Working Paper Series

THE FEDERAL RESERVE BANK OF RICHMOND

RICHMOND ■ BALTIMORE ■ CHARLOTTE

Market-Based Regulation

and the Informational Content of Prices[1]

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Abstract

Various laws and policy proposals call for regulators to make use of the information reflected in market prices. We focus on a leading example of such a proposal, namely that bank supervision should make use of the market prices of traded bank securities. We study the theoretical underpinnings of this proposal in light of a key problem: if the regulator uses market prices, prices adjust to reflect this use and potentially become less revealing. We show that the feasibility of this proposal depends critically on the information gap between the market and the regulator. Thus, there is a strong complementarity between market information and the regulator's information, which suggests that regulators should not abandon other sources of information when learning from market prices. We demonstrate that the type of security being traded matters for the observed equilibrium outcome and discuss other policy measures that can increase the ability of regulators to make use of market information.

1 Introduction

A basic premise in economics is that prices in financial markets aggregate useful information. This premise underlies several recent laws and policy proposals that call for regulatory agencies to use the information in market prices when making various decisions. One example is the Sarbanes-Oxley Act of 2002, which changed how publicly traded corporations are governed in the United States. Section 408 of the act calls for the Securities and Exchange Commission to consider market data – namely, share price volatility and price-to-earnings ratios – when deciding whether to review the legality of a firm's disclosures. Another example is class action securities litigation. Courts in the United States use share price changes as a guide for determining damages.[1]

Despite the attraction of such laws and proposals, policymakers must confront the following problem: if regulators use market prices, prices adjust to reflect this use and might become less revealing. In this paper, we analyze the extent to which the feedback from regulatory actions to market prices prevents effective inference in equilibrium. Our analysis focuses on arguably the most important policy proposal of this type, namely, that supervision of banks should make use of the information conveyed by the market prices of traded bank securities. We demonstrate that the feasibility of this policy proposal depends critically on the information gap between the market and the bank regulator. Implementing a successful policy of learning from market prices requires that the regulator has reasonably precise information from other sources.

An important responsibility of bank supervisors is to assess the probability of a bank failing and, if needed and possible, to take actions to reduce it. Presently, a supervisor's main tool for assessing the risk of failure is periodic direct examination. Supervisors evaluate all the important facets of a bank, including financial measures such as the strength of its loan portfolio and balance sheet, but also non-financial measures such as the quality of its management. This direct supervision is expensive. In the United States, the federal and state governments spent nearly 3 billion dollars in 2005 supervising banks and similar

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[1]See, e.g., Cooper Alexander (1994).

institutions such as thrifts and credit unions.[2] There are also limits to direct supervision. Evaluating a bank's balance sheet is more complex than it used to be, and the technical skills for such evaluation are in short supply. In addition, the supervisor may not be able to obtain relevant information held by other market participants. If market prices contain information on a bank's fundamentals, then the supervisor may be able to learn this information from market prices, saving substantial expenses and improving the quality of its information.

For these reasons, many recent proposals call for bank supervisors to use prices of traded bank securities. For example, one of the three pillars of the Basel II reform of capital regulations is the use of market discipline, one form of which is the use of information in market prices.[3] A specific proposal, most recently advocated by Evanoff and Wall (2000), would require a bank to regularly issue subordinated debt, partly so that supervisors could use the price of it to monitor the health of the issuing bank.[4] Finally, many policymakers emphasize the benefits of using information conveyed by market prices. Speaking in 2001, Gary Stern, President of the Federal Reserve Bank of Minneapolis, argued,[5]

> Market data are generated by a very large number of participants. Market participants have their funds at risk of loss. A monetary incentive provides a perspective on risk taking that is difficult to replicate in a supervisory context. Unlike accounting-based measures, market data are generated on a nearly con-

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[2] Number is from authors' calculations. It does not include costs borne by the examined banks themselves. Including these indirect costs would increase the estimated cost substantially.

[3] More generally, market discipline is the reliance on and use of private counterparty supervision to monitor and limit bank risk. Information produced by market participants and reflected in market prices provides one form of market discipline.

[4] Closely related, the Gramm-Leach-Bliley Act (1999) mandates that large banks wishing to engage in certain activities possess at least one outstanding issue of debt receiving a high rating from a credit rating agency (Section 5136A). This rule allows supervisors to make use of the assessment of one type of market participant, namely, credit rating agencies.

[5] See http://www.minneapolisfed.org/pubs/region/01-09/stern.cfm

tinuous basis and to a considerable extent anticipates future performance and conditions. Raw market prices are nearly free to supervisors. This characteristic seems particularly important given that supervisory resources are limited and are diminishing in comparison to the complexity of large banking organizations.

Similarly, according to Alan Greenspan (2001),[6]

> The Federal Reserve and other regulatory agencies already monitor subordinated debt yields and issuance patterns in evaluating the condition of large banking organizations. ... This use of subordinated debt is one example of the effort supervisors should undertake to employ data from a variety of markets.[7]

We evaluate the theory behind these proposals by studying a rational expectations model with a financial market that trades the securities of a bank and with a regulator who can take costly actions that improve a bank's health. Such actions affect the value of the bank's securities. The price that is set in the financial market reflects the expected value of the securities given the information available in the market. The bank regulator decides on his action based on his own information about the bank's fundamentals, and that which he can infer from the prices of the bank's securities.

A key problem in implementing a supervision policy that is based on market prices is the following. If the regulator plans to act on the information the security prices convey about the bank's fundamentals, then the prices will not only reflect information about the fundamentals, but also the effect of the regulator's resulting action. Prices will simultaneously *affect* and *reflect* the regulator's action, so both the prices and the price-dependent

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[6]See: http://www.minneapolisfed.org/pubs/region/01-09/greenspan.cfm

[7]Federal Reserve use of market data is described in Feldman and Schmidt (2003). They surveyed Federal Reserve supervisors to determine whether they used market data. They found that market signals were frequently used by supervisors to help form their overall opinion of a bank as well as to assess the quality of a bank's borrowers. The Federal Deposit Insurance Corporation's use of market data is described in Burton and Seale (2005). They report that market signals are used and that these signals are frequently used in off-site surveillance of banks and can be used to help target more detailed exams. For both bank supervisors, however, the degree to which market signals were used varied across supervisory personnel.

action of the regulator will need to be consistent. This consistency condition can interfere with the ability to infer information in equilibrium. For example, a low price may indicate that the fundamentals are bad and thus call for the regulator's intervention. It may also indicate that the fundamentals are not bad enough to justify intervention, in which case the price is low just because no intervention is expected (this assumes that intervention has a positive effect on the value of the security).

Our analysis highlights that the ability of the regulator to make use of market information depends critically on the quality of his own information. When the regulator has relatively precise information, he is able to learn from market prices and implement his preferred intervention rule as a unique equilibrium. When the regulator's information is moderately precise, additional undesirable equilibria exist in which the regulator intervenes either too much, or too little. Interestingly, in this range, the type of equilibrium – i.e., whether there is too much or too little intervention – depends on whether the traded security has a convex or a concave payoff. Thus, our model generates different implications for learning from the price of equity than for learning from the price of debt. Finally, when the regulator's information is imprecise, he is unable to implement his preferred intervention strategy in equilibrium. Overall, our results suggest that there is a strong complementarity between the regulator's information and the market's information. Thus, regulators should not completely abandon direct examination and should use it together with inference from market prices.

We discuss additional policy measures that help to implement the desired market-based intervention policy, even when the information gap between the market and the regulator is not small. These measures include observing the prices of multiple traded securities, improving the transparency on the side of the regulator, introducing a security that pays off in the event that the regulator intervenes, and taxing security holders to change the effect of the regulator's action on the value of their securities.

As mentioned in our opening paragraph, our analysis is based on the idea that market prices provide useful information to regulators. The usual justification for this is that markets gather information from many different participants who trade on their private

information on bank fundamentals and who do not communicate with the regulators outside the trading process. This idea goes back to Hayek (1945), who argues that markets provide an efficient mechanism for information production and aggregation. The ability of financial markets to produce information that accurately predicts future events has been demonstrated empirically. For example, Roll (1984) shows that private information of citrus futures traders regarding weather conditions is impounded into citrus futures prices, so that prices even improve upon public weather forecasts. In the corporate-finance literature, papers by Luo (2005), Chen, Goldstein and Jiang (2006), and Bakke and Whited (2006) provide evidence that information in prices guides real investment decisions. In the banking literature, many papers document that bank security prices reflect underlying risk (see Flannery [1998] and Furlong and Williams [2006] for surveys) and that markets have information that regulators do not have. For example, works by Krainer and Lopez (2004) and others find that market prices can forecast ratings downgrades by bank supervisors.[8]

Our analysis is also based on the idea that market prices reflect the expected result of the subsequent regulatory action. DeYoung, Flannery, Lang, and Sorescu (2001) find that the market prices of bank securities indeed reflect the likely regulatory actions that result from information generated by bank supervisors. Papers by Covitz, Hancock, and Kwast (2004) and Gropp, Vesala, and Vulpes (2004) find that the connection between market prices and risk depends on the regulatory regime, i.e., if government support of debt holders is more likely, then a weak connection between the price of debt and risk is observed.

Our model belongs to a class of models recently developed in the finance literature, in which an economic agent seeks to glean information from a firm's market price and then takes an action that affects the firm's value. Models of this type were analyzed by Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Subrahmanyam and Titman (1999), Goldstein and Guembel (2005), Bond and Eraslan (2006), and Dow, Goldstein and Guembel (2006). The difficulty in these models stems from the fact that prices *affect* and *reflect* firm

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[8] This effect is not found, however, in all markets; Gilbert, Vaughan, and Meyer (2003) find that prices in the jumbo CD market does not contain information not already contained in the supervisory surveillance model.

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value at the same time – a feature that is missing from the vast majority of models of financial markets, where the value of the firm is assumed to be exogenous.[9] Our model deals with a problem that is absent from existing models of this class: inferring information from the price is complicated by the fact that one price may be consistent with different fundamentals. This feature arises because, unlike in other models in this class, the agent in our model (regulator) acts to maximize his own objective function rather than the value of the traded security.

Three existing papers – namely, Sunder (1989), Bernanke and Woodford (1997), and Birchler and Facchinetti (2004) – note that market-based regulation is prone to an inference problem. They do not, however, analyze and characterize equilibrium outcomes as a function of the information gap between the market and the regulator or of the type of security being traded. This equilibrium analysis along with the resulting policy implications are the main results of our paper. Other papers study different dimensions of market-based regulation. Rochet (2004) and Lehar, Seppi, and Strobl (2005) study how market prices can help the regulator commit to an optimal regulatory policy. Faure-Grimaud (2002) points out that the availability of free stock price information reduces expropriation by the regulator. Morris and Shin (2005) argue that transparency by the central bank may be detrimental as it reduces the ability of the central bank to learn from the market.

The remainder of the paper is organized as follows. In Section 2, we present the model. Section 3 defines an equilibrium in our model. In Section 4, we characterize equilibrium outcomes as a function of the information gap between the market and the regulator and of the type of security being traded. Section 5 discusses policy measures that may restore optimal intervention in equilibrium. Section 6 concludes. All proofs are relegated to the Appendix.

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[9] For example, in papers such as Grossman and Stigltiz (1980), prices may reveal the information possessed by some investors to other investors, but the value of the asset being traded remains unaffected.

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2 The model

The model has one bank, a regulator, and a financial market that trades the bank's securities. There are three dates, $t = 0, 1, 2$. At date 0, the prices of bank securities are determined in the market. At date 1, the regulator may intervene in the bank's operations. Finally, at date 2, all security holders are paid.

2.1 The bank

In the absence of regulatory intervention, the bank's assets generate a gross cash flow of $\theta + \varepsilon - T$ at date 2. The component θ is stochastic and is realized at date 0. It represents the information available in the economy at date 0 regarding the bank's future cash flows. We will often refer to θ as the *fundamental* of the bank. The component ε is also stochastic and is realized at date 2. It represents the component of the bank's cash flow about which no information is available at date 0. The component T is deterministic and can be interpreted as an amount stolen by the bank's manager or any other inefficiency involved in the management of the bank. Throughout, we assume that the fundamental θ is drawn uniformly from some interval $[\underline{\theta}, \bar{\theta}]$. The shock ε is drawn from a single-peaked density function g (the cumulative distribution function is G) with mean 0.

The bank has two types of securities: insured deposits and uninsured claims. We let D denote the face value of deposits. This amount is insured by the regulator, i.e., if the bank does not generate enough cash flow to pay D to depositors, the regulator will provide the resources to ensure that depositors are paid in full. Other security holders are not insured. They receive payment from the bank at date 2 if the bank has resources left after paying D to depositors. The order in which they get paid is determined by a prespecified priority rule.

2.2 The regulator

In practice, a bank regulator who believes that a bank is performing poorly can take several actions to attempt to improve the bank's health. He can directly improve a bank's cash

flow by limiting capital distributions. He can also change its operations by restricting the bank's growth. He can dismiss existing board members and managers and require the hiring of new employees. If the regulator believes the bank is suffering from temporary liquidity problems, he can offer to provide funding at a below-market rate.

Formally, in our model the regulator has an opportunity to intervene in the bank's business at date 1. For tractability, we assume that whatever the specific form of intervention, the effect is to increase the bank's date 2 cash flow by an amount T. Specifically, if the regulator intervenes, the bank's date 2 cash flow is $\theta + \varepsilon$ instead of $\theta + \varepsilon - T$.[10]

When deciding whether to intervene, the regulator weighs the cost against the benefit. We assume a fixed cost of intervention C. The benefit of intervention is that it reduces the regulator's expected payment to insured depositors. Let $\Gamma(\theta)$ denote the expected payment to insured depositors *absent* regulatory intervention as a function of the fundamental θ. It is given by

$$\Gamma(\theta) = E\left[\max\left\{0, D - (\theta + \varepsilon - T)\right\}\right] = G(T - \theta) D + \int_{T-\theta}^{D+T-\theta} (D - (\theta + \varepsilon - T)) g(\varepsilon) d\varepsilon. \tag{1}$$

Then, using $V(\theta)$ to denote the gain to the regulator from intervention, we get

$$V(\theta) \equiv \Gamma(\theta) - \Gamma(\theta + T). \tag{2}$$

If the regulator were fully informed about θ, it would intervene when $V(\theta) > C$ and not when $V(\theta) < C$. Lemma 1 establishes an important property of the function $V(\theta)$:

Lemma 1 *The value of regulatory intervention $V(\theta)$ is first increasing and then decreasing in the fundamental θ.*

Intuitively, at low fundamentals the regulator pays out on its deposit insurance obligation regardless of whether or not it intervenes, while at high fundamentals the bank has

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[10] We assume that the type of intervention performed by the regulator cannot be replicated by the banks' security holders. We make this assumption because bank supervisors have legal powers that enable them to control bank actions. Governance by security holders, on the other hand, is limited, as emphasized by the literature on corporate governance.

enough resources to pay depositors in full, again independent of whether the regulator intervenes. Thus, the value of intervention is maximized at intermediate values of θ, where intervention is most likely to reduce the regulator's deposit insurance obligations. For tractability and to make the problem of economic interest, we assume that

$$V\left(\underline{\theta}\right) - C > 0 > V\left(\bar{\theta}\right) - C. \tag{3}$$

It follows that there exists a unique $\hat{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$ at which the regulator is indifferent between intervening and not intervening, i.e., $V(\hat{\theta}) = C$. For fundamentals below (above) $\hat{\theta}$, a fully informed regulator would strictly prefer to intervene (not intervene).

It is worth stressing that our main results would be qualitatively unchanged if the intervention cost C and/or the gross intervention benefit T were allowed to depend on θ. In particular, our results would still hold if T exceeds C only when bank fundamentals are poor, but C exceeds T when bank fundamentals are good. The key ingredient for our analysis is the existence of a threshold fundamental $\hat{\theta}$, above (below) which the net benefit of intervention to the regulator is negative (positive).

Finally, our focus in this paper is on a regulator that attempts to obtain his preferred outcome (minimizing the costs involved in deposit insurance), as opposed to the maximization of a social welfare function. It is important to note, however, that the two maximization problems coincide when the regulator's funds are raised by distortionary taxes and when the regulator is obliged to provide deposit insurance, for reasons exogenous to the model (such as preventing bank runs).

2.3 The value of bank securities

While the regulator may intervene in the bank to reduce the expected cost of a bail-out, the intervention also affects the value of the uninsured securities. Some uninsured securities may be traded in the financial market at date 0, in which case their price reflects their expected equilibrium value. Before turning to the equilibrium analysis, we characterize the value of securities with and without regulatory intervention.

We let $X\left(\theta\right)$ denote the expected value of a security absent regulatory intervention.

We focus on securities whose value $X(\theta)$ is strictly increasing in θ. A key property for our equilibrium results will be whether $X(\cdot)$ is concave or convex.[11] Two leading examples of uninsured bank securities are equity and debt. The expected value of equity absent intervention is

$$X(\theta) \equiv \int_{D+T-\theta} (\varepsilon - (D+T-\theta))\, g(\varepsilon)\, d\varepsilon,$$

which is easily shown to be strictly increasing and convex in the fundamental θ.[12] Likewise, the expected value of uninsured debt with a face value of B is

$$X(\theta) \equiv \int_{D+T-\theta}^{D+B+T-\theta} \varepsilon - (D+T-\theta)\, g(\varepsilon)\, d\varepsilon + (1 - G(D+B+T-\theta))\, B,$$

which can be shown to be strictly increasing, convex for low fundamentals, and concave for high fundamentals. Economically, the convex then concave shape arises because debt is junior to deposits but senior to residual equity claims. In what follows, we will sometimes refer to a convex security as equity and to a concave security as debt (assuming that we are in the range where debt is concave). We conduct most of our analysis for the case in which the regulator observes the price of only one traded security. In Section 5.2.1, we consider the case in which multiple security prices can be observed.

We denote the expected value of regulatory intervention for investors holding the security by $U(\theta)$. Our paper deals primarily with the case in which intervention affects the value of the security only through its effect on the bank's cash flows. Most of the forms of intervention discussed above fall within this category. In this case, the effect of regulatory intervention is equivalent to an exogenous addition of T to the bank's date 2 cash flow, so

$$U(\theta) = X(\theta + T) - X(\theta). \tag{4}$$

In Section 5.2.4, we discuss a case in which intervention has an additional effect on the value of the security, e.g., by taxing the security holders. As we demonstrate there, such a form

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[11] Our analysis applies equally to securities whose expected value is strictly decreasing in the fundamental θ. In our analysis, a decreasing concave (convex) security is equivalent to an increasing convex (concave) security.

[12] If the bank has also issued debt with a face value of B, the expected value of equity absent intervention is given by an analogous expression, with D simply replaced by $D+B$.

of intervention can resolve the inference problems analyzed in our paper, and this leads to one of our policy implications.

2.4   Information

A key point in our analysis is that the regulator does not know θ, and may learn it from market prices. We assume that the realization of θ is known in the market at date 0, and that it serves as a basis for the price formation. In addition, at date 0, the regulator observes a private noisy signal of θ: $\phi = \theta + \xi$. We assume that ξ, the noise with which the regulator observes the fundamental, is uniformly distributed over $[-\kappa, \kappa]$.

One limitation of our informational structure is that it assumes that the regulator always knows less than the information collectively possessed by market participants (i.e., the information of market participants aggregates to θ, while the regulator only observes a noisy signal of θ). To assess the robustness of the model, we have also analyzed an extension in which the regulator sometimes has more information than the market. We model this by assuming that the regulator sometimes observes $\theta + \varepsilon$ (recall that ε is not observed by the market). Our analysis indicates that this extension does not affect the qualitative results of the model. Details of this analysis are available upon request.

3   Equilibrium

3.1   Market prices

Before making its intervention decision, the regulator possesses two pieces of information: its own signal ϕ and the observed prices of market securities P (if there are several traded securities, P is a vector). An intervention policy is thus a function $I(P, \phi)$, where $I \in [0, 1]$ is the probability of intervention.

For a given intervention policy $I(\cdot, \cdot)$, the price of a traded security incorporates the intervention probability. Specifically, the equilibrium pricing function P satisfies the rational expectations equilibrium (REE) condition

$$P(\theta) = X(\theta) + E_\phi [I(P(\theta), \phi) | \theta] U(\theta) \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}]. \tag{5}$$

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The first component in this expression is the expected value of the security absent regulatory intervention given the fundamental θ. The second component is the additional value stemming from the possibility of regulatory intervention, the probability of which depends on the price $P(\theta)$ and the regulator's own signal ϕ.

3.2 Time consistency

For most of the analysis in the paper (Section 5.2.5 is the exception), we also require the regulator's intervention policy to constitute a "best response" to the market price. That is, we require the intervention policy to be *time consistent*. This implies that the regulator intervenes with probability 1 when the expected benefit from intervention is greater than the cost and intervenes with probability 0 when the expected benefit is smaller than the cost. Thus, under time consistency, I is either 0 or 1, except for the case in which the expected benefit is exactly equal to the cost. In this case, the regulator may choose to play a mixed strategy.

For a given pricing function $P(\cdot)$, the observation of a particular price P tells the regulator that the market observed a fundamental θ' such that $P(\theta') = P$. Formally, the intervention policy $I(\cdot, \cdot)$ is time consistent given a pricing function $P(\cdot)$, when for all equilibrium realizations $\left(\tilde{P}, \tilde{\phi}\right)$ of the price-signal pair,

$$I\left(\tilde{P}, \tilde{\phi}\right) = \begin{cases} 1 & \text{if } E_\theta\left[V(\theta)\,|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}\right] > C \\ 0 & \text{if } E_\theta\left[V(\theta)\,|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}\right] < C \end{cases}. \tag{6}$$

(Note that if the expected value of intervention exactly equals the cost C, any probability of intervention is time consistent.)

3.3 Equilibrium definition

The formal definition of an equilibrium is as follows:

Definition 1 *A pricing function $P(\cdot)$ and an intervention policy $I(\cdot, \cdot)$ together constitute an equilibrium if they satisfy the REE condition (5) and the time-consistency condition (6).*

4 Market-based intervention

We now explore the equilibrium outcomes when there is one traded security and the regulator attempts to learn the fundamental θ from the price of this security. Along the way, we will make a distinction between the equilibrium outcomes for a convex security (e.g., equity) and those for a concave security (e.g., debt). We start by defining an important class of equilibria for which the regulator can perfectly infer the market's information.

Definition 2 *A fully revealing equilibrium is an equilibrium in which each price is associated with one fundamental, and thus the fundamental can be inferred from the price. An equilibrium is essentially fully revealing if when a price is associated with more than one fundamental the regulator can still distinguish among the different fundamentals based on his private information.*

In both fully revealing and essentially fully revealing equilibria, the regulator chooses the optimal action based on θ: he intervenes when θ is less than the cutoff value $\hat{\theta}$ and does not intervene when θ is above $\hat{\theta}$. Thus, we will sometimes refer to fully revealing and essentially fully revealing equilibria as equilibria with *optimal intervention*. Clearly, any other equilibrium does not feature optimal intervention since in such equilibrium, the combination of the price and the regulator's private signal does not enable the regulator to infer θ perfectly.

From (4), the price function for the security under optimal intervention is given by

$$
P\left(\theta\right) = \begin{cases} X\left(\theta + T\right) & \text{if } \theta < \hat{\theta} \\ X\left(\theta\right) & \text{if } \theta > \hat{\theta} \end{cases}.
\tag{7}
$$

This function is graphically depicted in Figure 1. (Note that Figure 1 and the other figures in the paper are only schematic. In particular, the functions X and $X + U$ need not be linear.)

Inspection of Figure 1 reveals the difficulty in obtaining an equilibrium with optimal intervention when the security holder's gain from intervention satisfies (4). The difficulty stems from the fact that under optimal intervention, the price function is non-monotone and

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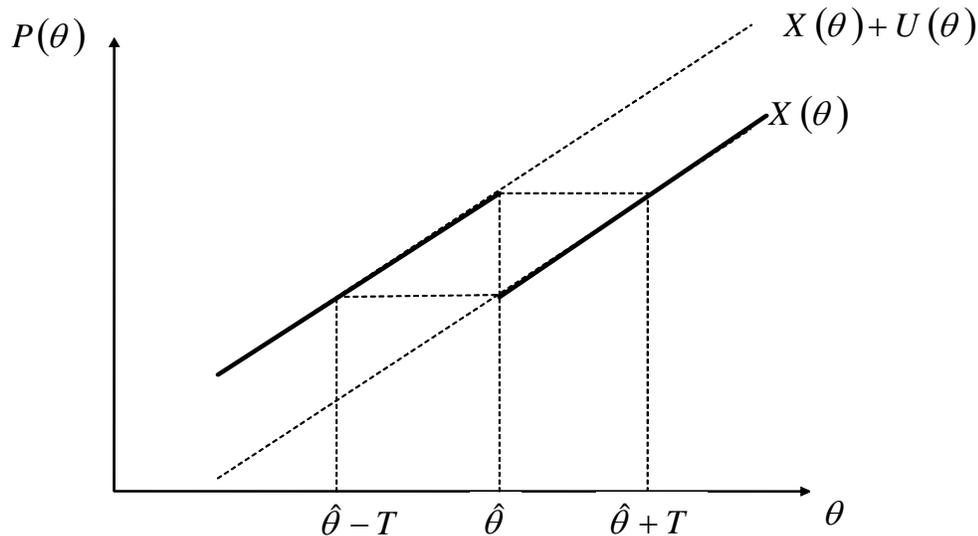
Figure 1: Security price under optimal intervention

that the non-monotonicity occurs around $\hat{\theta}$. This is a result of the optimal intervention rule employed by the regulator — to intervene only when the fundamentals are below $\hat{\theta}$ — and the increase in the value of the security from intervention. As a result of this non-monotonicity, fundamentals on both sides of $\hat{\theta}$ have the same price, so the regulator can infer neither the level of the fundamental, nor the the optimal action, from the price alone. Essentially, the fact that the price reflects the expected reaction of the regulator to the price makes learning from the price more difficult. Following this logic, the possibility of obtaining the optimal intervention rule in equilibrium depends on the precision of the regulator's private signal. A precise private signal will enable the regulator to distinguish between different fundamentals that have the same price. We now provide a complete analysis of equilibrium outcomes based on the precision of the regulator's signal.[13]

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[13] The above discussion should make clear that the key force that makes inference hard in our model is the non-monotonicity of the value of bank securities under optimal intervention. In our model, non-monotonicity arises in part from the discreteness of the intervention decision, but this feature is certainly not necessary for non-monotoncity.

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4.1 The regulator's signal is precise: unique equilibrium

We start with the case in which the regulator's signal ϕ is relatively precise. Our first result is as follows:

Proposition 1 *For $\kappa < T/2$, an equilibrium with optimal intervention exists.*

The intuition behind this result is that under the optimal intervention rule, there are at most two fundamentals associated with each price. Moreover, these fundamentals are at a distance T from each other (see Figure 1). Since the regulator's signal is relatively precise, the regulator can use the signal to perfectly infer the realization of the fundamental when the price is consistent with two different fundamentals. Thus, he can follow the optimal intervention rule. It is worth stressing that in this equilibrium, both the price and the private signal serve an important role: the price tells the regulator that one of two different fundamentals may have been realized, while the private signal enables the regulator to differentiate between these two fundamentals. Thus, the regulator uses both the price and the private signal to infer the underlying fundamental.

Our next result shows that if the regulator's signal is sufficiently accurate whenever two fundamentals have the same equilibrium price, the regulator's signal is sufficient to distinguish them. As such, the optimal intervention equilibrium is the only equilibrium. Although intuitive, the proof of this result is involved. The key difficulty is the need to rule out equilibria in which there are an infinite number of fundamentals associated with the same price.

Proposition 2 *There is a $\bar{\kappa} > 0$, such that regardless of whether the traded security is convex or concave, when $\kappa \leq \bar{\kappa}$ the optimal intervention equilibrium is the unique equilibrium.*

4.2 The regulator's signal is moderately precise: multiple equilibria

As the regulator's information precision worsens, in the sense that κ increases beyond the point $\bar{\kappa}$ defined by Proposition 2 and approaches $T/2$, the optimal intervention equilibrium remains an equilibrium. However, additional and less desirable equilibria emerge. In such
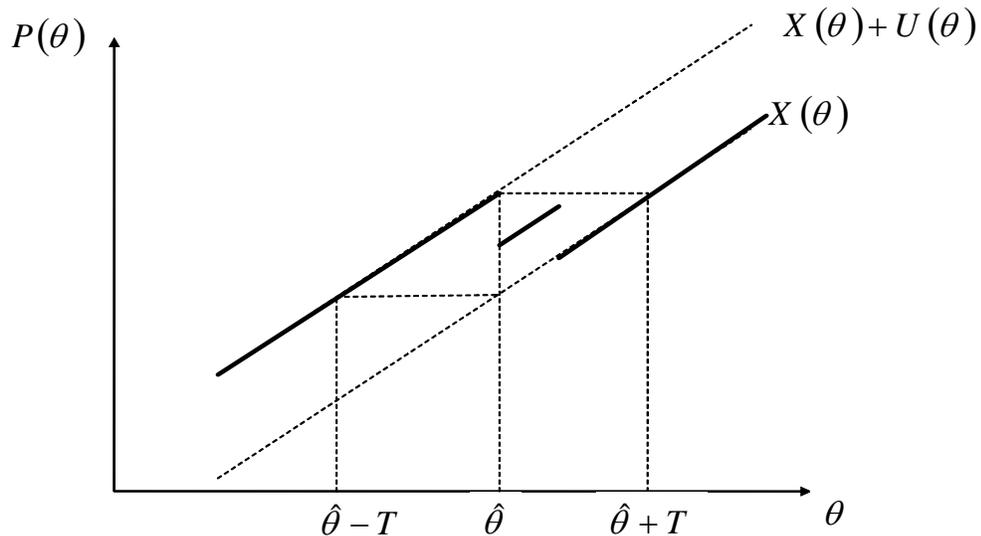
Figure 2: Security price in an equilibrium with too much intervention

equilibria, the regulator cannot perfectly infer the fundamental from market prices. As we will establish, the form of these non-optimal intervention equilibria depends critically on whether the expected security payoff X is convex or concave. Specifically, if X is convex, then these alternate equilibria feature excessive intervention, while if X is concave, the reverse is true.

4.2.1 The case of a convex security: equilibrium with too much intervention

We consider first the case in which the regulator observes the market price of a convex security. We establish that, in this case, there are equilibria in which the regulator intervenes too much relative to the optimal intervention equilibrium depicted in Figure 1. Figure 2 depicts an example of such an over-intervention equilibrium.

As we can see in Figure 2, in this equilibrium, the regulator always intervenes when the fundamentals are below $\hat{\theta}$ (the left line in the price function), never intervenes at *some* fundamentals above $\hat{\theta}$ (the right line in the price function), and intervenes with positive probability for other fundamentals above $\hat{\theta}$ (the middle line in the price function). This

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last feature reflects that the regulator intervenes too much in this equilibrium. This happens because, at the fundamentals associated with the middle line, the price does not fully reveal that the fundamentals are above $\hat{\theta}$, even after the price is combined with the regulator's private signal. That is, the distance between the middle line and the left line is too small for the regulator to be able to differentiate between fundamentals associated with these lines by using its private signal, since $\kappa > \bar{\kappa}$. Because at fundamentals associated with the middle line the regulator cannot rule out that the fundamental is below $\hat{\theta}$, he intervenes with positive probability. This increases the price at these fundamentals. In turn, it is this price increase that pushes the left and middle lines closer together, preventing the regulator from being able to distinguish them.

We now formally prove the existence of an equilibrium with too much intervention. The main result here is Proposition 3, which establishes the existence of such an equilibrium and provides a full characterization of it. We start with some algebra leading to the proposition. In an equilibrium with too much intervention, over-intervention occurs for fundamentals lying in some interval to the right of $\hat{\theta}$. As a first step, consider what is required to generate too much intervention at a point $\hat{\theta}_+$ infinitesimally to the right of $\hat{\theta}$. Over-intervention occurs if the market price at $\hat{\theta}_+$ is the same as the market price at some $\theta \in \left(\hat{\theta}_+ - T, \hat{\theta}\right)$. The probability of intervention at $\hat{\theta}_+$ in this case is the probability that the regulator receives a signal ϕ that is consistent with the fundamental θ:

$$\Pr\left(\phi \leq \theta + \kappa | \hat{\theta}_+\right) = \frac{\theta + \kappa - \hat{\theta}_+ - (-\kappa)}{2\kappa} = 1 - \frac{\hat{\theta}_+ - \theta}{2\kappa}.$$

As such, the market prices at $\theta < \hat{\theta}$ and $\hat{\theta}_+$ coincide if and only if

$$X\left(\hat{\theta}_+\right) + \left(1 - \frac{\hat{\theta}_+ - \theta}{2\kappa}\right) U\left(\hat{\theta}_+\right) = X\left(\theta\right) + U\left(\theta\right). \tag{8}$$

We first show that there is a fundamental θ satisfying (8) whenever the security is convex, $2\kappa < T$, and 2κ is sufficiently close to T:

Lemma 2 *For $\kappa < T/2$ sufficiently close to $T/2$, there exists a unique $\theta_{01} < \hat{\theta}$ such that (8) holds.*

Lemma 2 establishes that too much intervention is possible at a point immediately to the right of $\hat{\theta}$. The convexity of the security is important here. To see why, let us inspect equation (8). Obviously, the LHS in (8) is equal to the RHS when $\theta = \hat{\theta}_+$. Moreover, because $2\kappa < T$, the LHS is smaller than the RHS when $\theta = \hat{\theta}_+ - 2\kappa$.[14] Since the LHS is linear and increasing in θ, a necessary condition for (8) to have a solution below $\hat{\theta}$ is that its RHS is convex in θ. This implies that the security has to be convex for too much intervention to occur.

Using the result in Lemma 2, the following proposition fully establishes and characterizes the equilibrium. To better understand the proposition, it is useful to look again at Figure 2, which depicts an equilibrium of the type described here. As Figure 2 shows, there are at most three fundamentals associated with each price. If one takes this property as given, it is straightforward to evaluate the intervention probabilities and to then show that multiple fundamentals are associated with the same price. The key difficulty that we deal with in the proof is to show that, in equilibrium, there are indeed at most three fundamentals associated with each price.

Proposition 3 *Suppose the regulator observes the price of a convex security. For $\kappa < T/2$ sufficiently close to $T/2$, there exist equilibria in which the regulator intervenes with positive probability at fundamentals above $\hat{\theta}$. In particular, there exists an interval $(\theta_{01}, \theta_{11})$ in which θ_{01} is as defined in Lemma 2 and $\theta_{11} < \hat{\theta}$ such that the following holds:*
For any set $Y_1 \subset [\theta_{01}, \theta_{11}]$, there exists a strictly increasing function $\theta_2^* : Y_1 \to \Re$ such that $\theta_2^*(\theta_{01}) = \hat{\theta}$, and such that the following is an equilibrium:

1. *[Optimal intervention below $\hat{\theta}$] If $\theta \leq \hat{\theta}$, the regulator intervenes with probability 1, and the price is $X(\theta) + U(\theta)$.*

2. *[Over-intervention for some $\theta > \hat{\theta}$] If $\theta \in \theta_2^*(Y_1)$ the regulator intervenes with probability $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa} > 0$, and the price is $X(\theta) + \left(1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}\right) U(\theta)$.*

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[14] The RHS equals $X\left(\hat{\theta}_+ - 2\kappa\right) + U\left(\hat{\theta}_+ - 2\kappa\right) = X\left(\hat{\theta}_+ - 2\kappa + T\right)$, while the LHS equals $X\left(\hat{\theta}_+\right)$, which is smaller.

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3. *[Optimal intervention for some $\theta > \hat{\theta}$] If $\theta > \hat{\theta}$ and $\theta \notin \theta_2^*(Y_1)$, the regulator never intervenes, and the price is $X(\theta)$.*

The result in Proposition 3 begs the question of whether, when the regulator observes the price of a convex security, there also exist equilibria in which too little intervention occurs. The following proposition provides a negative answer to this question. Any equilibrium entails too much intervention in the following sense:

Proposition 4 *Suppose the regulator observes the price of a convex security, and $\kappa < T/2$. Then any equilibrium other than the optimal intervention equilibrium entails a strictly positive probability of intervention at some fundamental $\theta > \hat{\theta}$.*

4.2.2 The case of a concave security: equilibrium with too little intervention

We now consider the case in which the regulator observes the market price of a concave security. Parallel but opposite results hold in this case relative to the case of a convex security. Specifically, we establish that in this case there are equilibria in which the regulator intervenes too little relative to the optimal intervention equilibrium depicted in Figure 1. Figure 3 depicts an example of such an under-intervention equilibrium.

As we can see, the equilibrium depicted in Figure 3 exhibits too little intervention. The regulator intervenes optimally at fundamentals associated with the left line and the right line of the pricing function, but intervenes too little at fundamentals associated with the middle line. This happens because at these fundamentals, the price does not fully reveal that the fundamental is below $\hat{\theta}$, even after it is combined with the regulator's private signal.

Proposition 5 establishes the existence of equilibria with too little intervention and provides a full characterization of them. The proof is parallel to the proof of Proposition 3. To clarify the mathematical intuition and the role of concavity for equilibria with too little intervention, we now go over the first steps leading to the proof. Consider what is required to generate too little intervention at a point $\hat{\theta}_-$ infinitesimally to the left of $\hat{\theta}$. Under-intervention occurs if the market price at $\hat{\theta}_-$ is the same as the market price at some $\theta \in \left(\hat{\theta}, \hat{\theta}_- + T\right)$. The probability of intervention at $\hat{\theta}_-$ in this case is the probability that
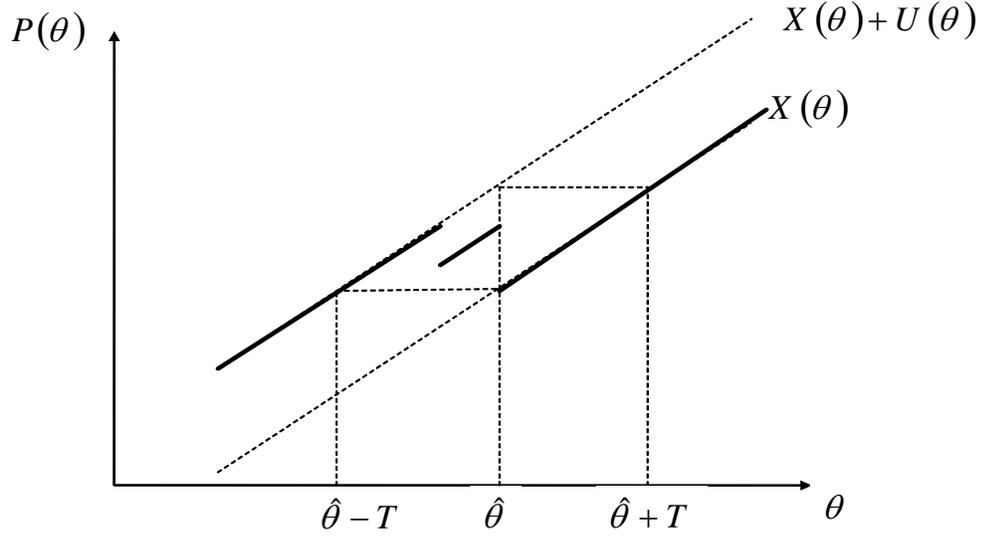
Figure 3: Security price in an equilibrium with too little intervention

the regulator receives a signal ϕ that is not consistent with the fundamental θ:

$$\Pr\left(\phi \leq \theta - \kappa | \hat{\theta}_-\right) = \frac{\theta - \kappa - \hat{\theta}_- - (-\kappa)}{2\kappa} = \frac{\theta - \hat{\theta}_-}{2\kappa}.$$

As such, the market prices at $\theta > \hat{\theta}$ and $\hat{\theta}_-$ coincide if and only if

$$X\left(\hat{\theta}_-\right) + \left(\frac{\theta - \hat{\theta}_-}{2\kappa}\right) U\left(\hat{\theta}_-\right) = X\left(\theta\right). \tag{9}$$

For this equation to hold, the value of the security has to be concave. To see why, note that the LHS in (9) is equal to the RHS when $\theta = \hat{\theta}_-$. Moreover, because $2\kappa < T$, the LHS is larger than the RHS when $\theta = \hat{\theta}_- + 2\kappa$. Since the LHS is linear and increasing in θ, a necessary condition for (9) to have a solution above $\hat{\theta}$ is that its RHS is concave in θ. This implies that the security has to be concave for too little intervention to occur. We now turn to the proposition.

Proposition 5 *Suppose the regulator observes the price of a concave security. For $\kappa <$ $T/2$ sufficiently close to $T/2$, there exist equilibria in which the regulator intervenes with*

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probability less than one at fundamentals below $\hat{\theta}$. In particular, there exists an interval $(\theta_{02}, \theta_{12})$ where $\theta_{02} > \hat{\theta}$ such that the following holds:

For any set $Y_2 \subset [\theta_{02}, \theta_{12}]$, there exists a strictly increasing function $\theta_1^* : Y_2 \to \Re$ such that $\theta_1^* (\theta_{12}) = \hat{\theta}$, and such that the following is an equilibrium:

1. *[Optimal intervention above $\hat{\theta}$] If $\theta \geq \hat{\theta}$, the regulator intervenes with probability 0, and the price is $X(\theta)$.*

2. *[Under-intervention for some $\theta < \hat{\theta}$] If $\theta \in \theta_1^*(Y_2)$ the regulator intervenes with probability $\frac{\theta_1^{*-1}(\theta) - \theta}{2\kappa} > 0$, and the price is $X(\theta) + \frac{\theta_1^{*-1}(\theta) - \theta}{2\kappa} U(\theta)$.*

3. *[Optimal intervention for some $\theta < \hat{\theta}$] If $\theta < \hat{\theta}$ and $\theta \notin \theta_1^*(Y_2)$, the regulator always intervenes, and the price is $X(\theta) + U(\theta)$.*

To complete the analysis of a concave security, the next proposition provides a result that is parallel to the one in Proposition 4. Essentially, for the case of a security with a concave payoff, any equilibrium that does not exhibit optimal intervention has too little intervention.

Proposition 6 *Suppose the regulator observes the price of a concave security, and $\kappa < T/2$. Then any equilibrium other than the optimal intervention equilibrium entails an intervention probability strictly less than 1 at some fundamental $\theta < \hat{\theta}$.*

4.2.3   Convex vs. concave: some intuition

The results we just described demonstrate that the shape of the security matters for the type of equilibrium one can expect. We now provide some intuition for these results.

Why does under-intervention never occur when the regulator tracks the price of a convex security? In general, under-intervention occurs when the regulator intervenes with probability less than 1 at some fundamental $\theta_1 < \hat{\theta}$, and probability 0 at some fundamental $\theta_2 > \hat{\theta}$ with a matching price. To gain intuition, it is useful to consider what is required for the intervention probability at θ_1 to equal $1/2$. First, observe that the intervention

probability at θ_1 is $\Pr(\theta_1 + \xi < \theta_2 - \kappa)$. Given our uniformity assumption, we can evaluate this expression explicitly as $\frac{\theta_2 - \theta_1}{2\kappa}$, and so the intervention probability at θ_1 is $1/2$ only if $\theta_2 - \theta_1 = \kappa < T/2$. Although the specifics of this calculation clearly depend on our distributional assumptions, the general point is that when the regulator's signal is relatively accurate, the fundamentals θ_1 and θ_2 must be close to each other for the intervention probability at θ_1 to equal $1/2$. But then, since θ_2 is relatively close to θ_1, if X were convex, $X(\theta_2)$ would certainly lie below the average of $X(\theta_1)$ and $X(\theta_1 + T)$. This implies that the prices at θ_1 and θ_2 do not coincide as the price at θ_1 in the proposed equilibrium is exactly the average of $X(\theta_1)$ and $X(\theta_1 + T)$, and the price at θ_2 is $X(\theta_2)$. As such, X must be concave for an equilibrium of this type to exist. More generally, an under-intervention equilibrium only exists if X is concave.

For the over-intervention case, a parallel argument applies. This time, consider what is required for an equilibrium with an intervention probability of 1 at $\theta_1 < \hat{\theta}$ and $1/2$ at $\theta_2 > \hat{\theta}$, with matching prices at the two fundamentals. Parallel to the previous analysis, θ_1 and θ_2 must lie at a distance of only $\kappa < T/2$ apart. For the prices to match, the average of $X(\theta_2)$ and $X(\theta_2 + T)$ must equal $X(\theta_1 + T)$. But this cannot occur if X is concave, for since $\theta_1 + T$ is relatively close to $\theta_2 + T$, certainly $X(\theta_1 + T)$ exceeds the average of $X(\theta_2)$ and $X(\theta_2 + T)$.

4.3 The regulator's signal is imprecise: no equilibrium

Finally, consider the case when $\kappa > T/2$, that is, when the regulator's signal is imprecise so that the information gap between the market and the regulator is large. The first thing to note is that when $\kappa > T/2$, optimal intervention cannot occur in equilibrium. To see this, look again at Figure 1. As we can see in the figure, in an equilibrium with optimal intervention, there are fundamentals at a distance of T from each other on both sides of $\hat{\theta}$ that have the same price. Since the regulator's signal is imprecise, i.e., since $2\kappa > T$, the signal does not enable the regulator to always distinguish between two fundamentals that have the same price. Thus, given a price that is associated with two fundamentals, it is impossible for the regulator to always intervene at one fundamental and never intervene at

the other and therefore optimal intervention cannot occur.

Our main result in this subsection is in fact much stronger. Proposition 7 shows that when $\kappa > T/2$, not only is there no equilibrium with optimal intervention, but there is also no other rational-expectations time-consistent equilibrium.

Proposition 7 *Suppose that the regulator's information is relatively poor ($\kappa > T/2$) and that it observes the price of a single security. Regardless of whether the security's expected payoff is convex or concave in the fundamental θ, no equilibrium exists.*

Although the proof of Proposition 7 is long and involved, in the limiting case in which the regulator receives no information at all (i.e., $\kappa \to \infty$) it is possible to give the following straightforward and intuitive proof. First, we claim that the only candidate equilibrium in this case is one with fully revealing prices. To see this, suppose instead that there is an equilibrium in which two fundamentals θ_1 and $\theta_2 \neq \theta_1$ are associated with the same price. Since the regulator has no information, its intervention policy must be the same at θ_1 and θ_2. But then the prices are not equal, giving a contradiction. (It is important to note that both Proposition 7 and this simple limit argument cover mixed strategies by the regulator.) However, there is no fully revealing equilibrium either: given time-consistency, a fully revealing equilibrium features optimal intervention, a possibility ruled out by the text preceding Proposition 7.

5   Policy implications: implementing market-based intervention

5.1   Main implication: the importance of the regulator's information

The results in the previous section demonstrate the difficulty in implementing an intervention policy that is based on the market price of a bank's security. The problem stems from the fact that the market price adjusts to reflect the expected regulator's action, and this reduces the extent to which the fundamental, which is what the regulator is trying to learn, can be inferred from the price.

A key determinant of whether optimal intervention can be implemented based on market price is the quality of the private signal that is observed by the regulator in addition to the price. We show that when the regulator has a very precise signal, optimal intervention is obtained as a unique equilibrium. When the regulator has a moderately precise signal, optimal intervention is still an equilibrium, but there are also other equilibria with suboptimal intervention. Finally, when the regulator's signal is imprecise, there is no equilibrium in the model. Of course, one has to be cautious when interpreting a no-equilibrium result. At the very least, however, we believe it points to the problem associated with implementing a market-based intervention when the quality of the regulator's own signal is poor.

For the environment studied in the previous section, these results imply that there is a strong complementarity between the market's information and the regulator's information. To be able to implement a successful market-based intervention policy, the regulator still needs to produce a reasonably precise signal of his own. Thus, market-based intervention cannot perfectly substitute for direct supervision but instead is a complement.

In the following subsection, we study whether there are alternatives to the regulator generating a precise private signal for which market-based intervention will work. The first alternative we consider is for the regulator to learn from the prices of multiple securities. The second alternative is to improve transparency by disclosing the regulator's signal to the market. The third alternative is to issue a security that directly predicts whether the regulator is going to intervene. The fourth alternative is to impose taxes on security holders that change their payoffs in case of regulatory intervention. Finally, we consider the possibility that the regulator can commit ex ante to a policy rule based on the realized price.

5.2 Other implications

5.2.1 Multiple securities

Thus far we have restricted attention to the case in which the regulator observes only one price, that of a convex security (e.g., equity), *or* that of a concave security (e.g., debt). A key question is whether it helps if both these securities trade publicly, and the regulator

learns from the prices of both.

It turns out that observing the prices of both securities resolves the problem of multiple equilibria when the regulator's signal is moderately precise, but does not solve the problem of no equilibrium when the regulator's signal is imprecise. We start by proving the first result.

Proposition 8 *Suppose that $\kappa < T/2$ and that the regulator observes the price of both a strictly concave and a strictly convex security. Then the optimal intervention equilibrium is the unique equilibrium.*

To gain intuition for this result, recall the results of the previous section. There, we showed that when the regulator's information is moderately precise, there may exist equilibria with too much or too little intervention, in addition to the equilibrium with optimal intervention. We also showed that an equilibrium with too much intervention requires that the security whose information the regulator observes be convex, while an equilibrium with too little intervention requires that the security be concave. Thus, in this range, observing both the price of a concave security and the price of a convex security eliminates the equilibria with suboptimal intervention and leaves the optimal intervention equilibrium as the unique equilibrium.

Although in theory the observation of multiple security prices helps the regulator infer the bank's fundamental, in practice the feasibility of this measure clearly depends on the existence of liquid and well-functioning markets in distinct securities. Additionally, even multiple security prices do not help the regulator when his own information is of low quality:

Proposition 9 *Suppose that $\kappa > T/2$ and that the regulator observes the price of both a concave and a convex security. Then no equilibrium exists.*

Essentially, once equilibrium fails to exist with one traded security, it will not be generated by adding another security.

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5.2.2 Transparency

We now return to the case of one traded convex or concave security and assume that the regulator makes public its own signal ϕ *before* the market price is formed. Our analysis implies that this form of transparency improves the regulator's ability to make use of market information. Specifically, transparency resolves the problem of multiple equilibria when the regulator's signal is moderately precise, but it does not solve the problem of no equilibrium when the regulator's signal is imprecise. The argument is as follows.

Under the "transparency" regime in which the regulator truthfully announces his signal ϕ, the equilibrium pricing function depends on both the fundamental θ and the regulator signal ϕ. Consider a specific realization ϕ^* of the regulator's signal, along with any pair of fundamentals θ_1 and θ_2 such that ϕ^* is possible after both. The prices at (θ_1, ϕ^*) and (θ_2, ϕ^*) must differ. If, instead, the prices coincided, the intervention decisions would also coincide, but in this case the prices would not be equal after all. It follows that all fundamentals θ for which the regulator's signal ϕ^* is possible must have different prices given realization ϕ^*, that is, given ϕ^* prices are fully revealing. This argument together with time-consistency implies that the only candidate equilibrium features optimal intervention. As such, transparency eliminates the suboptimal equilibria of Propositions 3 and 5. The intuition is that these equilibria were based on the market not knowing the regulator's action, a problem that is solved once the regulator discloses its signal truthfully.

Now, when $\kappa < T/2$, optimal intervention is indeed an equilibrium, with prices $P(\theta, \phi) = X(\theta) + U(\theta)$ for $\theta \leq \hat{\theta}$ and $P(\theta, \phi) = X(\theta)$ for $\theta > \hat{\theta}$. On the other hand, when $\kappa > T/2$, optimal intervention is not an equilibrium. To see this, if we suppose to the contrary that it was an equilibrium, then there exist fundamentals θ_1 and θ_2 and a regulator's signal realization $\phi \in [\theta_1 - \kappa, \theta_1 + \kappa] \cap [\theta_2 - \kappa, \theta_2 + \kappa]$ such that (θ_1, ϕ) and (θ_2, ϕ) have the same price, in contradiction to above. It follows that for $\kappa > T/2$, there is no equilibrium.

Although a policy of transparency improves the regulator's ability to infer bank fundamentals from market prices, in practice there may be limits to its viability. In particular, if a bank knows that the regulator will make its information public, it may be less inclined to grant easy access to the regulator in the first place. In this sense, it is possible that

transparency would serve to increase κ, potentially making the regulator's inference problem worse instead of better.

5.2.3 Regulator security

Neither of the measures discussed so far allows the regulator to infer the bank's fundamental when its own information is poor $(\kappa > T/2)$. The next possibility we discuss is the creation of an "event market" in which market participants trade a security that pays 1 if the regulator intervenes, and 0 otherwise. We will refer to such a security as a *regulator security*. Clearly such a market is feasible only if the regulator's intervention is publicly observable and verifiable — a condition that is not required in any of our analysis to this point, and in practice may fail to hold. However, if such a market could be created, its existence would render optimal intervention as the unique equilibrium *irrespective of the quality of the regulator's information*.

More formally, suppose that in addition to a standard security market, an event market of the type described is feasible and exists. Let Q be the price of the regulator security, with P being the price of the standard security as before. The regulator's intervention policy I can now depend on Q in addition to P and his own signal ϕ. The rational expectations equilibrium pricing condition for the regulator security is

$$Q(\theta) = E_\phi \left[I \left(P \left(\theta \right), Q \left(\theta \right), \phi \right) | \theta \right].$$

Under these conditions we obtain:

Proposition 10 *If the market trades both a standard bank security and the regulator security, then for all κ the unique equilibrium of the economy features optimal intervention.*

The intuition behind this result is the following: a regular bank security may have the same price for different fundamentals because the probability of intervention is different across these fundamentals. But, once the regulator security is traded, the probability of intervention can be inferred from its price, and thus the fundamental can be inferred from the combination of its price and the prices of regular bank securities. This implies that the regulator will intervene optimally in equilibrium.

Note that the regulator security specified here is not a standard security, i.e., it is not a security typically traded in financial markets. This is because the price of this security is not linked to the value that goes to any class of bank claim holders. Instead, the regulator security aggregates information regarding whether the regulator is going to intervene in the bank or not.[15] The result in Proposition 10 implies that issuing such a unique security is a very powerful tool: with this security in place, the unique equilibrium of the model entails optimal intervention for all κ.

5.2.4 Taxation

The next policy tool we consider is taxation. Suppose that there is one security, whose value absent intervention is $X(\theta)$ and with intervention is $X(\theta + T)$. The regulator would like to intervene if and only if $\theta < \hat{\theta}$, but is unable to do so because the value of the security under this intervention policy is not monotonic in θ, and so is not fully revealing. In principle, the regulator could restore monotonicity by taxing the investors who hold this security when it intervenes. A simple way to do this is to tax the investors by an amount T when intervention takes place. If the regulator intervenes if and only if $\theta < \hat{\theta}$, the market price of the security is then $X(\theta) + U(\theta) - T$ for $\theta < \hat{\theta}$, and $X(\theta)$ otherwise. Since this price is monotone in θ, the regulator is indeed able to learn the fundamentals perfectly from the price and to intervene precisely when fundamentals are below $\hat{\theta}$.[16]

The above argument establishes that when intervention-contingent taxation is possible, there is always an equilibrium with intervention if and only if $\theta < \hat{\theta}$, regardless of the precision of the regulator's information. In fact, we now show that when the regulator observes the price of a convex security, this equilibrium is unique:

Proposition 11 *Suppose the regulator observes the price of a convex security and imposes a tax of T whenever intervention occurs. Then the unique equilibrium is intervention if*

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[15] In this sense, it resembles securities traded in prediction markets, which try to predict the probability of an event.

[16] More generally, the price is monotone under this intervention policy if the regulator imposes a tax of at least $X(\theta + T) - X(\theta)$ whenever intervention occurs.

and only if $\theta < \hat{\theta}$.

It is important to note that, relative to the other measures discussed in this section, imposing an intervention-dependent tax on bank security holders is a more drastic policy response. Taxation of this form clearly has distributional consequences and is likely to be costly to implement — both in terms of direct costs, and in terms of other distortions. Moreover, intervention-contingent taxation is possible only if intervention is publicly observable — under which circumstances a regulator security of the type discussed above would also solve the regulator's inference problem.

5.2.5 Commitment

Thus far in the paper, we have assumed that the regulator's intervention decision is ex post efficient, i.e., the regulator does what is optimal for him to do given the prices and its private signal. A natural question is whether the regulator can achieve optimal intervention by committing ex ante to a policy rule as a function of the realized price. To answer this question, we reconsider the case of one traded security (convex or concave), and assume that the regulator can commit ex ante to an intervention policy that is a function of the price only. This last assumption is natural given that committing to a policy rule that is based on the publicly observed price may be feasible, while committing to a policy rule that is based on a privately observed signal is probably not. In view of the regulator's commitment, for this subsection only we drop the requirement that the time-consistency condition (6) must be satisfied in equilibrium.

The main thing to note about this case is that an equilibrium under regulatory commitment must entail fully revealing prices, i.e., in such an equilibrium every fundamental must be associated with a different price. This is because the regulator's intervention decision is now based only on the price. As a result, if two fundamentals had the same price, they would also have the same probability of intervention, and this would generate different prices. Thus, finding the optimal commitment policy boils down to finding the price function that maximizes the regulator's ex ante value function, subject to the constraint that the price function is fully revealing of the fundamentals.

The fact that the price function must be fully revealing implies that the regulator cannot achieve optimal intervention under commitment. This is because, as we saw in Figure 1, optimal intervention generates a price function that is not fully revealing – it has different fundamentals associated with the same price. The following proposition establishes a stronger result on the effectiveness of commitment. It says that, under commitment, the regulator will end up deviating from the benchmark full-information optimal intervention policy over a set of fundamentals that is at least of size T. Thus, commitment is not very effective in solving the problems raised in this paper.

Proposition 12 *If the regulator commits ex ante to an intervention policy based on the realization of the price of one security, it will not be able to achieve optimal intervention. The set of fundamentals at which the regulator deviates from the full-information optimal intervention policy is at least of size T.*

6   Conclusion

We study a rational expectations model of regulatory supervision of banks based on the market prices of bank securities. Because prices reflect bank fundamentals and expectations of regulatory actions at the same time, the regulator cannot always extract information from the price to make an efficient intervention decision. The ability of the regulator to extract such information depends on the gap between the information available to the market and the information available to the regulator. When the gap is large, our model has no rational-expectations equilibrium. When the gap is moderate, there are multiple equilibria, some of which exhibit sub-optimal intervention. When the gap is small, there is a unique equilibrium with optimal intervention. These results imply that a regulator cannot rely on market-based intervention alone, but instead should use market prices to supplement his own signal. Moreover, our analysis demonstrates that the equilibrium outcome under market based regulation depends on the type of security whose price the regulator observes. Convex securities lead to too much intervention and concave securities to too little.

We also analyze four policy measures that improve the ability of the regulator to use

market information for bank supervision. Two of these measures – learning from multiple securities and disclosing the regulator's information – resolve the problem of multiple equilibria when the regulator's information is moderately precise, but do not solve the problem of no equilibrium when the regulator's information is imprecise. The other two measures — issuing a regulator security and taxation — solve both problems. As we have noted, the implementation of each of these policy measures requires nontrivial conditions to be met. Finally, we study the case in which the regulator can commit ex ante to a policy rule that depends on the price. We show that this is not effective in obtaining optimal intervention.

Empirical exploration of the ideas presented in the paper is likely to revolve around the relation between market prices and the regulators' intervention decisions – two key variables in our model that are easily observable. Our theoretical analysis points out that such empirical exploration should take into account two key features of the model. First, if the regulator uses the market price in his intervention decision, there will be dual causality between the two variables: market prices will reflect the regulator's action and affect it at the same time. A similar setting, in which prices affect and reflect an action of market participants, has been empirically analyzed by Bradley, Brav, Goldstein, and Jiang (2006). Second, when the information that the regulator has outside the financial market is not precise enough, our model generates equilibrium indeterminacy, which might make the relation between the two variables more difficult to detect.

Finally, we noted before that the insights from our analysis can be applied to many other settings in which regulators intervene on the basis of information they glean from market prices. In fact, our analysis is more general than that; it can apply to other settings in which agents use information from market prices to take actions that affect the value of the security. Importantly, the analysis requires that, under optimal intervention, the value of the security is non monotone in the fundamentals. As we argued before, this is the result of a setting where the agent acts to maximize his own objective function rather than the value of the security. Examples that satisfy this criterion and to which our analysis can apply include the decision of bond holders on whether to monitor the firm based on information gleaned from its stock price; the decision of lenders on whether to lend money to the firm

based on the same information; and the design and inclusion of market-price triggers in debt covenants.

Mathematical Appendix

We start with a couple of preliminaries. First, we have favored expositional clarity over mathematical precision whenever no loss is associated with the former. Specifically, even though each combination of a fundamental and regulator signal has measure zero, we routinely evaluate the conditional probability $\Pr(\theta|\theta \in \{\theta_1, \theta_2\})$ as $1/2$ if $\theta \in \{\theta_1, \theta_2\}$ (recall that the fundamental θ is distributed uniformly). Likewise, we evaluate $\Pr(\theta|\theta \in \Theta) = 0$ if $\theta \notin \Theta$. We use parallel calculations for conditional expectations. The only proof in which it is important for us to proceed more formally in taking conditional probabilities and expectations is that of Proposition 2 (see the proof for the relevant details).

Second, it is convenient to state the following straightforward result separately.

Lemma 3 *In any equilibrium,*

$$\Pr(I|\theta) = \begin{cases} 1 & \text{if } \theta < \max\left\{\hat{\theta} - 2\kappa, \hat{\theta} - T\right\} \\ 0 & \text{if } \theta > \min\left\{\hat{\theta} + 2\kappa, \hat{\theta} + T\right\} \end{cases}.$$

Proof of Lemma 3: Consider a fundamental $\theta < \hat{\theta} - 2\kappa$. At this fundamental, the regulator observes only signals below $\hat{\theta} - \kappa$. Such signals are never observed after any fundamental $\tilde{\theta} \geq \hat{\theta}$. As such, when the fundamental is θ the regulator knows that the fundamental lies to the left of $\hat{\theta}$. By time-consistency, it intervenes with probability 1.

Next, consider a fundamental $\theta < \hat{\theta} - T$. In any equilibrium the price at θ is bounded above by $X(\theta) + U(\theta) = X(\theta + T) < X(\hat{\theta})$. Moreover, any fundamental $\tilde{\theta} \geq \hat{\theta}$ has a price that satisfies $P(\tilde{\theta}) \geq X(\tilde{\theta}) \geq X(\hat{\theta})$. Thus, in any equilibrium, if $\theta < \hat{\theta} - T$ then θ cannot share a price with *any* fundamental above $\hat{\theta}$. Again, by time-consistency the regulator intervenes with probability 1. The argument is similar for $\theta > \hat{\theta} + 2\kappa$ and $\theta > \theta + T$, and the result follows. ∎

Proof of Lemma 1: Observe first that

$$\begin{aligned} \Gamma'(\theta) &= -(G(D + T - \theta) - G(T - \theta)) < 0, \\ \Gamma''(\theta) &= g(D + T - \theta) - g(T - \theta). \end{aligned}$$

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This implies that $-\Gamma'(\theta)$ is single-peaked since (given that g is itself single-peaked) $-\Gamma''(\theta)$ is negative if and only if $D + T - \theta$ is below some cutoff level, i.e., if and only if θ is above some cutoff level. Second, since

$$V'(\theta) = -\Gamma'(\theta + T) - \left(-\Gamma'(\theta)\right)$$

it follows that V is itself single-peaked, since $V'(\theta)$ is first positive and then negative as a function of θ. ∎

Proof of Proposition 1: In an optimal intervention equilibrium there is a cutoff fundamental, $\hat{\theta}$, for which the regulator only intervenes for $\theta \leq \hat{\theta}$. What we need to check is that this policy is feasible. Under this intervention policy, $P(\theta) = X(\theta) + U(\theta)$ for $\theta \leq \hat{\theta}$, and $P(\theta) = X(\theta)$ for $\theta > \hat{\theta}$. As such, there are at most two fundamentals related to each price. For prices that are related to just one fundamental, the equilibrium price is trivially fully revealing. For prices that are related to two fundamentals, e.g., $\theta_1 < \hat{\theta} < \theta_2$,

$$X(\theta_1) + U(\theta_1) = X(\theta_2).$$

But from (4), this means that $X(\theta_1 + T) = X(\theta_2)$, and thus $\theta_2 = \theta_1 + T$. Since $T > 2\kappa$, the regulator can distinguish between θ_1 and θ_2 with its own signal and follow its optimal intervention rule. ∎

Proof of Proposition 2: We prove the result for the case in which X is convex. The case in which X is concave follows similarly.

The proof requires us to be more mathematically precise in our treatment of probabilities and expectations than is the case elsewhere in the paper. In particular, unlike elsewhere in the paper, we must assign conditional expectations and probabilities in cases where the conditioning set has infinitely many members yet is still null. Formally, let $\mathcal{B}$ denote the Borel algebra of $\left[\underline{\theta}, \bar{\theta}\right]$, so that $\left(\left[\underline{\theta}, \bar{\theta}\right], \mathcal{B}\right)$ is a measurable space. Let $\mu : \mathcal{B} \to [0, 1]$ be the probability measure associated with the uniform distribution on $\left[\underline{\theta}, \bar{\theta}\right]$.

Let $\bar{\kappa} > 0$ be such that $\frac{U(\theta)}{2\bar{\kappa}} - X'(\theta + T) > 0$ for all $\theta \in \left[\hat{\theta}, \hat{\theta} + T\right]$, and fix an arbitrary $\kappa \in [0, \bar{\kappa}]$. We will show that in any equilibrium optimal intervention occurs almost surely. The proof is by contradiction: suppose to the contrary that there exists an equilibrium in

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which the regulator intervenes suboptimally over a non-null set of fundamentals. Clearly suboptimal intervention can only occur at non-revealing prices; and by Lemma 3, suboptimal intervention and non-revealing prices can only occur in $\left[\hat{\theta} - 2\kappa, \hat{\theta} + 2\kappa\right]$.

Throughout the proof we use the following definitions. Let $\mathcal{P}$ be the set of non-revealing prices. For each non-revealing price $P \in \mathcal{P}$ let Θ_P be the set of fundamentals associated with that price. Let $\Theta = \bigcup_{P \in \mathcal{P}} \Theta_P$ be the set of all fundamentals with a non-revealing price. By hypothesis, Θ has strictly positive measure.

Claim A: In an equilibrium, in which the regulator intervenes suboptimally over a non-null set of fundamentals, $\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$ has strictly positive measure.

Proof of Claim A: Consider the conditional probability $\Pr\left(\Theta_P \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_P\right)$. Clearly it equals $\Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_P\right)$. Moreover,

$$\int_{\theta \in \Theta} \Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_{P(\theta)}\right) \mu(d\theta) = \Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta\right).$$

Suppose that contrary to the claim $\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$ is null. In this case, $\Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_{P(\theta)}\right) = 0$ for almost all θ in Θ. But then the regulator would intervene optimally for almost all $\theta \in \Theta$: it would intervene with probability 1 at almost all $\theta \in \Theta$, since almost all members of Θ lie below $\hat{\theta}$. Since suboptimal intervention can potentially happen only at $\theta \in \Theta$, this contradicts an equilibrium, in which the regulator intervenes suboptimally over a non-null set of fundamentals, and completes the proof of Claim A. ∎

For any signal realization ϕ, the regulator knows the true fundamental lies in the interval $[\phi - \kappa, \phi + \kappa]$. As such, for a price $P \in \mathcal{P}$ and signal ϕ the regulator's expected payoff (net of costs) from intervention is

$$v(P, \phi) \equiv E_\theta \left[V(\theta) - C | \theta \in \Theta_P \cap [\phi - \kappa, \phi + \kappa]\right]. \tag{10}$$

The heart of the proof lies in establishing:

Claim B: In an equilibrium of the kind described above, for any $P \in \mathcal{P}$: (1) $\sup \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right] = \hat{\theta}$ and (2) $v(P, \phi = \theta + \kappa) \geq 0$ for any $\theta \in \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right]$.

Proof of Claim B: Let θ_1 and $\theta_2 \in (\theta_1, \theta_1 + 2\kappa]$ be an arbitrary pair of members of Θ_P such that $\theta_1 \leq \hat{\theta}$ and $\theta_2 \geq \hat{\theta}$ (clearly all members of Θ_P cannot lie to the same side of $\hat{\theta}$,

and at least one such pair must lie within 2κ of each other). Since θ_1 and θ_2 have the same price

$$X\left(\theta_1\right) + \frac{U\left(\theta_1\right)}{2\kappa} \int_{\theta_1-\kappa}^{\theta_1+\kappa} I\left(P,\phi\right) d\phi = X\left(\theta_2\right) + \frac{U\left(\theta_2\right)}{2\kappa} \int_{\theta_2-\kappa}^{\theta_2+\kappa} I\left(P,\phi\right) d\phi.$$

By convexity of X, $U\left(\theta_2\right) > U\left(\theta_1\right)$. It follows that

$$X\left(\theta_1\right) + U\left(\theta_1\right) \leq X\left(\theta_2\right) + \frac{U\left(\theta_2\right)}{2\kappa} \left(\int_{\theta_2-\kappa}^{\theta_2+\kappa} I\left(P,\phi\right) d\phi + \int_{\theta_1-\kappa}^{\theta_1+\kappa} \left(1 - I\left(P,\phi\right)\right) d\phi \right).$$

Equivalently,

$$X\left(\theta_1 + T\right) \leq X\left(\theta_2\right) + \frac{U\left(\theta_2\right)}{2\kappa} \left(\theta_1 - \theta_2 + 2\kappa + \int_{\theta_1-\kappa}^{\theta_2-\kappa} \left(1 - I\left(P,\phi\right)\right) d\phi + \int_{\theta_1+\kappa}^{\theta_2+\kappa} I\left(P,\phi\right) d\phi \right).$$
(11)

Define $\theta_1^* = \sup \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right]$ and $\theta_2^* = \inf \Theta_P \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$.

Suppose that either $v\left(P, \phi = \theta_1 + \kappa\right) < 0$ or $\theta_1^* < \hat{\theta}$. In the former case, $v\left(P, \phi\right) < 0$ for any signal ϕ above $\theta_1 + \kappa$ (since $v\left(P, \phi\right)$ is monotonically decreasing in ϕ). In the latter case, any signal ϕ above $\theta_1^* + \kappa$ rules out that $\theta \leq \hat{\theta}$. As such, by time-consistency $I\left(P, \phi\right) = 0$ for all $\phi > \theta_1 + \kappa$ in the former case, and $\phi > \theta_1^* + \kappa$ in the latter case. Since both sides of (11) are continuous in θ_1 and θ_2, it follows that

$$X\left(\theta + T\right) \leq X\left(\theta_2^*\right) + \frac{U\left(\theta_2^*\right)}{2\kappa} \left(\theta - \theta_2^* + 2\kappa + \int_{\theta-\kappa}^{\theta_2^*-\kappa} \left(1 - I\left(P,\phi\right)\right) d\phi \right)$$

for $\theta = \theta_1$ in the former case, and $\theta = \theta_1^*$ in the latter case. Certainly $I\left(P, \phi\right) = 1$ for all $\phi < \theta_2^* - \kappa$, since for these signal values the regulator knows that the fundamental lies to the left of $\hat{\theta}$. Thus the function Z defined by

$$Z\left(\theta, \theta_2\right) \equiv X\left(\theta_2\right) + \frac{U\left(\theta_2\right)}{2\kappa} \left(\theta - \theta_2 + 2\kappa\right) - X\left(\theta + T\right)$$

is weakly positive at $\left(\theta, \theta_2\right) = \left(\theta_1, \theta_2^*\right)$ in the former case, and at $\left(\theta_1^*, \theta_2^*\right)$ in the latter case. However,

$$\begin{aligned} Z\left(\theta_2^*, \theta_2^*\right) &= X\left(\theta_2^*\right) + U\left(\theta_2^*\right) - X\left(\theta_2^* + T\right) = 0 \\ Z_1\left(\theta_2^*, \theta_2^*\right) &= \frac{U\left(\theta_2^*\right)}{2\kappa} - X'\left(\theta_2^* + T\right) > 0, \end{aligned}$$

where the strict inequality follows since $\theta_2^* \leq \hat{\theta} + T$ (see Lemma 3) and $\kappa \leq \bar{\kappa}$. Since Z is concave in its first argument, it follows that $Z\left(\theta, \theta_2^*\right) < 0$ for all $\theta < \theta_2^*$, which contradicts

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$Z\left(\theta_1, \theta_2^*\right) \geq 0$ in the former case, and $Z\left(\theta_1^*, \theta_2^*\right) \geq 0$ in the latter case. This completes the proof of Claim B. ∎

We are now ready to complete the proof. By Claim B, for any $\varepsilon > 0$ and any $P \in \mathcal{P}$ there exists $\theta_{P,\varepsilon} \in \Theta_P \cap \left[\hat{\theta} - \varepsilon, \hat{\theta}\right]$ such that $v\left(P, \phi = \theta_{P,\varepsilon} + \kappa\right) \geq 0$. As such, the integral

$$\int_{\cup_{P\in\mathcal{P}}\left(\Theta_P\cap\left[\theta_{P,\varepsilon},\theta_{P,\varepsilon}+2\kappa\right]\right)} v\left(P\left(\theta\right), \phi = \theta_{P(\theta),\varepsilon} + \kappa\right) \mu\left(d\theta\right) \tag{12}$$

is weakly positive. Since v is a conditional expectation (see its definition (10)), the integral is also equal to

$$\int_{\cup_{P\in\mathcal{P}}\left(\Theta_P\cap\left[\theta_{P,\varepsilon},\theta_{P,\varepsilon}+2\kappa\right]\right)} \left(V\left(\theta\right) - C\right) \mu\left(d\theta\right).$$

The domain of the integral (12) can be expanded as

$$\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa - \varepsilon\right]\right) \cup \bigcup_{P\in\mathcal{P}} \left(\Theta_P \cap \left[\theta_{P,\varepsilon}, \hat{\theta}\right]\right) \cup \bigcup_{P\in\mathcal{P}} \left(\Theta_P \cap \left[\hat{\theta} + 2\kappa - \varepsilon, \theta_{P,\varepsilon} + 2\kappa\right]\right).$$

The term $V\left(\theta\right) - C$ is strictly negative over the first set above, with the single exception of at $\hat{\theta}$. For all ε small enough and by Claim A, the first set has strictly positive measure, while the other two have measures that approach zero. As such, the integral in expression (12) is strictly negative for ε small enough. The contradiction completes the proof. ∎

Proof of Lemma 2: Define the function

$$\begin{aligned}
Z\left(\theta_1, \theta_2\right) &= X\left(\theta_2\right) + \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) U\left(\theta_2\right) - X\left(\theta_1\right) - U\left(\theta_1\right) \\
&= X\left(\theta_2\right) + \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) U\left(\theta_2\right) - X\left(\theta_1 + T\right),
\end{aligned}$$

where $\theta_2 \geq \theta_1$. Intuitively, this is the difference between the price at fundamental θ_2 given an intervention probability $1 - \frac{\theta_2 - \theta_1}{2\kappa}$, and the price at fundamental θ_1 given an intervention probability 1.

Observe that Z has the following properties:

$$\begin{aligned}
Z_{11}\left(\theta_1, \theta_2\right) &< 0, \\
Z_{12}\left(\theta_1, \theta_2\right) &= \frac{U'\left(\theta_2\right)}{2\kappa} > 0, \\
Z\left(\theta, \theta\right) &= 0.
\end{aligned}$$

Moreover, for any θ,

$$Z(\theta - 2\kappa, \theta) = X(\theta) - X(\theta - 2\kappa + T) < 0.$$

We know that $Z\left(\hat{\theta} - 2\kappa, \hat{\theta}\right) < 0$ and $Z\left(\hat{\theta}, \hat{\theta}\right) = 0$. Since $Z_{11} < 0$, the result follows provided $Z_1\left(\hat{\theta}, \hat{\theta}\right) < 0$. We know that,

$$
\begin{aligned}
Z_1\left(\hat{\theta}, \hat{\theta}\right) &= \frac{U\left(\hat{\theta}\right)}{2\kappa} - X'\left(\hat{\theta} + T\right) \\
&= \frac{X\left(\hat{\theta} + T\right) - X\left(\hat{\theta}\right)}{2\kappa} - X'\left(\hat{\theta} + T\right) \\
&= \frac{1}{2\kappa} \int_{\theta = \hat{\theta}}^{\hat{\theta} + T} \left(X'(\theta) - \frac{2\kappa}{T} X'\left(\hat{\theta} + T\right) \right) d\theta.
\end{aligned}
$$

Since X is a convex function, $Z_1\left(\hat{\theta}, \hat{\theta}\right) < 0$ for all 2κ close enough to T. ∎

Proof of Proposition 3: Define Z as in the proof of Lemma 2. Observe first that since $Z\left(\theta_{01}, \hat{\theta}\right) = Z\left(\hat{\theta}, \hat{\theta}\right) = 0$, and $Z_{11} < 0$, then $Z\left(\theta_1, \hat{\theta}\right) > 0$ for any $\theta_1 \in \left(\theta_{01}, \hat{\theta}\right)$. Moreover, $Z\left(\cdot, \hat{\theta}\right)$ is single-peaked. Let $\hat{\theta}_{11} \in \left(\theta_{01}, \hat{\theta}\right)$ be its maximum. Since for any $\theta_1 \in \left(\theta_{01}, \hat{\theta}_{11}\right)$, $Z\left(\theta_1, \hat{\theta}\right) > 0$ and $Z(\theta_1, \theta_1 + 2\kappa) < 0$, by continuity there exists some $\theta_2 > \hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. We define a function, $\theta_2^*(\theta_1)$, where θ_2^* is the smallest θ_2, above $\hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. Economically, $\theta_2^*(\theta_1)$ is the fundamental which has the same market price as θ_1. We know that $\theta_2^*(\theta_{01}) = \hat{\theta}$.

We know $\theta_2^*(\theta_1)$ is a strictly increasing function. To see this, note that

$$Z(\theta_1, \theta_2) = Z\left(\theta_1, \hat{\theta}\right) + \int_{\hat{\theta}}^{\theta_2} Z_2(\theta_1, y)\, dy.$$

Since $Z\left(\theta_1, \hat{\theta}\right)$ is increasing over the range $[\theta_{01}, \hat{\theta}_{11}]$, and $Z_{12} > 0$, it follows that for any $\theta_2 \geq \hat{\theta}$, $Z(\theta_1, \theta_2)$ is increasing in θ_1 over $[\theta_{01}, \hat{\theta}_{11}]$. Thus, the smallest θ_2, at which $Z(\theta_1, \theta_2) = 0$, is strictly increasing in θ_1, implying that $\theta_2^*(\theta_1)$ is a strictly increasing function.

Since $\theta_2^*(\theta_{01}) = \hat{\theta}$, the function $\frac{V(\theta_1) + V(\theta_2^*(\theta_1))}{2}$ is strictly positive at $\theta_1 = \theta_{01}$. Define θ_{11} as the minimum of $\hat{\theta}_{11}$ and the infimum value such that $\frac{V(\theta_1) + V(\theta_2^*(\theta_1))}{2} \leq C$. As such, $\theta_2^*(\cdot)$ is increasing and $\frac{V(\cdot) + V(\theta_2^*(\cdot))}{2} > C$ over the interval $[\theta_{01}, \theta_{11}]$.

We have now defined the values θ_{01} and θ_{11} of the proposition statement. It remains to show that there is an equilibrium of the type described. This requires showing that the prices are rational given the intervention probabilities, and that the intervention probabilities result from the regulator's optimal behavior given the information in the price and its own private signal. It is immediate to show that the prices in the proposition statement are rational given the corresponding intervention probabilities. Thus, we turn to show that the intervention probabilities result from the regulator's optimal behavior. We will do this by analyzing different ranges of the fundamentals separately.

For a fundamental $\theta \le \hat{\theta}$ and $\theta \notin Y_1$, the price is $X(\theta) + U(\theta) = X(\theta + T)$. The same price may be observed at the fundamental $\theta + T$. Since $2\kappa < T$, the regulator's private signal will indicate for sure that the fundamental is θ and not $\theta + T$. Hence, the regulator will optimally choose to intervene, generating intervention probability of 1, as is stated in the proposition. Note that the same price cannot be observed at any fundamental below $\theta + T$. Observing such a price at a fundamental below $\theta + T$ would imply that the fundamental belongs to the set $\theta_2^*(Y_1)$, but this contradicts the fact that $\theta \notin Y_1$.

For a fundamental $\theta \le \hat{\theta}$ and $\theta \in Y_1$, the price is again $X(\theta) + U(\theta)$. As before, the same price may be observed at the fundamental $\theta + T$ without having an effect on the decision of the regulator to intervene at θ, given that $2\kappa < T$. Here, however, the same price will also be observed at the fundamental $\theta_2^*(\theta)$. This is because the fundamental $\theta_2^*(\theta) \in \theta_2^*(Y_1)$ generates a price of $X(\theta_2^*(\theta)) + \left(1 - \frac{\theta_2^*(\theta) - \theta}{2\kappa}\right) U(\theta_2^*(\theta))$, which by construction is equal to $X(\theta) + U(\theta)$. (Note that the same price will not be observed at any other fundamental in the set $\theta_2^*(Y_1)$, since $X(\theta) + U(\theta)$ and $\theta_2^*(\theta)$ are strictly increasing in θ.) Thus, at the fundamental θ, the regulator observes a price that is consistent with both θ and $\theta_2^*(\theta)$, and may observe a private signal that is also consistent with both of them. If this happens, given the uniform distribution of noise in the regulator's signal, the regulator will intervene as long as $\frac{V(\theta) + V(\theta_2^*(\theta))}{2} \ge C$. By construction, this is true for all $\theta \in Y_1$, and thus, at the fundamental θ, the regulator will intervene with probability 1, as is stated in the proposition.

For a fundamental $\theta > \hat{\theta}$ and $\theta \notin \theta_2^*(Y_1)$, the price is $X(\theta) = X(\theta - T) + U(\theta - T)$. The same price may be observed at the fundamental $\theta - T$ and also at some $\theta' > \hat{\theta}; \theta' \in \theta_2^*(Y_1)$.

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Since $2\kappa < T$, the regulator's private signal at the fundamental θ will indicate for sure that the fundamental is not $\theta - T$. Hence, the regulator will know that the fundamental is above $\hat{\theta}$, and will optimally choose not to intervene, generating intervention probability of 0, as is stated in the proposition.

Finally, for a fundamental $\theta > \hat{\theta}$ and $\theta \in \theta_2^* (Y_1)$, the price is $X(\theta) + \left(1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa} \right) U(\theta)$. As follows from the arguments above, the same price will be observed at the fundamental $\theta_2^{*-1}(\theta)$, and also may be observed at some fundamental $\theta'' > \hat{\theta}$; $\theta'' \notin \theta_2^*(Y_1)$. (As argued before, two fundamentals in the set $\theta_2^*(Y_1)$ cannot have the same price.) As also follows from the arguments above, the regulator will optimally choose to intervene if and only if its signal is consistent with both θ and $\theta_2^{*-1}(\theta)$ (the signal cannot be consistent with both $\theta_2^{*-1}(\theta)$ and θ''). Due to the uniform distribution of noise in the regulator's signal, this generates an intervention probability of $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}$, as is stated in the proposition. ∎

Proof of Proposition 4: Suppose to the contrary that there exists an equilibrium without optimal intervention and in which the probability of intervention for all $\theta > \hat{\theta}$ is 0. In this equilibrium there must exist some $\theta_1 < \hat{\theta}$ such that $E[I|\theta_1] < 1$. Because $\theta_1 < \hat{\theta}$, it follows that there must exist $\theta_2 \in (\hat{\theta}, \theta_1 + 2\kappa]$ with the same price as θ_1. Moreover, because $E[I|\theta] = 0$ for all $\theta > \hat{\theta}$, the fundamental θ_2 is the unique fundamental to the right of $\hat{\theta}$ with the same price as θ_1. So the intervention policy I in this equilibrium must satisfy

$$
I(P(\theta_1), \phi) = \begin{cases} 0 & \text{if } \phi \in (\theta_2 - \kappa, \theta_2 + \kappa) \\ 1 & \text{if } \phi \in (\theta_1 - \kappa, \theta_1 + \kappa) \text{ and } \phi \notin (\theta_2 - \kappa, \theta_2 + \kappa) \end{cases}.
$$

As such, the expected intervention probability at θ_1 is

$$
E[I|\theta_1] = \Pr\left((\theta_1 + \xi) \in (\theta_1 - \kappa, \theta_2 - \kappa) \right) = \frac{\theta_2 - \theta_1}{2\kappa}.
$$

Define a function

$$
Z(\theta) = X(\theta_1) + \frac{\theta - \theta_1}{2\kappa} U(\theta_1) - X(\theta).
$$

On the one hand, observe that $Z(\theta_2) = X(\theta_2) + E[I|\theta_1] U(\theta_1) - X(\theta_2) = 0$, since by hypothesis θ_1 and θ_2 have the same price. But on the other hand, $Z(\theta_1) = 0$, $Z(\theta_1 + 2\kappa) = X(\theta_1 + T) - X(\theta_1 + 2\kappa) > 0$, and Z is concave since X is convex. As such, there is no

value of $\theta \in (\theta_1, \theta_1 + 2\kappa]$ for which $Z(\theta) = 0$. The resultant contradiction completes the proof. ∎

Proof of Proposition 5: The proof is omitted, and is available from the authors. It is parallel to the proof of Proposition 3. ∎

Proof of Proposition 6: The proof is omitted, and is available from the authors. It is parallel to the proof of Proposition 4. ∎

Proof of Proposition 7: We prove the result for the case in which X is concave. The case in which X is convex follows similarly.

Suppose to the contrary that an equilibrium exists. Let $P(\cdot)$ be the equilibrium price function. We know that there cannot be a fully-revealing equilibrium (see the main text immediately prior to the proposition statement). Define Θ^* to be the non-empty set of fundamentals at which the price is not fully-revealing, i.e.,

$$\Theta^* = \left\{ \theta : \exists \theta' \neq \theta \text{ such that } P(\theta) = P(\theta') \right\}.$$

Given Θ^*, define $\theta^* = \sup \Theta^*$. We prove the following claims.

Claim 1: If two fundamentals θ' and θ'' have the same price, i.e., $P(\theta') = P(\theta'')$, then $|\theta' - \theta''| \leq T < 2\kappa$.

Proof of Claim 1: Let θ' and $\theta'' > \theta'$ be two fundamentals with the same price. We know that $P(\theta'') \geq X(\theta'')$ and

$$X(\theta' + T) = X(\theta') + U(\theta') \geq X(\theta') + E(I|\theta') U(\theta') = P(\theta').$$

So if $\theta'' > \theta' + T$ then $P(\theta'') > X(\theta' + T) \geq P(\theta')$, a contradiction. Thus $\theta'' \leq \theta' + T$.

Claim 2: If $\theta > \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) = X(\theta)$; and if $\theta \leq \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) \leq X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$.

Proof of Claim 2: By definition, if $\theta > \theta^*$ the price is fully-revealing. So if $\theta > \hat{\theta}$ also, the regulator does not intervene, and $P(\theta) = X(\theta)$. So for any $\theta \in \left(\max\left\{\theta^*, \hat{\theta}\right\}, \infty\right)$, the price is $X(\theta)$.

Next, suppose that contrary to the claim $P(\theta') > X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$ for some $\theta' \leq \max\left\{\theta^*, \hat{\theta}\right\}$. But then there exists $\theta > \max\left\{\theta^*, \hat{\theta}\right\} \geq \theta^*$ such that $P(\theta) = P(\theta')$, contradicting the fact that $\theta^* = \sup \Theta^*$. This completes the proof of Claim 2.

Claim 3: $\theta^* > \hat{\theta}$.

Proof of Claim 3: Suppose to the contrary that $\theta^* \leq \hat{\theta}$, so that $\max\left\{\theta^*, \hat{\theta}\right\} = \hat{\theta}$. By Claim 2, $P(\theta) = X(\theta)$ if $\theta > \hat{\theta}$, and $P(\theta) \leq X\left(\hat{\theta}\right)$ for $\theta \leq \hat{\theta}$. As such, whenever the true fundamental is strictly below $\hat{\theta}$ the regulator knows either that the fundamental is strictly below $\hat{\theta}$; or that the fundamental is either strictly below $\hat{\theta}$ or equal to $\hat{\theta}$, with a positive probability of both. So the regulator intervenes with probability one for any $\theta < \hat{\theta}$. But then the price is not below $X\left(\hat{\theta}\right)$ for any θ close to $\hat{\theta}$. This contradiction completes the proof of the Claim 3.

Claim 4: $P(\theta^*) = X(\theta^*)$, and so $E(I|\theta^*) = 0$.

Proof of Claim 4: From Claims 2 and 3, $P(\theta) \leq X(\theta^*)$ for $\theta \leq \theta^*$. The claim follows since certainly $P(\theta^*) \geq X(\theta^*)$.

Now, consider first the case where $\theta^* \in \Theta^*$. By construction, there exists a fundamental $\theta' < \theta^*$ such that: $P(\theta') = X(\theta') + E(I|\theta')U(\theta') = X(\theta^*)$. By Claim 1 and by $T < 2\kappa$, $\theta^* < \theta' + 2\kappa$. Since $E(I|\theta^*) = 0$, the regulator does not intervene at signals above $\theta^* - \kappa$. Thus, $E(I|\theta') \leq \Pr(\theta' + \xi \leq \theta^* - \kappa) = \frac{\theta^* - \theta'}{2\kappa}$. Define the function $Z(\theta', \theta^*)$ as follows:

$$Z(\theta', \theta^*) \equiv X(\theta') + U(\theta')\frac{\theta^* - \theta'}{2\kappa} - X(\theta^*).$$

By the above arguments, in the proposed equilibrium, $Z(\theta', \theta^*) \geq 0$. We know that $Z(\theta', \theta') = 0$, and that $Z(\theta', \theta' + 2\kappa) = X(\theta' + T) - X(\theta' + 2\kappa) < 0$. Since the security is concave, $Z_{22} > 0$. Thus, there are no θ' and $\theta^* \in (\theta', \theta' + 2\kappa)$ for which $Z(\theta', \theta^*) \geq 0$. This is a contradiction to the proposed equilibrium.

Suppose now that $\theta^* \notin \Theta^*$. There exists some sequence $(\theta_i)_{i=0}^{\infty} \subset \Theta^*$ that converges to θ^*. Moreover, by Claims 2, 3, and 4, $E(I|\theta_i) \to 0$ as $i \to \infty$: for if this is not true, there is a $\theta_i \leq \theta^*$ at which the price is above $X(\theta^*)$. For each θ_i in this sequence there exists at least one fundamental, θ_i', at which the price is the same and which lies to the left of $\hat{\theta}$. (If instead all fundamentals with price $P(\theta_i)$ were to the right of $\hat{\theta}$, no intervention would occur, and they could not have the same price.) So $X(\theta_i') + E(I|\theta_i')U(\theta') = X(\theta_i) + E(I|\theta_i)U(\theta_i)$.

Note that $\theta_i - \theta_i'$ is bounded away from 0 as $i \to \infty$ since $\theta_i \to \theta^* > \hat{\theta}$. We know that

$$
\begin{aligned}
E\left(I|\theta_i'\right) &= \int_{\theta_i'-\kappa}^{\theta_i'+\kappa} I\left(P\left(\theta_i\right),\phi\right)\frac{1}{2\kappa}d\phi \\
&\leq \int_{\theta_i'-\kappa}^{\theta_i-\kappa} I\left(P\left(\theta_i\right),\phi\right)\frac{1}{2\kappa}d\phi + \int_{\theta_i-\kappa}^{\theta_i+\kappa} I\left(P\left(\theta_i\right),\phi\right)\frac{1}{2\kappa}d\phi \\
&\leq \frac{\theta_i-\theta_i'}{2\kappa} + E\left(I|\theta_i\right).
\end{aligned}
$$

Define the function $Z\left(\theta_i',\theta_i\right)$ as follows:

$$
Z\left(\theta_i',\theta_i\right) \equiv X\left(\theta_i'\right) + U\left(\theta_i'\right)\frac{\theta_i-\theta_i'}{2\kappa} - X\left(\theta_i\right) + E\left(I|\theta_i\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right).
$$

By the above arguments, in the proposed equilibrium, $Z\left(\theta_i',\theta_i\right) \geq 0$. We use ε_i to denote $E\left(I|\theta_i\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right)$. We know that ε approaches 0 (the value of intervention, $U\left(\theta\right)$, is bounded above by T). We know that $Z\left(\theta_i',\theta_i'\right) = \varepsilon_i$, and that $Z\left(\theta_i',\theta_i'+2\kappa\right) = X\left(\theta_i'+T\right) - X\left(\theta_i'+2\kappa\right) + \varepsilon_i < 0$ for all i large enough. Since the security is concave, $Z_{22} > 0$. Thus, for any θ_i between θ_i' and $\theta_i'+2\kappa$, $Z\left(\theta_i',\theta_i\right) \leq \varepsilon_i + \frac{\left(\theta_i-\theta_i'\right)\left(X\left(\theta_i'+T\right)-X\left(\theta_i'+2\kappa\right)\right)}{2\kappa}$. This implies that $Z\left(\theta_i',\theta_i\right) \geq 0$ can hold only if $\theta_i' \leq \theta_i \leq \theta_i' + \frac{2\kappa\varepsilon_i}{X\left(\theta_i'+2\kappa\right)-X\left(\theta_i'+T\right)}$. Then, since ε_i approaches 0, there are no θ_i' and θ_i that are bounded away from each other, for which $Z\left(\theta_i',\theta_i\right) \geq 0$. This is a contradiction to the proposed equilibrium. ∎

Proof of Proposition 8: Suppose the regulator observes the price of securities A and B, where security A is strictly convex and security B is strictly concave. The heart of the proof is the following straightforward claim:

Claim: For any pair of fundamentals θ_1 and $\theta_2 \neq \theta_1$ there is no probability $q \in (0,1)$ such that

$$
X_s\left(\theta_1\right) + qU_s\left(\theta_1\right) = X_s\left(\theta_2\right) \text{ for securities } s = A,B \tag{13}
$$

or

$$
X_s\left(\theta_1\right) + qU_s\left(\theta_1\right) = X_s\left(\theta_2+T\right) \text{ for securities } s = A,B. \tag{14}
$$

Proof of Claim: Observe that

$$
X_s\left(\theta_1\right)+qU_s\left(\theta_1\right) = (1-q)X_s\left(\theta_1\right)+qX_s\left(\theta_1+T\right) \begin{Bmatrix} > \\ < \end{Bmatrix} X_s\left(\theta_1+qT\right) \text{ if security } s \text{ is } \begin{Bmatrix} \text{convex} \\ \text{concave} \end{Bmatrix}
$$

Since X_s is monotone strictly increasing for both securities, it is immediate that neither (13) nor (14) can hold. ∎

The proof of the main result applies this Claim. Consider any equilibrium, and let Θ be the set of fundamentals that share the same price vector as a fundamental at which intervention is suboptimal. Suppose that (contrary to the claimed result) the set Θ is non-empty. Let θ^* be its supremum. Clearly if $\theta^* \leq \hat{\theta}$ then for all equilibrium prices associated with fundamentals Θ the regulator would know the true fundamental lies below $\hat{\theta}$, and would intervene optimally. So $\theta^* > \hat{\theta}$. Moreover, by Lemma 3, $\theta^* \leq \hat{\theta} + 2\kappa < \hat{\theta} + T$.

By construction, for fundamentals $\theta > \theta^*$ the regulator intervenes optimally, so $P(\theta) = X(\theta)$. Therefore, for all fundamentals $\theta \in \Theta$ the equilibrium price vector satisfies $P(\theta) \leq X(\theta^*)$. Consider an arbitrary sequence $\{\theta_i\} \subset \Theta$ such that $\theta_i \to \theta^*$. The intervention probabilities converge to zero along this sequence, $E[I|\theta_i] \to 0$ (otherwise, the equilibrium price would strictly exceed $X(\theta^*)$ for some θ_i). There are two cases to consider:

Case A: On the one hand, suppose there exists some $\varepsilon > 0$ and some infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each θ_j there is a fundamental $\theta_j' \neq \theta_j$ with the same price, and $E[I|\theta_j'] \in [\varepsilon, 1-\varepsilon]$. It follows that there is a subsequence $\{\theta_k\} \subset \{\theta_j\}$ such that for each θ_k there is a fundamental $\theta_k' \neq \theta_k$ with the same price, and $E[I|\theta_k']$ converges to $q \in [\varepsilon, 1-\varepsilon]$ as $k \to \infty$. Since for all k

$$X_s(\theta_k) + E[I|\theta_k]U_s(\theta_k) = X_s(\theta_k') + E[I|\theta_k']U_s(\theta_k')$$

for securities $s = A, B$, and the left-hand side converges to $X_s(\theta^*)$, it follows that $\{\theta_k'\}$ must converge also, to θ' say. Thus $X_s(\theta^*) = X_s(\theta') + qU_s(\theta')$ for securities $s = A, B$, directly contradicting the above Claim.

Case B: On the other hand, suppose that Case A does not hold. So there exists an infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each fundamental θ_j' possessing the same price as θ_j the intervention probability $E[I|\theta_j']$ is either less than $1/j$ or greater than $1 - 1/j$. It follows that for j large, all fundamentals with the same price vector as θ_j are close to either θ^* (if the intervention probability is close to 0) or $\theta^* - T$ (if the intervention probability is close to 1): formally, there exists some sequence ε_j such that $\varepsilon_j \to 0$ and such

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that $\theta'_j \in [\theta^* - T - \varepsilon_j, \theta^* - T + \varepsilon_j] \cup [\theta^* - \varepsilon_j, \theta^*]$. But for j large enough, $\theta^* - \varepsilon_j > \hat{\theta}$, $\theta^* - T + \varepsilon_j < \hat{\theta}$, and $(\theta^* - \varepsilon_j) - (\theta^* - T + \varepsilon_j) = T - 2\varepsilon_j > 2\kappa$. That is, for j large, if the regulator observes price vector $P(\theta_j)$ and its own signal, it knows with certainty which side of $\hat{\theta}$ the fundamental lies. As such, it intervenes optimally, giving a contradiction. ∎

Proof of Proposition 9: Exactly as in Proposition 7 a fully-revealing equilibrium cannot exist. Suppose a non-fully revealing equilibrium exists. So at some set of fundamentals Θ^* the prices of both the concave and convex securities must be the same for at least two distinct fundamentals. That is, the set

$$\Theta^* \equiv \left\{ \theta : \exists \theta' \neq \theta \text{ such that } P_i(\theta) = P_i(\theta') \text{ for all securities } i \right\}$$

is non-empty. The proof of the concave half of Proposition 7 applies, and gives a contradiction. ∎

Proof of Proposition 10: First, in any equilibrium where there exist $\theta_1 < \theta_2$ with the same price, the expected intervention probabilities $E[\theta_1|I]$ and $E[\theta_2|I]$ must differ (otherwise prices would not be identical). Given that the probability of intervention can be directly inferred from $Q(\theta)$, then the regulator can always infer θ based on $P(\theta)$ and $Q(\theta)$. Then, the regulator will choose to intervene when $\theta \leq \hat{\theta}$, and not intervene otherwise. The same is true if the equilibrium is fully revealing. Thus, if there is an equilibrium, it must feature optimal intervention.

Now, let us show that optimal intervention is indeed an equilibrium. In such an equilibrium, the price of any bank security is $X(\theta + T)$ for $\theta < \hat{\theta}$ and $X(\theta)$ for $\theta > \hat{\theta}$. The regulator security has a price of 1 for $\theta < \hat{\theta}$ and 0 for $\theta > \hat{\theta}$. Then, independent of the regulator's private signal, the regulator will choose to intervene below $\hat{\theta}$ and not intervene above $\hat{\theta}$. This is indeed consistent with the prices, so optimal intervention is an equilibrium. ∎

Proof of Proposition 11: Suppose that contrary to the claimed result there is another equilibrium. It cannot entail fully revealing prices. Let Θ be a set of (at least two) fundamentals that are all associated with the same price. Clearly the sets $\Theta \cap \left[\underline{\theta}, \hat{\theta}\right]$ and $\Theta \cap (\hat{\theta}, \bar{\theta}]$ are both non-empty, since otherwise all fundamentals in Θ lie to the same side

of $\hat{\theta}$ — in which case intervention occurs at all members of Θ with the same probability (either 0 or 1), and so the fundamentals cannot be associated with the same price.

Let θ^* be the highest fundamental in Θ that is still below $\hat{\theta}$, i.e., $\theta^* = \sup \Theta \cap \left[\underline{\theta}, \hat{\theta}\right]$. Take $\theta' \in \Theta \cap (\hat{\theta}, \bar{\theta}]$. Let (θ_i) be a sequence in Θ converging to θ^* such that $\theta_i \leq \theta^*$. (The degenerate case in which $\theta^* \in \Theta$ and $\theta_i = \theta^*$ for all i is a special case.)

Observe that for any θ_i in the sequence,

$$
\begin{aligned}
E\left[I|\theta'\right] &= \int_{\theta'-\kappa}^{\theta'+\kappa} \frac{1}{2\kappa} I\left(P\left(\Theta\right), \phi\right) d\phi \\
&= \int_{\theta^*+\kappa}^{\theta'+\kappa} \frac{1}{2\kappa} I\left(P\left(\Theta\right), \phi\right) d\phi + \int_{\theta_i+\kappa}^{\theta^*+\kappa} \frac{1}{2\kappa} I\left(P\left(\Theta\right), \phi\right) d\phi \\
&\quad + \int_{\theta_i-\kappa}^{\theta_i+\kappa} \frac{1}{2\kappa} I\left(P\left(\Theta\right), \phi\right) d\phi - \int_{\theta_i-\kappa}^{\theta'-\kappa} \frac{1}{2\kappa} I\left(P\left(\Theta\right), \phi\right) d\phi.
\end{aligned}
$$

Since there is no intervention at regulator signals above $\theta^* + \kappa$, it follows that

$$
E\left[I|\theta'\right] \leq E\left[I|\theta_i\right] + \varepsilon_i,
$$

where $\varepsilon_i \to 0$. In the conjectured equilibrium, for all i

$$
X\left(\theta'\right) - X\left(\theta_i\right) = E\left[I|\theta_i\right]\left(U\left(\theta_i\right) - T\right) - E\left[I|\theta'\right]\left(U\left(\theta'\right) - T\right).
$$

The LHS is strictly positive, and is bounded away from 0. Since $U\left(\theta'\right) - T \leq 0$, the RHS is bounded above by

$$
E\left[I|\theta_i\right]\left(U\left(\theta_i\right) - T\right) - \left(E\left[I|\theta_i\right] + \varepsilon_i\right)\left(U\left(\theta'\right) - T\right) = E\left[I|\theta_i\right]\left(U\left(\theta_i\right) - U\left(\theta'\right)\right) - \varepsilon_i\left(U\left(\theta'\right) - T\right).
$$

As $i \to \infty$ this upper bound converges to $E\left[I|\theta_i\right]\left(U\left(\theta_i\right) - U\left(\theta'\right)\right)$. This term is weakly negative since X is convex and so U is increasing. This gives the required contradiction and completes the proof. ∎

Proof of Proposition 12: Denote the size of the set of parameters in $\left[\hat{\theta} - T, \hat{\theta}\right]$ over which the regulator intervenes optimally as λ^-, and the size of the set of parameters in $\left[\hat{\theta}, \hat{\theta} + T\right]$ over which the regulator intervenes optimally as λ^+.

By the shape of the price function under optimal intervention (see Figure 1), every fundamental $\theta \in \left[\hat{\theta} - T, \hat{\theta}\right]$ that exhibits optimal intervention decision implies that the

intervention decision at $\theta + T \in \left[\hat{\theta}, \hat{\theta} + T\right]$ is suboptimal. This is because optimal intervention at both θ and $\theta + T$ implies that the two fundamentals have the same price, but this is impossible in a commitment equilibrium. Thus, the set of fundamentals with optimal intervention in $\left[\hat{\theta} - T, \hat{\theta}\right]$ cannot be greater than the set of fundamentals with suboptimal intervention in $\left[\hat{\theta}, \hat{\theta} + T\right]$. That is, $\lambda^- \leq T - \lambda^+$, which implies that $\lambda^- + \lambda^+ \leq T$. This completes the proof. ∎

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