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Abstract

We study a model of interbank credit where physical and informational frictions limit the opportunities for intertemporal trade among banks and outside investors. Banks obtain loans in an over-the-counter market (involving search, bilateral matching, and negotiations over the terms of the loan) and hold assets of heterogeneous quality that in turn determine their ability to repay those loans. When asset quality is not observable by outside investors, information about the actions taken by a bank in the loan market may influence prices in the asset market. In particular, under some conditions, borrowing from the central bank can be regarded as a negative signal about the quality of the borrower's assets and banks may be willing to borrow in the market at rates higher than the one offered by the central bank.

Keywords: Discount window, signaling, search, bargaining, private information, banking.
JEL Clasification Numbers: G21, E58.

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1 Introduction

Occasionally, some banks in the U.S. have borrowed in the interbank market for loans (the fed funds market) at a rate higher than the one they would pay to borrow from the central bank's discount window (Peristiani, 1998, Furfine, 2001, Darrat el al. 2004). This phenomenon is commonly explained as the result of a *stigma* effect attached to borrowing from the discount window. The general argument is that market participants may eventually identify which bank(s) have borrowed at the discount window and take this activity as a sign of weakness in the financial condition of the borrowing institution(s).

While most policymakers and empirical researchers consider the stigma hypothesis plausible, no formal treatment of the issue has ever been provided in the literature.[1] In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; (2) physical and informational frictions limit those trade opportunities; and (3) under some conditions a stigma effect like the one commonly associated with the regularities in the data can arise in the model.

The objective in this paper is not to assess the extent to which stigma is prevalent in the US fed funds market. Rather, we take as a starting point that policymakers consider stigma a cause of concern. As an example, while still serving as Fed governor, Kohn (2010) said: "The problem of discount window stigma is real and serious." In this paper, we intend to provide a detailed formal analysis of the microeconomic foundations of an instance of stigma that is consistent with the more informal explanations commonly provided in policy circles. Among other things, this analytical approach allows us to identify essential elements that form the basis for those explanations.

Understanding the apparent reluctance of banks to use the discount window is crucial to be able to address many important policy issues. For example, the prevalence of stigma may limit the ability of the central bank to effectively implement a "hard ceiling" on the range of interest rates observable in the interbank market. Partly in an effort to address such issues the Federal Reserve completely revised the terms of operation of its credit facilities in 2003. In spite of such efforts, evidence suggesting the presence of stigma could still be found in the data after the change (Furfine, 2005).

More recently, the reluctance of banks to borrow from the window bedeviled the central bank's attempts to provide funding support to institutions during the crisis. As explained by Chairman Bernanke (2009), "In August 2007, ... banks were reluctant to rely on discount window credit to address their funding needs. The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness – the so-called stigma problem. The perceived stigma of borrowing at the discount window threatened to prevent the Federal Reserve

[1] Only recently, Philippon and Skreta (2011) provide an alternative framework that could be used to study stigma. They develop a model of adverse selection in the provision of credit in which participation in government programs can influence the terms of trade available to agents. Their focus is on the design of optimal government interventions, not in the origin or implications of stigma.

from getting much-needed liquidity into the system."

The creation of the Term Auction Facility (TAF), and some of its particular organizational features, can be regarded as an attempt to limit the possibility of stigma associated with accessing this source of central bank liquidity.[2] In this paper, we will discuss specific conditions under which stigma may arise in the context of a formal model. We believe that the resulting insights are useful for evaluating alternative arrangements and policy options aimed at reducing the incidence of stigma in the interbank market.

Banks in our model obtain loans in an over-the-counter market, involving search, bilateral matching, and negotiations over the terms of the loans.[3] To repay those loans, banks sell assets of heterogeneous quality to outside investors. When asset quality is observable by loan counterparties but not by investors, information about the experience of the bank in the loan market may influence the price at which the bank can sell in the asset market. In this situation, discount window borrowing may become a negative signal of the quality of the borrower's assets. When this happens, some of the banks in our model – just as in the data – are willing to borrow in the interbank market at a higher rate than the one they would pay at the discount window.

Aside from the possibility of stigma, our model generates other interesting outcomes even when discount window borrowing is not possible. For example, we find that there is a strong equilibrium interconnection between the outcomes in the interbank market and the asset market. In particular, when participants in the interbank loan market expect disruptions in the asset market (due to adverse selection) they will not be willing to lend to other banks and, as a result, the interbank market effectively shuts down.[4]

We make some simplifying assumptions in our model. It is fairly easy to see that many of them could be readily generalized. However, our main objective here is to formalize in as simple a framework as possible an argument that is often used to explain certain apparently abnormal trading patterns in the U.S. interbank market for funds. Abstracting from some features of reality allows us to better capture the basic mechanism at play and to identify the main components of the logic involved. Some of these components may not have been fully appreciated before; for example, market frictions and bilateral negotiations play a critical role in our formal explanation of the phenomenon but do not often appear in policy discussions. We believe that highlighting these important components is one of the main contributions of our paper.

The model we develop combines several elements that are commonly regarded as important in

[2] "The TAF, apparently because of its competitive auction format and the certainty that a large amount of credit would be made available, appears to have overcome the stigma problem to a significant degree." (Bernanke, 2008). See also Armantier, Krieger, and McAndrews (2008) and Armantier, Ghysels, Sarkar and Shrader (2011).

[3] Bartolini et al. (2005) and Bech and Klee (2011) convincingly argue that the relative bargaining power of borrowers and lenders plays a significant role in the determination of interest rates in the fed funds market.

[4] There is now a large literature providing formal treatment of various issues related to the functioning of the interbank market. Some prominent examples are Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas and Holthausen (2004). More recent contributions include Acharya et al. (2008), Freixas and Jorge (2008), Allen et al. (2009), Freixas et al. (2011) and Bolton et al. (2011). For a good discussion of this literature see the introduction of Allen et al. (2009).

explaining the nature of financial (and, in particular, interbank) market outcomes. First, as in Freeman (1996) and the large literature that followed, spatial separation plays a key role in limiting the ability of some agents (banks) to trade with other agents (outside investors) at a certain point in time. Second, search and bilateral negotiations determine the terms of trade in the interbank market, as in Afonso and Lagos (2011).[5] Third, informational asymmetries and asset-quality heterogeneity play a crucial role in determining equilibrium interest rates and prices (as in, for example, Eisfeldt, 2004). Furthermore, the theory in this paper is in line with the long tradition, launched by Leland and Pyle (1977), of studying the role of signaling in financial markets.

The paper is organized as follows. In the next section we discuss some evidence that has often been regarded as indicating the presence of stigma attached to borrowing from the Fed's discount window. This evidence is examined in more detail in the referenced literature. Then, in Section 3 we introduce our basic model and in Section 4 we study equilibrium when the discount window is not available to banks. This section is intended to provide a description of the essential economic forces at work in the model. In Section 5 we introduce the discount window and study an equilibrium in which discount window lending becomes a negative signal and, hence, results in stigma. At the end of this section, we also discuss other possible equilibrium outcomes. Finally, in Section 6 we conclude.

2 The empirical case for stigma

During the 1980s and 1990s, U.S. Federal Reserve Banks provided discount window loans to banks at a rate below the fed funds target rate (i.e., the rate announced as the target for monetary policy). Supervisory scrutiny was used to control the amount borrowed by banks. On January 9, 2003, the Federal Reserve System dramatically changed its discount window policy and started to operate a standing facility, offering loans to eligible depository institutions at an interest rate higher than the fed funds target rate (at the time, the spread was set at 100 basis points). For banks in good financial conditions, no other restriction or special supervision is associated with borrowing from the discount window. In principle, under this new regime (a so-called Lombard-type facility), the rate at the discount window (plus the implicit cost of collateral) should act as a ceiling for the fed funds market rates. However, there is extensive evidence contradicting this presumption.

Furfine (2003), for example, studies the period immediately after the change in discount window policy and finds that trade in the market at rates higher than the discount window rate was significant. He concludes that banks were extremely reluctant to borrow from the Fed during that time.[6] Of course, one could argue that, at the time, banks still believed that tapping the discount window

[5] Ashcraft and Duffie (2007) argue that these are realistic features of the US interbank market for funds. See Duffie et al. (2005) and Lagos and Rocheteau (2009) for studies of the general implications of these frictions in financial markets.

[6] Furfine (2001) studies the operation of a temporary standing (Lombard-type) facility around the turn of the century in the U.S. and finds similar results.

was likely to trigger some extra scrutiny from supervisors. It may take time to change the culture and the perceptions of participants. The extent to which this is a factor still today remains an open question (see Duke, 2010). However, Artuç and Demiralp (2010) provide evidence in support of the argument that the Federal Reserve has been effective at reducing the fear of regulatory scrutiny since the change in policy in 2003.

In August 2007, as a response to the incipient financial crisis, the Fed lowered the spread in the discount window rate and started to allow eligible institutions to borrow funds at longer terms (instead of just overnight, as was usually the case). The change generated little to no additional borrowing. Furthermore, Klee (2011) extensively documents that the discount window rate did not provide an effective ceiling for rates arranged in the market during the crisis.

In December 2007, the Fed created the TAF, a biweekly (uniform-price) auction of a fixed amount of 28-day funding for depository institutions eligible to obtain (unrestricted/primary) credit at the discount window. For all practical purposes, eligibility and collateral terms were equivalent at the TAF and the discount window. In contrast with the discount window, though, borrowing at the TAF was in high demand since its creation.

Several features of the TAF may have made it less likely to generate stigma. The auction of a fixed, large amount of funds with a cap on individual bids guaranteed participation by multiple bidders and made them more likely to remain anonymous. Furthermore, a period of three days was set between the auction day and the settlement day (when the funds were transferred to the winners of the auction). This delay might have helped to decrease the perception that participants were in desperate need of funding (Bernanke, 2009).

Armantier et al. (2010) postulate that there was no significant stigma attached to borrowing from the TAF and interpret bids above the discount window rate as the result of discount window stigma. They show that for most of 2008, at least 60 percent of the bids in each auction were higher than the discount window rate. In fact, the stop-out rate (the rate at which funds were effectively allocated) was higher than the discount window rate also for most of the auctions during that year. Armantier et al. (2010) take these facts as providing conclusive evidence of the existence of discount window stigma.

While comparing the interest rates in the fed funds market with the discount window rate is complicated by the fact that the former is uncollateralized, ample evidence suggests that there was a significant amount of trading at rates much higher than the discount window rate during the height of the crisis (see Klee, 2011). In line with the earlier research by Furfine, this evidence can also be interpreted as suggesting the existence of stigma.

3 The basic model

Regardless of how persuasive the empirical case for stigma is considered to be, it seems clear that the concept has played a key role in the evaluation of many important policy measures. If only for this

reason, it seems relevant to try to improve our understanding of the economic mechanism behind the idea. In this section, we describe a simple environment that can be used for this purpose. The objective is not to provide a close representation of the U.S. interbank market, nor to exhaust all the specific situations in which stigma may arise in that market. Rather, we are interested here in clearly specifying, and then studying, a particular set of basic components that are consistent with the possibility of such a phenomenon. In the rest of the section, we describe the environment and then discuss some of the assumptions in more detail.

3.1 The environment

The economy lasts for three periods, $t = 1, 2, 3$. There are two groups of banks, liquid and illiquid, and a large number of investors. Banks are in a continuum and each group has measure one. Both banks and investors are risk neutral and do not discount the future.

Illiquid banks need to make a payment of size 1 at the end of period 1 and own a long-term asset that pays a return v in period 3 if held to maturity (and zero in periods 1 and 2). The return v can take one of two possible values, R or 0. If the return of the asset in period 3 is R we say that the asset is high quality. If it is 0, then we say that the asset is low quality. The probability that the asset is high quality is $p \in (0, 1)$ and with probability $1 - p$ the asset is low quality. Asset quality is realized in period 1 even though the return only becomes available in period 3. Illiquid banks who are unable to make their required payment in period 1 suffer a non-pecuniary penalty ρ, with $1 < \rho < R$.[7]

Liquid banks have 1 unit of funds in period 1 and do not need to make any payments at that time, nor do they own any assets. All banks can costlessly store funds from one period to the next.

The assets owned by illiquid banks have some degree of specificity and generate a much lower expected return if sold to another bank. Investors, in contrast, have the ability to manage the asset appropriately and, hence, can buy the asset without affecting its return. Assume that investors have deep pockets and have access to the same storage technology as banks.

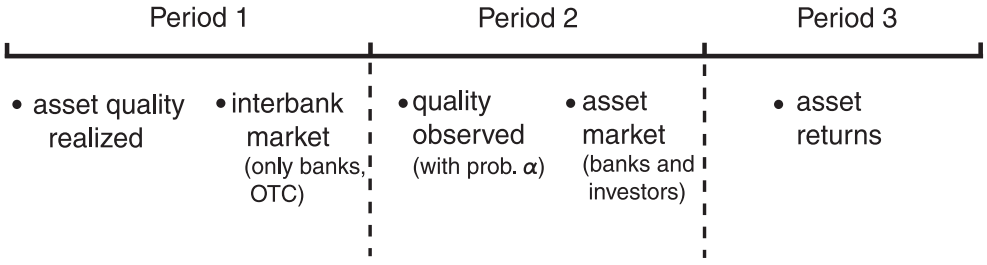

Figure 1: Timing

[7] As will become clear later, the assumption that ρ is less than R simplifies payments feasibility, but it is not essential for the results.

In period 1, after the quality of assets is realized, liquid and illiquid banks can interact in an over-the-counter market for funds. Illiquid banks search for liquid banks to obtain immediate funding. Investors cannot participate in this market. An illiquid bank finds a liquid bank with probability $\sigma \in (0,1)$. The matching technology is such that illiquid banks can only match with liquid banks (and vice versa) and the probability of more than one match is zero.[8]

When two banks match, the liquid bank can costlessly verify the quality of the asset held by its illiquid counterparty. The two banks in the match then decide whether or not to enter a lending agreement with each other and, finally, bargain over the terms of the loan. The outcome of the negotiations is determined according to Nash bargaining with θ being the bargaining power of the lender. Loan maturity is one period and, at the time of repayment (i.e., in period 2), if a bank is not able to pay back the loan in full, it is still obligated to sell the asset and transfer the proceeds to the lender. Investors cannot observe whether a bank has borrowed from another bank.

In period 2, banks and investors participate in a centralized market in which participants can trade funds and assets with each other, and make payments to each other. Each bank has a probability $\alpha \in (0,1)$ that the quality of its asset becomes publicly observable at the beginning of period 2. With probability $1-\alpha$, the quality of the asset remains unknown to investors. At the end of period 2 all banks and investors part ways and, consequently, there are no possible business interactions in the economy during period 3.

3.2 Discussion

The set of frictions that characterize our environment are designed to capture a situation where some banks own illiquid assets but have an immediate need for funding. While, in principle, there are enough funds in the economy (on investors' hands) to cover all immediate needs, banks cannot access such liquidity directly. Instead, in the short run, illiquid banks can only trade with other banks in a market with frictions. Trade in this market is based on the premise that banks will have access to investors' funds in the medium term. In summary, illiquid banks have resources in period 3 that they need in period 1. They effectively transfer (at least part of) those period-3 resources to period 2 by trading with investors and to period 1 by taking loans from liquid banks (see Figure 1). We are interested in studying the implications of private information for this process of intertemporal reallocation of funds via borrowing and asset trading.[9]

It is possible to provide further microfoundation for the liquidity position of banks by introducing, for example, a set of depositors with random withdrawal demands, as in Bhattacharya and Gale

[8] These assumptions rule out the possibility of fund intermediation in the interbank market and, in this way, keep the analysis simple. Afonso and Lagos (2011) study intermediation by allowing the possibility that a bank experiences multiple matches during a period in a model that shares many features with the specification here.

[9] See Acharya et al. (2008) for another model where the interaction between the interbank market and the asset market plays a critical role. In the terminology of Bolton et al. (2011), we assume that there are two distinct sources of outside liquidity, bank loans and investors funds, and no inside liquidity. While bank loans are available on short notice, access to investors funds takes time and involves the sale of assets of (possibly) uncertain quality. We study the interaction between these two markets for outside liquidity.

(1987) and Allen et al. (2009). Clouse and Dow (2002) go a step further and describe a model of the demand for liquidity by banks that is intended to capture more closely the many specific features of the U.S. banking system.

We broadly interpret the penalty ρ as representing the costs (explicit and implicit) for the bank of not being able to fund a preestablished commitment through the 'normal' funding channels. The premium from incurring an overnight overdraft in the bank's account at the central bank is one (pecuniary) component of this cost, but there are many other (non-pecuniary) components that are just as important (such as loss of reputation and increased scrutiny by regulators; see Clouse and Dow, 2002, p. 1792). For simplicity, we assume that the penalty ρ is entirely non-pecuniary in our model.

We have assumed that banks cannot buy and manage effectively the assets of another bank. This assumption generates a role for interbank loans in the model. We could consider the case where some banks are good managers of purchased assets as long as this is not the case for all banks. In other words, as long as some liquid banks prefer to lend rather than to buy the assets of illiquid banks, our results still apply.

Note that the setup is too simple to differentiate between collateralized and uncollateralized lending. When a bank is not able to fully repay the loan, liquidation of the bank (and, hence, the asset) takes place in order to fulfill, to the extent possible, the bank's obligation to its creditor. While lending in the fed funds market is generally uncollaterilized, lending at the discount window requires collateral. It would certainly be interesting to study how this property of the lending contracts influences the outcomes in the market. We leave this issue for future research.

It is important in the model that lenders have information about the financial conditions of their counterparties in the interbank market. The fact that this information is perfect is not essential, but simplifies the trading protocols significantly. Including information frictions at the lending stage would be an interesting extension to consider. For an early contribution in this direction, see Flannery (1996). To limit the flow of information between markets, we combine perfect information in the interbank market with the assumption that investors cannot observe if a bank has borrowed from another bank in period 1. In general, it would be interesting to study alternative informational structures. Here, to maintain the focus of our analysis, we make these simplifying assumptions that allow us to isolate the possible signaling role of discount window lending.

4 Equilibrium in the basic model

We solve for a (symmetric pure-strategy) Perfect Bayesian Equilibrium of this economy. To do so, it is helpful to proceed going backward in time, beginning with the last period. To start, then, we compute asset prices in period 2 given investors' beliefs about the trading strategies of banks. Then, in period 1, illiquid banks look for liquid banks and, when matched, negotiate over the terms of a loan taking into account their prediction about asset prices in period 2. In equilibrium, the

interactions in period 1 confirm the predictions of investors about prices in period 2.

Asset price determination in this model is fairly simple. When the quality of the asset is observed by investors, two prices are possible. If the asset is high quality, then competition among investors determines that the asset can be sold at a price $P_H = R$. On the other hand, since low quality assets give zero return in period 3, investors are not willing to pay any positive amount for low quality assets in period 2 and its price P_L is equal to zero. The more interesting case is when the quality of the asset is not observed. In this case, pricing in period 2 will depend on the beliefs of investors about the relative prevalence of high and low quality assets in the market (as in Eisfeldt, 2004). Let q denote the (equilibrium) probability investors assign to the event that a given asset of unobserved quality being sold in the market in period 2 is high quality. Then, the price of this asset will be given by the function $P_U(q) = qR$. Since $q \leq 1$ we have that $P_H \geq P_U(q) \geq P_L = 0$.

In period 1, depending on the quality of the asset held by the bank and on equilibrium asset prices, an illiquid bank that finds a liquid bank may be able to obtain a loan of size 1 from the latter. More specifically, a loan will be granted whenever there is a feasible interest rate such that both parties (the borrower and the lender) are left not worse off by entering the lending agreement. To evaluate this possibility, we need to specify the net payoff to the parties from agreeing to a loan.

Consider first the case when the illiquid bank is holding a high quality asset. In this case, the net payoff to the lender from granting a loan at (gross) interest rate r_H is given by:

$$S_l(r_H, q) = \alpha r_H + (1-\alpha)\min\{P_U(q), r_H\} - 1,$$

where we are using $r_H \leq P_H$, since there is no point in arranging an interest rate greater than the maximum that a borrower could possibly pay. Note, also, that we are subtracting from the payoff S_l the lender's opportunity cost of funds, which in this simple environment is equal to one.

The net payoff to the borrower from getting a loan at rate r_H is given by:

$$S_b(r_H, q) = \alpha(P_H - r_H) + (1-\alpha)\max\{P_U(q) - r_H, 0\} - (R-\rho).$$

The last term $R-\rho$ represents the payoff to the illiquid bank from not accepting the loan, incurring the penalty ρ, and keeping the asset until period 3.

The total surplus from a loan in this case is given by:

$$TS_H(q) = S_l(r_H, q) + S_b(r_H, q) = \alpha P_H + (1-\alpha)P_U(q) - (R-\rho) - 1.$$

When the illiquid bank is holding a low quality asset it can only repay a loan in the event that its quality does not get observed by investors and the price P_U is positive. Clearly, the interest rate arranged must be lower than P_U. Then, the net payoff to the lender from entering a loan arrangement is given by

$$S_l(r_L) = (1-\alpha)r_L - 1, \tag{1}$$

and the net payoff to the borrower is

$$S_b(r_L) = \rho - (1-\alpha)\, r_L$$

where we are using the fact that a bank holding an asset of low unobserved quality in period 2 would be selling the asset in the market whenever P_U is greater than zero, even if it has not obtained a loan in period 1 (the alternative is to hold the asset and get zero).

Note that $TS_L = S_l(r_L) + S_b(r_L) = \rho - 1 > 0$. Even though the total surplus from a loan is always positive in this case, when $P_U(q) < 1/(1-\alpha)$ the parties will not enter a loan agreement. To see this, note that it only makes sense to consider values of r_L less than or equal to $P_U(q)$, since this is the maximum amount that a lender could obtain in period 2 from a borrower holding a low quality asset. It is clear, then, that the lender, in expected terms, cannot get more than $(1-\alpha)P_U(q)$ from the borrower, and if this quantity is less than 1, then the lender would not agree to participate in the loan.[10]

We now need to determine the possible equilibrium values of q. The first thing to notice is that whenever $q > 0$ all banks holding assets of low unobserved quality will want to sell their assets in the market. What makes the determination of equilibrium beliefs nontrivial is the actions in the loan market of illiquid banks holding a high quality assets. These banks may or may not take a loan in period 1 depending on the value of q. In turn, whether these banks take a loan or not determines the relative prevalence of high quality assets in the market and, hence, the values of q consistent with equilibrium.

Proposition 1. When $TS_H(0) < 0$ there is an equilibrium with $q = 0$.

Proof: We will show that when $q = 0$ no loans are made in period 1. The reason for this is the following: If an illiquid bank with a high quality asset does not take a loan in period 1, its payoff is $R - \rho$. Hence, this bank should get at least as much in expected terms from entering a loan contract. Since $q = 0$ the maximum expected payoff obtainable from the asset in period 2 is αR. Then, a lender can get a maximum expected repayment equal to $\alpha R - (R - \rho)$, but he can get 1 from not making the loan. Note that $TS_H(0) = \rho - 1 - (1-\alpha)R$ and $TS_H(0) < 0$ implies that the maximum expected repayment to a lender is less that what it can get by not making the loan. So, if banks expect that investors will not be willing to pay for an asset of unobserved quality (i.e., if $q = 0$), it is not possible to have the liquid and illiquid banks agreeing on a feasible loan contract. But then, since illiquid banks with assets of high unobserved quality do not have a loan to repay, they have no reason to sell their assets (they get zero from doing so, instead of R if they do not). Therefore,

[10] Recall that ρ is a cost incurred by the bank if it cannot fund its liquidity needs in period 1. When the surplus from the loan agreement is positive and the loan does not happen, the illiquid bank would like to be able to use some of the resources dedicated to cover ρ to make a payment to the potential lender. We have assumed that ρ is a non-pecuniary cost. So, there are no resources to make that kind of payments. Alternatively, we could assume that (some of) the cost ρ is pecuniary but the emergency funding used to pay for it is restricted and cannot be used in period 2 for the purpose of debt repayment.

only low quality assets can be expected to be for sale in period 2, which is consistent with the belief expectation $q = 0$. ■

The proposition gives us a condition under which the asset market in period 2 shuts down and, in anticipation of that fact, illiquid banks get excluded from the loan market in period 1 even when they are holding a high quality asset. It is interesting to note that the condition is more likely to hold when the probability α that the quality of an asset will become observable in period 2 is low; that is, when the information frictions in the asset market are expected to be large.[11]

This "no credit" equilibrium does not exist if $TS_H(0) > 0$. Furthermore, even if the condition of the proposition is satisfied, another equilibrium with credit in the interbank market may be possible. Suppose that $q > 0$ in equilibrium. Then, we know that in period 2 there will be $(1-\alpha)(1-p)$ low unobserved quality assets in the market. Furthermore, for $q > 0$ to be (part of) an equilibrium, it must be true that the high quality assets of illiquid banks that manage to obtain a loan in the interbank market are put for sale in period 2 (otherwise q would be equal to zero, as we saw in Proposition 1). Then, the total amount of assets of unobserved quality in the market that are high quality will be equal to $p\sigma(1-\alpha)$ and the consistent equilibrium beliefs are given by:

$$q^* \equiv \frac{p\sigma}{1-p+p\sigma}.$$

When a liquid bank enters a lending relationship with an illiquid bank holding a high quality asset, the interest rate they will agree upon solves the following Nash bargaining problem:

$$\max_{r_H \leq R} S_l(r_H, q^*)^\theta S_b(r_H, q^*)^{1-\theta}$$

subject to $S_l(r_H, q^*) \geq 0$ and $S_b(r_H, q^*) \geq 0$. As is clear from the expressions of the payoff functions, the solution to this problem depends on whether the borrowing bank will be able to repay the loan when the quality of its asset is unobserved in period 2. Hence, the expression for the solution of the Nash problem depends on whether the solution is less than or greater than $P_U(q^*)$. To characterize the solution, then, define two functions $\widetilde{r}_H(\theta, q^*) \equiv 1 + \theta\, TS_H(q^*)$ and $\widehat{r}_H(\theta, q^*) \equiv \frac{1}{\alpha}\left[1-(1-\alpha)P_U(q^*)\right] + \frac{\theta}{\alpha} TS_H(q^*)$. We have that the solution to the Nash bargaining problem is given by:

$$r_H(\theta, q^*) = \begin{cases} \widetilde{r}_H(\theta, q^*) & \text{if} \quad \theta < \theta^T(q^*) \\ \\ \widehat{r}_H(\theta, q^*) & \text{if} \quad \theta \geq \theta^T(q^*) \end{cases}$$

where $\theta^T(q^*) = \max\{0, \min\{(P_U(q^*) - 1)/TS_H(q^*), 1\}\}$. Note that when $\theta^T(q^*) \in (0,1)$ we have

[11] Changes in the quality of the asset, as reflected by changes in the return R, have two opposing effects. On the one hand, an increase in R increases the availability of funds for repayment; but, on the other hand, it increases the outside option for the potential borrower, reducing his incentive to take the loan. In our setup, the second effect dominates and, as a consequence, increases in R make the possibility of a shut-down of the interbank credit market compatible with a larger set of values for the other relevant parameters.

that $\widetilde{r}_H(\theta^T(q^*), q^*) = \widehat{r}_H(\theta^T(q^*), q^*) = P_U(q^*)$. Figure 3 plots the payoff functions $S_l(r_H, q^*)$ and $S_b(r_H, q^*)$, which have a kink at $r_H = P_U(q^*)$. In the figure, we also plot the objective function of the Nash bargaining problem for a particular value of θ, to illustrate how the interest rate is determined.

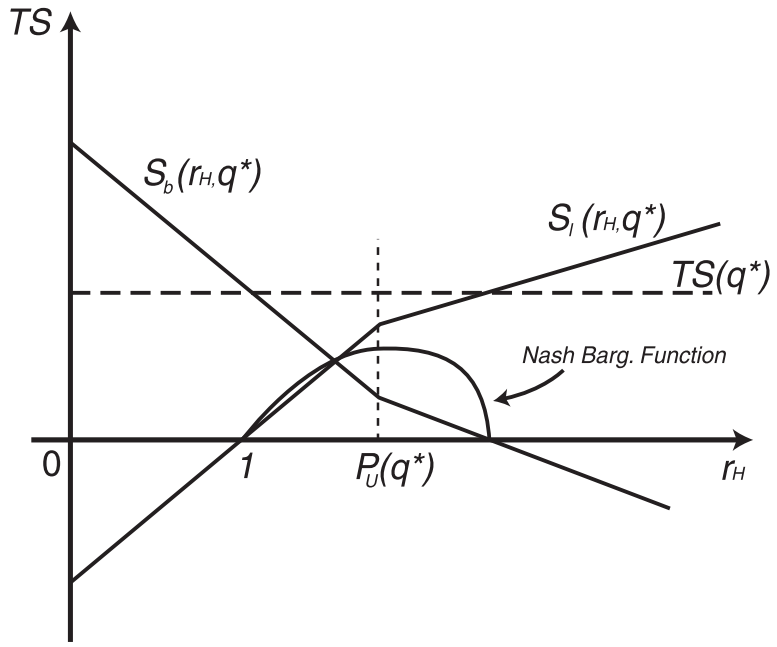

Figure 2: The Nash bargaining solution

Replacing the expression for $r_H(\theta, q^*)$ in the payoff functions for the banks, it can be shown after some algebra that the $S_b(r_H, q^*) = (1-\theta)TS_H(q^*)$ and $S_l(r_H, q^*) = \theta TS_H(q^*)$ for all θ. Furthermore, since $R > \rho$ we have that $r_H(\theta, q^*) \leq r_H(1, q^*) < R$ for all (θ, q^*) and, in consequence, $r_H(\theta, q^*)$ is always feasible in the sense that there are at least some borrowers that are able to pay as much for a loan. From this, we conclude that when $TS_H(q^*)$ is positive, both the liquid and illiquid banks in a match will agree to participate in a loan agreement. For concreteness, let us assume that when indifferent, banks will also enter a loan relationship. We then have the following proposition.

Proposition 2. When $TS_H(q^*) \geq 0$ there is an equilibrium with interbank credit.

Proof: Suppose that investors conjecture that a proportion q^* of the unobserved quality assets for sale in period 2 are high quality. Then, the expected price of unobserved quality assets in period 2 is q^*R. Since $q^* > 0$ all banks holding a low unobserved quality asset will sell it in period 2. Similarly, if a bank holding a high unobserved quality asset in period 2 has taken a loan in period 1, then its asset will be sold in the market in period 2. Also, since $q^* < 1$, banks holding a high quality asset and no loan will not sell their asset in period 2. Finally, since $TS_H(q^*) \geq 0$, illiquid banks holding a high quality asset take a loan from liquid banks whenever they find a counterparty in period 1. We have, then, that all the assets of low unobserved quality and the assets of high unobserved quality

held by banks with a loan to repay, will be sold in the market in period 2. This implies that the expected value of assets of unobserved quality being sold in period 2 is q^*R, which is consistent with the investors' initial conjecture. ■

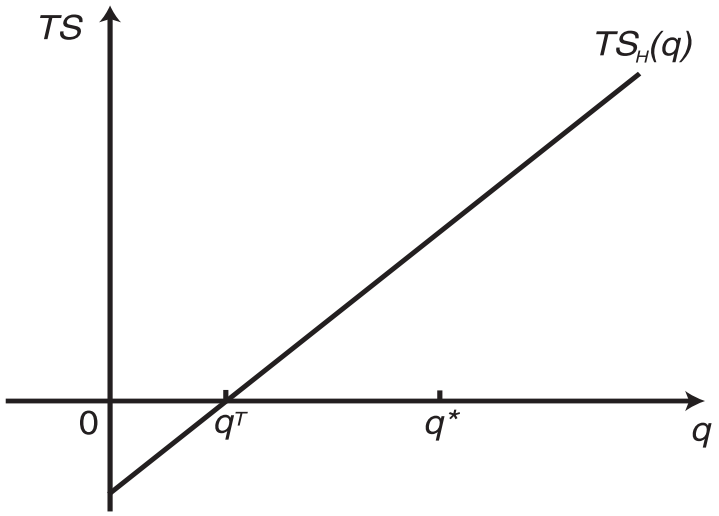

Figure 3: The surplus function

Note that whether an illiquid bank holding a low quality asset takes a loan in the interbank market when it finds a counterparty does not influence outcomes in the asset market. Whenever $(1-\alpha)\,q^*R > 1$, we have that in the equilibrium of Proposition 2 all banks that find a counterparty take a loan, regardless of the quality of the asset that they are holding. If, instead, $(1-\alpha)\,q^*R < 1$ then only banks holding a high quality asset are able to secure a loan in the interbank market of period 1.

Let us now define the threshold value q^T as:

$$q^T \equiv -\frac{TS_H(0)}{(1-\alpha)R}.$$

Combining Propositions 1 and 2 we have the following two corollaries (see Figure 3).

Corollary 1. When $q^* \geq q^T > 0$ there are two symmetric pure-strategy equilibria, one with interbank credit and one where interbank credit shuts down.[12]

Proof: Since the parameters determining q^* are different from those determining q^T, we can always find parameters such that the conditions in the corollary hold. In such a case, since $q^T > 0$ implies that the condition in Proposition 1 holds, we have that a "no credit" equilibrium exists. Furthermore,

[12] There is also a mixed-strategy equilibrium in which illiquid banks that find a counterparty and are holding a high quality asset take a loan from their counterparty only with probability $\eta = q^T(1-p) / \left[\left(1-q^T\right)p\sigma\right]$ and the surplus from those matches is exactly equal to zero (i.e., $TS_H\left(q^T\right) = 0$).

given that $TS_H(q)$ is strictly increasing in $q \in (0,1)$, by definition of q^T we have that $TS_H(q) \geq 0$ for all $q \geq q^T$. Then, since $q^* \geq q^T$, Proposition 2 implies that an equilibrium with interbank credit also exists. ■

This corollary tells us that, for a set of the parameter values, the model is consistent with multiple equilibria. Furthermore, these equilibria have significantly different implications for the outcome of the interbank market for loans. If banks expect that pessimistic investors will be pricing the assets of uncertain quality in period 2, then those banks may not be willing to enter into loan relationships in period 1. This reduction in activity in the loan market will, in turn, result in a selective reduction of bank participation in the asset market (i.e., high-quality-asset holders will be out of the market), justifying in this way the (rational) pessimism of investors.

Corollary 2. When $q^T < 0$ there is a unique equilibrium and the equilibrium has interbank credit.

Proof: By definition of q^T we have that $TS_H(0) > 0$, and since $TS(q)$ is strictly increasing in $q \in (0,1)$ we have that $TS(q) \geq 0$ for all $q \in [0,1]$. Then, high quality illiquid banks that find a match in the interbank credit market always enter a loan relationship. This implies that the assets of those banks will be put for sale in the market in period 2 and, hence, that the only consistent equilibrium value of q is q^*. ■

In this section, we have studied the functioning of an interbank market for funds in the presence of frictions that limit the ability of banks to trade with each other. As a result of these frictions, it is possible that banks are not able to borrow during period 1 even when they are holding high quality assets that have a present value larger than the face value of the loans that the banks seek to obtain. A natural question to ask is how equilibrium outcomes would change if a backup source of funding in the form of a central bank lending facility (i.e., a discount window) is available to banks in this environment. In the next section, we extend the model to allow for discount window lending and describe equilibrium in that case.

5 Discount window lending

Assume now that banks have access in period 1 to a central bank credit facility; i.e., a discount window.[13] Assume also that the interest rate at the discount window is $r_W > 1$ and that, after a bank borrows from the window in period 1, investors in period 2 become aware of such activity with probability $\delta \in (0,1)$. Before we turn to the study of equilibrium in this case, we briefly discuss how these assumptions relate to some features of the U.S. system.

[13] In Acharya et al. (2008) discount window loans play a related funding role by reducing illiquid banks' exposure to the risk of having to sell their assets in the market at a very significant loss. Here, the market for assets is actually closed at the time the illiquid bank needs funding in period 1. In a sense, our assumption is an extreme version of that considered in Acharya et al. (2008).

The U.S. Federal Reserve regularly monitors the financial condition of banks with access to the discount window.[14] In principle, the Fed does not intend to provide primary credit to institutions with non-negligible repayment risk. However, the only way that discount window borrowing can become a negative signal of the financial condition of a bank, as is required in the explanation of stigma being explored here (Bernanke, 2009), is if there is at least a perception in the market that imperfect screening is happening at the window. Assuming, as we do here, that all banks in the model have access to the window produces imperfect screening without introducing additional complications. We could introduce a less coarse categorization of assets and have only some banks accessing the discount window. The only essential component is the possibility of imperfect screening.

In practice, each Federal Reserve Bank reports only the total amount of discount window lending granted the previous two weeks. These announcements constitute a noisy signal of the participation of particular banks in the reported discount window activity. Under certain circumstances, however, market participants may be able to put together various pieces of additional information (like market funding requests by a particular institution) which, in combination with the Fed's reported numbers, may actually reveal (with some certainty) the identity of the borrowing banks (Furfine, 2001, Duke, 2010). In line with this situation, we allow for the possibility that investors in the model obtain accurate information about banks' activities at the discount window with some probability. The essential point here is that a theory of stigma based on signaling requires that agents perceive this probability to be positive.

5.1 Equilibrium

We again solve for a symmetric pure-strategy Perfect Bayesian equilibrium and start by identifying possible outcomes in the market for assets during period 2. As before, when the quality of an asset becomes observable, the price of the asset is either R or zero depending on whether the asset is of high or low quality, respectively.

When the quality of the asset is not observable, things become more complicated. Observing that a bank has borrowed at the discount window could be an informative signal about the quality of the asset that the bank is trying to sell. This possibility is the result of two important assumptions in our model. On the one side, banks in the interbank market have accurate information about the quality of the asset held by counterparties, which in turn influences their lending behavior. On the other side, sometimes investors are not able to observe directly the quality of the asset being traded, nor the seller's private dealings in the interbank market, but do get to observe the seller's

[14] Depository institutions in the US have access to three types of discount window credit: primary credit, secondary credit, and seasonal credit. Primary credit is available to depository institutions that were categorized by supervisors as in sound financial condition. Its provision is associated with minimal administrative requirements and its usage is essentially unrestricted. Secondary credit is available to depository institutions that are not eligible for primary credit. It is provided only in particular situations and the institutions borrowing from the secondary credit program are more closely monitored by the Fed. Seasonal credit is provided to assist small depository institutions to manage seasonal swings in loans and deposits.

transactions with the central bank.

As before, investors in period 2 form beliefs about the quality of the assets of unobserved quality that are put for sale in the market. Those beliefs depend on whether or not buyers can see that the seller of the asset has borrowed at the discount window. Let q_W be the belief probability that the asset is high quality when the seller of the asset is known to have borrowed at the window; and let q_N be the corresponding probability (of high quality) if the seller is not known to have borrowed at the window. Then, there are two possible prices for assets of unobserved quality in period 2, which we denote by $P_U(q_j) = q_j R$ with $j = N, W$. These prices depend on equilibrium beliefs that must be consistent with the decisions taken by banks, given those beliefs. We study bank decisions next.

Since liquid banks have access to funds at an opportunity cost equal to one, they have no incentive to take loans from the discount window at rates higher than one. In other words, the lend-to-borrow strategy is not profitable for liquid banks.

Illiquid banks that do not find a liquid counterparty in the interbank market have to decide whether or not to borrow from the discount window. For this decision, the bank compares the payoff of taking each possible action. To calculate this payoff, we assume that all banks that have borrowed at the window sell their asset in period 2 to pay back the loan (in full or partially).[15] Define the auxiliary functions

$$\pi(q_j, r_i) \equiv \max\{P_U(q_j) - r_i, 0\} \text{ with } j = N, W \text{ and } i = H, L, W.$$

Then, an illiquid bank that has not found a counterparty and is holding a high quality asset will borrow at the window if:

$$\alpha(P_H - r_W) + (1-\alpha)\left[\delta\pi(q_W, r_W) + (1-\delta)\pi(q_N, r_W)\right] \geq -\rho + R; \tag{2}$$

and a bank holding a low quality asset will borrow at the window if:

$$(1-\alpha)\left[\delta\pi(q_W, r_W) + (1-\delta)\pi(q_N, r_W)\right] \geq -\rho + (1-\alpha)P_U(q_N). \tag{3}$$

The following lemma demonstrates that, in equilibrium, if an illiquid bank holding a high quality asset borrows at the discount window when it does not find a counterparty, then so does an illiquid bank that is holding a low quality asset and also does not find a counterparty.

Lemma 1. Inequality (2) implies inequality (3).

Proof: The proof follows from the fact that $r_W > 0$ and $q_N \leq 1$. ■

[15] It could be interesting to consider alternative treatments of those borrowers that cannot repay discount window loans in full. In the simple case we study here, loans from the discount window differ from loans granted by private counterparties only in the way the interest rate is determined. At the window, the rate is exogenously set and is not contingent on asset quality.

In what follows, we will restrict attention to the case in which inequality (2) holds. In other words, we will consider the case in which the discount window rate r_W is such that banks that do not find a counterparty borrow from the window. We consider this the empirically relevant case. One of the stated purposes of the discount window is to serve as a "backup source of short-term funds for generally sound depository institutions." In our model, illiquid banks that do not find a counterparty and are holding a high quality asset seem to be the most natural candidates for discount window credit, given this stated objective.

Illiquid banks that do find a counterparty in period 1 must decide among three possible alternatives: they could borrow from the liquid bank, from the window, or not at all. Recall that if a bank borrows in period 1, it will have to sell its asset in period 2 to repay (all or some of) the loan. Then, the net payoff for an illiquid bank holding a high quality asset and taking a private loan is given by:

$$\begin{aligned} S_b(r_H, q_N, q_W) &= \alpha(P_H - r_H) + (1-\alpha)\pi(q_N, r_H) - \\ &\quad \max\left\{\alpha(P_H - r_W) + (1-\alpha)\left[\delta\pi\left(q_W, r_W\right) + (1-\delta)\,\pi\left(q_N, r_W\right)\right], -\rho + R\right\}. \end{aligned}$$

Correspondingly, the net payoff from taking a private loan for an illiquid bank holding a low quality asset is given by:

$$\begin{aligned} S_b(r_L, q_N, q_W) &= (1-\alpha)\pi(q_N, r_L) - \\ &\quad \max\left\{(1-\alpha)\left[\delta\pi\left(q_W, r_W\right) + (1-\delta)\,\pi\left(q_N, r_W\right)\right], -\rho + (1-\alpha)P_U\left(q_N\right)\right\}. \end{aligned}$$

Of course, a bank will agree to take a private loan if and only if the net payoff from doing so is non-negative.

Finally, we need to consider the decision of liquid banks upon entering a match with an illiquid bank. If the illiquid bank is holding a high quality asset, then the net payoff for the liquid bank that agrees to make a loan is:

$$S_l(r_H, q_N) = \alpha r_H + (1-\alpha)\min\left\{P_U\left(q_N\right), r_H\right\} - 1.$$

Similarly, when the illiquid bank is holding a low quality asset, the liquid bank net payoff from making the loan is $S_l(r_L, q_N) = S_l(r_L)$ as defined in expression (1) of Section 4, with $r_L \leq P_U\left(q_N\right)$.

Define the total surplus in a match as $TS_j(q_N, q_W) = S_b(r_j, q_N, q_W) + S_l(r_j, q_N)$ for $j = H, L$. When banks agree to enter a lending relationship, they negotiate over the interest rate. The outcome of such negotiation is the solution to the following Nash problem for $j = L, H$:

$$\max_{r_j} S_l(r_j, q_N)^{\theta} S_b(r_j, q_N, q_W)^{1-\theta},$$

subject to $S_l(r_j, q_N) \geq 0$, $S_b(r_j, q_N, q_W) \geq 0$, $r_H \leq R$, and $r_L \leq P_U\left(q_N\right)$. Call the solution to this problem, if it exists, $r_j(\theta, q_N, q_W)$ for $j = L, H$.

In period 2, those banks that have taken a (private or discount window) loan in period 1 will sell their asset in the market. If $q_N > 0$ then all banks holding a low quality asset will sell their asset even if they do not have a loan to repay. If $q_N < 1$ then banks that do not have a loan to repay and are holding a high quality asset will not sell their asset. These cases exhaust all the possibilities.

A Perfect Bayesian equilibrium, then, can be characterized by a set of beliefs (q_N, q_W), loan agreements with the corresponding interest rates, and asset sales and prices such that: (1) all agents make optimal lending and asset sale decisions given those beliefs; (2) asset prices reflect those beliefs; and (3) the agents' decisions validate those equilibrium beliefs in the sense that they are the result of applying Bayes rule on equilibrium outcomes (i.e., a fixed point in beliefs).

The first thing to notice is that the availability of discount window credit rules out the type of equilibrium described in Proposition 1, where the price of assets of unobserved quality is equal to zero in period 2. Even if all banks are borrowing at the discount window, the prices of unobserved quality assets in period 2, $P_U(q_j)$ with $j = N, W$, will be both positive.

5.2 Stigma

The objective in this section is to study, in the context of the model, the empirical and theoretical arguments discussed in the introduction of the paper. In particular, we want to construct an equilibrium in which stigma is associated with lending from the discount window and, for this reason, some banks take loans in the interbank market at rates higher than the discount window rate r_W. In the process, we identify specific conditions under which such a situation is theoretically possible and draw some conclusions about its empirical plausibility.

Since we are assuming that inequality (2) holds, we know that banks that do not find a counterparty will borrow at the window. Hence, we only need to consider the decision of banks that find a counterparty. Of these banks, some of them may borrow at the window and some of them may borrow in the interbank market. To have borrowing at the window constitute a negative signal in equilibrium, we need some degree of "negative selection" in the sense that there is a higher proportion of holders of low quality assets borrowing at the window than there is borrowing in the interbank market. This requirement narrows down the possibilities to the case where the holders of high quality assets borrow in the interbank market when they find a counterparty and the holders of low quality assets borrow at the window even when they find a counterparty. We now discuss conditions under which this equilibrium configuration is possible.

We restrict attention to only those cases in which $\pi(q_j, r_i) > 0$ for all $j = N, W$ and $i = H, L, W$. In other words, repayment risk is only associated with banks that hold a low quality asset and such risk only materializes in those situations in which the quality of the asset is actually observed by investors in period 2. This situation allows us to capture more fully the tradeoffs associated with borrowing at the window.[16]

[16] One of the benefits of borrowing in the market is that the bank does not have to pay the penalty attached to the

Define the following two auxiliary functions:

$$\zeta(p) \equiv \frac{p\sigma + (1-\delta)\, p\,(1-\sigma)}{p\sigma + (1-\delta)\,(1-p\sigma)},$$

and

$$\xi\,(p) \equiv \frac{p\,(1-\sigma)}{1-p\sigma}.$$

It is easy to see that both functions are increasing in p and that they are both equal to zero when p is equal to zero and equal to one when p equals one. Furthermore, we have that $\zeta\,(p) > p > \xi\,(p)$ for all $p \in (0,1)$.

Proposition 3. Given all other parameter values, there exists a threshold value $p^T < 1$ such that for all $p \in (p^T, 1)$ we have that we can find a value $r_W^T\,(p) > 1$ such that for all $r_W \in \left(1, r_W^T\,(p)\right)$ there is an equilibrium with both interbank credit and discount window lending in which $q_W = \xi(p)$, $q_N = \zeta(p)$, and $r_H = 1 + \theta\, T S_H(q_N, q_W)$.

Proof: See the Appendix. ■

Even though in this equilibrium all banks borrowing in the market hold a high quality asset, the price of the assets sold by banks that show no evidence of having borrowed at the window still comprises a discount. The reason for this discount is the imperfect observability of discount window activity. Note, however, that since $\xi(p) < \zeta(p)$ the discount in the price is larger for banks that can be identified as having borrowed from the window. In other words, evidence that a bank has borrowed at the window in period 1 constitutes a negative signal about the quality of its asset when trading in the market of period 2. This effect is how our model captures the idea of stigma formally.

In the equilibrium of Proposition 3 only illiquid banks that find a counterparty in the interbank market and are holding a high quality asset borrow from another bank. Illiquid banks that do not find a counterparty, plus those banks that do but are holding a low quality asset, borrow from the discount window. Note that this configuration does not necessarily imply unrealistic levels of borrowing at the discount window. In fact, if both p and σ are close to unity, as they are likely to be in the empirically relevant case, then most banks actually borrow from the interbank market in this equilibrium.

The following corollary considers the relevant comparison between the equilibrium interest rate in the interbank market and the discount window rate.

Corollary 3. In the equilibrium of Proposition 3, for θ close enough to unity, $r_H > r_W$.

Proof: Note that θ appears only in the condition that determines the surplus splitting rule between

discount window rate. However, if the bank cannot fully repay the discount window loan in some state (for example, if $\pi\,(q_W, r_W) = 0$) then the benefits of not borrowing at the window are reduced because with positive probability some of the penalty will not be paid. When the bank is holding a low quality asset and this quality is observed by investors, the bank does not repay its loan; but in this case it does not repay the loan regardless of whether it borrowed from the window or from the market.

the liquid and the illiquid bank holding a high quality asset. Then, the existence of the equilibrium described in Proposition 3 is independent of the value of θ and the equilibrium exists for any value of θ, including those arbitrarily close to one. Since such an equilibrium has $r_H = 1+\theta\, TS_H(q_N, q_W) = 1 + \theta\,[r_W - 1 + (1-\alpha)\delta\,(q_N - q_W)\,R]$, which approaches $r_W + (1-\alpha)\,(q_N - q_W)\,R > r_W$ as θ approaches unity, the corollary clearly holds. ■

This corollary demonstrates that when the bargaining power of lenders is high, the equilibrium of the model may have some banks willing to take a loan in the interbank market at rates higher than the rate that they could obtain at the discount window. That is, under certain conditions, our model predicts the pattern on interest rates that has been previously highlighted in the empirical literature (see Section 2).

The equilibrium premium of the observed private rate r_H over the discount window rate r_W is given by:

$$r_H - r_W = \theta(1-\alpha)(q_N - q_W)R - (1-\theta)\,(r_W - 1)\,,$$

and, hence, is actually decreasing in $r_W - 1$ which is a measure of the premium (penalty) paid for borrowing at the discount window. Policies that lower this premium, then, could actually increase the positive spread between the market rate and the discount window rate. This is an interesting finding. A common reaction by policymakers to the reluctance by banks to borrow from the window has been to lower the penalty spread at the discount window.[17] Our model predicts that such a change could increase the extent by which observed market rates are higher than the discount window rate as a result of stigma.

More generally, changes in the incidence of stigma are likely to be associated with changes in the composition of banks borrowing at the window (in terms of the quality of their assets) relative to the composition of banks borrowing in the interbank market.[18] The model we have presented here allows for only abrupt changes in this composition (either low quality banks that find a counterparty borrow at the window or they do not) and, for this reason, it may not be particularly suitable to conduct comparative statics on certain policy parameters. Generalizing the patterns of heterogeneity in asset qualities could make those exercises more meaningful. In such an extension of the model it seems likely that policy actions that increase the prevalence at the window of borrowers holding assets of relatively high quality would tend to decrease stigma.

In the equilibrium of Proposition 3, the level of the interest rate that is observed in the market is given by r_H, which does not involve any repayment risk. Banks with low quality assets (which could be regarded as the risky ones in our setup) do not receive loans from private banks. Furthermore, all

[17] From 2003 to 2007, the Fed provided discount window credit at a rate 100 basis points over the target fed funds rate. The Fed lowered the spread significantly during the financial crisis, to 50 basis points in August 2007 and to 25 basis points in March 2008. In February 2010, the spread was increased back to 50 basis points.

[18] Klee (2011) discusses the possibility that selection due to heterogeneity across bank in their perception of stigma could also explain how market rates responded to changes in the discount window spread during the recent financial crisis.

banks borrowing from other banks pay the same interest rate. In the U.S. system, however, on any given day there is a distribution of rates observed in the market. We could generalize the model to capture these different rates by, for example, introducing heterogeneity in the bargaining power of different banks. This modification may, in fact, be realistic (Ashcraft and Duffie, 2007 and Afonso and Lagos, 2011). For example, it seems plausible that, on any given day, some banks in need of funds may find that their usual counterparty has no funds available that day. In that case, they need to search in the market for alternative counterparties and, depending on their network connections, they may find that their bargaining power is much reduced. In our model, illiquid banks with low bargaining power will pay higher interest rates. In fact, the resulting heterogeneity will be consistent with the fact that, in the U.S., most of the time only some banks pay interest rates that are higher than the one they could obtain at the discount window.

5.3 Other possible equilibrium outcomes

The model is consistent with a rich variety of possible outcomes depending on parameter values. If we restrict attention to only equilibria where condition (2) holds and $\pi(q_j, r_i) > 0$ for all $j = N, W$ and $i = H, L, W$, then the only other possible equilibrium configuration is the one in which all banks that find a counterparty borrow from the market and all banks that do not find a counterparty borrow from the discount window. There is a non-empty set of parameter values for which this equilibrium exists. The following proposition provides the formal result.

Proposition 4. Assume that $\rho > 1/(1-\alpha)$. Given all other parameter values, there exists a threshold value $p^{T'} < 1$ such that for all $p \in (p^{T'}, 1)$ we have that we can find a value $r_W^{T'}(p) > 1/(1-\alpha)$ such that for all $r_W \in \left(1/(1-\alpha), r_W^{T'}(p)\right)$ there is an equilibrium with both interbank credit and discount window lending in which $q_W = q_N = p$, and $r_H = \theta r_W + (1-\theta) < r_L = \theta r_W + (1-\theta)/(1-\alpha) < r_W$.

Proof: See the Appendix. ■

Since $q_W = q_N$, in this equilibrium there is no stigma attached to borrowing from the discount window. Furthermore, we have that $r_H < r_L < r_W$, so there is some dispersion in the interest rates observed in the interbank market and all interest rates arranged in the market are below the rate offered at the discount window.[19]

The set of parameter values identified in Proposition 4 is disjoint from the set of parameter values identified in Proposition 3. In other words, if one of the equilibria exists, the other does not. The issue of stigma is often discussed in relation to the possibility of multiple equilibria. Loosely speaking, the multiplicity logic refers to an activity that "everybody does" because there is low stigma, and there is low stigma because "everybody does it" (see, for example, the tax evasion example of Kim,

[19] King (2008), using data from the US interbank market, finds some evidence that banks with higher repayment risk tend to obtain credit from private counterparties at higher interest rates than banks with lower repayment risk. The equilibrium of Proposition 4 is consistent with such finding.

2003). This is not the idea behind the stigma effects that we have discussed in this paper. In the model, stigma does not always decrease when participation at the window increases. In fact, when moving from the equilibrium in Proposition 4 to the equilibrium in Proposition 3, the proportion of banks borrowing at the discount window becomes higher and stigma actually increases.

Another equilibrium configuration that seems natural to consider is the case in which all banks borrow from the discount window. In such a case, again, equilibrium beliefs would imply that $q_W = q_N = p$ and, hence, there would be no stigma. Furthermore, if $pR > 1$ then an equilibrium of this sort does not exist. The key to this result is the fact that when $pR > 1$ we have that $TS_H(p, p)$ is increasing in r_W for all $r_W > 1$ and $TS_H(p, p) = r_W - 1$ for $r_W \in (1, pR)$. Then, it follows that $TS_H(p, p) > 0$ for all $r_W > 1$ and illiquid banks that find a counterparty and are holding a high quality asset would prefer to borrow from the market. Of course, this action makes the proposed system of beliefs inconsistent with actual equilibrium behavior.[20]

The remaining cases to be considered as candidate equilibrium configurations involve situations where condition (2) is not satisfied. These are cases in which banks that are not able to find a counterparty in the market and are holding a high quality assets do not access the discount window. For this situation to occur, the interest rate charged at the discount window must be relatively high. We maintain that these cases are unlikely to be empirically relevant given the stated objective of the discount window. For this reason, we do not provide here a detailed description of the analytical derivations corresponding to those cases.

6 Conclusion

In this paper, we have provided a formal model of the interaction between the interbank market for funds and the asset market. Our model is capable of reproducing certain trading patterns by banks that are consistent with a situation in which outside investors attach some degree of stigma to the activity of borrowing from the central bank's discount window. The intention was to introduce a minimum number of elements in the model to allow us to capture this stigma effect. In this kind of signaling setting there is a delicate balance between information and frictions, which needs to be maintained so that, while some information flows to the market, the equilibrium does not become fully revealing. Achieving this balance in a parsimonious way is a motivation for many of our simplifying assumptions.

The main components of our model are the following. On one hand, participants in the interbank market have (some) information about the quality of their counterparty's assets. On the other hand,

[20] When $pR < 1$ it is possible to have $TS_H(p, p) < 0$. In this case, the bank may repay the discount window, in expected value, less than the opportunity cost of funds (if $\alpha r_W < 1$) making the window a more attractive deal than private loans where this is not possible. This situation implies sure losses for the discount window and seems at odds with common practice. Also, this case may be considered less interesting to the extent that $pR < 1$ implies that projects of unknown quality (loosely speaking, from an "ex ante" perspective) have a negative expected net present value.

private dealings in the interbank market are not observable by third parties. Actions in the interbank market, if observable, could reveal asset quality and, hence, pin down asset prices independent of any signal.

Repayment risk is (to a certain degree) endogenous in the model and depends on the equilibrium in the asset market. In turn, banks' activities in the asset market depend on their ability to borrow in the interbank market. In particular, some banks in the asset market may be selling their assets *because* they need to repay their interbank loans. Other banks, however, may be selling their assets because they know that their assets are low quality and that, at the prevailing equilibrium prices, they are effectively overpriced.

When we introduce the possibility of borrowing from the central bank, bank participation at the discount window may be subject to (adverse) selection and, as a consequence, stigma may arise. We make information about borrowing activity at the window partially observable by outside investors. The idea of stigma clearly relies on some degree of observability; the underlying logic is that information held by participants in the interbank market, which would otherwise remain private, can flow to asset market participants through reported activity at the discount window.

Another important aspect associated with the possibility of stigma is that loan repayment risk, while partly endogenous, also depends on the true quality of the assets held by the banks. This fact is what implies that, in equilibrium, banks holding low quality assets are less likely to obtain loans in the interbank market and, hence, are more likely to be borrowing at the discount window.

This is a paper on positive economics. We make no attempt to address commonly discussed policy issues. For instance, if some amount of discount window lending is optimal and stigma makes banks reluctant to access such liquidity (as it has often been argued), then the effectiveness of policy may be seriously impaired as a result. While addressing policy issues like this one is, of course, very important, we think that an essential first step in the process of reaching reliable conclusions is to develop a better understanding of the fundamental mechanisms that give rise to particular forms of stigma in the interbank market. Taking this first step was the objective of this paper.

7 Appendix

Proposition 3. Given all other parameter values, there exists a threshold value $p^T < 1$ such that for all $p \in (p^T, 1)$ we have that we can find a value $r_W^T(p) > 1$ such that for all $r_W \in \left(1, r_W^T(p)\right)$ there is an equilibrium with both interbank credit and discount window lending in which $q_W = \xi(p)$, $q_N = \zeta(p)$, and $r_H = 1 + \theta\, TS_H(q_N, q_W)$.

Proof of Proposition 3: Consider the following three conditions on parameters:

$$\begin{aligned} \rho - (1-\alpha)\left[1 - \delta\xi(p) - (1-\delta)\zeta(p)\right]R &> 1, \\ \xi(p)\,R &> 1, \\ 1/(1-\alpha) - \delta\left[\zeta(p) - \xi(p)\right]R &> 1. \end{aligned}$$

Given all other parameter values, there exists a threshold value p^T such that for all $p \in \left(p^T, 1\right)$ these three conditions hold. To see this, note that as p gets close to one, both $\zeta(p)$ and $\xi(p)$ also get close to one. Then, since ρ, R and $1/(1-\alpha)$ are greater than one, for p sufficiently close to one the three conditions hold.

Now, consider any $p \in \left(p^T, 1\right)$. There exists a threshold value $r_W^T(p)$ such that for all $r_W \in \left(1, r_W^T(p)\right)$ the following three conditions hold:

$$\begin{aligned} \rho - (1-\alpha)\left[1 - \delta\xi(p) - (1-\delta)\zeta(p)\right]R &> r_W, \\ \xi(p)\,R &> r_W, \\ 1/(1-\alpha) - \delta\left[\zeta(p) - \xi(p)\right]R &> r_W. \end{aligned} \tag{A1}$$

Then, for any $p \in (p^T, 1)$ and any $r_W \in \left(1, r_W^T(p)\right)$ we have that:
(1) $\zeta(p)\,R > \xi(p)\,R > r_W$ and if in equilibrium $q_W = \xi(p)$ and $q_N = \zeta(p)$ then $\pi(q_W, r_W)$ and $\pi(q_N, r_W)$ are both positive.
(2) Banks that do not find a counterparty in period 1 and are holding a high quality asset borrow from the discount window. To see this, note that we can re-write condition (2) as

$$\alpha(R - r_W) + (1-\alpha)\left[\delta\left(q_W R - r_W\right) + (1-\delta)\left(q_N R - r_W\right)\right] \geq -\rho + R$$

which is implied by the first inequality in (A1).
(3) By Lemma 1, we have that banks that do not find a counterparty and are holding a low quality asset also borrow from the discount window.
(4) Since $r_W > 1$ and $\zeta(p) > \xi(p)$ we have that $TS_H(q_N, q_W) = r_W - 1 + (1-\alpha)\,\delta\,(q_N - q_W)\,R > 0$ when $q_W = \xi(p)$ and $q_N = \zeta(p)$. Then, we have that banks that find a counterparty and are holding a high quality asset borrow in the interbank market. It is easy to show that the Nash problem implies that $S_b(r_H, q_N, q_W) = (1-\theta)\,TS_H(q_N, q_W)$ and $S_l(r_H, q_N) = \theta\,TS_H(q_N, q_W)$. Since $S_b(r_W, q_N, q_W)$ is decreasing in r_H and, under the hypothesis of the proposition, $S_b(q_N R, q_N, q_W) < 0$ we have that the equilibrium value of r_H is lower than $q_N R$ for all values of θ. Then, using the Nash solution, it is easy to verify that $r_H = 1 + \theta TS_H(q_N, q_W)$ for all θ.
(5) Using the definitions of $S_b(r_L, q_N, q_W)$ and $S_l(r_L, q_N)$ we have that in equilibrium $TS_L(q_N, q_W) = (1-\alpha)\,r_W - 1 + (1-\alpha)\,\delta\,(q_N - q_W)\,R$ and by the last condition in (A1) we have that $TS_L(q_N, q_W) < 0$ which implies that banks that finds a counterparty in the interbank market but are holding low

quality assets are not able to get private loan. Instead, these banks have to borrow from the discount window.[21]

(6) We then have that in equilibrium $1-p\sigma$ banks borrow from the discount window and $p\sigma$ banks borrow in the interbank market. Of those borrowing from the window only $p(1-\sigma)$ are holding a high quality asset. Hence, when in period 2 an investor observes that a given bank has borrowed from the discount window, his equilibrium belief about the probability that such a bank is holding a high quality asset is given by $\xi(p)$. In other words, $q_W = \xi(p)$ in equilibrium.

Similarly, the total measure of banks that show no evidence of having borrowed from the discount window is given by the sum of those that did not borrow from the window and those that borrowed from the window without this information becoming public in period 2; that is, $p\sigma+(1-\delta)(1-p\sigma)$. Of this total measure of banks, only $p\sigma+(1-\delta)p(1-\sigma)$ of them are holding a high quality asset. Hence, by Bayes rule, the equilibrium belief about the probability that a bank showing no evidence of borrowing at the window holds a high quality asset is given by $\zeta(p)$. In other words, the equilibrium value of q_N is equal to $\zeta(p)$. ■

Proposition 4. Assume that $\rho > 1/(1-\alpha)$. Given all other parameter values, there exists a threshold value $p^{T'} < 1$ such that for all $p \in (p^{T'}, 1)$ we have that we can find a value $r_W^{T'}(p) > 1/(1-\alpha)$ such that for all $r_W \in \left(1/(1-\alpha), r_W^{T'}(p)\right)$ there is an equilibrium with both interbank credit and discount window lending in which $q_W = q_N = p$, and $r_H = \theta r_W + (1-\theta) < r_L = \theta r_W + (1-\theta)/(1-\alpha) < r_W$.

Proof: Consider the following two conditions on parameters:

$$\rho - (1-\alpha)(1-p)R > \frac{1}{1-\alpha},$$

$$pR > \frac{1}{1-\alpha}.$$

Since we are assuming that $R > \rho > 1/(1-\alpha)$ we have that there exists a threshold value $p^{T'} < 1$ such that for all $p \in (p^{T'}, 1)$ these two conditions hold. Now consider any value of $p \in (p^{T'}, 1)$. There exists a threshold value $r_W^{T'}(p) > 1/(1-\alpha)$ such that for all $r_W \in \left(1/(1-\alpha), r_W^{T'}(p)\right)$ the following three conditions hold:

$$\rho - (1-\alpha)(1-p)R > r_W > \frac{1}{1-\alpha},$$

$$pR > r_W.$$

Then, for any $p \in (p^{T'}, 1)$ and any $r_W \in \left(1/(1-\alpha), r_W^{T'}(p)\right)$ we have that:
(1) $\pi(p, r_W) > 0$ and $\alpha(R - r_W) + (1-\alpha)\pi(p, r_W) = [\alpha + (1-\alpha)p]R - r_W > -\rho + R$ which implies

[21] It can be shown that whenever $TS_L(q_N, q_W) > 0$ then $q_N R > 1/(1-\alpha)$ and hence we cannot have a situation like the one described in Section 4 in which TS_L is positive and still banks cannot agree on a loan.

that condition (2) holds and illiquid banks that do not find a counterparty and are holding a high quality asset borrow from the discount window. Then, by Lemma 3 again, condition (3) also holds and illiquid banks that do not find a counterparty and are holding a low quality asset also borrow from the window.

(2) $TS_H(p,p) = r_W - 1 > 0$ which implies that banks that find a counterparty and are holding a high quality asset borrow from the market. Similarly, $TS_L = (1-\alpha) r_W - 1 > 0$ and, hence, banks holding a low quality asset also borrow in the market if they are able to find a counterparty.

(3) In summary, we have that $(1-\sigma)$ banks borrow from the discount window and σ banks borrow from the interbank market. Of the banks borrowing from the discount window, a proportion p of them are holding a high quality asset. Hence, $q_W = p$ in equilibrium. To obtain the equilibrium value of q_N we need to consider the likelihood that a bank that shows no evidence of borrowing from the window is holding a high quality asset. The probability that a bank shows no evidence of borrowing from the window is the sum of the probability that the bank actually did not borrow from the window, σ, plus the probability that having borrowed from the window, such information has not been revealed. In each case, with probability p the bank is holding a high quality asset. Then, $q_N = [\sigma p + (1-\delta)(1-\sigma) p] / [\sigma + (1-\delta)(1-\sigma)] = p$.

(4) Bargaining over the terms of the loan result in $r_H = 1 + \theta TS_H = \theta r_W + (1-\theta)$ and $r_L = \frac{1}{1-\alpha}(1 + \theta TS_L) = \theta r_W + (1-\theta)\frac{1}{1-\alpha}$. Clearly, then, $r_W > r_L > r_H$. ■

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