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Optimal bonuses and deferred pay for bank employees: implications of hidden actions with persistent effects in time

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Abstract

We present a sequence of two-period models of incentive-based compensation in order to understand how the properties of optimal compensation structures vary with changes in the model environment. Each model corresponds to a different occupation within a bank, such as credit line managers, loan originators, or traders. All models share a common trait: the effects of hidden actions are persistent, and hence are revealed over time. We characterize the corresponding optimal contracts that are consistent with prudent risk taking. We compare the contracts by ranking them according to the average wage, the proportion of deferred compensation, and the structure and importance of variable pay (bonuses). We also compare these characteristics of the models with persistence with those of a standard repeated moral hazard. We find that small changes in the structure of asymmetric information have important implications for the characteristics of optimal pay, and that persistence does not necessarily imply a higher proportion of deferred pay.

Key Words: Moral Hazard, Persistence, Bonus, Deferred Pay, Banking.

Journal of Economic Literature Classification Numbers: D80, D82, D86, G21, G28.

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1 Introduction

This paper works through a sequence of two-period models of incentive-based compensation in order to understand how the properties of optimal compensation structures vary with changes in the model environment. We place particular emphasis on deriving the time profile of compensation when an agent's actions have persistent effects on returns. The purpose is to further develop a useful model for analyzing compensation at firms, and banks in particular.

Many actions taken by an employee of a bank, or any other firm, have persistent effects on returns. For example, a mortgage loan originated by a loan officer need not default right away. Similarly, a commercial loan officer who offers a credit line to a new borrower will generate profits or losses for the bank for the entire duration of the contract, rather than only when it was signed. A trader who purchases assets, particularly illiquid ones, will affect bank profits until the assets mature, default, or are sold.

The degree to which pay for an employee should depend on the long-term results of his action is an important question, not only for a bank who employs him, but also for regulators who are concerned that pay practices at a bank can significantly contribute to its risk profile. This concern is reflected in an excerpt from the press release that accompanied the issuance of the final guidance in incentive compensation by the Federal Reserve, OCC, OTS, and FDIC in June 21, 2010:

> "Many firms are using deferral arrangements to adjust for risk, but they are taking a "one-size-fits-all" approach and are not tailoring these deferral arrangements according to the type or duration of risk [...]"

Whether banks are properly deferring compensation is unclear. The discussion that emerged following the recent financial crisis does make clear, however, that there is a need to understand the optimal incentive arrangements in situations in which information about past actions of bankers is revealed over time. This article attempts to improve this understanding.

The models we present differ in the particular type and timing of asymmetric information present. As we shall see, this will have important implications for the characteristics of the contract that implements prudent lending practices.

We consider two main environments. In the first one, based on Hopenhayn and Jarque (2010), potential borrowers are filtered by the bank's risk management based on objective criteria and delivered to the agent. The agent's action simply improves repayment probabilities through suitably tailoring the lending contract. We refer to this model as the *producer* model to stress that the agent is not evaluating borrowers on their risk characteristics. This environment tries to model occupations such as a credit line manager or a mortgage servicer.

In the second environment, based on Jarque and Prescott (2009), the agent can take a costly action to evaluate a borrower's risk. The agent's action and evaluation are not verifiable by the bank, so the agent first needs an incentive to evaluate a borrower and then, if the borrower is a poor risk, to not make the loan to him. Throughout, there is a pool of safe, but less profitable, borrowers to whom the agent can also lend. We refer to this model as the *screening* model to convey that the agent conducts the risk evaluation. This environment tries to model occupations such as a commercial loan or mortgage originator or an underwriter.

Figure 1 provides timelines for the two main models.

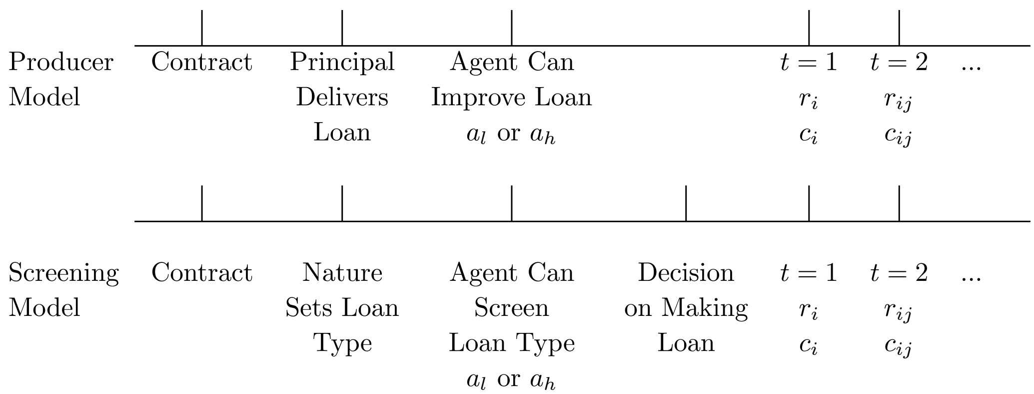

Figure 1: Timelines for the two models.

In all environments, the agent is compensated through a series of payments contingent on repayment realizations over time. We analyze the optimal time and state contingent payments that implement high effort and sorting of loans in equilibrium. Hence, our analysis focuses on compensation schedules that are consistent with prudent lending.

We modify the producer benchmark to consider the effect of joint production in a team. Our analysis suggests that the degree of labor substitutability in team production has a substantial effect on the optimal sensitivity of pay to performance. We also consider a variation of the producer model in which the agent needs to input an action in the second period, after having observed the first period output (based on Mukoyama and Sahin, 2005). We study the case in which this second action has a much smaller effect on the repayment probabilities and conclude that incentives are harder to provide in this setting due to smaller informational content of output realizations.

We also consider a variation of the screener model in which the agent can prove that he chose the right action when deciding to approve a risky loan (Phelan, 2009). This variation results generally in easier incentive provision, because the relevant deviation for the agent is to lie about the type of the loan, as opposed to not exerting effort, and this implies higher informational content in the repayment observations when compared to the benchmark screener model.

All five models that we present share a common characteristic: the effects of the hidden action are persistent in time, and hence information about the quality of action is revealed over time. However, the particular structure of the incentive problem varies slightly from model to model, and these slight variations have important implications for compensation. We compare some important characteristics of the optimal contracts: the level of compensation, the proportion of pay that is deferred, and the structure of base salaries and bonuses.

In our comparisons, we find that the type of occupations described in the producer benchmark model— a pure hidden action problem with signals about effort arriving over time— tend to exhibit lower wages, and a lower proportion of deferred pay, and tend to be less reliant on variable pay. Furthermore, if the moral hazard problem changes by making a subsequent action available to the agent, the wages, deferred pay and bonus importance all increase. Also, team production in this type of occupation intensifies the incentive problem if the actions of the different producer agents in the team are substitutes. This leads to higher wages, more deferred compensation and more reliance on variable pay.

In the screener models, which have some hidden information in addition to a hidden action, the optimal contract exhibits higher wages, a higher portion of deferred pay, and more reliance on variable pay than the producer contracts. The severity of the hidden information problem, which is higher in the benchmark screener model, increases all three measures.

An important difficulty in understanding the implications of persistence for the structure of optimal contracts is the fact that any dynamic optimal contract in the presence of persistence shares some important characteristics with dynamic contracts without persistence. In order to clarify the implications of persistence from the more standard implications due to the commitment to long–term contracts, we provide a numerical comparison of our five models with persistence to a standard repeated moral hazard model (see Rogerson, 1985). We find that the proportion of deferred pay is not necessarily higher in the setting with persistence. Hence, being able to recognize the persistence of the effects of hidden actions over time in a particular job assignment may not necessarily lead us to expect extraordinary levels of deferred pay in the optimal pay arrangement.

The next section introduces the notation and assumptions common to the models. Section 3 presents the producer models. Section 4 presents the screener models. Section 5 compares the characteristics of the optimal contracts across the five models, and to a standard repeated moral hazard. Numerical examples of all the models are presented and discussed. Section 6 concludes.

2 Common framework

There are two periods. Both the agent and the principal discount the future at a rate $\beta \leq 1$. In most of the models, the agent only takes an initial action that affects the loan's pay off in the first and second periods. In some of the models, however, the agent also takes a second period action that affects the loan's pay off in the second period.

Borrowers are either risky or risk free. When a loan is granted to a risky borrower, the return to the firm each period is normalized to an amount of either $r_l = 0$ (no repayment), or $r_h = 1$ (repayment). Among the risky borrowers, a fraction γ are good risks who repay with probability π_g each period. The remaining fraction $(1-\gamma)$ are bad risks who repay with probability π_b each period. We will interchangeably use the following notation:

$$\begin{array}{ll} f(r_h|a_h) = \pi_g, & f(r_h|a_l) = \pi_b, \\ f(r_l|a_h) = (1-\pi_g), & f(r_l|a_l) = (1-\pi_b) \end{array}.$$

Risk free borrowers repay with probability one each period, but the repayment amount is y where

$$\pi_g > y > \pi_b.$$

Whether a borrower is safe or risky, of good or bad type, is influenced by the agent. Actions, or effort levels, can take on two values $A = \{a_l, a_h\}$ with the disutility of a_l denoted $v(a_l)$ and normalized to 0, and that of a_h denoted by $v(a_h)$ and equal to $a > 0$. In the production models, borrowers are inherently risky, of bad type, unless the agent takes the high action. In the screening models, the agent identifies the risky borrowers' type if he takes the high action. We use a_1 to refer to the first period action. For the model with actions in the second period, we use a_2 to refer to the second-period action. The agent receives utility from compensation in both periods. Let c_1 indicate compensation in period 1 and c_2 indicate compensation in period 2. The agent's utility is $U(c_1)+\beta U(c_2)-v(a_1)$ for the models where there is only an initial action. Utility is $U(c_1)+\beta U(c_2)-v(a_1)/(1+\beta)-\beta v(a_2)/(1+\beta)$ for the model with a second period action. The principal is risk neutral.

In all the models discussed in this paper, compensation under full information will be characterized by a constant wage over the two periods. This contract is a useful benchmark for the numerical examples that we provide in section 5. We assume that $U(c)=\frac{c^{1-\sigma}}{1-\sigma}$ and we set $\sigma = 1/2$ for most of the analysis. Then, if $\bar{U}$ is the agent's participation level of utility and the high action is taken, the full-information compensation level in each period will be

$$c^* = \left(\frac{\bar{U}+a}{2(1+\beta)}\right)^2.$$

In the next two sections we present the producer and the screener models in detail, both in their benchmark version and in their variations. Throughout the paper, we index each environment by $m \in \{PB, PT, PV, SB, SV\}$, corresponding to each specification in the next sections.

3 Producer-Agent: hidden action

We present two versions of a model in which the agent can change the probability of repayment of a given loan with his hidden action. We refer to this type of agent as a "producer." These models are meant to capture the incentive problem inherent in positions within a bank such as a credit line manager. A credit line manager needs to work out the conditions of a credit line that optimize the probability of repayment, and this is costly. When lending conditions change or the situation of the borrower evolves, a reevaluation of the credit line conditions may be needed, with some extra costly effort involved on the part of the agent.

3.1 Producer Benchmark (PB): one hidden action with persistence

The first producer-agent model follows Hopenhayn and Jarque (2010). The action is only taken in the first period, but it affects the distribution of the returns in periods 1 and 2. Taking the low action means a borrower repays with probability π_b and taking the high action means a borrower repays with probability π_g. Our interpretation is that the agent's action involves activities like properly setting the terms of the loan and providing other assistance to the borrower.

We only consider the case where the principal wants the agent to take the high action, so we can just consider the problem of finding the minimal cost to implement the high action. Let c_i, $i = l, h$, be compensation in the first period where $i = l$ corresponds to $r_l = 0$ and $i = h$ corresponds to $r_h = 1$. Let c_{ij} be compensation in the second period as a function of first period repayment, i, and second period repayment j. Here and in the rest of the paper we use the standard relabeling of $u_i = U(c_i)$, and $u_{ij} = U(c_{ij})$, and denote $h(\cdot) = U^{-1}(\cdot)$. We use utility levels as choice variables to simplify the analysis.

The principal's optimization problem is

$$\min_{u_i, u_{ij}} \sum_i f(r_i|a_h) \left[h(u_i) + \beta \sum_j f(r_j|a_h) h(u_{ij}) \right] \tag{1}$$

subject to the participation constraint

$$\sum_i f(r_i|a_h) \left[u_i + \beta \sum_j f(r_j|a_h) u_{ij} \right] - a \geq \bar{U} \tag{2}$$

and the incentive constraint

$$\sum_i f(r_i|a_h) \left[u_i + \beta \sum_j f(r_j|a_h) u_{ij} \right] - a \geq \sum_i f(r_i|a_l) \left[u_i + \beta \sum_j f(r_j|a_l) u_{ij} \right] \tag{3}$$

and the domain constraints

$$u_i, u_{ij} \geq 0 \ \ \forall i, j. \tag{4}$$

The program is a strictly concave programming problem, so it has a unique solution. The proofs for the PC and the IC being binding are also standard in a static moral hazard problem, so they are omitted here.

Let LR be the likelihood ratio of a history of repayment realization. These ratios are

$$\begin{aligned} LR_i &= \frac{f(r_i|a_l)}{f(r_i|a_h)}, \quad i = l, h, \\ LR_{ij} &= \frac{f(r_i|a_l)f(r_j|a_l)}{f(r_i|a_h)f(r_j|a_h)}, \quad i, j = l, h. \end{aligned}$$

The first order conditions of this problem are

$$\begin{aligned} \frac{1}{U'(c_i)} &= \lambda + \mu \left(1 - LR_i\right), \ \ \forall i, \\ \frac{1}{U'(\tilde{c}_{ij})} &= \lambda + \mu \left(1 - LR_{ij}\right), \ \ \forall i, j, \end{aligned}$$

where $\lambda \geq 0$ is the multiplier on the participation constraint, (2), and $\mu \geq 0$ is the multiplier on the incentive constraint, (3). The following proposition follows from these conditions.

Proposition 1 (Hopenhayn and Jarque (2010)) *For any utility specification that satisfies $u' > 0$, $u'' < 0$, and assuming the domain constraint for utility (4) does not bind, compensation levels in the optimal contract in the PB model are ranked by*

$$c_{ll} < c_l < c_{lh} = c_{hl} < c_h < c_{hh}.$$

In particular, for our utility specification, when the lower bound of utility is not binding we can solve for the closed form solution for the contingent utilities.[1]

$$u_s^{PB} = \frac{1}{1+\beta} \left[\bar{U} + a \left(1 + t_s^{PB}\right)\right], \tag{5}$$

[1] Note that if the domain constraint in (4) were to bind, some of the strict inequalities in Prop. 1 would become equalities.

where $s \in \{l, h, ll, lh, hl, hh\}$ and t_s^{PB} is defined for each history as:

$$
\begin{aligned}
t_l^{PB} &= \frac{-\pi_g}{(\pi_g - \pi_b)} \frac{v_1}{\bar{v}}, \\
t_h^{PB} &= \frac{1-\pi_g}{(\pi_g - \pi_b)} \frac{v_1}{\bar{v}}, \\
t_{ll}^{PB} &= \frac{\pi_g}{(\pi_g - \pi_b)} \frac{v_1}{\bar{v}} \frac{(\pi_g + \pi_b - 2)}{(1-\pi_g)}, \\
t_{lh}^{PB} &= t_{hl}^{PB} = \frac{1}{(\pi_g - \pi_b)} \frac{v_1}{\bar{v}} (1 - \pi_g - \pi_b), \\
t_{hh}^{PB} &= \frac{1-\pi_g}{(\pi_g - \pi_b)} \frac{v_1}{\bar{v}} \frac{(\pi_g + \pi_b)}{\pi_g},
\end{aligned}
\tag{6}
$$

where v_t denotes the variance of the likelihood ratios in period t and $\bar{v}$ is the discounted average variance across the two periods. It is simple to show that

$$v_1 \equiv Var\left[LR\left(r_1\right)|a_h\right] = \frac{\left(\pi_g - \pi_b\right)^2}{\pi_g(1-\pi_g)},$$

and

$$\bar{v} \equiv \frac{v_1 + \beta v_2}{1+\beta} = \frac{v_1\left[1 + \beta\left(2 + v_1\right)\right]}{(1+\beta)}.$$

Note that $u_{lh} = u_{hl}$ and that the structure of utility does not depend on the period when the transfer takes place except through the term t_i^{PB}. For the purpose of the comparison across the different models in the article, it is useful to introduce the following notation: let π^m and $\widehat{\pi}^m$ denote the probabilities of repayment on and off the equilibrium path, respectively, and v_1^m and $\bar{v}^m$ the variances of the likelihood ratios, corresponding to model $m \in \{PB, PT, PV, SB, SV\}$. Then the terms defined above can be written as

$$
\begin{aligned}
t_l^{PB} &= \frac{-\pi^{PB}}{\left(\pi^{PB} - \widehat{\pi}^{PB}\right)} \frac{v_1^{PB}}{\bar{v}^{PB}}, \\
t_h^{PB} &= \frac{1-\pi^{PB}}{\left(\pi^{PB} - \widehat{\pi}^{PB}\right)} \frac{v_1^{PB}}{\bar{v}^{PB}}, \\
t_{ll}^{PB} &= \frac{\pi^{PB}}{\left(\pi^{PB} - \widehat{\pi}^{PB}\right)} \frac{v_1^{PB}}{\bar{v}^{PB}} \frac{\left(\pi^{PB} + \widehat{\pi}^{PB} - 2\right)}{(1-\pi^{PB})}, \\
t_{lh}^{PB} &= t_{hl}^{PB} = \frac{1}{\left(\pi^{PB} - \widehat{\pi}^{PB}\right)} \frac{v_1^{PB}}{\bar{v}^{PB}} \left(1 - \pi^{PB} - \widehat{\pi}^{PB}\right), \\
t_{hh}^{PB} &= \frac{1-\pi^{PB}}{\left(\pi^{PB} - \widehat{\pi}^{PB}\right)} \frac{v_1^{PB}}{\bar{v}^{PB}} \frac{\left(\pi^{PB} + \widehat{\pi}^{PB}\right)}{\pi^{PB}},
\end{aligned}
\tag{7}
$$

with $\pi^{PB} = \pi_g$ and $\widehat{\pi}^{PB} = \pi_b$.[2]

An important property of the optimal contract is that it never implies perfect insurance in the first period; information is used as soon as it becomes available. Incentives are more high-powered, however, in the second period, after a low return in the first period. That is, if we define $\Delta_0 = u_h - u_l$ and $\Delta_i = u_{ih} - u_{il}$ for $i = l, h$, it is easy to see that, for any arbitrary functional form of the utility of consumption,

$$0 < \Delta_h < \Delta_0 < \Delta_l.$$

A numerical example of this optimal contract and its comparison to the next models is provided in section 5.

3.2 Producer in a Team (PT)

In this section, we expand the PB model to incorporate actions supplied by more than one person.[3] This model belongs to a class of problems usually referred to as team production models.[4] They are common to many production processes. For example, the person who originates a commercial loan is usually different than the person who performs a workout if the loan has trouble repaying. Similarly, for mortgage origination the originator is different than the underwriter.

We work with a two period model, where two agents exert an action simultaneously in period one. Both actions affect repayment in the two periods. The first agent takes action $a_1 \in \{a_l, a_h\}$, and the second agent takes action $b_1 \in \{b_l, b_h\}$. Both actions are on the same support, so $a_l = b_l$ and $a_h = b_h$. Actions are also private information. Not only does the principal not observe the actions, but each agent does not observe the other agent's action. The probability of repayment is $f(r_h|g(a_1, b_1))$, where $g(a_1, b_1)$ is increasing in each of its arguments and f is increasing in g. We will consider particular specifications for $g(\cdot)$ and $f(r_h|g(\cdot))$ later in this section. Utility for agent one is $U(c_a) - V(a_1)$ and utility for agent two is $U(c_b) - V(b_1)$. We consider the symmetric case in which $V(a_l) = V(b_l) = 0$ and $V(a_h) = V(b_h) = a$. The outside utility of each agent is equal to $\bar{U}$. These choices keep

[2] The necessary and sufficient condition on the parameters for the non–negativity constraint on square root utility not to be binding is:

$$\frac{\bar{U}}{a} + 1 \geq \max_m \frac{1}{\bar{v}^m} \left[\left(\frac{1 - \widehat{\pi}^m}{1 - \pi^m} \right)^2 - 1 \right],$$

where π^m and $\widehat{\pi}^m$ denote the probabilities of repayment on and off the equilibrium path, respectively, and $\bar{v}^m$ the average discounted variance of the likelihood ratios, corresponding to model $m \in \{PB, PT, PV, SB, SV\}$.

[3] Indeed, one of the main reasons for the existence of a firm is the difficulty in monitoring and measuring the contribution of an individual to total output. See Alchian and Demsetz (1972) for a statement of this view.

[4] Holmström (1982) is the first formulation of moral hazard in team models.

the compensation of each agent comparable to that of the individual producer agent in the benchmark model of the previous section. Note that, even if we do not model payouts to the principal explicitly, we interpret this team production technology as having a higher output for the firm than the individual production technology. The focus of our analysis, however, is on the differences in information extraction between the individual and the team production, and its implications for the properties of the optimal contract. For this purpose, we again assume that the principal wants to implement the high action from both agents, and study the cost minimization problem.

The participation constraint of agent 1 reads

$$\sum_i f(r_i|g(a_h,b_h))\left[u_i+\beta\sum_j f(r_j|g(a_h,b_h))u_{ij}\right]-a\geq\bar{U}, \tag{8}$$

and it coincides with that of agent 2. The incentive constraint of agent 1 reads

$$\begin{aligned}
&\sum_i f(r_i|g(a_h,b_h))\left[u_i+\beta\sum_j f(r_j|g(a_h,b_h))u_{ij}\right]-a \\
\geq\ &\sum_i f(r_i|g(a_l,b_h))\left[u_i+\beta\sum_{ij} f(r_j|g(a_l,b_h))\right]u_{ij}.
\end{aligned} \tag{9}$$

Agent's 2 incentive constraint takes a similar form but considers the effect of deviating to b_l taking as given $a_1=a_h$. We will consider two specifications of $g(\cdot)$, both symmetric, i.e., $f(r_i|g(a_l,b_h))=f(r_i|g(a_h,b_l))$ for all i. Hence, agent's 2 incentive constraint also coincides exactly with 9. The principal's optimization problem is

$$\min_{\substack{u_{1i},u_{1ij}\\u_{2i},u_{2ij}}}\sum_i f(r_i|g(a_h,b_h))\left[h(u_{1i})+h(u_{2i})+\beta\sum_{ij} f(r_j|g(a_h,b_h))\left[h(u_{1ij})+h(u_{2ij})\right]\right].$$

subject to the participation constraint in 8 and the incentive constraint in 9. Concavity of the utility function and the symmetry of the agents immediately implies $u_{1i}=u_{2i}$, and $u_{1ij}=u_{2ij}$. Hence, the principal solves independently the incentive problem of each agent. This problem has the same structure as in the PB model, stated in (1), with probabilities on and off the equilibrium path of:

$$\begin{aligned}
\pi^{PT} &= f(r_h|g(a_h,b_h)) \\
\widehat{\pi}^{PT} &= f(r_h|g(a_l,b_h)).
\end{aligned}$$

This similarity in the structure implies that payments in the PT model will satisfy the ordering characterized in Prop. 1, for any concave utility function. Whenever $U(c)=2\sqrt{c}$, the utility solutions will follow the form in (5). The terms t_s^{PT}, in turn, will be as in (7), but their value will depend on the particulars of the probability function f and on g. We consider two functional forms of $g(a_1, b_1)$. In the first one, actions are substitutes, with $g(a_1, b_1) = \frac{a_1+b_1}{2}$. In the second one, actions are Leontief, with $g(a_1, b_1) = \min\{a_1, b_1\}$. The Leontief production technology is an extreme form of complementarity in production. We now explore the implications of each of these technologies.

Consider first the case of Leontief production. Assume the following $f_L(g(\cdot))$ function, which assumes the same effect for the minimum effort in team production as was assumed for individual effort in the PB model:

$f_L(r_h \vert a_1, b_1)$	$b_1 = a_l$	$b_1 = a_h$
$a_1 = a_l$	π_b	π_b
$a_1 = a_h$	π_b	π_g

We denote this first case of the producer in a team model by $PT(L)$. This matrix implies $\pi^{PT(L)} = \pi_g$ and $\widehat{\pi}^{PT(L)} = \pi_b$, the same probabilities as in the PB model. Hence, the variance of the likelihood ratios is the same as in the PB model: $v_1^{PT(L)} = v_1$. It follows that $t_s^{PT(L)} = t_s$ for all s, and the optimal contract for the producer in a team under Leontief production coincides with the optimal contract in the PB model of an individual producer.

Consider now the case of both efforts being substitutes. Assume the following $f_S(g(\cdot))$ function, which assumes that the probability changes linearly with the average action level of the two agents, from a minimum of π_b when both agents choose the low action to a maximum of π_g when they both choose the high action:

$f_S(r_h \vert a_1, b_1)$	$b_1 = a_l$	$b_1 = a_h$
$a_1 = a_l$	π_b	$\pi_g - \frac{\pi_g - \pi_b}{2}$
$a_1 = a_h$	$\pi_g - \frac{\pi_g - \pi_b}{2}$	π_g

We denote this case of the producer in a team model by $PT(S)$. In this case, the $t_s^{PT(S)}$ terms will be determined by $\pi^{PT(S)} = \pi_g$ and $\widehat{\pi}^{PT(s)} = \frac{\pi_g+\pi_b}{2}$. This implies

$$v_1^{PT(S)} = \frac{1}{4}\frac{(\pi_g - \pi_b)^2}{\pi_g(1-\pi_g)} = \frac{1}{4}v_1 < v_1.$$

Hence we have

$$\frac{v_1^{PT(S)}}{\bar{v}^{PT(S)}} = \frac{1+\beta}{1+\beta\left(2+\frac{1}{4}v_1\right)} > \frac{v_1}{\bar{v}},$$

which implies the following result.

Proposition 2 *The differences in utility in the optimal contract corresponding to the PT model satisfy*

$$\Delta_0^{PT(L)} = \Delta_0, \qquad \Delta_L^{PT(L)} = \Delta_L, \qquad \textit{and } \Delta_H^{PT(L)} = \Delta_H$$

under the Leontief production function, and

$$\Delta_0^{PT(S)} > \Delta_0, \qquad \Delta_L^{PT(S)} > \Delta_L, \qquad \text{and } \Delta_H^{PT(S)} > \Delta_H$$

under the substitutability production function.

The result implies that incentives are more high-powered when inputs are substitutable than when they are Leontief. Under substitutability, each agent can free ride off of each other. Providing the lower action has a smaller effect on the probability of repayment and a non–repayment event is less informative. In contrast, with the Leontief production function a deviation by one agent drastically lowers the probability of success, which allows compensation in the event of non–repayment to be higher without violating incentive compatibility.

While monitoring by the principal is not in the model, the results predict that monitoring would be more likely when actions are substitutes, i.e., when there is more of a free rider problem, than when actions are complements. The model also predicts heavy correlation of compensation within teams.

We conclude that the degree of labor substitutability in team production has a substantial effect on the optimal sensitivity of pay to performance. However, in practice, production processes are more complicated than the stylized model presented here and include other features like job rotation as well as dealing with other sources of private information. For example, Hertzberg, Liberti, and Paravisini (forthcoming) study the use of loan officer rotation in a large international bank as a device for eliciting private information for loan officer. For an extension of this model that incorporates a theory of job rotation, see Prescott and Townsend (2006).

3.3 Producer Variation (PV): repeated hidden action with persistence

This section considers a variation on the PB model in which the agent takes an action in each of the two periods. It follows Mukoyama and Sahin (2005).[5] In this framework the agent's first period action still determines first period return, but first and second period actions determine the second period return. However, the first-period action has a larger effect on output than the second period action.

The probability of return in the first period is the same as in the PB model. The probability distribution of return in the second period is

$f(r_h \mid a_1, a_2)$	$a_2 = a_l$	$a_2 = a_h$
$a_1 = a_l$	π_b	$\pi_g - \alpha_1 (\pi_g - \pi_b)$
$a_1 = a_h$	$\pi_g - \alpha_2 (\pi_g - \pi_b)$	π_g

[5] See also Kwon (2006).

where $\alpha_t \in (0,1)$ represents how big of a decrease in the probability of repayment is triggered by a deviation in period t, and $\alpha_1 > \alpha_2$.

We assume that the high action is equally costly in the two periods and that the action disutility is $v(a_1)/(1+\beta) + \beta v(a_2)/(1+\beta)$, which scales the total disutility of the action over the two periods to the same level as in the previous model. For example, total disutility is equal to a if $a_1 = a_2 = a_h$ and equal to zero if $a_1 = a_2 = a_l$.

Again, we do not model the profit maximization of the principal, but rather the cost minimization problem. By doing this, we implicitly assume that the effect of the second period action on the probability of repayment is large enough (i.e., α_2 is large enough) compared with the disutility, so that the principal will always find it profitable to implement the high action in both periods. We can write the principal's problem as the following minimization problem.

$$\min_{u_i, u_{ij}} \sum_i f(r_i|a_h)h(u_i) + \beta \sum_{ij} f(r_i|a_h)f(r_j|a_h, a_h)h(u_{ij})$$

subject to the participation constraint

$$\sum_i f(r_i|a_h)u_i - a/(1+\beta) + \beta \sum_{ij} f(r_i|a_h)f(r_j|a_h, a_h)u_{ij} - \beta a/(1+\beta) \geq \bar{U}$$

the two second-period incentive constraints along the equilibrium path

$$\forall i, \quad \sum_j f(r_j|a_h, a_h)u_{ij} - a/(1+\beta) \geq \sum_j f(r_j|a_h, a_l)u_{ij},$$

and first-period incentive constraints

$$\begin{aligned} &\sum_i f(r_i|a_h)u_i - a/(1+\beta) + \beta \sum_{ij} f(r_i|a_h)f(r_j|a_h, a_h)u_{ij} - \beta a/(1+\beta) \\ &\geq \sum_i f(r_i|a_l)u_i + \beta \sum_{ij} f(r_i|a_l)f(r_j|a_l, z(r_i))u_{ij} - \beta v(z(r_i))/(1+\beta), \quad \forall z : r_i \to A. \end{aligned}$$

The first-period incentive constraints prevent four deviations by the agent:

1. Take $a_1 = a_l$ and then take $a_2 = a_l$ if r_l and $a_2 = a_l$ if r_h;
2. Take $a_1 = a_l$ and then take $a_2 = a_l$ if r_l and $a_2 = a_h$ if r_h;
3. Take $a_1 = a_l$ and then take $a_2 = a_h$ if r_l and $a_2 = a_l$ if r_h;
4. Take $a_1 = a_l$ and then take $a_2 = a_h$ if r_l and $a_2 = a_h$ if r_h.

There are six incentive constraints in total. Fortunately, the number of incentive constraints can be reduced under our assumptions. Proposition 4 in Mukoyama and Sahin (2005) shows that the following conditions are necessary and sufficient for the first period incentive constraints not to be binding:

$$\beta\left[f(r_h|a_h,a_h)-f(r_h|a_l,a_h)\right] \geq f(r_h|a_h,a_h)-f(r_h|a_h,a_l) \tag{10}$$
$$\beta\left[f(r_h|a_h,a_l)-f(r_h|a_l,a_l)\right] \geq f(r_h|a_h,a_h)-f(r_h|a_h,a_l). \tag{11}$$

When deviations in the first period are not binding, we have $c_l = c_h$. Under our assumptions for the probability distribution of the return, both (10) and (11) are satisfied under some restrictions on β and α, summarized in the following lemma.

Lemma 1 *The optimal contract in the PV model with two efforts exhibits perfect insurance in the first period if and only if* $\beta \geq \max\left\{\frac{\alpha_2}{\alpha_1}, \frac{\alpha_2}{1-\alpha_2}\right\}$.

Note that, when $\beta < 1$, the condition in this proposition is only satisfied if $\alpha_1 > \alpha_2$ and $\alpha_2 < 1/2$ both hold: The inequality $\alpha_1 > \alpha_2$ says that the decrease in $f(r_h|a_1,a_2)$ should be larger if there is a deviation in the first period action than if there is a deviation in the second period only, while the inequality $\alpha_2 < 1/2$ puts a limit on $f(r_h|a_h,a_h) - f(r_h|a_h,a_l)$. In words, Lemma 1 says that the agent needs to care enough about second period payoffs, in relation with the relative effectiveness of second period effort versus first period effort. In the rest of the paper, we assume that the condition in Lemma 1 holds. Hence, optimal compensation satisfies

$$\begin{aligned} u_l &= u_h \equiv u_1, \\ u_{lh} &= u_{hh} \equiv u_{2h}, \\ u_{ll} &= u_{hl} \equiv u_{2l}. \end{aligned}$$

Optimal compensation is characterized by perfect insurance in the first period, i.e., a constant wage with no incentive pay. All variable compensation is deferred until the second period. As a corollary, there is also no difference in continuation utility after observing r_l or r_h in the first period. In this specification, incentives for the first-period action are provided entirely by incentives for the second-period action in the second period. The second-period action is not very effective at increasing the probability of a repayment, so implementing the high action in the second period requires high-powered incentives. However, the first-period action is very effective in increasing the probability of repayment in the second period, so the agent works hard in the first period to earn a high pay in the second. Incentives in the first period come "for free" in this setup.

For simplicity, we can set $\alpha_1 = 1$, so that $f(r_h|a_l,a_h) = \pi_b$. Hence, the condition in Lemma 1 can be restated as:

$$\beta \geq \max\left\{\alpha_2, \frac{\alpha_2}{1-\alpha_2}\right\} = \frac{\alpha_2}{1-\alpha_2}. \tag{12}$$

We can denote α_2 simply as α. The constraints of the problem simplify to:

$$\begin{aligned} u_1 + \beta\left[\pi_g u_{2h} + (1-\pi_g)u_{2l}\right] - a &= \bar{U} \\ \pi_g u_{2h} + (1-\pi_g)u_{2l} - a &= \left[\pi_g - \alpha(\pi_g - \pi_b)\right] u_{2h} \\ &\quad + \left[1 - \pi_g + \alpha\left(\pi_g - \pi_b\right)\right] u_{2l} - a/\left(1+\beta\right). \end{aligned}$$

For $U(c) = 2\sqrt{c}$, these constraints and the first-order conditions imply that

$$\begin{aligned} u_1 &= \frac{\bar{U}+a}{(1+\beta)} \\ u_{2l} &= \frac{\bar{U}+a}{(1+\beta)} - \frac{\pi_g}{\alpha\left(\pi_g - \pi_b\right)}\frac{a}{(1+\beta)} \\ u_{2h} &= \frac{\bar{U}+a}{(1+\beta)} + \frac{(1-\pi_g)}{\alpha\left(\pi_g - \pi_b\right)}\frac{a}{(1+\beta)}. \end{aligned}$$

In this model, the average variance of the likelihood ratios is an average of $v_1^{PV} = 0$ in the first period and $v_2^{PV} = \frac{\alpha^2(\pi_g - \pi_b)^2}{\pi_g(1-\pi_g)}$ in the second:

$$\bar{v}^{PV} = \frac{0 + \beta v_2^{PV}}{1+\beta} = \frac{\beta}{1+\beta} v_2^{PV} = \frac{\beta}{1+\beta}\alpha^2 v_1.$$

In this variation of the producer agent model, all incentive provision, and hence any form of variable compensation, are deferred until the second period of the contract. This is in sharp contrast with the structure of the optimal contract in our benchmark producer model. In terms of our earlier notation, we have that, for any arbitrary functional form of the utility of consumption,

$$\begin{aligned} \Delta_0^{PV} &= 0 \\ \Delta_h^{PV} &= \Delta_l^{PV} > 0. \end{aligned}$$

A numerical example of this optimal contract and its comparison to the rest of the models is provided in section 5. Before that, the next section introduces the screener agent models.

4 Screener–Agent: hidden action and information

In this section we present two versions of a model in which the agent cannot change the probability of repayment of a given loan, but rather simply discover it, with his hidden action. We refer to this type of agent as a "screener." With this modified environment we try to capture the incentive problem that is present in jobs within the bank such as a credit line originator, a mortgage originator, or an investor. These agents need to exert some costly effort to discover and/or evaluate the quality of prospective borrowers. To the extent that their effort is not observable, this creates a moral hazard problem. However, there is a second layer of asymmetric information in their jobs, which we model as well: since the quality of the prospective borrower remains their private information, they can choose to lend to low quality borrowers that imply a loss for the bank with respect to the safe lending alternative.

4.1 Screener Benchmark (SB): unverifiable action

This section is based on Jarque and Prescott (2009). In this model, the agent screens loans. The agent can evaluate the riskiness of the borrower and then decide whether to lend to the borrower or lend to a different borrower who is safe. The principal neither observes whether the agent evaluated the borrower nor the agent's assessment of the borrower. The principal does know, however, if the agent lends to the safe borrower. Figure 1 shows the timeline.

Recall that a risky borrower of the good type brings a higher expected stream of payments to the principal than a safe one, and a safe one in turn brings higher (and certain) payments than the expected from a risky borrower of the bad type. We assume that the principal wants the agent to evaluate the risky borrower, to lend if the risky borrower is good, and to lend to the safe borrower if the risky borrower is bad.

The agent has four possible deviating strategies. They are

1. Do not evaluate, claim that the risky borrower is good, and lend to him, i.e., pool the two types of mortgages.

2. Do not evaluate and make a loan to a safe borrower, i.e., take the safe lending always.

3. Having evaluated, if the risky borrower is good, do not lend to him.

4. Having evaluated, if the risky borrower is bad, still lend to him.

4.1.1 One period version of SB

For the purpose of characterizing the optimal compensation scheme, a one-period version of this model is sufficient. Later, we extend the model to a two–period framework comparable to the producer models presented previously, with two repayment observations.

To distinguish the solution in this one period version from the solution in the two period version, we denote optimal utility levels here by $\widetilde{u}$. Let $\widetilde{u}_0$ be the compensation to the agent if a loan is made to a safe borrower. Since safe borrowers always repay, the compensation to the agent is uncontingent if he chooses to lend to the safe borrower. If a loan is made to a risky borrower instead, the payment to the agent depends on whether the loan repays. Let $\widetilde{u}_l$ be the compensation if the loan does not repay and $\widetilde{u}_h$ be the compensation if it does.

We define the following notation. For a risky borrower of random quality, the probability of repayment is π_p, where

$$\pi_p = \gamma\pi_g + (1-\gamma)\pi_b.$$

Also, let $E\widetilde{U}_g$ be the agent's utility from taking the recommended strategy, that is,

$$E\widetilde{U}_g \equiv \gamma\left[\pi_g \widetilde{u}_h + (1-\pi_g)\widetilde{u}_l\right] + (1-\gamma)\widetilde{u}_0 - a.$$

In a similar way, the agent's utility from pooling (deviation 1), is

$$E\widetilde{U}_{pool} \equiv \pi_p \widetilde{u}_h + (1-\pi_p)\widetilde{u}_l,$$

and from lending always to a safe borrower (deviation 2), is

$$E\widetilde{U}_{safe} \equiv \widetilde{u}_0.$$

The principal's problem is

$$\min_{u_0,u_l,u_h} \gamma\left[\pi_g h\left(\widetilde{u}_h\right) + (1-\pi_g) h\left(\widetilde{u}_l\right)\right] + (1-\gamma) h\left(\widetilde{u}_0\right)$$

$$\begin{aligned}
\text{s.t.} \qquad E\widetilde{U}_g &\geq \bar{U}, && (13)\\
E\widetilde{U}_g &\geq E\widetilde{U}_{pool}, && (14)\\
E\widetilde{U}_g &\geq \widetilde{u}_0, && (15)\\
\pi_g \widetilde{u}_h + (1-\pi_g)\widetilde{u}_l &\geq \widetilde{u}_0. && (16)\\
\widetilde{u}_0 &\geq \pi_b \widetilde{u}_h + (1-\pi_b)\widetilde{u}_l. && (17)
\end{aligned}$$

Equation (13) is the participation constraint. Incentive constraints (14) to (17) prevent deviations (1) to (4), correspondingly. It can be shown that constraint (14) implies constraint (17), and that constraint (15) implies constraint (16). It is also the case that (13), (14) and (15) bind. Hence, the optimal contract is simply the solution to the system of three equations and three unknowns:

$$\begin{aligned}
\widetilde{u}_l &= \bar{U} + \frac{a}{\gamma} - \frac{a}{\gamma}\frac{\pi_g}{(\pi_g - \pi_p)}, && (18)\\
\widetilde{u}_h &= \bar{U} + \frac{a}{\gamma} + \frac{a}{\gamma}\frac{1-\pi_g}{(\pi_g - \pi_p)}, && (19)\\
\widetilde{u}_0 &= \bar{U}. && (20)
\end{aligned}$$

4.1.2 Two–period version of SB

To distinguish from the one–period version, let EU_g be the agent's utility from taking the recommended strategy in the two–period model. This utility is now

$$\begin{aligned}
EU_g \equiv \gamma &\left\{\pi_g\left(u_h + \beta\left[\pi_g u_{hh} + (1-\pi_g)u_{hl}\right]\right)\right.\\
&\left. + (1-\pi_g)\left(u_l + \beta\left[\pi_g u_{lh} + (1-\pi_g)u_{ll}\right]\right)\right\}\\
&+ (1-\gamma)\left(1+\beta\right) U(c_0) - a
\end{aligned}$$

The binding incentive constraints are the two–period version of constraints (14) and (15). Define the following notation for the expected utility (over the two periods) of following deviation 1:

$$EU_{pool} \equiv \pi_p \left\{ u_h + \beta \left[\pi_p u_{hh} + (1-\pi_p) u_{hl} \right] \right\} \\ + (1-\pi_p) \left\{ u_l + \beta \left[\pi_p u_{lh} + (1-\pi_p) u_{ll} \right] \right\}$$

In the same way, define the expected utility of deviation 2 as:

$$EU_{safe} \equiv u_0 + \beta u_0.$$

Given the parallel structure across the two durations, it is clear that, for the PC and the incentive constraints to hold in the two–period problem, the expected utilities from the equilibrium strategy and from the deviations must be equal across the two durations:

$$EU_g = E\tilde{U}_g, \tag{21}$$
$$EU_{pool} = E\tilde{U}_{pool}, \tag{22}$$
$$EU_{safe} = E\tilde{U}_{safe}. \tag{23}$$

Hence, substituting into (21)-(23) the $E\tilde{U}_g$, $E\tilde{U}_{pool}$ and $E\tilde{U}_{safe}$ implied by the solutions for the one–period problem given in (18)-(20), we have,

$$\begin{aligned} EU_g &= \bar{U} + \frac{a}{\gamma} - a, \\ EU_{pool} &= \bar{U}, \\ EU_{safe} &= \bar{U}. \end{aligned}$$

It is immediate that $EU_{safe} = \bar{U}$ implies

$$u_0 = \bar{U} / (1+\beta).$$

In order to solve for the rest of the contingent compensations, we can define an auxiliary problem: the principal needs to provide the agent, at the minimum cost possible, with an expected utility of $\bar{U} + \frac{a}{\gamma}$ if he chooses the equilibrium strategy of sorting mortgages, and an expected utility of $\bar{U}$ if he chooses instead to pool them. Formally, the auxiliary problem is:

$$\min_{\substack{u_l, u_{ll}, u_{lh}, \\ u_h, u_{hl}, u_{hh}}} \left\{ \begin{array}{c} \pi_g \left\{ h(u_h) + \beta \left[(1-\pi_g) h(u_{hl}) + \pi_g h(u_{hh}) \right] \right\} \\ +(1-\pi_g) \left\{ h(u_l) + \beta \left[\pi_g h(u_{lh}) + (1-\pi_g) h(u_{ll}) \right] \right\} \end{array} \right\}$$
$s.t.$

$$\begin{aligned} \bar{U} + \frac{a}{\gamma} - a &= \pi_g \left\{ u_h + \beta \left[(1-\pi_g) u_{hl} + \pi_g u_{hh} \right] \right\} \\ & \quad +(1-\pi_g) \left[u_l + \pi_g u_{lh} + (1-\pi_g) u_{ll} \right] - a \end{aligned} \tag{24}$$

$$\begin{aligned}\bar{U} &= \pi_p \{u_h + \beta [(1-\pi_p) u_{hl} + \pi_p u_{hh}]\} \\ &\quad + (1-\pi_p)\{u_l + \beta [\pi_p u_{lh} + (1-\pi_p) u_{ll}]\}\end{aligned} \tag{25}$$

Note that Eq. 25 can be combined with Eq. 24 into an expression of the form of an incentive constraint:

$$\begin{aligned}& \pi_g \{u_h + \beta [(1-\pi_g) u_{hl} + \pi_g u_{hh}]\} \\ & + (1-\pi_g)\{u_l + \beta [\pi_g u_{lh} + (1-\pi_g) u_{ll}]\} - \frac{a}{\gamma} \\ = \; & \pi_p \{u_h + \beta [(1-\pi_p) u_{hl} + \pi_p u_{hh}]\} \\ & + (1-\pi_p)\{u_l + \beta [\pi_p u_{lh} + (1-\pi_p) u_{ll}]\}.\end{aligned}$$

Also, Eq. 24 can be interpreted as the participation constraint of this auxiliary problem. Hence, we can view the problem of optimally choosing the contingent consumptions in the screener problem as a producer problem with promised utility $\bar{U}$, effort disutility a/γ, and π_g and π_p as the probabilities on and off the equilibrium path. This is exactly the structure of the PB model. Hence, the ordering of consumptions characterized in Prop. 1 applies to the compensations in the contingent branch of this problem. Moreover, we can use the derivations in the PB model to find the expressions for the utilities in this auxiliary problem. Recall that the utility levels in models of this structure are always of the form :

$$u_s^m = \frac{1}{1+\beta}\left[\bar{U} + a^m\left(1 + t_s^m\right)\right], \tag{26}$$

where we had $a^{PB} = a$ but we have now that $a^{SB} = a/\gamma$. Also, the terms t_s^{SB} differ accordingly with the off equilibrium probability and the variance of the likelihood ratio. The terms reported in Eq. (6) define the solution for the SB model with $\pi^{SB} = \pi_g,\ \widehat{\pi}^{SB} = \pi_p$ and

$$\begin{aligned} v_1^{SB} &= \frac{\{\pi_g - [\gamma \pi_g + (1-\gamma)\pi_b]\}^2}{\pi_g(1-\pi_g)} \\ &= \frac{(1-\gamma)^2(\pi_g - \pi_b)^2}{\pi_g(1-\pi_g)} \\ &= (1-\gamma)^2 v_1, \end{aligned}$$

as well as

$$\bar{v}^{SB} = \frac{v_1^{SB}\left(1 + \beta\left(2 + v_1^{SB}\right)\right)}{(1+\beta)}.$$

We conclude from the extension of PB to a two period setting that when more than one repayment realization is observed, more information is available, and this means that payments can be made contingent on more signals. In particular, second period observations are more informative. This can be shown from the likelihood ratios, which get more dispersed.[6]

[6] See Hopenhayn and Jarque (2010).

Punishments can be allocated in histories that are less likely to happen on the equilibrium path, and hence the expected utility that the contract establishes for each strategy can be provided in a more efficient way than in the one–period case. A numerical example of the optimal contract for the two–period version is provided in section 5.

4.2 Screener Variation (SV): action is verifiable when the loan is granted to a risky borrower

Consider this slight variation on the previous model that is based on Phelan (2009). Assume that risky borrowers are not readily available, but that the agent needs to take the high action to find them. (In the previous model they arrived on their own.) Once the agent takes the action, he *also* observes the risky borrower's type. In this framework the incentive constraints are slightly different than in the SB model: the deviation described in (1) above (always pool) is not available to the agent. Unlike in the SB model, however, whenever the agent makes a loan to a risky borrower, he proves to the principal that he took the high action. The analysis of this model reveals that that there is less private information than in the SB model, making the incentive problem less severe here.

4.2.1 One period version of SV

The analysis is similar to that of the SB model. The participation constraint and objective function are unchanged and there is one less incentive constraint. The principal's problem is

$$\min_{u_0,u_l,u_h} \gamma\left[\pi_g h\left(\widetilde{u}_h\right) + (1-\pi_g) h\left(\widetilde{u}_l\right)\right] + (1-\gamma) h\left(\widetilde{u}_0\right)$$

subject to (13), (15), (16), and (17). As in the previous analysis, constraint (15) implies constraint (16). In this case, it can be showed that (13), (15) and (17) bind. The relevant expected utilities of each deviation are:

$$\begin{aligned} E\tilde{U}_g &= \gamma\left(\pi_g u_h + (1-\pi_g) u_l\right) + (1-\gamma) u_0 - a, \\ E\tilde{U}_{safe} &= u_0, \\ E\tilde{U}_{bad} &= \pi_b u_h + (1-\pi_b) u_l. \end{aligned}$$

Note that, while $E\tilde{U}_g$ is the expected utility from an ex–ante strategy of lending only to good loans, $E\tilde{U}_{bad}$ is the expected utility of accepting a loan —instead of rejecting it as the equilibrium stategy mandates— once the type is known to be "bad"; hence, the effort disutility is not taken into account by the agent when calculating $E\tilde{U}_{bad}$.

The solution to the system of equations defined by the binding constraints satisfies:

$$\widetilde{u}_l = \bar{U} + \frac{a}{\gamma} - \frac{a}{\gamma} \frac{\pi_g}{(\pi_g - \pi_b)}, \tag{27}$$

$$\widetilde{u}_h = \bar{U} + \frac{a}{\gamma} + \frac{a}{\gamma} \frac{1 - \pi_g}{(\pi_g - \pi_b)}, \tag{28}$$

$$\widetilde{u}_0 = \bar{U}. \tag{29}$$

There is better information in the second model and this shows up when comparing the right hand sides of (18) and (19) with (27) and (28). In particular,

$$\pi_g - \pi_b > \pi_g - \pi_p = \gamma (\pi_g - \pi_b),$$

which implies that the spread in utilities is smaller in the second model, that is, incentives need to be less high powered. Furthermore, this translates in the cost to the principal of implementing the high action being lower in the second model, as we will show in a proposition in section 5, when we compare all the models. A numerical example of the optimal contract in the two-period extension will also be provided.

4.2.2 Two–period version of SV

We follow the same strategy as in section 4.1.2 to derive the solution for the two–period version. It is clear that the expected utilities from the equilibrium strategy and from the two binding deviations must be equal across the two durations:

$$EU_g = E\tilde{U}_g, \tag{30}$$

$$EU_{safe} = E\tilde{U}_{safe}, \tag{31}$$

$$EU_{bad} = E\tilde{U}_{bad}, \tag{32}$$

where $E\tilde{U}_{bad}$ is the utility from following deviation 4, calculated using the off–the–equilibrium–path probability of lending to a bad borrower, π_b. This differs from the benchmark screener model, where the deviation of pooling dominated the deviation of lending to a bad risky borrower, making π_p (the probability of repayment of a random quality loan) the relevant off–the–equilibrium–path probability.

Using the above solutions for the one–period problem, we have,

$$EU_g = \bar{U} + \frac{a}{\gamma} - a,$$

$$EU_{safe} = \bar{U},$$

$$EU_{bad} = \bar{U}.$$

It is immediate that

$$\bar{U}(c_0) = \bar{U} / (1 + \beta).$$

In order to solve for the contingent consumptions, we can define an auxiliary problem: the principal needs to provide the agent, at the minimum cost possible, with an expected utility of $\bar{U}+\frac{a}{\gamma}$ if he chooses the equilibrium strategy of sorting mortgages, and an expected utility of $\bar{U}$ if he chooses instead to lend to bad risky borrowers as well as good ones. Formally, the auxiliary problem is:

$$\min_{\substack{u_l,u_{ll},u_{lh},\\ u_h,u_{hl},u_{hh}}} \left\{ \begin{array}{c} \pi_g \left\{ h\left(u_h\right)+\beta\left[(1-\pi_g)h\left(u_{hl}\right)+\pi_g h\left(u_{hh}\right)\right]\right\} \\ +(1-\pi_g)\left\{ h\left(u_l\right)+\beta\left[\pi_g h\left(u_{lh}\right)+(1-\pi_g)h\left(u_{ll}\right)\right]\right\} \end{array} \right\}$$

$s.to$

$$\begin{aligned} \bar{U}+\frac{a}{\gamma} &= \pi_g\left\{u_h+\beta\left[(1-\pi_g)u_{hl}+\pi_g u_{hh}\right]\right\} \\ &\quad +(1-\pi_g)\left\{u_l+\beta\left[\pi_g u_{lh}+(1-\pi_g)u_{ll}\right]\right\} \end{aligned} \tag{33}$$

$$\begin{aligned} & \pi_g\left\{u_h+\beta\left[(1-\pi_g)u_{hl}+\pi_g u_{hh}\right]\right\} \\ & +(1-\pi_g)\left\{u_l+\beta\left[\pi_g u_{lh}+(1-\pi_g)u_{ll}\right]\right\}-\frac{a}{\gamma} \\ = \; & \pi_b\left\{u_h+\beta\left[(1-\pi_b)u_{hl}+\pi_b u_{hh}\right]\right\} \\ & +(1-\pi_b)\left\{u_l+\beta\left[\pi_b u_{lh}+(1-\pi_b)u_{ll}\right]\right\}. \end{aligned} \tag{34}$$

Just like in the SB model, we rewrite the original constraints: (15) becomes (33), the participation constraint in the auxiliary problem, and (17) becomes (34), the incentive constraint in the auxiliary problem. Note that the individual output probabilities in this auxiliary problem are as in the benchmark producer model. Hence, here we will have that the variance of the likelihood ratios coincides with v_1, and the average with $\bar{v}$. The effort disutility is normalized, however. So the solution for the utilities in the branch in which the loan is accepted, which we find by using the auxiliary problem, are again of the form in Eq. 26, with $a^{SV}=a/\gamma$, with $v_1^{SV}=v_1$ and with $t_s^{SV}=t_s$ as in the PB model, stated in page 8. Once again, this common structure with the PB model implies that the ordering of consumptions characterized in Prop. 1 applies to the compensations in the contingent branch of this problem.

5 Implications for deferred compensation and bonuses

All five models that we have presented so far share a common characteristic: the effects of the hidden action are persistent in time, and information about the quality of action thus gets revealed over time. However, the particular structure of the incentive problem varies slightly from model to model. To illustrate how those differences may affect pay practices in real life, we now compare some important characteristics of the series of optimal contracts: we look at the level of compensation, the proportion of pay that is deferred, and the structure of base salaries and bonuses.

There is a clear interest in the policy debate on understanding the optimal incentive arrangements in situations in which information about past actions of bankers is revealed over time, i.e., understanding the implications of persistence for optimal contracts. Our models should be useful to that objective. However, it is important to separate the implications of persistence from the more standard implications due to the commitment to long term contracts. As will become clear from our analysis below, these different features (persistence and commitment) do sometimes translate into similar characteristics of the optimal contract (for example, deferred compensation). To clarify this issue, we include in our comparisons a related repeated moral hazard problem without persistence (denoted RMH from now on.)[7]

In our RMH model, the agent takes the high action in equilibrium in each of the two periods. The probability of repayment is π_g if the action is high and π_b if the action is low, i.i.d. across periods. Disutility of the action is $a/(1+\beta)$, normalized so that total disutility of effort is a, as in the models with persistence presented earlier; this normalization and the common probabilities and outside utility, $\bar{U}$, make the RMH model comparable to the producer and screener models.

When possible, we present analytical comparisons. For the comparisons that cannot be established analytically, we turn to numerical examples. We use the following baseline parametrization to compute numerical solutions to the different models in the article:

a	π_g	π_b	$\bar{U}$	β	γ	α
1	0.6	0.3	16	0.8	0.5	0.4

(35)

We start by providing a graphical comparison of the optimal contracts. Figure 2 depicts the solution for six different models, under the baseline parametrization in (35). The first subplot shows the full-information contract and the second subplot shows the repeated moral hazard contract. The next two rows show the solutions to the producer model. The first one in row 2 is for the PB model, and to the right of it is the $PT(S)$ model. The third row includes the $PT(L)$ model, which solution coincides with the solution to the PB model, and the repeated action variation, the PV model. The final row of subplots shows solutions to the screener-agent models. The left one is the benchmark, SB, and the right one is the variation version, SV. Looking at the RMH plot in the right cell of the first row, the two crosses in the first period represent the compensation given to the agent if r_l is observed, c_l, and if r_h is observed, c_h. In the second period, contingent on each possible two-period history, there are four possible compensation levels, c_{ll}, c_{lh}, c_{hl}, and c_{hh}. The dotted lines connecting the compensation levels have been included to help track the possible histories of compensation levels (for example, it is not possible for the agent to consume c_l in the first period and c_{hl}

[7] This is a standard textbook model. For details, see Rogerson (1985).

in the second period, and hence no dotted line connects these two compensation levels.) All the plots follow this pattern of lines and crosses.

Throughout the following three subsections, we report in a series of tables the numerical comparison of the five models with persistence and the RMH. We provide the comparison under the baseline parametrization, and we perform comparative statics by changing, one parameter at a time, to the following alternative parametrization:

$$\begin{array}{|c|c|c|c|c|cc} a & \pi_b & \bar{U} & \beta & \gamma & \alpha & \alpha \\ \hline 0.2 & 0.38 & 11 & 1 & 0.6 & 0.3 & 0.2 \end{array}. \tag{36}$$

For the sake of brevity, we only report the comparison under an alternative parameter if the ranking changes with respect to the ranking under the baseline parametrization.

5.1 The level of compensation

We now use the derivations in the previous sections to compare the level of pay predicted for each different model.

From the closed form solution derived in section 3.1 we can get an expression for the (discounted) average per period cost of the contract for the PB model:

$$k^{PB} \equiv \frac{E\left[c_1\right] + \beta E\left[c_2\right]}{1+\beta} = \frac{1}{4\left(1+\beta\right)^2}\left[\left(\bar{U}+a\right)^2 + \frac{a^2}{\bar{v}}\right].$$

For the purpose of the comparison across models, the measure k^m produces the same ranking as the average payments. We use it because it has a closed form that is convenient for the analytical comparisons.

As we discussed in section 3.2, the optimal contract in the $PT(L)$ model, the case of Leontief team production, corresponds to the one in the PB. For the substitute effort production, however, the contract differs and it implies an average cost of

$$k^{PT(S)} = \frac{1}{4\left(1+\beta\right)^2}\left[\left(\bar{U}+a\right)^2 + \frac{a^2}{\bar{v}^{PT}}\right].$$

Since we have established that $\bar{v}^{PT(L)} = \bar{v}$ and $\bar{v}^{PT(S)} < \bar{v}$, the comparison stated in the next proposition follows.

Proposition 3 *The average payments to the agent in the PB model are (i) of the same magnitude as in the PT model with Leontief production, i.e., $k^{PB} = k^{PT(L)}$, and (ii) lower than those in the PT model with substitute efforts, i.e., $k^{PB} < k^{PT(S)}$.*

Effort substitutability implies that agents can free ride on their team mate's effort; this implies that it is more difficult to statistically discriminate deviations, translating into higher cost.

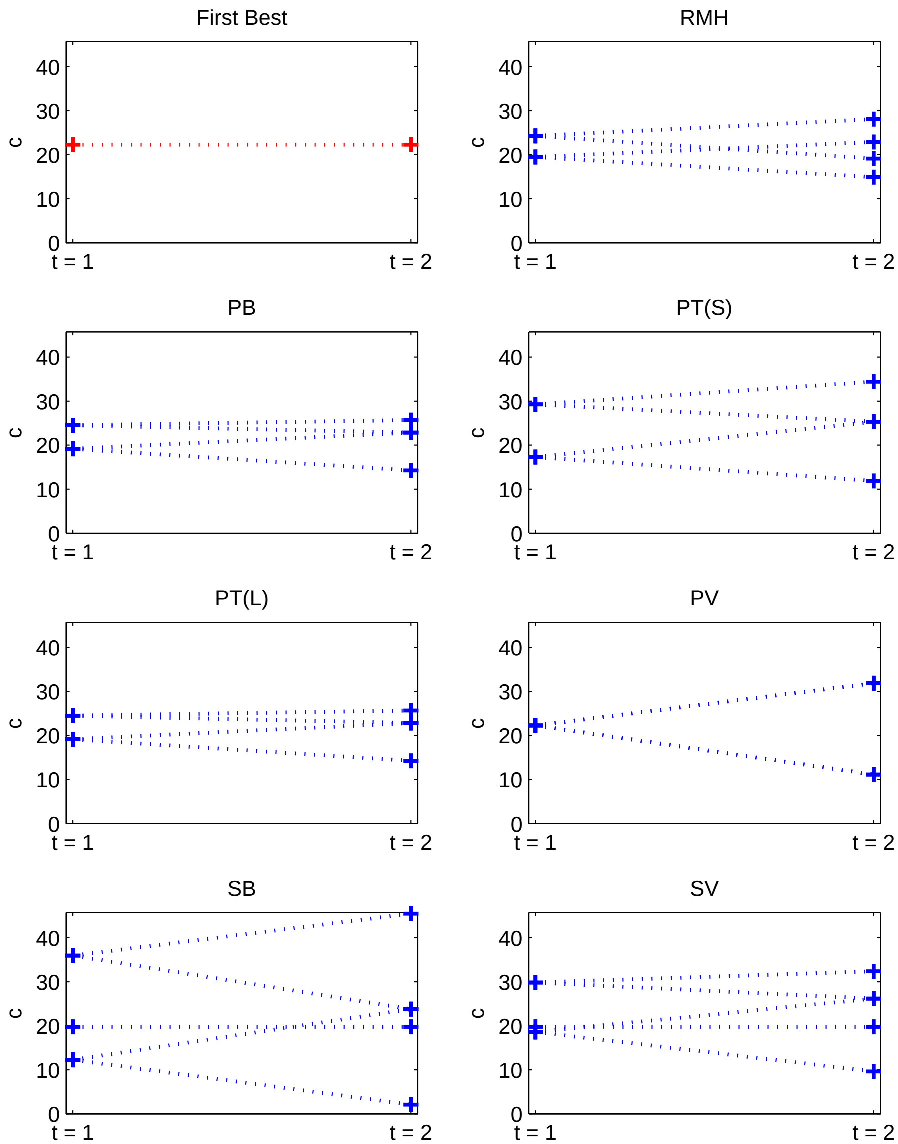

Figure 2: In the first row, First Best is the full information model. RMH is the repeated moral hazard model. The second and third row of subplots are the producer–agent models. The fourth row of subplots are the screener–agent models.

Using the derivations in section 3.3, for the PV model the average discounted cost can be written as:

$$k^{PV} = \frac{1}{4\left(1+\beta\right)^{2}} \left[\left(\bar{U}+a\right)^{2} + a^{2} \left(\frac{\beta}{1+\beta}\right)^{2} \frac{1}{\bar{v}^{PV}} \right].$$

As we previously discussed, all incentive provision, and hence any form of variable compensation, is deferred until the second period of the contract in the variation. This is in sharp contrast with the structure of the optimal contract in our benchmark producer model. This perfect insurance in the first period, together with the fact that only one effort needs to be incentivized (with a lower effort disutility,) suggest that the level of payments may be lower in this variation model. However, there is less information overall in the contract and that may lead to higher variance of consumption in the second period. Whether these factors result in higher or lower expected cost of one contract versus another is characterized in the next proposition.

Proposition 4 *The average payments to the agent in the PB model are lower than those in the PV model, i.e., $k^{PB} < k^{PV}$.*

In the PV model the agent can affect the probability of repayment with his second period action, even if only slightly (α may be arbitrarily small). Since the cost of this action is of similar magnitude to that of the first period action, this translates into a considerable burden for the principal, even if the first period incentives come for free as discussed earlier. In other words, under the condition $\alpha < \frac{\beta}{1+\beta}$, the lower cost of per period effort is not enough to compensate the lower informational content of the signals. We conclude that the extra freedom of action available to the agent in the variation model increases the expected payments to the agent.

We now compare the level of payments in the PB model with those in the screener models. As we first noted in section 4.1.2, the utilities in the contingent branch of both screener models can be written in the general form of Eq. 26. With the solution specifics stated in sections 4.1.2 and 4.2.2, we can derive the expression for the average per period cost of the contract for the screener models.

The average payments in the auxiliary SB model are:

$$k_{aux}^{SB} = \frac{1}{4\left(1+\beta\right)^{2}} \left[\left(\bar{U}+\frac{a}{\gamma}\right)^{2} + \frac{\left(a/\gamma\right)^{2}}{\bar{v}^{SB}} \right].$$

With this, it is simple to see that the average per period cost of the whole contract is:

$$\begin{aligned} k^{SB} &= (1-\gamma)\left(\frac{u_0}{2}\right)^{2} + \gamma k_{aux}^{SB} \\ &= \frac{1}{4\left(1+\beta\right)^{2}} \left[\bar{U}^{2} + \gamma \left(2\frac{a}{\gamma}\bar{U} + \left(\frac{a}{\gamma}\right)^{2} + \frac{\left(a/\gamma\right)^{2}}{\bar{v}^{SB}} \right) \right]. \end{aligned}$$

The average per period cost of the contract for the auxiliary SV model is:

$$k_{aux}^{SV} = \frac{1}{4(1+\beta)^2} \left[\left(\bar{U} + \frac{a}{\gamma} \right)^2 + \frac{(a/\gamma)^2}{\bar{v}} \right].$$

The average per period cost of the whole contract is:

$$k^{SV} = \frac{1}{4(1+\beta)^2} \left[\bar{U}^2 + \gamma \left(2\frac{a}{\gamma}\bar{U} + \left(\frac{a}{\gamma} \right)^2 + \frac{(a/\gamma)^2}{\bar{v}} \right) \right].$$

It is simple to compare the average payments across the two screener models:

Proposition 5 *The expected payments to the agent in the SB model are higher than those in the SV model, i.e. $k^{SB} > k^{SV}$.*

It is intuitive that the possibility of pooling risky loans without exerting effort in the SB model gives an extra informational advantage to the agent, translating into higher cost.

We can also compare the cost of the screener models to that of the PB model.

Proposition 6 *The expected payments to the agent in the SV model are higher than those in the PB model, i.e. $k^{SV} > k^{PB}$.*

The information content in the observations of the contingent branch of the SV model is the same as in the PB, since the contract is trying to discriminate good versus bad loans in both models. In the producer model, when the agent exerts effort the loan is of good quality with probability one. However, in the screener model, when the agent exerts effort the loan is of good quality only with probability γ. This means that the screener agent will sometimes exert effort and not "use" it, making effort effectively more costly and hence the incentive problem more severe. This translates into higher utility spreads, higher average payments, and hence higher cost to the principal.

We conclude from the last two propositions that, for the three models PB, SB and SV, we have a complete ranking of the average per period cost, summarized in the following corollary.

Corollary 1 *The expected payments are always lower for a benchmark producer agent than for a screener agent:*

$$k^{SB} > k^{SV} > k^{PB}.$$

It is apparent that the extra level of information asymmetry in the screener models implies that higher payments to the agent are needed. Moreover, the ability to accept pooled risky loans in the SB model implies more difficulty in the statistical discrimination of deviations,

k^m (section)	RMH	Producer PB (3.1)	$PT(S)$ (3.2)	PV (3.3)	Screener SB (4.1)	SV (4.2)
baseline	2	1	5	4	6	3
$\alpha = 0.3$	2	1	4	5	6	3
$\alpha = 0.2$	2	1	4	6	5	3
$\beta = 1$	2	1	4	5	6	3

Table 1: Ranking of the average payments in the optimal compensation schedules.

translating into higher payments, on average, to the screener in the benchmark model than in the variation.

For the comparisons with the PV model, which depend on the particular parametrization used, we turn to numerical examples. Table 1 reports the comparisons for average payments across all five models, and the RMH. The results in the propositions of this section are confirmed. Moreover, in our baseline parametrization the RMH model ranks as the second lowest payments, following the PB model; the PV model has the fourth highest payments, second only to the $PT(S)$ and the SB models. The $PT(S)$ contract is cheaper than the SB contract in all parametrizations. When we lower α to 0.3, the PV contract surpasses the $PT(S)$ contract in cost. When we lower α further, to 0.2, the PV contract becomes the most expensive one, surpassing even the PB contract. Increasing β to 1 reverses the original ranking between the $PT(S)$ contract and the PV contract, making the PV contract the most expensive of the two.

5.2 Deferred compensation

We interpret the extra expected compensation in the second period, over first period expected pay, as the percentage of deferred compensation, i.e. there is deferred compensation whenever:

$$\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]} > 1.$$

It is important to emphasize that any two–period RMH optimal contract will optimally make payments in the second period depend both on first and second period observations. The dependence of second period payments on first period outcomes allows for smoothing of incentives over time. In particular, future compensation can be used as part of the incentives, which helps reduce the variance of compensation in the short term. In the presence of persistence, this effect is still present, but it is also the case that second period signals contain information about first period actions, which also affects the compensation structure.

We now discuss the three main determinants of deferred compensation in long term contracts, with and without persistence: the level of risk aversion and the smoothing of incentives, and the structure of asymmetric information. Then we compare deferred compensation across all the models.

Risk aversion and deferred compensation

In each of the above discussed frameworks, including the RMH, whether there is compensation deferral (i.e., whether $\frac{E[c_2|a_h]}{E[c_1|a_h]}$ is greater than 1 or not) depends exclusively on the curvature of the utility function of the agent. This is determined by the inverse Euler equation, also known as the Rogerson condition, first discussed in Rogerson (1985). This condition can be derived from the first-order condition of each of the problems above, and it reads:

$$\frac{1}{u'(c_i)} = E\left[\frac{1}{u'(c_{ij})}|a_h\right] \quad \forall i,j. \tag{37}$$

For the specification of utility that we have used in this article, $u(c) = 2\sqrt{c}$, equation (37) implies that

$$E[c_i|a_h] < E[c_{ij}|a_h] \quad \forall i,j.$$

Within the class of CRRA utility functions, of the form $\frac{c^{1-\sigma}}{1-\sigma}$, coefficients of relative risk aversion (σ) of one or less imply an increasing path for expected compensation. However, coefficients of one or more imply a decreasing path, and hence for highly risk averse agents compensation is front–loaded rather than deferred (i.e., $\frac{E[c_2|a_h]}{E[c_1|a_h]}$ is smaller than 1.) In our examples, we fix the level of risk aversion by assuming $U(c) = 2\sqrt{c}$, and we compare the *size* of the deferral across the different models.

There is an important implication that stems from equation (37), and hence is common to all the models discussed here: the optimal contract is such that the agent is always left with a desire to save (assuming that the savings rate is the inverse of the discount rate, $1/\beta$.) This is easily seen because equation 37 implies that the Euler equation of the agent is violated as follows:

$$E[u'(c_i)] > E[u'(c_{ij})] \quad \forall i,j.$$

Hence, even if the path for expected compensation is increasing (as it is in our example with $U(c) = 2\sqrt{c}$,) the optimal contract is such that the agent would like to save some of his income in the first period. The theory implies that the principal wants to *limit* the ability of the agent to save, and hence most of the results in this article depend on the assumption that the principal can, in fact, control the savings of the agent. In conclusion, it is important to note that deferred compensation does not imply that the agent would like to borrow against his future income.

The structure of asymmetric information and deferred compensation

It is our objective to understand how persistence of effort, the fact that better information gets revealed over time, and the particular structure of the incentive problem inherent in a given job assignment affect the optimal provision of incentives over time. With this objective in mind, we derive the expression for deferred compensation in each of the five models:

$$\begin{aligned}
\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PB} &= 1+\frac{\left(\frac{a}{\bar{v}}\right)^2 v_1\left(1+v_1\right)}{\left(\bar{U}+a\right)^2+\left(\frac{a}{\bar{v}}\right)^2 v_1}, \\
\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PT(S)} &= 1+\frac{\left(\frac{a}{\bar{v}^{PT(S)}}\right)^2 v_1^{PT(S)}\left(1+v_1^{PT(S)}\right)}{\left(\bar{U}+a\right)^2+\left(\frac{a}{\bar{v}^{PT(S)}}\right)^2 v_1^{PT(S)}}, \\
\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PV} &= 1+\frac{a^2}{\alpha^2 v_1\left(\bar{U}+a\right)^2}, \\
\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{SB} &= 1+\frac{\left(\frac{a/\gamma}{\bar{v}}\right)^2 v_1^{SB}\left(1+v_1^{SB}\right)}{\left(\bar{U}+a/\gamma\right)^2+\left(\frac{a/\gamma}{\bar{v}^{SB}}\right)^2 v_1^{SB}}, \\
\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{SV} &= 1+\frac{\left(\frac{a/\gamma}{\bar{v}}\right)^2 v_1\left(1+v_1\right)}{\left(\bar{U}+a/\gamma\right)^2+\left(\frac{a/\gamma}{\bar{v}}\right)^2 v_1}.
\end{aligned}$$

Proposition 7 *The proportion of pay deferred is smaller in the PB model than in the PV model:*

$$\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PB} < \left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PV}$$

This result is intuitive, since in the PV model all variable pay takes place in the second period and $U(c) = 2\sqrt{c}$ implies that expected utility should be equal across the two periods. Risk aversion then implies the result.

Proposition 8 *The proportion of pay deferred is smaller in the PB model than in the SV model:*

$$\left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{PB} < \left[\frac{E\left[c_2|a_h\right]}{E\left[c_1|a_h\right]}\right]^{SV}$$

We have already discussed how the information content in the observations of the contingent branch of the SV model is the same as in the PB. The higher "effective" cost of effort in the screener model translates into higher average payments; through the information structure implied by persistence, where more high powered incentives are given in the second period, this in turn translates into higher deferred compensation.

For the comparisons that cannot be established analytically, we turn to numerical examples. Table 2 calculates the ranking of each optimal contract in terms of the percentage of

$\frac{E[c_2]}{E[c_1]}$ (section)	RMH	Producer PB (3.1)	Producer PT(S) (3.2)	Producer PV (3.3)	Screener SB (4.1)	Screener SV (4.2)
baseline	2	1	4	6	5	3
$\pi_b = 0.4$	3	1	4	6	5	2
$\beta = 1$	3	1	4	6	5	2
$\gamma = 0.6$	3	1	4	6	5	2

Table 2: Ranking of the deferred compensation in the optimal compensation schedules for five models.

deferred pay. We confirm in the examples the results in this section. In our baseline parametrization, we find that the PV model is in fact the one with most deferred compensation, higher also than the RMH and the two screener models. The PB is always the one with the least deferred compensation. The $PT(S)$ model ranks fourth in all parametrizations, always exhibiting less deferred compensation than the PV and the SB models. Within the screener models, the benchmark has more deferred compensation. Finally, the RMH ranks second in the benchmark parametrization, with only the PB model having less deferred compensation. The ranking between the RMH and the SV gets exchanged (deferred compensation becomes larger in the RMH) when: i) π_b increases, ii) β increases, or iii) γ increases. The rest remain unchanged.

These comparisons imply that deferred compensation is not always higher in the presence of persistence. One may conjecture that the incentive smoothing motive would imply that more costly incentive problems may tend to rely more on deferred compensation. However, our numerical comparisons show that, even if the RMH model ranked lower than the screener models in terms of cost in table 1 for all our parametrizations, implying a less severe incentive problem, in table 2 the proportion of deferred pay for the RMH model is higher than that of the SV contract with persistence in several of our parametrizations. This suggests that incentive smoothing interacts with the structure of asymmetric information in nontrivial ways.

5.3 Salaries and bonuses

In this section we take a closer look at the structure of pay, within and across periods, in order to understand the implications of the structure of asymmetric information for compensation instruments used in real life contracts such as base salaries and bonuses.

Before we discuss in detail the differences across models, we map the optimal contracts to actual compensation schemes. The optimal contracts characterized in this article so far

base salary (section)	RMH	Producer PB (3.1)	PT(S) (3.2)	PV (3.3)	Screener SB (4.1)	SV (4.2)
baseline	6	5	4	3	1	2
$\alpha = 0.3$	6	5	4	2	1	3
$\alpha = 0.2$	6	5	4	2	1	3

Table 3: Ranking of the base salary in the optimal compensation schedules for five models.

take the form of payments contingent on a history of realizations. In practice, compensation contracts of bankers take a much simpler form, usually consisting of a base salary and a bonus payment, some of it deferred. We propose the following mapping: identify the lowest compensation level throughout the two periods of the contract as the base salary, and the extra compensation at every other node as a bonus contingent on performance.

For the screener models, we need to establish whether c_{ll} or c_0, and c_{hh} or c_0, represents the smallest and largest consumption level, correspondingly.

Lemma 2 *In the SB and SV models, the minimum consumption is* c_{ll} *and the maximum is* c_{hh}.

Tables 3 and 4 provide the ranking of the base salary and the maximum consumption, respectively, for all five models. In all parametrizations, the base salary is smallest in the SB. In our baseline parametrization, it is followed by the SV, the PV, the $PT(S)$ and the PB base salary, in this order. The RMH contract has the highest base salary in all of our parametrizations. Only when we decrease α do we find a change in the ranking: the base salary in the PV decreases, becoming only the second highest, below the base salary in the SV contract.

As for the maximum payment, the PB contract always has the lowest, followed by the RMH, the PV, the SV, and the $PT(S)$. The SB contract always has the highest. (Since SB is also the contract with the lowest base salary, as discussed in the previous table, this makes it the contract with the highest absolute variation in compensation. We discuss this measure in the next table.) When we decrease α the ranking between the maximum payment in the PV contract increases to surpass both the SV and the $PT(S)$ payments. When we increase γ the ranking between the SV and the PV payments gets reversed, with the PV one being now larger.

Bonuses

We define bonuses as the difference between a contingent payment and the base salary in the contract:

$$b_i^m = c_i^m - c_{ll}^m \quad \forall m, i.$$

max(c)	RMH		Producer		Screener	
(section)		PB (3.1)	PT(S) (3.2)	PV (3.3)	SB (4.1)	SV (4.2)
baseline	2	1	5	3	6	4
$\alpha = 0.3$	2	1	4	5	6	3
$\alpha = 0.2$	2	1	4	5	6	3
$\gamma = 0.6$	2	1	5	4	6	3

Table 4: Ranking of the maximum compensation in the optimal compensation schedules for five models.

We denote the maximum bonus as:

$$\bar{b}^m = c^m_{hh} - c^m_{ll},$$

with

$$\begin{aligned}
\bar{b}^{PB} &= \frac{a}{1+\beta}\left(t_{hh} - t_{ll}\right), \\
\bar{b}^{PV} &= \frac{a}{1+\beta}\left(t^{PV}_{hh} - t^{PV}_{ll}\right), \\
\bar{b}^{SB} &= \frac{a/\gamma}{1+\beta}\left(t^{SB}_{hh} - t^{SB}_{ll}\right), \\
\bar{b}^{SV} &= \frac{a/\gamma}{1+\beta}\left(t_{hh} - t_{ll}\right).
\end{aligned}$$

Proposition 9 *The maximum bonus payout in the PB model is lower than in the PV model.*

In line with our result on the expected payments being lower in the PB than in its variation model, we find that, in spite of the poorer informational content of second period signals in the variation model, the contract needs to rely on them more heavily than for the benchmark model.

We can as well compare the screener models among themselves and with the PB model.

Proposition 10 *The maximum bonus payout in the SV model is lower than in the SB model, i.e. $\bar{b}^{SV} < \bar{b}^{SB}$.*

Proposition 11 *The maximum bonus payout in the PB model is lower than in the SV model, i.e. $\bar{b}^{PB} < \bar{b}^{SV}$.*

For these three models, we have a complete ranking of the maximum bonus, summarized in the following corollary.

$\bar{b}$ (section)	RMH	Producer PB (3.1)	Producer PT(S) (3.2)	Producer PV (3.3)	Screener SB (4.1)	Screener SV (4.2)
baseline	2	1	4	3	6	5
$\pi_b = 0.38$	2	1	5	3	6	4
$\overline{U} = 11$	2	1	5	3	6	4
$\alpha = 0.3$	2	1	3	5	6	4
$\alpha = 0.2$	2	1	3	5	6	4
$\gamma = 0.6$	2	1	5	4	6	3

Table 5: Ranking of the maximum bonus payout in the optimal compensation schedules for five models.

Corollary 2 *The maximum bonus payment is always lower for a benchmark producer agent than for a screener agent:*

$$\bar{b}^{PB} < \bar{b}^{SV} < \bar{b}^{SB}.$$

We conclude that more asymmetry of information always translates into more variable pay. The results in this section help one to understand the ranking of expected payments provided in section 5.1: for risk averse agents, the need of variable pay for incentives translates into higher average pay levels. We report the comparative statics and the extra ranking in Table 5. In all parametrizations, the PB contract has the lowest $\bar{b}$, the RMH contract has the second lowest, while the SB model has the highest. In the benchmark parametrization we have $\bar{b}^{PV} < \bar{b}^{PT(S)} < \bar{b}^{SV}$. Increasing π_b or decreasing U reverses the comparison between the $PT(S)$ and the SV bonuses. Decreasing α changes the ordering to $\bar{b}^{PT(S)} < \bar{b}^{SV} < \bar{b}^{PV}$. Increasing γ, instead, changes it to $\bar{b}^{SV} < \bar{b}^{PV} < \bar{b}^{PT(S)}$.

Guaranteed bonuses

As a related point to our measures of the importance of variable pay, it is worth discussing guaranteed bonuses. A guaranteed bonus is one that is not contingent on performance measures. These type of bonuses are common in the banking industry for new hires, and typically are in place only for the first year of the contract. One justification often suggested for guaranteed bonuses is that they serve to attract new hires, by compensating an employee for the loss of deferred compensation from a previous employer. Another possible function could be to provide some insurance against the uncertainty that comes with any new job. We are not explicitly modelling here the change of employment decision, but rather simply assuming an outside utility that should be met by the optimal contract. In spite of the simplicity of our framework, however, our optimal contracts exhibit guaranteed bonuses. We define a guaranteed bonus as the difference between the lowest payment in the first

b_g (section)	RMH	PB (3.1)	PT(S) (3.2)	PV (3.3)	SB (4.1)	SV (4.2)
		Producer			Screener	
baseline	1	2	3	6	5	4
$\pi_b = 0.38$	2	1	3	6	4	5
$e = 0.2$	1	2	3	5	6	4
$\overline{U} = 11$	1	2	3	6	4	5

Table 6: Ranking of the deferred compensation in the optimal compensation schedules for five models.

period and the base salary:

$$b_g^m \equiv c_l^m - c_{ll}^m \quad \forall m.$$

The rationale for the existence of these guarantees in our context is that, in all of the environments that we consider, it is desirable to allow for second-period compensation to be less than first period compensation. This is optimal, as we discussed above, both because of incentive smoothing (see the RMH plot in the first row of Figure 2), and because of the better quality of information in the second period.

Table 6 reports the numerical comparison of the guaranteed bonuses. In the baseline parametrization, the RMH has the smallest guaranteed bonus, followed by the PB, the $PT(S)$, the SV, the SB and finally the PV contract, with the highest guaranteed bonus. Increasing π_b reversed the ranking between the RMH and the PB bonuses, and also between the SB and the SV bonuses. Decreasing e reversed the ranking between the PV and the SB bonuses. Decreasing U reversed the ranking between the SB and the SV bonuses.

Importance of variable pay

We consider a final measure that captures the importance of variable compensation in each setup: the average proportion of pay that is in the form of bonuses:

$$\frac{E\left[b_i^m\right]}{c_{ll}^m}.$$

The ranking is reported in Table 7. The baseline parametrization indicates that the RMH model and the persistent, one-action case of the PB model rely the least on variable compensation. This contrasts with the SB model, which relies the most. As is clear from Figure 2, in the optimal contract for the SB model the agent receives the lowest base salary of all the contracts considered. If the agent claims that the loan is a bad type and hence should not be accepted, he receives a constant bonus payment in both periods. This bonus is higher than the one he receives if he accepts a good loan but it does not repay in the first period. However, if the loan repays in the second period he receives a higher bonus payment. The

$\frac{E\left[b_i^m\right]}{c_{ll}^m}$ (section)	RMH	Producer PB (3.1)	PT(S) (3.2)	PV (3.3)	Screener SB (4.1)	SV (4.2)
baseline	1	2	3	4	6	5
$\alpha = 0.3$	1	2	3	5	6	4
$\alpha = 0.2$	1	2	3	5	6	4
$\gamma = 0.6$	1	2	3	5	6	4

Table 7: Ranking of the importance of variable pay in the optimal compensation schedules for five models.

high variable compensation in this model is what makes this contract the most expensive environment.

In table 7 we see that the ranking is fairly stable across our comparative statics. In all parametrizations, the RMH contract relies the least on variable compensation, followed by the PB, and the $PT(S)$. The SB contract relies the most. In the baseline parametrization, the PV contract ranks fourth and the SV contract fifth. Decreasing α or increasing γ had the same effect: they made the comparison between PV and SV reverse, with PV now having a higher proportion of pay implemented through bonuses.

6 Conclusion

We have studied the characteristics of optimal contracts for various jobs in banking on which, without exception, hidden actions affect output over several periods. We conclude from our analysis that the level of pay, the proportion of pay that should be deferred, as well as the importance of performance-based bonuses, vary greatly with the production process, that is, the sources of private information.

In particular, we have established that occupations of the type described in the producer benchmark model— a pure hidden action problem with signals about effort arriving over time— are the most likely to exhibit lower wages and lower proportion of deferred pay and to rely less on variable pay. In particular, if the moral hazard problem changes by making a consequent action available to the agent, the wages, deferred pay, and bonus importance all increase. Also, team production in this type of occupation increases the incentive problem if the actions of the different producer agents on the team are substitutes. This leads to higher wages, more deferred compensation, and more reliance on variable pay.

In the screener models we have analyzed occupations that entail not only a hidden action but also hidden information (i.e., the agent retains some control over the outcome realization after exerting his action). We conclude from our analysis that the optimal contract in these types of occupations exhibits higher wages, a higher portion of deferred pay, and more reliance on variable pay than the producer contracts. The severity of the hidden information problem, which is higher in the benchmark screener model, increases all three measures.

An interesting conclusion from comparing the models with persistence to a standard repeated moral hazard model without persistence is that deferred compensation is not always higher in the presence of persistence. Hence, being able to recognize the persistence of the effects of hidden actions over time in a particular job assignment may not necessarily lead us to expect extraordinary levels of deferred pay in the optimal pay arrangement.

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7 Appendix

Proof of Lemma 1. The conditions in Mukoyama and Sahin become:

$$\begin{aligned}
\beta\left\{\pi_g-\left[\pi_g-\alpha_1\left(\pi_g-\pi_b\right)\right]\right\} &\geq \pi_g-\left[\pi_g-\alpha_2\left(\pi_g-\pi_b\right)\right], \\
\beta\left\{\left[\pi_g-\alpha_2\left(\pi_g-\pi_b\right)\right]-\pi_b\right\} &\geq \pi_g-\left[\pi_g-\alpha_2\left(\pi_g-\pi_b\right)\right].
\end{aligned}$$

Simplifying, the first condition becomes:

$$\begin{aligned}\beta\alpha_1(\pi_g-\pi_b) &\geq \alpha_2(\pi_g-\pi_b)\\ \beta &\geq \frac{\alpha_2}{\alpha_1},\end{aligned}$$

and the second condition becomes:

$$\begin{aligned}\beta(1-\alpha_2)(\pi_g-\pi_b) &\geq \alpha_2(\pi_g-\pi_b)\\ \beta &\geq \frac{\alpha_2}{1-\alpha_2}.\end{aligned}$$

■

Proof of Prop. 4. We compare the two expressions for the cost:

$$\begin{aligned}k^{PB} &< k^{PV}\\ \frac{1}{4(1+\beta)^2}\left[(\bar{U}+a)^2+\frac{a^2}{\bar{v}}\right] &< \frac{1}{4(1+\beta)^2}\left[(\underline{U}+a)^2+a^2\left(\frac{\beta}{1+\beta}\right)^2\frac{1}{\bar{v}^{PV}}\right]\\ \frac{1}{\bar{v}} &< \left(\frac{\beta}{1+\beta}\right)^2\frac{1}{\bar{v}^{PV}}\\ \frac{1+\beta}{v_1(1+\beta(2+v_1))} &< \frac{\beta}{1+\beta}\frac{1}{\alpha^2 v_1}\\ \frac{1+\beta}{1+\beta(2+v_1)} &< \frac{\beta}{1+\beta}\frac{1}{\alpha^2} \qquad (38)\\ \alpha^2 &< \frac{\beta(1+\beta(2+v_1))}{(1+\beta)^2}. \qquad (39)\end{aligned}$$

By Lemma 1, we have

$$\begin{aligned}\beta &\geq \frac{\alpha}{1-\alpha}\\ \frac{\beta}{1+\beta} &\geq \alpha,\end{aligned}$$

and since $\alpha<1$, we have $\alpha^2<\alpha$. If the maximum value of α satisfies inequality 38, we are done. We now show that this is the case:

$$\frac{\beta}{1+\beta}\leq\frac{\beta(1+\beta(2+v_1))}{(1+\beta)^2}$$

$$1+\beta\leq 1+\beta(2+v_1).$$

This holds trivially since $v_1>0$. ■

Proof of Prop. 5. It is simple to see that $k^{SB} > k^{SV}$ by comparing the two expressions:

$$\begin{aligned}
\bar{U}^2 + \gamma\left[2\frac{a}{\gamma}\bar{U} + \left(\frac{a}{\gamma}\right)^2 + \frac{(a/\gamma)^2}{\bar{v}^{SB}}\right] &> \bar{U}^2 + \gamma\left[2\frac{a}{\gamma}\bar{U} + \left(\frac{a}{\gamma}\right)^2 + \frac{(a/\gamma)^2}{\bar{v}}\right] \\
\frac{1}{\bar{v}^{SB}} &> \frac{1}{\bar{v}} \\
\frac{1+\beta}{(1-\gamma)^2 v_1\left[1+\beta\left(2+(1-\gamma)^2 v_1\right)\right]} &> \frac{1+\beta}{v_1\left[1+\beta\left(2+v_1\right)\right]} \\
1+\beta\left(2+v_1\right) &> (1-\gamma)^2\left[1+\beta\left(2+(1-\gamma)^2 v_1\right)\right],
\end{aligned}$$

which is always true since $\gamma \in (0,1)$ and $v_1 > 0$. ■

Proof of Prop. 6. It is simple to see that $k^{SV} > k^{PB}$ by comparing the two expressions:

$$\begin{aligned}
\frac{1}{4(1+\beta)^2}\left\{\bar{U}^2 + \gamma\left[2\frac{a}{\gamma}\bar{U} + \left(\frac{a}{\gamma}\right)^2 + \frac{(a/\gamma)^2}{\bar{v}}\right]\right\} &> \frac{1}{4(1+\beta)^2}\left[\left(\bar{U}+a\right)^2 + \frac{a^2}{\bar{v}}\right] \\
\bar{U}^2 + 2a\bar{U} + \frac{a^2}{\gamma} + \frac{a^2}{\gamma}\frac{1}{\bar{v}} &> \bar{U}^2 + 2a\bar{U} + a^2 + \frac{a^2}{\bar{v}} \\
\frac{a^2}{\gamma} + \frac{a^2}{\gamma}\frac{1}{\bar{v}} &> a^2 + \frac{a^2}{\bar{v}} \\
\frac{1}{\gamma}\left(1+\frac{1}{\bar{v}}\right) &> \left(1+\frac{1}{\bar{v}}\right) \\
1 &> \gamma,
\end{aligned}$$

which is always true. ■

Proof of Corollary 1. Follows from Prop. 5 and Prop. 6. ■

Proof of Prop. 7. We can simply compare the expressions:

$$\begin{aligned}
1 + \frac{\left(\frac{a}{\bar{v}}\right)^2 v_1\left(1+v_1\right)}{\left(\bar{U}+a\right)^2 + \left(\frac{a}{\bar{v}}\right)^2 v_1} &< 1 + \frac{a^2}{\alpha^2 v_1\left(\bar{U}+a\right)^2} \\
\left(\frac{a}{\bar{v}}\right)^2 v_1\left(1+v_1\right)\alpha^2 v_1\left(\bar{U}+a\right)^2 &< a^2\left[\left(\bar{U}+a\right)^2 + \left(\frac{a}{\bar{v}}\right)^2 v_1\right] \\
\alpha^2\left(\frac{v_1}{\bar{v}}\right)^2\left(1+v_1\right)\left(\bar{U}+a\right)^2 &< \left(\bar{U}+a\right)^2 + \left(\frac{a}{\bar{v}}\right)^2 v_1 \\
\alpha^2\frac{(1+\beta)^2\left(1+v_1\right)}{\left[1+\beta\left(2+v_1\right)\right]^2}\left(\bar{U}+a\right)^2 &< \left(\bar{U}+a\right)^2 + \left(\frac{a}{\bar{v}}\right)^2 v_1 \qquad (40)
\end{aligned}$$

Note that the restrictions on the parameters say $\alpha \leq \frac{\beta}{1+\beta}$, so

$$\alpha^2\frac{(1+\beta)^2\left(1+v_1\right)}{\left[1+\beta\left(2+v_1\right)\right]^2} < \frac{\beta^2}{(1+\beta)^2}\frac{(1+\beta)^2\left(1+v_1\right)}{\left[1+\beta\left(2+v_1\right)\right]^2}$$

If we can show that the right hand side of this expression is smaller than 1, then 40 holds trivially. It is easy to see that that is the case:

$$\begin{aligned}
\frac{\beta^2(1+v_1)}{\left[1+\beta(2+v_1)\right]^2} &< 1 \\
\beta^2+\beta^2 v_1 &< 1+2\beta(2+v_1)+\beta^2\left(4+4v_1+v_1^2\right) \\
0 &< 1+4\beta+2\beta v_1+3\beta^2+3\beta^2 v_1+\beta^2 v_1^2.
\end{aligned}$$

■

Proof of Prop. 8. We can simply compare the expressions:

$$\begin{aligned}
1+\frac{\left(\frac{a}{\bar v}\right)^2 v_1(1+v_1)}{\left(\bar U+a\right)^2+\left(\frac{a}{\bar v}\right)^2 v_1} &< 1+\frac{\left(\frac{a/\gamma}{\bar v}\right)^2 v_1(1+v_1)}{\left(\bar U+a/\gamma\right)^2+\left(\frac{a/\gamma}{\bar v}\right)^2 v_1} \\
\left(\frac{a}{\bar v}\right)^2 v_1(1+v_1)\left[\left(\bar U+a/\gamma\right)^2+\left(\frac{a/\gamma}{\bar v}\right)^2 v_1\right] &< \left(\frac{a/\gamma}{\bar v}\right)^2 v_1(1+v_1)\left[\left(\bar U+a\right)^2+\left(\frac{a}{\bar v}\right)^2 v_1\right] \\
\left(\bar U+a/\gamma\right)^2+\left(\frac{a/\gamma}{\bar v}\right)^2 v_1 &< \frac{1}{\gamma^2}\left[\left(\bar U+a\right)^2+\left(\frac{a}{\bar v}\right)^2 v_1\right] \\
\gamma^2\bar U^2+\gamma^2 2\bar U\frac{a}{\gamma}+\gamma^2\frac{a^2}{\gamma^2}+\gamma^2\frac{a^2}{\gamma^2\bar v^2}v_1 &< \bar U^2+2\bar U a+a^2+\left(\frac{a}{\bar v}\right)^2 v_1 \\
\gamma^2\bar U^2+\gamma 2\bar U a &< \bar U^2+2\bar U a.
\end{aligned}$$

■

Proof of Lemma 2. To show the first part of the result, we need to prove $u_0 > u_{ll}$. But we have

$$u_0=\frac{\bar U}{1+\beta}$$

and also

$$\begin{aligned}
EU_{pool} \equiv\ & \pi_p\left(u_h+(1-\pi_p)\beta u_{hl}+\pi_p\beta u_{hh}\right) \\
& +(1-\pi_p)\left(u_l+(1-\pi_p)\beta u_{ll}+\pi_p\beta u_{lh}\right)=\bar U.
\end{aligned}$$

It follows that we cannot have all consumptions in the contingent branch be greater than c_0, and since c_{ll} is the minimum, we have $c_0 < c_{ll}$. In the same way,

$$\begin{aligned}
EU_g \equiv\ & \gamma\left[\pi_g\left(u_h+(1-\pi_g)u_{hl}+\pi_g u_{hh}\right)\right. \\
& \left.+(1-\pi_g)\left(u_l+\pi_g u_{lh}+(1-\pi_g)u_{ll}\right)\right]+(1-\gamma)u_0-a=\bar U+\frac{a}{\gamma}.
\end{aligned}$$

Hence, at least one consumption is greater than c_0, and we know that c_{hh} is the greatest one of them. The same argument goes through for the SV model. ■

Proof of Prop. 9.

$$\begin{aligned}
a\left(t_{hh}-t_{ll}\right) &< a\left(t_{hh}^{PV}-t_{ll}^{PV}\right) && (41)\\
\frac{v_1}{\bar{v}}\frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)}{\left(\pi_g-\pi_b\right)\pi_g\left(1-\pi_g\right)} &< \frac{1}{\alpha\left(\pi_g-\pi_b\right)}\\
\alpha &< \frac{1+\beta\left(2+v_1\right)}{1+\beta}\frac{\pi_g\left(1-\pi_g\right)}{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)}. && (42)
\end{aligned}$$

We have

$$\frac{\pi_g\left(1-\pi_g\right)}{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)}>\frac{1}{2}$$

and

$$\frac{1+\beta\left(2+v_1\right)}{1+\beta}>1,$$

so

$$\alpha<\frac{1}{2}$$

implies that inequality 41 is always satisfied. ■

Proof of Prop. 10.

$$\frac{a}{\gamma}\left(t_{hh}-t_{ll}\right)<\frac{a}{\gamma}\left(t_{hh}^{SB}-t_{ll}^{SB}\right).$$

We can simplify

$$\begin{aligned}
t_{hh}-t_{ll} &= \frac{1-\pi_g}{\left(\pi_g-\pi_b\right)}\frac{v_1}{\bar{v}}\frac{\left(\pi_g+\pi_b\right)}{\pi_g}-\frac{\pi_g}{\left(\pi_g-\pi_b\right)}\frac{v_1}{\bar{v}}\frac{\left(\pi_g+\pi_b-2\right)}{\left(1-\pi_g\right)}\\
&= \frac{v_1}{\bar{v}}\frac{1}{\left(\pi_g-\pi_b\right)}\left[\frac{\left(1-\pi_g\right)\left(\pi_g+\pi_b\right)}{\pi_g}-\frac{\pi_g\left(\pi_g+\pi_b-2\right)}{\left(1-\pi_g\right)}\right]\\
&= \frac{v_1}{\bar{v}}\frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)}{\left(\pi_g-\pi_b\right)\pi_g\left(1-\pi_g\right)}.
\end{aligned}$$

In the same way,

$$\begin{aligned}
t_{hh}^{SB}-t_{ll}^{SB} &= \frac{\left(1-\gamma\right)^2 v_1}{\bar{v}^{SB}}\frac{\pi_g\left(1-\gamma\pi_g-\left(1-\gamma\right)\pi_b\right)+\left(\gamma\pi_g+\left(1-\gamma\right)\pi_b\right)\left(1-\pi_g\right)}{\left(\pi_g-\gamma\pi_g-\left(1-\gamma\right)\pi_b\right)\pi_g\left(1-\pi_g\right)}\\
&= \frac{\left(1-\gamma\right)^2 v_1}{\bar{v}^{SB}}\frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)-\gamma\left(\pi_g-\pi_b\right)\left(2\pi_g-1\right)}{\left(1-\gamma\right)\left(\pi_g-\pi_b\right)\pi_g\left(1-\pi_g\right)}.
\end{aligned}$$

So we need to show

$$\begin{aligned}
t_{hh}-t_{ll} &< t_{hh}^{SB}-t_{ll}^{SB}\\
\frac{v_1}{\bar{v}}\frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)}{\left(\pi_g-\pi_b\right)\pi_g\left(1-\pi_g\right)} &< \frac{\left(1-\gamma\right)^2 v_1}{\bar{v}^{SB}}\frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)-\gamma\left(\pi_g-\pi_b\right)\left(2\pi_g-1\right)}{\left(1-\gamma\right)\left(\pi_g-\pi_b\right)\pi_g\left(1-\pi_g\right)}\\
\frac{\left(1-\gamma\right)\left[\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)\right]}{1+\beta\left(2+v_1\right)} &< \frac{\pi_g\left(1-\pi_b\right)+\pi_b\left(1-\pi_g\right)-\gamma\left(\pi_g-\pi_b\right)\left(2\pi_g-1\right)}{1+\beta\left(2+\left(1-\gamma\right)^2 v_1\right)}.
\end{aligned}$$

Since

$$1 + \beta\,(2 + v_1) > 1 + \beta\left(2 + (1-\gamma)^2\, v_1\right),$$

it suffices to show that the numerator in the left hand side is smaller that the numerator in the right hand side:

$$\begin{aligned}
\left\{\begin{array}{c} \pi_g\,(1-\pi_b) + \pi_b\,(1-\pi_g) \\ -\gamma\left[\pi_g\,(1-\pi_b) + \pi_b\,(1-\pi_g)\right] \end{array}\right\} &< \left\{\begin{array}{c} \pi_g\,(1-\pi_b) + \pi_b\,(1-\pi_g) \\ -\gamma\,(\pi_g - \pi_b)\,(2\pi_g - 1) \end{array}\right\} \\
-\pi_g\,(1-\pi_b) - \pi_b\,(1-\pi_g) &< -\,(\pi_g - \pi_b)\,(2\pi_g - 1) \\
-\pi_g + 2\pi_g\pi_b - \pi_b &< -2\pi_g^2 + \pi_g + 2\pi_g\pi_b - \pi_b \\
0 &< -2\pi_g^2 + 2\pi_g \\
0 &< \pi_g\,(1-\pi_g)\,,
\end{aligned}$$

which is always true. ■

Proof of Prop. 11. Trivially, since:

$$a\,(t_{hh} - t_{ll}) < \frac{a}{\gamma}\,(t_{hh} - t_{ll})\,.$$

■

Proof of Corollary 2. Follows from Prop. 10 and Prop. 11. ■