probability distribution $F(\nu,\phi)$, which is common knowledge. The random variable ν takes values on the unit interval $I \equiv [0,1]$. The random variable ϕ is discrete and takes values on $\Phi = \{\phi_l, \phi_h\}$.

Agents are not able to observe the realizations of those variables, but they receive informative signals about the tree. Each signal can be either good or bad. Every agent receives n number of signals. The realizations of the signals are drawn from a Binomial distribution with parameter ν. As the number of signals increases, the information that agents receive becomes more accurate. The case where ν is common knowledge corresponds to an infinite number of signals.

The assumption described in the previous paragraph deserves two comments. First, we acknowledge that other mechanisms exist that can be used to model the transmission of information besides the binary signals structure. We choose the latter for the sake of simplicity. Second, the paper does not explain how the number of signals is determined. In order to circumvent this limitation, the model can be interpreted as a case where agents decide the number of signals they purchase.[6] The process of acquisition of information is not the main focus of this paper and is not modelled explicitly.

All the action takes place in a single period. Markets open in the morning, the tree pays off in the afternoon, and agents consume at the end of the day. In the absence of trade, agents consume their endowments and dividends paid by the tree. Actually, this is the equilibrium allocation if there is no heterogeneity across agents. This is not the case in the present framework. Poor agents have a stronger preference for consumption smoothing than wealthy individuals, so there are gains from trade.[7]

Agents can transfer resources freely across the two states of nature that can be realized, i.e., whether the tree pays high or low dividends. This means that consumers can trade in two Arrow-Debreu securities. One of them pays 1 unit of the consumption good if the high dividends state is realized. Otherwise, it pays zero. The other security only pays (1 unit) in the low dividends state. There is only one price to be determined: the relative price between these two securities.

This market structure would be enough to attain an efficient allocation in an economy with full

[6] See Verrecchia [36] for a case where agents decide the precision of the information acquired.

[7] This result is true if the utility function is concave and shows a decreasing coefficient of absolute risk aversion. The latter is defined as $\frac{-u''(c)}{u'(c)}$. The utility function assumed in the present paper (logarithmic) satisfies both properties.

information. This is not true in the present example. As will become clear in the next section, an extra asset (and, hence, another market) is necessary to attain an efficient allocation with the information structure assumed above. The present paper studies the case where this is not possible (markets are incomplete).

The equilibrium price depends on ν and ϕ. A higher value of ν means that the high dividends state is more likely to occur, which makes the contingent claim paying in that state more valuable. A higher value of ϕ implies that a small fraction of agents need insurance, reducing the demand for contingent claims paying in the low state.

The critical assumption in this paper is that agents are fully rational and use the information pooled by the equilibrium price to update their beliefs. Agents not only learn from their private signals, but also understand how the equilibrium price is determined. This allows them to make inferences about the realizations of ν and ϕ once they have observed the market price. In addition, the endowment realization also conveys valuable information, as will be described below. Finally, the paper assumes that agents do not behave strategically. They take the price and everyone else's behavior as given. This is justified on grounds that there is a "large" number of agents. Thus, a single individual does not exert any influence on aggregate variables.

3.1 Definition of equilibrium

Agents maximize their expected utility of consumption. They do not know the actual state probabilities, so they use the information contained in the private signals, the endowment, and the market price to refine their beliefs about ν. The belief consists of the expectation of ν conditional on the price and the agents' private information. Formally, let $\mathcal{I}_i$ denote the private information set of agent i, specifically, the signals and endowment received. His belief about ν is denoted by $\tilde{\nu}^i$ where:

$$\tilde{\nu}^i (\mathcal{I}, p) = E [\nu \mid \mathcal{I}_i, p] .$$

Given the equilibrium price function $p(\cdot)$, the above expectation is a straightforward application of Bayes' rule: the agent knows that ν and ϕ are drawn according to a joint distribution F and that p is random and given by a function $p(\nu, \phi)$. The next subsection describes in more detail how agents compute their beliefs in the class of economies we analyze. Note, of course, that these beliefs are endogenous here: they depend on the pricing function $p(\cdot)$.

A type i consumer solves the following optimization problem:

$$\begin{aligned} &\underset{c_h, c_l}{Max} \left\{ \tilde{\nu}^i (\mathcal{I}, p)\, u(c_h) + \left(1 - \tilde{\nu}^i (\mathcal{I}, p)\right) u(c_l) \right\} \\ &\text{subject to} \quad (1-p)c_l + pc_h = W = a^i + (1-p)d_l + pd_h \\ &c_l, c_h \geq 0, \end{aligned} \tag{7}$$

where $u(c)$ denotes the utility function; c_j denotes planned consumption in state j; a^i denotes the riskless endowment of a type i agent; d_j denotes the dividends paid by the tree in state j; and W denotes individual wealth. The sum of the prices of the Arrow-Debreu securities is normalized to 1.

If each agent receives n signals, there are $2(n+1)$ different combinations: an agent can receive either a high or low riskless endowment combined with n+1 different signal realizations. For simplicity, this paper assumes that the distribution of signals in the population is independent from the distribution of riskless endowments. Denote by $\mu^i(\nu, \phi, n)$ the measure of agents i in the population. For instance, the model implies that there is a fraction $(1-\phi)\binom{n}{j}\nu^j(1-\nu)^{(n-j)}$ of agents with a low riskless endowment and j good signals.

Let $Y_j(\phi)$ denote the overall aggregate resources in state j, $c_j^i(p)$ denote the optimal consumption of agent i in state j, and $Z_i(p, \mu)$ denote the aggregate demand in state j. The latter is computed as follows:

$$Z_j(p, \mu) = \sum_{i=1}^{2(n+1)} c_j^i(p)\, \mu^i(\nu, \phi, n).$$

We are now ready to define a competitive equilibrium for this class of economies.

Definition 2 *A rational expectations equilibrium (REE) consists of a measurable price function* $p : \mathrm{I} \times \Phi \to [0,1]$ *and individual demands* $\left\{c_l^i(p), c_h^i(p)\right\}_{i=1}^{i=2(n+1)}$ *such that:*

(1) $\left\{c_l^i(p), c_h^i(p)\right\}$ *solves consumer i's problem* $\forall\, i = 1, \ldots, 2(n+1)$ *given that consumers use the equilibrium price function* $p(\nu, \phi)$ *when they update their beliefs; and*

(2) *Markets Clear:* $Z_j\left(p\left(\nu,\phi\right),\mu\left(\nu,\phi\right)\right)=Y_j\left(\phi\right)\ \forall\ j=l,h$ *and* $\nu\in\mathrm{I},\ \phi\in\Phi.$

Radner [32] provides a more general definition of the equilibrium concept defined above.[8] An important assumption implicit in Definition 2 is that the individuals' perceived price function coincides with the actual equilibrium function. Agents fully understand how prices are determined and take this information into account when updating their beliefs. Note that, in general, finding a solution to the previous problem requires solving for a fixed point functional equation: the price function perceived by the agents must coincide with the price function generated by their behavior.

In what follows, we consider a simplified version of the framework described above. This reduces the generality of the results but allows us to characterize the equilibrium. Even though the conclusions depend on the specific assumptions we make, the economies analyzed do not belong to a negligible set, i.e., the results are robust to any perturbation of the primitives: utility function, dividends and endowment process, or joint distribution of ν and ϕ.

3.2 Finding the equilibrium: a particular case

We choose a logarithmic utility function because it has the advantage that individual demands are linear in wealth. The optimal consumption rules are specified in equation (8). It is assumed for simplicity that the random variables ν and ϕ are independent. The random variable ν is drawn from a uniform distribution with support $[0,1]$, while ϕ takes a high value (ϕ_h) with probability π and a low value (ϕ_l) with probability $1-\pi$.

$$c_h^i\left(p,p\left(\cdot\right)\right)=\tilde{\nu}^i\left(p,p\left(\cdot\right)\right)\frac{W^i}{p}\qquad c_l^i\left(p,p\left(\cdot\right)\right)=\left(1-\tilde{\nu}^i\left(p,p\left(\cdot\right)\right)\right)\frac{W^i}{1-p}\tag{8}$$

In order to understand how the model works, we assume for the moment that each agent receives only one signal. This implies that there are four different types of agents in the model. They are listed below with their corresponding measure.

- $\nu\phi$ agents with high endowment and a good signal (denoted by $\bar{1}$),
- $(1-\nu)\phi$ agents with high endowment and a bad signal (denoted by $\bar{0}$),

[8]Dubey et al. [12] criticize the REE approach because it assumes implicitly that prices pool individuals' private information before they trade. Nonetheless, the approach has been extensively used in the literature, showing that, despite its limitations, it constitutes a useful tool to analyze problems with asymmetric information.

- $\nu(1-\phi)$ agents with low endowment and a good signal (denoted by $\underline{1}$),
- $(1-\nu)(1-\phi)$ agents with low endowment and a bad signal (denoted by $\underline{0}$).

In equilibrium, aggregate planned consumption for the high state must equal aggregate resources in that state. If that equality holds, by Walras' law, the other market is also in equilibrium. The market clearing condition is formally stated in equation (9):

$$\phi\left[\nu c_h^{\bar{1}} + (1-\nu)\, c_h^{\bar{0}}\right] + (1-\phi)\left[\nu c_h^{\underline{1}} + (1-\nu)\, c_h^{\underline{0}}\right] = \phi\bar{a} + (1-\phi)\,\underline{a} + d_h, \tag{9}$$

where $\bar{a}$ denotes the high value of the riskless endowment and $\underline{a}$, the low value.

The equilibrium price is obtained after replacing individual demands into the market clearing condition:

$$p(\nu,\phi) = \frac{\phi(\bar{a}+d_l)\left[\nu\tilde{\nu}^{\bar{1}} + (1-\nu)\tilde{\nu}^{\bar{0}}\right] + (1-\phi)(\underline{a}+d_l)\left[\nu\tilde{\nu}^{\underline{1}} + (1-\nu)\tilde{\nu}^{\underline{0}}\right]}{\phi\bar{a} + (1-\phi)\,\underline{a} + d_h + \left\{\phi\left[\nu\tilde{\nu}^{\bar{1}} + (1-\nu)\tilde{\nu}^{\bar{0}}\right] + (1-\phi)\left[\nu\tilde{\nu}^{\underline{1}} + (1-\nu)\,\tilde{\nu}^{\underline{0}}\right]\right\}(d_l - d_h)}. \tag{10}$$

It is easy to show that this model does not possess a fully revealing equilibrium. The reasoning is that the agents' private signals and endowments do not convey enough information to reveal the realization of (ν,ϕ). Thus, the only way agents can infer the values of those variables is if in equilibrium there is a one to one mapping between (ν,ϕ) and the equilibrium price. In other words, for prices to be fully revealing, there must be only one possible realization of ν consistent with a given price and value of ϕ. The equilibrium relationship between ν and the last two variables in the fully revealing case is described in equation (11).[9]

$$\nu(p,\phi) = \frac{p\left[\phi\bar{a} + (1-\phi)\,\underline{a} + d_h\right]}{\phi\bar{a} + (1-\phi)\,\underline{a} + (1-p)\,d_l + p d_h} \tag{11}$$

It is apparent that there is more than one combination of ν and ϕ consistent with a given price. This contradicts the hypothesis that prices are fully revealing. Furthermore, it suggests that the equilibrium is pair wise revealing: the market price reveals that the probability of high dividends can take one of two possible values. Thus, individual beliefs consist of a weighted sum of those values. The weights are determined by the signal and endowment received.

[9]Equation 11 is obtained after replacing individual beliefs $\tilde{\nu}^i$ in equation (10) with the actual realization of ν.

In order to understand how the price depends on ν and ϕ, it is helpful to consider again the case where both variables are common knowledge. In that case, the equilibrium price function is given by

$$p^{FR}(\nu, \phi) = \frac{\nu\left[\phi \bar{a} + (1-\phi)\underline{a} + d_l\right]}{\phi \bar{a} + (1-\phi)\underline{a} + d_h - \nu\left(d_h - d_l\right)}. \tag{12}$$

The relative price trivially increases with ν. As the high state becomes more likely, agents demand more contingent claims paying in that state. It can easily be shown that the equilibrium price also increases with ϕ. We have already mentioned that "poor" agents (with a low riskless endowment) are de facto more risk averse than "rich" agents, so the former buy insurance from the latter. That is, agents with a low endowment transfer consumption from the high dividends state to the low dividends state.[10] As the fraction of rich individuals (ϕ) increases, less agents demand contingent claims that pay off in the low state, so the relative price decreases, i.e., p increases.

Presumably, the equilibrium price in the economy with differential information is also increasing in both arguments. We found this to be true in all the simulations performed.

3.2.1 Beliefs' updating scheme

The equilibrium price specified in equation (10) takes the values of beliefs as given. However, as explained before, the latter are a function of the market price and the price function itself. This section explains in more detail how agents compute their beliefs.

Figure 6 shows how agents extract the information pooled by the market price. Every agent is assumed to know the price function. When they observe a particular price, say p_0 in the figure, they infer that only two values of ν could have been realized: $\nu(p_0, \phi_l)$ or $\nu(p_0, \phi_h)$. The first one corresponds to the value of ν consistent with a price p_0 and a low fraction of highly endowed agents. The second

[10] Let θ_h denote the net demand of contingent claims that pay only if the high state is realized. The agent is endowed with $a + d_h$ of this asset. It can be shown that

$$\frac{\partial \theta_h}{\partial a} > 0 \Longleftrightarrow \frac{-u''(c_h)}{u'(c_h)} < \frac{-u''(c_l)}{u'(c_l)},$$

where

$$c_i = a + d_i + \theta_i \quad i = l, h.$$

From the individual first order conditions and the aggregate resource constraint, it transpires that $c_h > c_l$ for every agent. Thus, a sufficient condition for the previous inequality to hold is that the coefficient of absolute risk aversion decreases with consumption. The utility function assumed in the present paper satisfies this property.

one corresponds to the value of ν consistent with a low fraction of poorly endowed agents. Since agents do not know the actual distribution of riskless endowments, they cannot distinguish which of the values corresponds to the actual realization of ν. In addition to learning from the market price, their private signals and endowments reveal information. An agent with a high endowment believes that it is more likely that the fraction of rich agents is ϕ_h rather than ϕ_l, so he assigns more weight to $\nu(p_0, \phi_h)$. An agent with a good signal believes that it is more likely that the highest ν was realized.

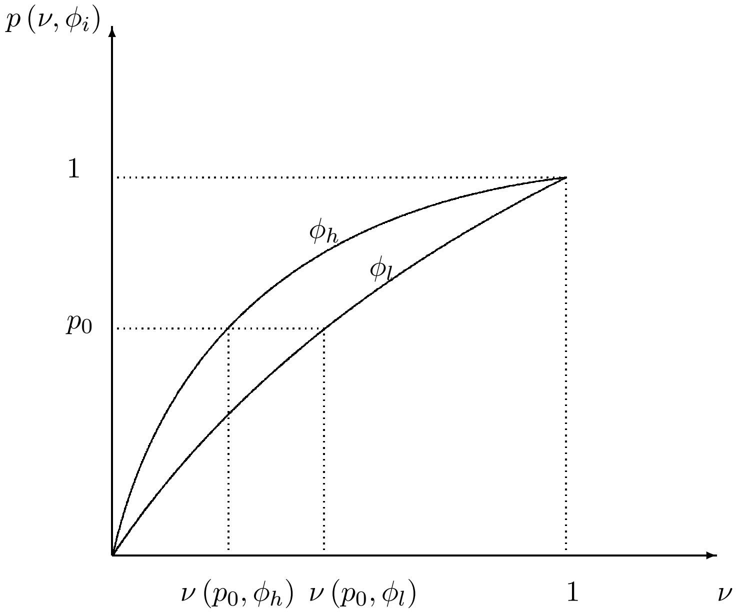

Figure 6: **Information revealed by the price function.**

We now formalize the previous argument taking the case of an agent who has received a high riskless endowment and a good signal. The updating of beliefs of the remaining agents follows the same logic.

The paper assumes that each agent's belief regarding the probability that the tree pays high dividends consists of the expectation of ν conditional on his private information and the market price, namely:

$$\begin{aligned} \tilde{\nu}^{\bar{1}}(p) &= E\left[Pr\left(tree\ pays\ d_h\right) \mid signal = 1, endowment = \bar{a}, price = p\right] \\ &= \nu(p, \phi_h)\, Pr\left(\nu(p, \phi_h) \mid 1, \bar{a}, p\right) + \nu(p, \phi_l)\, Pr\left(\nu(p, \phi_l) \mid 1, \bar{a}, p\right). \end{aligned}$$

The second equality takes into account that once the agent has conditioned on the price, the probability ν has a dichotomous distribution. Then we apply the law of conditional probabilities to the last expression and use the fact that once we condition on ϕ, the following events are mutually independent: the tree pays high dividends, the agent receives a good signal, and the agent receives a high riskless endowment. The result is the following equation:

$$\tilde{\nu}^{\bar{1}}(p) = \frac{\left\{ \begin{array}{c} \nu(p,\phi_h)\, Pr(1 \mid p,\phi_h)\, Pr(\bar{a} \mid p,\phi_h)\, Pr(p \mid \phi_h)\, Pr(\phi_h) + \\ \nu(p,\phi_l)\, Pr(1 \mid p,\phi_l)\, Pr(\bar{a} \mid p,\phi_l)\, Pr(p \mid \phi_l)\, Pr(\phi_l) \end{array} \right\}}{\left\{ \begin{array}{c} Pr(1 \mid p,\phi_h)\, Pr(\bar{a} \mid p,\phi_h)\, Pr(p \mid \phi_h)\, Pr(\phi_h) + \\ Pr(1 \mid p,\phi_l)\, Pr(\bar{a} \mid p,\phi_l)\, Pr(p \mid \phi_l)\, Pr(\phi_l) \end{array} \right\}}.$$

Finally, equation (13) is obtained after replacing the probabilities in the last expression by their actual values. Recall that the probability of receiving a good signal and a high riskless endowment coincides with the actual realizations of ν and ϕ, respectively.

$$\tilde{\nu}^{\bar{1}}(p) = \frac{\nu(p,\phi_h)\,\nu(p,\phi_h)\,\phi_h g(p \mid \phi_h)\,\pi + \nu(p,\phi_l)\,\nu(p,\phi_l)\,\phi_l g(p \mid \phi_l)(1-\pi)}{\nu(p,\phi_h)\,\phi_h g(p \mid \phi_h)\,\pi + \nu(p,\phi_l)\,\phi_l g(p \mid \phi_l)(1-\pi)} \tag{13}$$

The function $g(p \mid \phi_i)$ denotes the density of the price conditional on ϕ_i, where

$$g(p \mid \phi) = f(\nu(p,\phi_i))\,\frac{\partial \nu(p,\phi_i)}{\partial p} = f(\nu_i(p))\,\nu_i'(p).$$

The last equality just simplifies the notation. The subindex i denotes the fraction of rich agents in the economy (ϕ_i), and $f(\cdot)$ denotes the density function of ν.[11] The intuition for the formula of the conditional density is that a price p is likely to be observed when the value of ν consistent with that price is likely to be drawn, i.e., $f(\nu)$ is high, or when the price function $p_i(.)$ is not sensitive to ν at $\nu_i(p)$. A heuristic description of the last argument is provided in the picture below. Consider a hypothetical case where it is known that the price belongs to the range $[p_0,\ p_1]$. Its actual value however, is not observed. In this case, agents infer that ν belongs to $[\nu_h(p_0),\ \nu_h(p_1)]$ if the fraction of rich agents is ϕ_h, and to $[\nu_l(p_0),\ \nu_l(p_1)]$ if the fraction is ϕ_l. In the case where ν is drawn from a uniform distribution, the probability of observing a price in $[p_0,\ p_1]$ consists of the length of the interval $[\nu_i(p_0),\ \nu_i(p_1)]$,

[11] The paper assumes a uniform distribution over the interval [0, 1], so the density is just the constant 1. However, it will assist the intuition to consider for the moment the more general case.

which is clearly higher for price function $p(\cdot, \phi_l)$. In the limit, as the length of the price range collapses to a single point, the likelihood of observing a particular price becomes inversely proportional to the derivative of the price function at that point, or directly proportional to $\nu_i'(q)$.

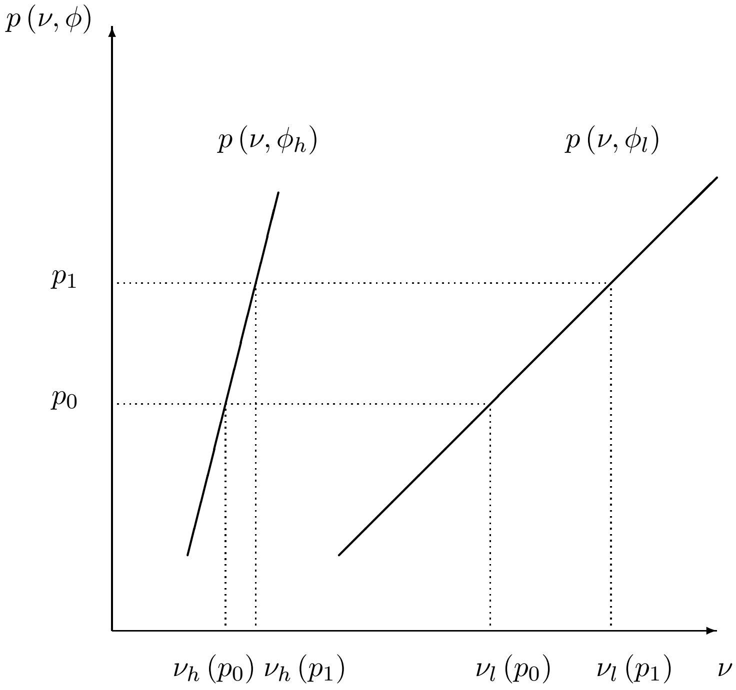

Figure 7: **The likelihoods of $\nu_h(p)$ and $\nu_l(p)$ are inversely proportional to the derivatives $\frac{\partial p(\nu,\phi_h)}{\partial \nu}$ and $\frac{\partial p(\nu,\phi_l)}{\partial \nu}$.**

The beliefs of the remaining types are described in (14) – (16):

$$\tilde{\nu}^{\bar{0}}(p) = \frac{\pi f(\nu_h)\nu_h'\phi_h(1-\nu_h)\nu_h + (1-\pi) f(\nu_l)\nu_l'\phi_l(1-\nu_l)\nu_l}{\pi f(\nu_h)\nu_h'\phi_h(1-\nu_h) + (1-\pi) f(\nu_l)\nu_l'\phi_l(1-\nu_l)} \tag{14}$$

$$\tilde{\nu}^{\underline{1}}(p) = \frac{\pi f(\nu_h)\nu_h'(1-\phi_h)\nu_h^2 + (1-\pi) f(\nu_l)\nu_l'(1-\phi_l)\nu_l^2}{\pi f(\nu_h)\nu_h'(1-\phi_h)\nu_h + (1-\pi) f(\nu_l)\nu_l'(1-\phi_l)\nu_l} \tag{15}$$

$$\tilde{\nu}^{\underline{0}}(p) = \frac{\pi f(\nu_h)\nu_h'(1-\phi_h)(1-\nu_h)\nu_h + (1-\pi) f(\nu_l)\nu_l'(1-\phi_l)(1-\nu_l)\nu_l}{\pi f(\nu_h)\nu_h'(1-\phi_h)(1-\nu_h) + (1-\pi) f(\nu_l)\nu_l'(1-\phi_l)(1-\nu_l)} \tag{16}$$

Note that the equilibrium price affects the beliefs in two ways. First, for a given market price p, agents use the equilibrium price function to retrieve the possible realizations of ν: $\nu_h(p)$ and $\nu_l(p)$.

Second, they use the derivative of the price function ($\nu_h'(p)$ and $\nu_l'(p)$) in order to assess how likely those points are.

The structure of the model is similar to Ausubel [2, 3]. He also analyzed an economy with partially revealing prices, where the state of the economy is characterized by two variables: one continuous and the other dichotomous. In our framework, the first one is represented by ν and the second one by ϕ. The difference is that he considers the case where a fraction of the population is fully informed while the rest is uninformed and must learn from the equilibrium price. This structure allows him to prove the existence and uniqueness of equilibrium. Ausubel [2] also obtains a closed-form solution for the equilibrium price using specific assumptions about the utility function and the distribution of the continuous variable. Unfortunately, his results do not extend to the present framework.

4 Parameter selection

Our model does not allow for a tractable analytical solution, but an approximate solution can be found using numerical techniques. The appendix provides a detailed description of the procedure followed to find the equilibrium. The model is highly stylized and has limited ability to replicate real data. Thus, the parameters that characterize the dividend and endowment processes are not chosen following a standard calibration exercise, i.e., they are not based on actual data. There are other reasons that motivate this choice. The assumption of a risky asset that lives for only one period cannot mimic the returns to any real-world aggregate stock index.[12] Moreover, in order to calibrate the process of the riskless endowment it would be necessary to consider not only the labor income of stockholders, but also other sources of income, like the returns to private businesses, which are not easy to obtain.

The strategy is to choose baseline parameters that will help to illustrate the effects the paper tries to emphasize. To that end, the bad realization of the riskless endowment is allowed to take a relatively low value. This magnifies the different attitudes toward risk of rich and poor agents, thus increasing the

[12] If the risk-free bond is taken as numéraire, the expected return of the tree for a given realization of ν and ϕ in an economy with full information is

$$R(\nu,\phi) = \frac{\nu d_h p}{d_l(1-p) + d_h p} + \frac{(1-\nu)\, d_l (1-p)}{d_l(1-p) + d_h p}, \quad where\ p = p(\nu,\phi).$$

The gross return is below 1 for almost all realizations of ν and ϕ. This implies that the model cannot generate positive net rates of returns of the risky asset, as it is observed in the data.

$d_h = 1$	$d_l = 0.1$
$\bar{a} = 1$	$\underline{a} = 0.5$
$\phi_h = 0.8$	$\phi_l = 0.2$
$\pi = 0.5$	$\nu \sim U(0,1)$

Table 2: **Parameter values**

sensitivity of the equilibrium to changes in the distribution of endowments (controlled by ϕ). Similarly, if the dividend dispersion was low, equilibrium state prices would lie close to the corresponding state probability, regardless of the realization of ϕ. In that case, agents' beliefs would tend to coincide with the actual realization of ν, and the economy would behave almost as if everyone were fully informed. A dispersed dividend realization is therefore a necessary ingredient. In summary, we restrict attention to the case where the lower realizations of the riskless endowment and dividends take small values compared to their higher counterparts. The parameters chosen are specified in the table below.

5 Results

Figure 8 compares the equilibrium prices between an economy with full information and an economy with partial information (agents receive one signal). The graph shows that the monotonicity property of the price function is preserved in the differential information framework. It also illustrates that when the economy is hit with a good endowment shock ($\phi = \phi_h$), the relative price of the high dividends state is higher in the partial information case compared to the full information case. The result is reversed when the economy is hit with a bad shock.

The explanation rests on the scheme agents use to update their beliefs. Consider again Figure 6 on page 22. We can interpret the picture as the price schedule in the case where all but a single agent are fully informed. The unlucky agent has to infer ν from the price observed in the market and his private information. If the values $\nu(p_0, \phi_l)$ and ϕ_l are realized, the agent's belief lays below the actual realization of ν. The equilibrium price is not affected because the single agent has measure zero, so his behavior does not influence aggregate variables. However, if the fraction of agents who are imperfectly

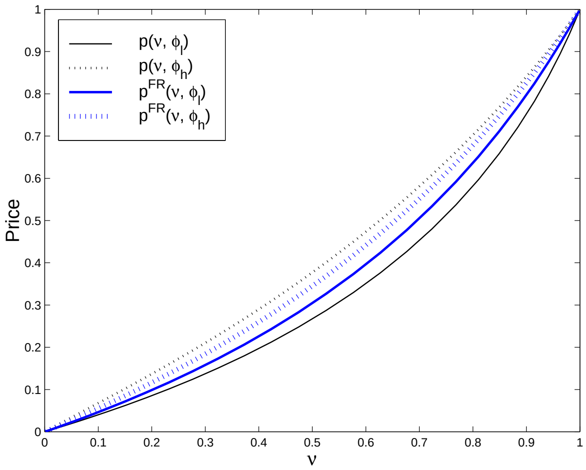

Figure 8: **Equilibrium price under full and partial information.**

informed increases, the average belief in the economy decreases and the equilibrium price falls, as can be deduced from equation (10). Eventually, if no agent is fully informed, the average belief is below the actual realization of ν. This implies that the equilibrium price is below its level in the full information economy, as Figure 8 shows. The previous argument holds for any realization of ν. Similar logic can be used to explain why the equilibrium price is higher in an economy with differential information and a high realization of ϕ.

Figures 9 and 10 illustrate the beliefs as a function of the price. It shows that the value of the riskless endowment conveys more information than the signal about the tree. Agents with low endowments are more optimistic than the rest, independent of the signal received. An agent hit with a low riskless endowment assigns more weight to the possibility that $\phi = \phi_l$ than a rich individual would. This means that receiving a low endowment can be taken as a signal that the actual ν is closer to $\nu_l(p)$ than $\nu_h(p)$. The first value is higher than the second one, explaining why poor agents tend to be more optimistic.

The heterogeneity in beliefs along with the difference in the endowments induce agents to trade. In Section 2 we stated that in an economy with full information, rich agents sell contingent claims that pay in the low state. This may not be true in the present case. Poor agents are more optimistic than wealthy individuals, so the former may now have an incentive to transfer resources to the high state.

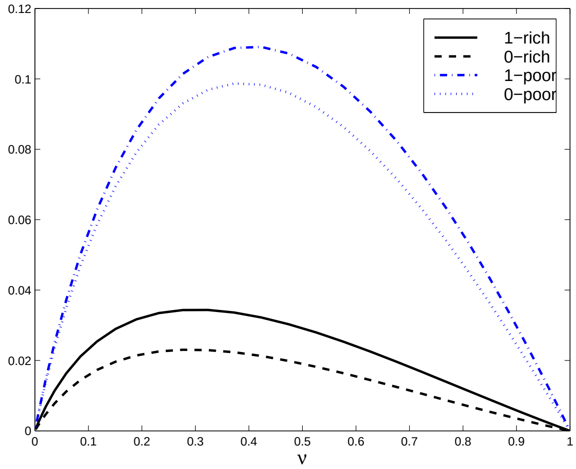

Figure 9: **Difference between individual beliefs and actual realizations of ν for the case $\phi = \phi_h$.**

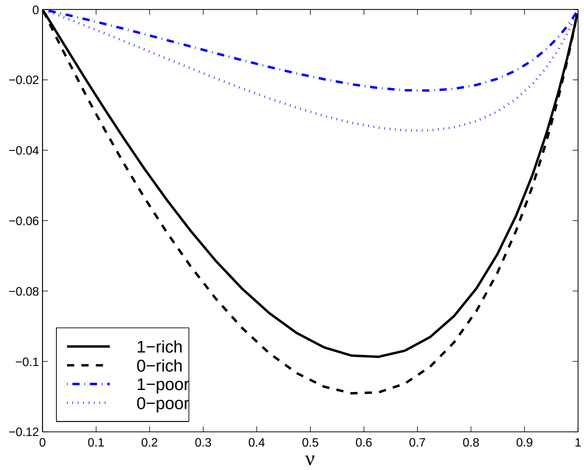

Figure 10: **Difference between individual beliefs and actual realizations of ν for the case $\phi = \phi_l$.**

5.1 Definitions of welfare

The objective of the paper is to assess, in a framework with differential information, whether the receipt of more accurate public information increases or reduces individual welfare. As Holmstrom and Myerson [22] point out, the heterogeneity in private information raises a question that is not present in an economy with complete information: what is the appropriate measure of welfare? For instance, expected utility can be computed as a function of each agent's private information or it can be computed prior to the receipt of any private information. Holmstrom and Myerson denote the first concept as *interim* utility and the second one as *ex ante* utility. This paper employs these two measures of welfare to present the results. Ex ante utility in the case where agents receive n signals is computed as follows:

$$E\left(u\right) = \sum_{i=1}^{2} \sum_{j=0}^{n} Pr\left(\phi_i\right) \int_0^1 \binom{n}{j} \nu^j \left(1-\nu\right)^{(n-j)} \left[\phi_i \mathcal{U}^{\bar{j}}\left(\nu, \phi_i\right) + \left(1-\phi_i\right) \mathcal{U}^{\underline{j}}\left(\nu, \phi_i\right)\right] d\nu, \tag{17}$$

where $\mathcal{U}^{\bar{j}}\left(\nu, \phi\right)$ denotes the expected utility of an agent with j good signals and high riskless endowment, conditional on the information possessed at the node (ν, ϕ). Similarly, $\mathcal{U}^{\underline{j}}\left(\nu, \phi\right)$ denotes the conditional expected utility of an agent with a low riskless endowment and j good signals. The conditional expected utility of an agent of type i is computed as follows:

$$\mathcal{U}^i\left(\nu, \phi\right) = E\left[u \mid \mathcal{I}^i, p\left(\nu, \phi\right)\right] = \nu u\left(c_h^i\left(\nu, \phi\right)\right) + \left(1-\nu\right) u\left(c_l^i\left(\nu, \phi\right)\right). \tag{18}$$

The decision rules that govern consumption in both states are the same as in equation (8) on page 19. Note that even though agents are not able to observe ν, the actual state probabilities are used to compute the expected utility of each type. It is easy to check that if those probabilities were replaced by agents' beliefs, the formula would yield the same level of ex ante utility. The formulation in equation (18) is chosen because it stresses that more precise information affects ex ante welfare only through its influence on the consumption allocation rules.

The model considered in this paper assumes that there is no ex ante heterogeneity. Agents differ only after they have received endowments and signals. Thus, ex ante utility consists of a scalar variable. Although the previous measure is informative and allows us to evaluate aggregate welfare, it limits the ability to compare our results with the previous literature (that assumes exogenous heterogeneity). For that reason, we also provide a measure of interim welfare. It is computed after each individual has received his riskless endowment but prior to observing any signal or the market price, namely:

$$\begin{aligned} E\left(u \mid a\right) &= \sum_{i=1}^{2} Pr\left(\phi_i \mid a\right) E\left[u \mid a, \phi_i\right], \qquad \text{where} \qquad (19) \\ E\left[u \mid \bar{a}, \phi_i\right] &= \sum_{i=0}^{n} \int_0^1 \binom{n}{i} \nu^i \left(1-\nu\right)^{(n-i)} \mathfrak{U}^{\bar{i}}\left(\nu, \phi_i\right) d\nu, \text{ and} \\ E\left[u \mid \underline{a}, \phi_i\right] &= \sum_{i=0}^{n} \int_0^1 \binom{n}{i} \nu^i \left(1-\nu\right)^{(n-i)} \mathfrak{U}^{\underline{i}}\left(\nu, \phi_i\right) d\nu. \end{aligned}$$

$Pr\left(\phi_i \mid a\right)$ is computed using Bayes' rule.

The formula above assumes that agents update the probability distribution of ϕ before computing their expected utility. The reason is twofold. First, it is consistent with the rest of the paper, i.e., agents are fully rational and use all the available information when they evaluate their expected utility. Second, and also related to the previous reason, it is consistent with the ex ante utility measure. If the unconditional probability distribution of ϕ had been used in equation (19), it would not have been possible to relate both welfare measures.

5.2 Welfare and the precision of information

Appendix B considers a simpler version of the model, where agents learn only from public signals. It shows that as the number of signals increases, information becomes more precise in Blackwell's sense (see Blackwell [7]). The information structure assumed in this paper is different, though. Instead of learning from a public signal, agents learn from the market price and private information. This leads to a richer model but has the disadvantage that it is no longer possible to apply the standard definition of "better" information used in the literature. The result in Appendix B may support the conjecture that the precision of the information increases with the number of signals. We provide a measure of how "far" the beliefs are with respect to the actual realizations of ν in order to compare two information structures. As will be described at the end of the section, that measure confirms the presumption that receiving more signals implies that the agents possess more accurate information.

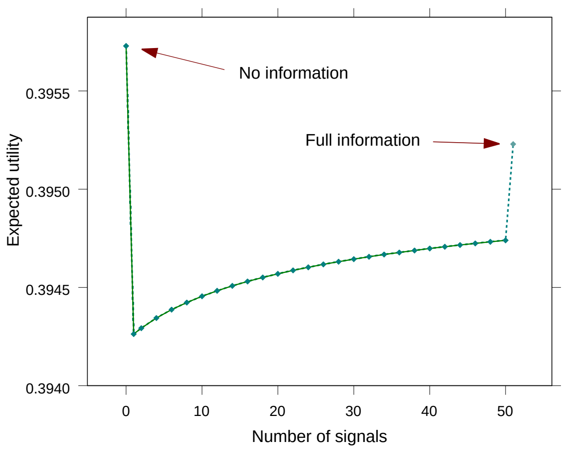

Figure 11: **Welfare of rich agents as a function of the number of signals.**

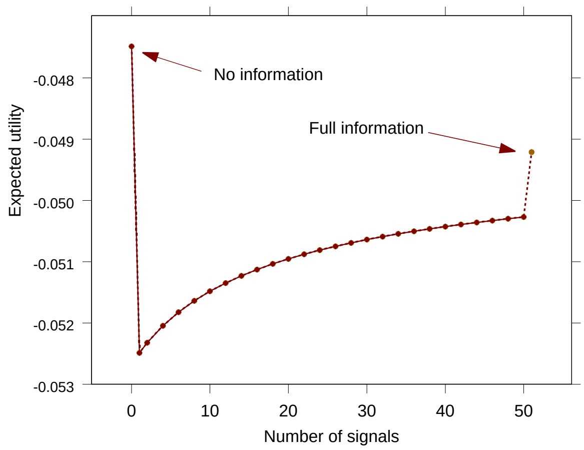

Figure 12: **Welfare of poor agents as a function of the number of signals.**

We are now ready to present the main result of the paper. Figure 11 and Figure 12 show that welfare may be non-monotonic in the precision of information. The relationship between welfare and the number of signals is described by a reverse J curve. This is independent of the endowment realizations, which implies that ex ante welfare follows the same pattern.

The values at the boundaries correspond to the null and complete information cases. It is not surprising that expected utility under full information is lower than under no information. The reason is the following. Uncertainty about ν and ϕ introduces an extra source of risk in the economy. Some agents would like to insure against "bad" realizations of those variables, while others could gain from selling insurance. We consider economies where that market is missing. Agents trade only after observing their private information and market price. Given the previous restriction, the best allocation is attained when agents trade knowing their riskless endowment but with no information about ν.[13] In that economy, even if there was another trading round after the value of ν has been disclosed, there would be no further net trade.[14] Prices would adjust in order to accommodate to the new "belief", but the consumption allocation would remain invariant. This result was first pointed out in Marshall [28]. On the other hand, if markets open after the value of ν has become common knowledge, there are less opportunities to share risk. For instance, if ν takes an extreme value (zero or one), one of the Arrow-Debreu securities is worthless and no trade takes place. This example illustrates a general principle: the better the information agents possess, the lower the possibility to insure against "bad news". This is known as the Hirshleifer effect.

Blackwell was the first to formalize the intuitive result that more information enhances welfare. In his framework, agents receive an informative signal about the state and then choose action a out of a set $\mathcal{A}$. He shows that, as the signal transmits more accurate information, agents enjoy a higher degree of freedom when they decide on the optimal action. In the event that the signal is fully informative, agents can condition their decision on the actual state. Blackwell's result though, relies on the fact that the set $\mathcal{A}$ does not change with the precision of information. Hirshleifer [21] shows that this assumption cannot be maintained in a general equilibrium model. In a competitive environment, the arrival of more precise information about the state of the economy affects the equilibrium prices and through this, it modifies each individual's budget constraint. In fact, as it is stated in the previous paragraph, better information limits the opportunities to share risk, which in turn offsets the Blackwell effect.

Several authors have studied the implications of the Hirshleifer effect in more general environments. More recently, Schlee [34] provides quite general conditions under which the receipt of better information leads to a Pareto-inferior allocation, meaning that no agent is better off, and at least some agents are

[13] The paper restricts attention to economies where agents cannot insure against endowment shocks.

[14] The endowment allocation prior to the second trading round consists of the equilibrium allocation obtained in the first trading round.

worse off (as long as the arrival of more precise information modifies the consumption allocation). Unlike previous papers, Schlee's conclusion does not depend on any specific assumption about agents' initial level of information: his result holds for any informational improvement, not only when agents receive partial information starting from no information.

Figures 11 and 12 point out that there exist endowment economies with competitive markets in which individual welfare may increase with the precision of information. The graphs show that both types of agents enjoy higher expected utility under full information compared to the situation where they receive one signal. The Hirshleifer effect is still present in our model, but the differential information assumption introduces an added adverse effect on trade that is not observed in Schlee's paper.

The dispersion of beliefs is such that it decreases agents' needs to participate in the market. In an equilibrium with homogeneous beliefs, individuals with a high riskless endowment provide insurance to poor agents by selling risk free bonds and purchasing shares of the tree. However, in the economy we study there is disagreement in the beliefs about ν. Rich agents are more pessimistic than poor agents. This feature is summarized in Figure 14. The picture shows that for any realization of ν, poor agents' average beliefs are above the actual ν, while rich agents' average beliefs are below ν. This induces a contraction in the demand and supply of insurance compared to the full information setting. Thus, an economy with partial information displays a lower volume of trade than the economy where ν is common knowledge. The latter accounts for the difference in welfare between the two cases.

More generally, Figures 11 and 12 summarize the interaction between two effects. At the corners we observe the pure Hirshleifer effect, while at the "interior" points the negative effect caused by the heterogeneity in beliefs plays a role. Both forces limit the opportunities to share risk. This explains why the arrival of better information always reduces welfare if agents initially have no information. However, the consequences of receiving better information starting from a situation of partial information are ambiguous. The receipt of more signals strengthens the Hirshleifer effect at the same time that it weakens the adverse effect due to the correlation between beliefs and endowments (as individual beliefs tend to concentrate, the correlation decreases). If the latter effect outweighs the former, we should observe a welfare increase following the arrival of more precise information. This is the situation described in the graph.

If the argument described in the previous paragraph is correct, we should expect welfare and trading

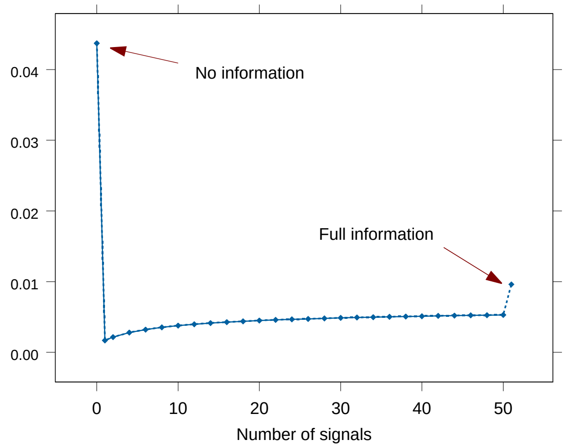

Figure 13: **Aggregate volume of trade as a function of the number of signals.**

volumes to be strongly correlated with each other. Figure 13 confirms this feature. The average volume of trade displays the same reverse J curve as welfare.

We cannot use Blackwell's criterion to compare the degree of precision between different information structures, so we are not able to assess a priori whether receiving more signals implies better information or not. As is expected, though, there is a positive *ex post* relationship between the number of signals received and the accuracy of the beliefs. This is confirmed by Figure 14, which shows that the distance between individual beliefs and the actual realizations of ν shrinks as the number of signals increases. In addition, this result seems to hold for every possible value of ν and ϕ. Figure 15 uses a simple measure to quantify the precision of the beliefs.[15] It also supports the conjecture that more signals imply better information.

[15]The graph summarizes the average distance between individual beliefs and realizations of ν using the following measure:

$$\left(\sum_{i=1}^{2}\sum_{j=0}^{n} Pr\left(\phi_i\right)\int_0^1 \binom{n}{j}\nu^j\left(1-\nu\right)^{n-j}\left[\phi_i\left(\tilde{\nu}^{\bar{j}}-\nu\right)^2+\left(1-\phi_i\right)\left(\tilde{\nu}^{\underline{j}}-\nu\right)^2\right]d\nu\right)^{\frac{1}{2}}.$$

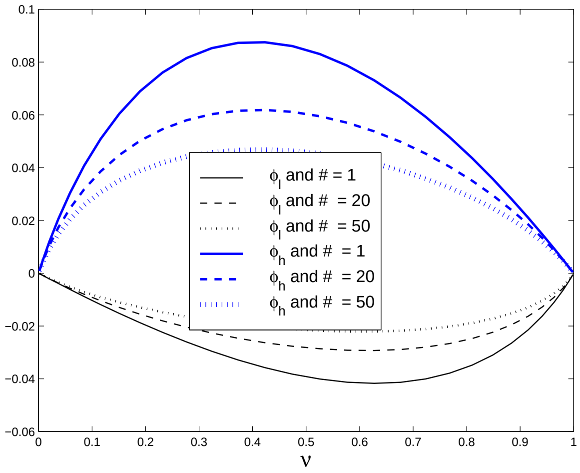

Figure 14: **Difference between average beliefs and actual realizations of ν as a function of the number of signals.**

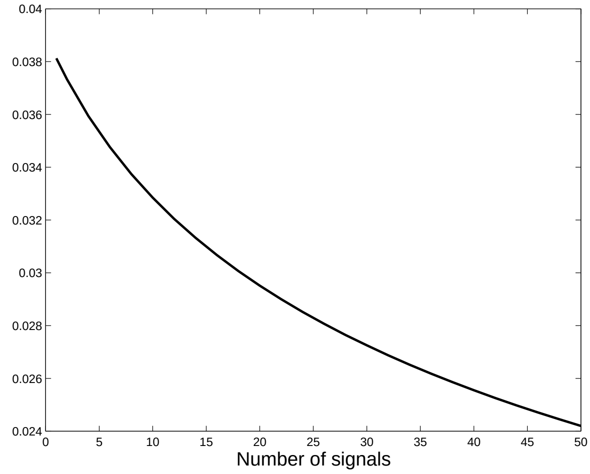

Figure 15: **Precision of beliefs as a function of the number of signals.**

5.3 Incentives to acquire information

We have shown how the receipt of informative signals can hurt everyone in the economy. This result relies on the assumption that agents actually use all the available information to compute their beliefs. This would not be an appealing assumption if each individual found that it is in his own best interest to ignore all or part of the information received. We presume this is not the case in our model. Recall that no agent can exert any influence on the aggregate. This implies that from the point of view of a single agent, the arrival of better information has a pure Blackwell effect, so it is welfare enhancing. The chart below illustrates the private incentives to use information in a particular case. It describes the expected utility of an individual who lives in our benchmark economy: the set of agents who use all the available information to update their beliefs has full measure. Again, agents receive only one signal. It is clear that choosing not to use all the available information leads to decreased welfare.

Similarly, consider an economy where no one uses any available information. In this case, agents' beliefs are homogeneous and coincide with the unconditional expectation of ν. However, if an individual decides to incorporate the information revealed by the signal to update his belief, he will enjoy an increase in welfare (expressed in terms of consumption) of 5.98 percent or 6.11 percent depending on

Individual welfare when different pieces of information are used [16]		
	Rich	Poor
Price, signals, and endowment	100.0000	100.0000
Price and endowment only	99.9966	99.9966
Price and signals only	99.8008	99.8008
Price only	99.7932	99.7931
Signal only	87.5619	87.5619
No information	82.7408	82.7408

Table 3: **Welfare gain associated to each piece of information.**

whether he has received a high or low riskless endowment, respectively.

Although the complexity of the model does not allow us to prove the conjecture that every individual has a private incentive to use all the available information, we show that this is true for the baseline parameterization. This result has an additional implication. If agents were required to pay for the signals and the costs were sufficiently low, every individual would effectively acquire a signal.[17] That would lead to an overacquisition of information due to the Hirshleifer and heterogeneous beliefs effects. A similar result is reported in Berk [5] using a model with strategic behavior.

6 Concluding remarks

Hirshleifer was the first to point out that in a general equilibrium framework, the arrival of better information may leave some agents worse off. Although the result is based on a simple example, his finding has proven to be robust. The present work stresses that those results depend crucially on the existence of homogeneous beliefs. We consider an economy with endogenous heterogeneity, differential information, and partially revealing prices. This paper identifies two effects through which

[16] Welfare is expressed in terms of certainty equivalent consumption. The latter is normalized to 100 in the case where the individual uses all the information available.

[17] For simplicity, we assume at the moment that agents can only purchase one signal.

information affects the equilibrium allocation: the well known Hirshleifer effect and an adverse effect induced by the negative correlation between individual beliefs and endowment realizations. From an ex ante perspective, both effects limit the possibility to share risk and therefore have a negative impact on welfare. In this setup, the arrival of more precise information strengthens the Hirshleifer effect at the same time that it weakens the adverse effect due to the presence of heterogeneous beliefs. This raises the possibility that welfare might be non-monotonic in the precision of information. The paper focuses on the last case. It analyzes an example in which the impact of information on welfare depends on the initial level of information available in the economy: more information increases welfare in an economy with partial information, but it decreases welfare in an economy with no information.

To the best of our knowledge, the mechanism explaining this result has not been explored in the literature. The closest antecedent of this work is Citanna and Villanacci [8]. They also consider an economy with differential information and partially revealing prices. As in this paper, they conclude that welfare may increase upon the arrival of more accurate information. They argue that the result is explained by wealth effects due to changes in relative prices (they study a multiple-goods economy).

The policy implications derived from the present work also differ from the previous literature. If the first best cannot be implemented (agents trade prior to the arrival of information), it may still be possible to find policies that lead to a Pareto-superior allocation. Those policies must induce agents to acquire more information.

From a security design perspective, the example studied in the paper favors a complete market structure. Introducing a new security in a partial information economy would complete the markets and allow agents to fully infer the value of ν. In that case, the second best allocation would be attained. However, a negative relationship between welfare and information may be observed for alternative parameter values or preference specifications.[18] In those economies the conclusion is reversed, and an incomplete financial structure is optimal.[19]

[18]That kind of relationship would be observed in economies where the dispersion of beliefs is not as strong as in our example.

[19]See Marín and Rahi [27] for more on this topic.

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A Technical Appendix

In order to study different properties of the equilibrium, we must find a price function $p(\nu, \phi)$ that satisfies certain conditions. The equilibrium is determined by a particular relationship among price, ν and ϕ, which means that any of these variables can be expressed as a function of the other ones. It simplifies the exposition to write ν as a function of p and ϕ, so we proceed this way. If agents receive only one signal, the function $\nu(\cdot)$ satisfies equation (A.1). The formula follows from the equilibrium price equation (10) on page 20. It also takes the beliefs as exogenously given.

$$
\begin{aligned}
\nu_i(p) &= \nu(p, \phi_i) = \frac{p\left[\phi \bar{a} + (1-\phi)\underline{a} + d_h\right] - \left[\phi \bar{W} \tilde{\nu}^{\bar{0}} + (1-\phi)\underline{W} \tilde{\nu}^{\underline{0}}\right]}{\phi \bar{W}\left(\tilde{\nu}^{\bar{1}} - \tilde{\nu}^{\bar{0}}\right) + (1-\phi)\underline{W}\left(\tilde{\nu}^{\underline{1}} - \tilde{\nu}^{\underline{0}}\right)} \qquad \text{(A.1)} \\
\underline{W} &= \underline{a} + d_l(1-p) + d_h p \\
\bar{W} &= \bar{a} + d_l(1-p) + d_h p
\end{aligned}
$$

More generally, if agents receive n signals, the equilibrium price is given by:

$$
p(\nu, \phi) = \frac{\phi(\bar{a} + d_l)\sum_{h=0}^{n}\binom{n}{h}\nu^h(1-\nu)^{n-h}\tilde{\nu}^{\bar{h}} + (1-\phi)(\underline{a} + d_l)\sum_{h=0}^{n}\binom{n}{h}\nu^h(1-\nu)^{n-h}\tilde{\nu}^{\underline{h}}}{\phi\bar{a} + (1-\phi)\underline{a} + d_h - \left(\sum_{h=0}^{n}\binom{n}{h}\nu^h(1-\nu)^{n-h}\tilde{\nu}^{\bar{h}} + \sum_{h=0}^{n}\binom{n}{h}\nu^h(1-\nu)^{n-h}\tilde{\nu}^{\underline{h}}\right)(d_h - d_l)},
$$

which is nonlinear in ν. The functional form precludes the possibility of obtaining a functional expression for $\nu(\cdot)$, though it is possible to approximate the last function using numerical techniques.

Before describing the numerical algorithm, notice that the object that needs to be found (the function $\nu(\cdot)$) has infinite dimension, whereas numerical techniques only allow us to solve for finite dimensional problems. Our strategy consists of parameterizing $\nu(\cdot)$ as the weighted sum of Chebychev polynomials. Namely,

$$
\begin{aligned}
\nu_j(p) &\simeq \hat{\nu}(p; \vec{a}_j) = \sum_{i=0}^{i=N} a_i^j T_i\left(2\frac{p - \underline{p}}{\bar{p} - \underline{p}} - 1\right) \\
\nu_j'(p) &\simeq \hat{\nu}'(p; \vec{a}_j) = \sum_{i=0}^{i=N} a_i^j T_i'\left(2\frac{p - \underline{p}}{\bar{p} - \underline{p}} - 1\right)\left(\frac{2}{\bar{p} - \underline{p}}\right) \\
for\ j &= l, h,
\end{aligned}
$$

where $\vec{a}_h = \left(a_h^0, a_h^1, ..., a_h^N\right)$ and $\vec{a}_l = \left(a_l^0, a_l^1, ..., a_l^N\right)$ are the corresponding weights and $T_i(\cdot)$ is the Chebychev polynomial of order i.

$$T_n(x) = \cos(n \cos^{-1} x)$$

With this approach, the choice of the polynomial family becomes an important issue. We use Chebychev polynomials because they are mutually orthogonal and allow for an efficient parameterization of $\nu(\cdot)$. Finally, notice that the problem simplifies now to find a finite number of parameters, instead of an entire function. The algorithm that solves the problem is laid down below.

1) A grid for p is defined using the expanded Chebychev array (see Judd [23], page 222). $\vec{p} = (p_0, p_1, ..., p_N)$, where $p_0 = 0$ and $p_N = 1$. The grid defines the points at which the functions $\nu(\cdot)$ and $\hat{\nu}(\cdot)$ are evaluated.

2) A closed form solution exists in the full information economy (see equation (11) on page 20). Thus, a first set of values for the parameters $\vec{a}_h$ and $\vec{a}_l$ is obtained after equating the approximate function $\hat{\nu}(p;\ \vec{a}_j)$ to the known function $\nu^{FR}(\cdot)$ at the N grid points. This gives $2N$ equations in $2N$ unknowns.

3) The values $\left\{a_i^h, a_i^l\right\}_{i=1}^{i=N}$ found in (2) are then used as an initial guess to solve for the system of equations defined by $G(\cdot)$:

$$G_{i\,j}(\vec{a}_h, \vec{a}_l) = \nu_j\left(\hat{\nu}(p_i; \vec{a}_h), \hat{\nu}(p_i; \vec{a}_l), \hat{\nu}'(p_i; \vec{a}_h), \hat{\nu}'(p_i; \vec{a}_l), p_i\right) - \hat{\nu}\left(p_i; \vec{a}^j\right), \quad i = 0, 1, ..., N, \ \ j = l, h \tag{A.2}$$

The function $\nu_i(\nu_h, \nu_l, \nu_h', \nu_l', p)$ refers to the same object as equation (A.1), but the arguments are different. The new formulation does not take the beliefs as exogenously determined. Instead, it includes as additional arguments the equilibrium values of ν consistent with a given price p (and the two possible realizations of ϕ) and the derivatives of the price function, captured by ν_i'. The last four variables are necessary to compute the beliefs, as can be seen in equations (13)-(16).

The system of equations (A.2) summarizes the fixed point problem. The function $G_i^j(\cdot)$ does the following. It takes the set of parameters that defines $\hat{\nu}_h(\cdot)$ and $\hat{\nu}_l(\cdot)$ as arguments. Then, it computes $\nu_h(p_i)$, $\nu_l(p_i)$ and their derivatives by evaluating the corresponding parameterized functions $\hat{\nu}(p;\ \vec{a}_j)$

and $\hat{\nu}'(p;\ \vec{a}_j)$ at each price p_i in the grid. If the approximate functions $\hat{\nu}(\cdot)$ are sufficiently close to the actual equilibrium functions, the evaluation of $\nu_j(\cdot, p_i)$ at the values described will yield a number similar to $\hat{\nu}\left(p_i; \vec{a}^j\right)$. That is, $G_i^j(\cdot)$ will be close to zero. Therefore, the purpose of the algorithm is to find the root of the system of equations defined above.

The root is found using a modified Powell hybrid algorithm and a finite-difference approximation to the Jacobian. This corresponds to routine NEQNF of the IMSL library. A root is defined as a set of parameters$\left\{a_i^j\right\}_{i=l,h\ \ j=0,1,\ldots,N}$ such that

$$\sum_{i=l,h}\sum_{j=0}^{N} G_i^j\left(\vec{a}_h, \vec{a}_l\right)^2 < 10^{-12}.$$

The solution was tested at prices outside the Chebychev nodes defined for q. The average value of $\left|G_i^j\left(\vec{a}_h, \vec{a}_l\right)\right|$ at these prices is never larger than 10^{-5}. The maximum deviation observed is less than 10^{-3} and is always located at prices close to the corners. This result suggests the numerical solution we obtained is very close to the actual one.

B Comparing information structures

Let us introduce the following modifications to the baseline model presented in the paper:

1. The signals about the tree are public.
2. There is no uncertainty regarding the distribution of riskless endowments, i.e., the fraction of highly endowed agents is common knowledge.

Given the above assumptions, the equilibrium price does not reveal any more information relative to the information conveyed by the public signals. The other difference with respect to the model considered in the main text, is that the economy is now characterized by the realization of one variable, ν. Agents try to infer the latter using the information contained in the public signal. More formally, we assume agents observe a signal s from a set $\mathcal{S}$. The latter has finitely many components: $\mathcal{S} = \{s_1, s_2, \ldots\}$. The components of $\mathcal{S}$ are defined as follows:

$s_1 = 1$, agents observe one good signal out of one;

$s_2 = 0$, agents observe one good signal out of one;

$s_3 = (1, 1)$, agents observe two good signals out of two;

$s_4 = (1, 0)$, agents observe one good and bad signal out of two;

$s_5 = (0, 0)$, agents observe two bad signals out of two;

$s_6 = (1, 1, 1,)$, agents observe three good signals out of three;

$\vdots$

Thus, each s_i corresponds to a particular realization of the binary signals structure. The conditional probability distribution over $\mathcal{S}$ is given by a vector $\boldsymbol{\pi}^n(\nu) = (\pi_1^n(\nu), \pi_2^n(\nu), \ldots)$, where

$$\boldsymbol{\pi}^n(\nu) = \begin{cases} (\nu,\ 1-\nu,\ 0,\ \ldots) & if \quad n = 1, \\ \left(0,\ 0,\ \nu^2,\ 2\nu(1-\nu),\ (1-\nu)^2,\ 0,\ \ldots\right) & if \quad n = 2, \\ \vdots & \\ \left(0,\ \ldots,\ 0,\ \binom{N}{N}\nu^N,\ \binom{N}{N-1}\nu^{(N-1)}(1-\nu),\ \ldots,\ \binom{N}{0}(1-\nu)^N,\ 0,\ \ldots\right) & if \quad n = N. \\ \vdots & \end{cases}$$

An information structure consists of a collection of signals and their corresponding probabilities for every possible realization of ν. In the present framework, an information structure is fully specified by the number of binary signals agents observe (n), and it is denoted by $\mathbf{I}^n$, where $\mathbf{I}^n = \{\mathcal{S}, \boldsymbol{\pi}^n(\nu) \quad for\ \nu \in [0,1]\}$. The advantage of this set up is that it allows us to apply Blackwell's criterion to compare the degree of informativeness of the two information structures.

Definition B.1 *Information structure* $\mathbf{I}^n$ *is more informative than* $\mathbf{I}^{n'}$ *if there is a transition probability function* $t(s', s) : \mathcal{S} \times \mathcal{S} \to [0,1]$, *such that* $\sum_{j=1}^{\infty} t(s_j, s) = 1 \quad \forall\ s \in \mathcal{S}$ *and* $\pi_i^{n'}(\nu) = \sum_{j=1}^{\infty} t(s_i, s_j)\, \pi_j^n(\nu) \quad \forall\, \nu \in [0,1]$.

The intuition behind the definition is simple. Each realization s_i under information structure $\boldsymbol{I}^{n'}$ can be interpreted as being obtained from $\boldsymbol{I}^n$ by adding some noise through a process of randomization. In other words, the more informative structure is sufficient for the less informative one.

Proposition B.2 $\boldsymbol{I}^n$ *is more informative than* $\boldsymbol{I}^{n'} \iff n > n'$

Proof. We prove first that $n > n' \Rightarrow \boldsymbol{I}^n$ is more informative than $\boldsymbol{I}^{n'}$. For the sake of simplicity, consider the case where $n' = n-1$. The argument can easily be extended to any $n' < n$. The objective is to find a transition probability matrix $t_{i,j}$ that replicates the probability distribution $\boldsymbol{\pi}^{n'}(\nu)$ after being applied to $\boldsymbol{\pi}^n(\nu)$ for all ν. It can be shown that only $n(n+1)$ coefficients need to be computed. The remaining ones trivially equal zero. The solution satisfies the following system of equations:

$$\begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n+1} \\ t_{21} & t_{22} & \dots & t_{2n+1} \\ \dots & \dots & \dots & \dots \\ t_{n1} & t_{n2} & \dots & t_{nn+1} \end{bmatrix} \begin{bmatrix} \binom{n}{n}\nu^n \\ \binom{n}{n-1}\nu^{n-1}(1-\nu) \\ \vdots \\ \binom{n}{0}(1-\nu)^n \end{bmatrix} = \begin{bmatrix} \binom{n-1}{n-1}\nu^{n-1} \\ \binom{n-1}{n-2}\nu^{n-2}(1-\nu) \\ \vdots \\ \binom{n-1}{0}(1-\nu)^{n-1} \end{bmatrix} \tag{B.1}$$

We let the reader check that the matrix

$$\begin{bmatrix} 1 & \frac{1}{n} & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1-\frac{1}{n} & \frac{2}{n} & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & 1-\frac{n-2}{n} & \frac{n-1}{n} & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & 1-\frac{n-1}{n} & 1 \end{bmatrix}$$

is a solution of system (B.1).

It remains to be shown that if $\boldsymbol{I}^n$ is more informative than $\boldsymbol{I}^{n'}$, then $n > n'$. Assume that $n' = n+1$. If the hypothesis is true, it must be the case then that there exists a solution for the following system of equations:

$$\begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n+1} \\ t_{21} & t_{22} & \dots & t_{2n+1} \\ \dots & \dots & \dots & \dots \\ t_{n+21} & t_{n+22} & \dots & t_{n+2n+1} \end{bmatrix} \begin{bmatrix} \binom{n}{n} \nu^n \\ \binom{n}{n-1} \nu^{n-1} (1-\nu) \\ \vdots \\ \binom{n}{0} (1-\nu)^n \end{bmatrix} = \begin{bmatrix} \binom{n+1}{n+1} \nu^{n+1} \\ \binom{n+1}{n} \nu^n (1-\nu) \\ \vdots \\ \binom{n+1}{0} (1-\nu)^{n+1} \end{bmatrix}.$$

The first equation implies that a linear combination of $\nu^0, \nu, \dots, \nu^{n-1}$ and ν^n must equal ν^{n+1} for all ν in the interval $[0, 1]$. This is clearly not possible. The same argument holds for any $n' > n+1$. ■

It is intuitive that a higher number of binary signals entails more precise information. We have shown above that this is true in a similar setting to the one described in the main text, as well as when Blackwell's criterion is used to compare information structures. Unfortunately, the former criterion cannot be applied to our baseline model. In that framework, the degree of informativeness is endogenously determined.