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Stepping Stone and Option Value in a Model of Postsecondary Education*

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Abstract

A stepping stone arises in risky environments with learning and transferable human capital. An example is the role played by academic two-year colleges in postsecondary education: Students, as they learn about the uncertain educational outcomes, can drop out or transfer up to harder and more rewarding schools, carrying a fraction of the accumulated human capital. A theory of education is built and contrasted empirically to find that i) option value explains a large part of returns to enrollment, ii) enrollment in academic two-year colleges is driven by the option to transfer up, and iii) the value of the stepping stone is small.

JEL classification: D83, I21, J24.
Keywords: stepping stone, investment under uncertainty, academic learning, postsecondary education, college education, returns to education.

1 Introduction

Option value is a feature of environments where an investment, or a decision, needs to be made under uncertainty, as in Abel (1983). When there is learning and capital is at least partially transferable, a stepping stone arises as in Jovanovic and Nyarko (1997). Notably, academic two-year colleges constitute one of these stepping stones; as they provide a cheaper learning technology, they act as a safer path toward more rewarding and demanding environments, namely, four-year colleges. This paper, by the means of counterfactual analysis, quantifies this stepping stone and the different sources of option value using a micro-estimated model of education.

Evidence from the *National Longitudinal Study of the High School Class of 1972* (NLS-72) is compelling both in terms of transitions and returns. Dropout rates are high in both academic two- and four-year colleges. Graduation rates are six times higher in four-year colleges and transfer rates are mostly one-sided from academic two- to four-year colleges. For students enrolling in academic two-year colleges, the return to graduation is negligible while that to transferring is large. Academic two-year colleges are cheaper, easier, and some fraction of the accumulated credits can be transferred to a four-year college. Students sort across institution types with respect to how uncertain is their likelihood of success.[1] For students with low expectations, academic two-year colleges are an ideal training ground. In the educational ladder, four-year colleges play the role of the step above academic two-year colleges.

This paper produces a theory of postsecondary education where academic two-year colleges act a stepping stone following from the availability of dropout and transfer options in both steps of the ladder. A calibrated version of the model is then used to evaluate the value-added of each option to each step, how substitutable the steps are, the effect to risk aversion on enrollment and its composition across types of colleges, and the effect of information acquisition on returns to education.

The model incorporates academic two-year colleges together with four-year colleges and work. High-school graduates are uncertain about their ability to accumulate human capital. Pessimistic agents join the workforce, optimistic agents enroll in four-year colleges, and those with intermediate beliefs enroll in academic two-year colleges. During their tenure as students, agents are presented with exams, which govern the accumulation of credits and provide information that updates beliefs about ability, inducing dropouts and transfers.

Academic two-year colleges are ideal for students with aspirations regarding graduation at four-year colleges but with low expectations about their ability to accumulate human capital.

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[1]Similar to the likelihood of investing by firms facing uncertain demand, as in Leahy and Whited (1996).

Depending on the evolution of their beliefs and accumulation of credits, students can decide to transfer to four-year colleges and carry with them a proportion of their stock of credits, implying that academic two-year colleges act as a stepping stone, similar to Jovanovic and Nyarko (1997), where workers move up in the work ladder once they acquire the necessary skills. Further, the model has features of bandit models as students learn about their innate ability to accumulate human capital, similar to Johnson (1978) and Miller (1984). Jovanovic and Nyarko (1997) evaluate the predictive power of bandit and stepping stone models in terms of job mobility and find that there is some evidence favoring a combination of both. Postsecondary education also presents features of bandit and stepping stone models.

The model relates to Miao and Wang (2007), which studies the problem of an investor facing two investment projects, one risky and one riskless. The riskiness follows from uncertainty regarding the true quality of the project. For agents investing in the risky project, the arrival of information might induce re-balancing the investment portfolio toward the riskless one. The model developed here has, as a limiting case, a configuration similar to the model in Miao and Wang (2007). When grades provide only information and graduation is random with poisson arrival rate, the model can be casted in a very similar fashion to Miao and Wang (2007) with the only difference being that in this model agents face three investment opportunities with two that are not absorbing, while in Miao and Wang (2007) they only face two with only one nonabsorbing. Even in this simplified setup, academic two-year colleges can be used to learn in a cheaper and less risky environment.

This paper also relates to Kircher et al. (2009). They document, using a large panel of workers from Denmark, that for a given occupation, high and low wage earners are more likely to switch occupations, the first to more rewarding occupations and the second to occupations with a lower mean wage. Later, they produce a theory of learning about the worker's own ability that explains the sorting and the transitions. In postsecondary education, and in the model developed here, the same pattern of sorting and transitions occurs.

The paper is also related to some empirical research that recognizes education as a sequential process. Altonji (1993) computes internal rates of return for a simple sequential model where agents are uncertain about future income flows and thus evaluation of expectations induces dropout behavior. Stange (2012) and ongoing research by Heckman-Urzua estimate models of educational choice where students are allowed to transfer and drop out, and therefore are able to produce estimates of their attached option value. This paper departs from these two in two important dimensions. First, both Stange (2012) and Heckman-Urzua assume that students learn about their own ability and nonpecuniary costs of education.[2]

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[2]This follows a large tradition in structural labor to use nonpecuniary costs of education to explain the low enrollment pattern observed in college in spite of the high returns to enrollment. See, for example, Keane

To make the model consistent with the data, their estimations provide that learning about nonpecuniary costs of education is important in explaining dropout behavior. In my paper, students learn only about their own ability and the model is still able to capture the main features of the data. The paper is also related to Pugatch (2012), which evaluates the option value of the re-enrollment option for students who drop out of school in South Africa, and to Arcidiacono (2004), which estimates the returns to the major choice in college. Moreover, my modeling assumption is consistent with Stinebrickner and Stinebrickner (2012), which found, using a panel study that learning about ability explains dropout behavior in deterrence of learning about nonpecuniary costs of education. Second, my model is highly tractable, allowing for a clean characterization of the optimal policy of students.

As briefly commented before, an important feature of the model is its tractability, which allows for a characterization of the optimal policy that governs enrollment, dropout, and transfer behavior. The model is calibrated using data from NLS-72 by assuming that observable measures of ability are correlated with the high school graduates initial beliefs. The model is consistent with the following facts: (1) among those initially enrolled in academic two-year colleges, more able agents are less likely to graduate, more likely to transfer, and less likely to dropout; (2) among those initially enrolled in four-year colleges, more able agents are more likely to graduate and less likely to dropout or transfer; (3) there is a higher concentration of high-ability students among transferees.

Using the calibrated version of the model, a counterfactual analysis of the value added of each option (dropout and transfer), the importance of risk aversion, the degree of substitutability across college types, and a measurement of the value of the slow arrival of information while in college are performed. To do this I analyze not only changes in the enrollment pattern, but also how the return to enrollment, relative to joining the workforce directly after high school graduation, is affected by each of these options.

The decomposition of returns to enrollment shows that the dropout option accounts for 31 percent of the full average return to enrolling in an academic two-year college while the transfer option accounts for 69 percent. No one would enroll in an academic two-year college if these options were not available. For four-year colleges, the dropout option accounts for 87 percent of the full average return to enrollment while the rest is explained by simple human capital accumulation that follows from enrollment until graduation. If students were to be risk neutral instead of risk averse, enrollment in academic two-year colleges would fall from nearly 16 percent to 10 percent, while enrollment in four-year colleges would rise from 27 percent to nearly 50 percent. The interaction of risk and option value proves to be an important force in postsecondary education.

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and Wolpin (1997).

The paper then turns to evaluate the degree of substitution between academic two-year colleges and four-year colleges, to find a high degree of substitutability. In accordance with this high degree of substitutability, the welfare effect of the availability of academic two-year colleges is at most moderate and is primarily driven by an increase in participation of nearly 7 percent. This result contrasts with the common wisdom that academic two-year colleges are a key part of the educational system as they train the students at the margin.

Finally, the paper explores how much returns to education are affected by the fact that, through the learning process, information reveals slowly to the student. Introducing an exam that fully reveals a student's ability level increases the annual return to the availability of postsecondary education by 6.3 percent, from a benchmark of 3.6 percent in the baseline economy.

2 Evidence

This section presents statistics on postsecondary educational patterns based on the NLS-72. The unit of analysis are high school graduates who join the workforce directly after leaving school (with no spells of postsecondary education) or join a postsecondary institution with no discontinuities in their educational spells. NLS-72 follows the educational histories of the senior class of 1972 up to 1980. A final wave in 1986 was performed to acquire long-run job market information.

The postsecondary educational system broadly consists of four-year colleges, academic two-year colleges, and vocational/technical school. Some institutions provide both academic and vocational programs; in this paper, each of these programs is understood as a separate entity. The goal of academic two-year colleges is to prepare students for transferring to four-year colleges, while vocational schools offer specialized education through terminal programs that require either two or three years to complete. In the model, vocational schools will not be present as the model will be a story of a stepping stone, which will be given by academic two-year colleges. For the calibration of the model (see Section 4), agents who enroll in a vocational school or program will be pooled with those who never pursued higher education. It will be clear in this section that individuals who enroll at a vocational school or program do not transfer to other types of institutions, and that the opposite is also true. Moreover, individuals who enroll in vocational schools or programs are sorted below those who enroll in the other types of institutions or programs.

Table 1 can be used to evaluate the dynamic pattern of postsecondary educational histories. Individuals are faced with an initial enrollment choice between four-year colleges, academic two-year colleges, vocational schools, or joining the workforce. The first spell

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of education can end in three different ways. First, a student can drop out and join the workforce. Second, a student can transfer to a different type of educational institution. Within-type transfers (e.g., four-year college to four-year college) are not understood here as transfers. Third, a student can graduate and join the workforce. A student that transfers holding a degree is counted as a transfer.

Table 1: Educational transitions

Type of school	Value
Vocational School (enrollment share: 0.09)	
Fraction that drops out	0.88
Fraction that graduates	0.06
Fraction that transfers to Ac. two-year c.	0.03
Fraction that transfers to four-year c.	0.03
Academic two-year colleges (enrollment share: 0.15)	
Fraction that drops out	0.59
Fraction that graduates	0.05
Fraction that transfers to Voc. school	0.04
Fraction that transfers to four-year c.	0.32
four-year colleges (enrollment share: 0.25)	
Fraction that drops out	0.41
Fraction that graduates	0.56
Fraction that transfers to Voc. school	0.03
Fraction that transfers to Ac. two-year c.	0.02

Only half of the sample pursues higher education directly after high school graduation (the total enrollment share is 0.49). Among them, nearly 20 percent enroll in vocational school, around 30 percent enroll in academic two-year colleges, and the rest enroll in four-year colleges. Transitions (i.e., dropout, graduation and transfer behavior) differ for students depending on their initial enrollment choice. Dropout rates are high in the three types of institutions but are higher in vocational schools and academic two-year colleges than in four-year colleges.[3] Transfer rates are important in academic two-year colleges: Approximately 32 percent of students who initially enroll in this type of institution eventually transfer to four-year colleges; only four-year colleges graduate a large percentage of their students.

It is also interesting to evaluate what happens to those students transferring from an academic two-year college to a four-year college. Fifty-six percent of them obtain a degree in a four-year college while the rest, i.e., 44 percent, eventually graduate.[4] A straightforward

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[3]Dropout rates at vocational schools are inflated since vocational schools have students who enroll in particular classes such as pottery, learning to use Excel, etc. Once they acquire the particular skill, these students leave the school and return to the workforce. These students don't get terminal degrees, and so they are recorded as dropouts.

[4]The proportion of students who transfer more than once is negligible and therefore this analysis is

comparison with the results in Table 1 shows that these values are similar for those initially enrolled at four-year colleges. This fact favors the idea that the initial enrollment choice does not hinder the probability of graduation at four-year colleges.[5]

Low enrollment and high attrition rates can be associated with the risk (possibly due to heterogeneity in returns) and costs attached to education. Costs include forgone earnings (income stream that a student "loses" by attending school) and direct costs of education that include tuition, fees, and housing. For example, because four-year colleges cost twice as much as academic two-year colleges, students might find it optimal to enroll in academic two-year colleges.[6]

Students who enroll in two-year colleges have observable measures of ability that lie between those of high school graduates who join the workforce directly and those of students who enroll in four-year colleges as noted by Grubb (1993) and Kane and Rouse (1999). Table 2 presents summary statistics for measures of ability correlated with enrollment decisions tabulated by initial enrollment choice, extending the analysis of Grubb (1993) and Kane and Rouse (1999) by splitting two-year colleges between vocational schools and academic two-year colleges. two-From left to right, the table shows that there is evidence of an ordered enrollment choice. For example, see the rank in the senior year of high school. The rank decreases monotonically with the enrollment choice.

2.1   Academic two-year colleges as a stepping stone

Ladders have been associated with the growth of skill as discussed in Jovanovic and Nyarko (1997). Lower steps of the ladder are characterized as stepping stones because they provide a less risky environment to learn compared with higher steps. As agents acquire the necessary skills, they move upward in the ladder. The process that starts after high school graduation and culminates with a four-year college graduation is a ladder with two steps. The first step, the stepping stone, is an academic two-year college. The second step is a four-year college. But there are also ways of going down. First, a student at four-year college can decide to transfer to an academic two-year college. Second, a student can decide to drop out.

In contrast with the characterization of the ladder discussed above where agents move on once they acquire the necessary skills, this ladder also presents features of bandit models as those discussed in Johnson (1978), Miller (1984), and Jovanovic and Nyarko (1997). Models of skill accumulation imply that agents should enroll first in the lower step of the ladder, as it

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reduced to only account for students who transfer at most once.

[5]Similar patterns are present in NELS:88 that correspond to a cohort that graduates from high school in 1992.

[6]See Table 15 for the tuition cost of the different types of colleges.

Table 2: Summary statistics for ability measures conditional on enrollment

	Work	V.S.	Ac. two-year C.	four-year C.
Male	0.49	0.4	0.54	0.52
	(0.5)	(0.5)	(0.5)	(0.5)
Black	0.11	0.13	0.08	0.10
	(0.3)	(0.3)	(0.3)	(0.3)
Socioeconomic status				
Low	0.42	0.29	0.19	0.17
	(0.5)	(0.5)	(0.4)	(0.4)
Medium	0.51	0.58	0.55	0.41
	(0.5)	(0.5)	(0.5)	(0.5)
High	0.08	0.12	0.25	0.43
	(0.27)	(0.33)	(0.43)	(0.49)
Education of father				
< than HS completion	0.52	0.39	0.29	0.21
	(0.5)	(0.5)	(0.4)	(0.4)
HS completion	0.32	0.40	0.34	0.29
	(0.5)	(0.5)	(0.5)	(0.4)
four-year college drop out	0.11	0.14	0.21	0.19
	(0.3)	(0.3)	(0.4)	(0.4)
four-year college graduate	0.05	0.06	0.15	0.31
	(0.2)	(0.2)	(0.4)	(0.5)
Rank	0.50	0.43	0.39	0.27
	(0.3)	(0.3)	(0.3)	(0.2)

Rank=rank in high school class. Socio-Status: Socioeconomic status of family at moment of high school graduation.

provides a less risky environment for learning and experimentation. Bandit models suggest that students should enroll in the harder step -the last step- since the learning technology provides more information about innate ability.

3   Model

The economy is populated by individuals who, upon high school graduation, decide whether to join the labor force or pursue a degree at a postsecondary educational institution. At $t = 0$ agents graduate from high school endowed with asset level a_0. Agents differ in their ability to accumulate human capital at college, which can either be low or high. Let $\mu \in \{0, 1\}$ denote the ability level, with $\mu = 0$ denoting low ability. The ability level μ is not observable by the agent. Instead, a high school graduate, at the moment where the decision to enroll in college is made, inherits a signal about her true type denoted by $p_0 \in [0, 1]$, where $p_0 = Pr(\mu = 1)$.

At any period in time an agent can either be working, studying at a four-year college

(or C) or at an academic two-year college (or A). Let $i \in I = \{A, C\}$ denote the type of institution. The cost of education per period of schooling is denoted by τ^i, with $\tau^C > \tau^A$. A student graduates from institution i after accumulating T^i credits, with $T^C > T^A$. The evolution of credits is closely tied to signals that arrive during an agent's tenure as student that are labeled by η.

Work is assumed to be an absorbing state with constant wage function $h(GS, i, \mu)$, where the first argument accounts for the graduation status of the agent, the second for the type of institution the agent graduated from (highest degree), and the third for her true ability level. Further, the wage function $h(GS, i, \mu)$ is specified as follows:

$$h(GS, i, \mu) = \begin{cases} h^w & \text{if GS=0,} \\ h^i(\mu) & \text{if GS=1,} \end{cases}$$

with $h^i(1) \geq h^i(0) > h^w$ for all i and $h^C(\mu) \geq h^A(\mu) > h^w$ for all μ. That is, for any ability level, graduation from a four-year college implies higher wage profiles than graduation from an academic two-year college and, for any institution i, wage profiles of graduates are increasing in their ability level.

The evolution of the asset level a is given by

$$a_{t+1} = \begin{cases} (1 + r)a_t - \tau^i - c_t & \text{if enrolled at } i, \\ (1 + r)a_t + h(GS, i, \mu) - c_t & \text{if working,} \end{cases}$$

where no borrowing constraints are present.[7]

During their tenure as students, agents receive signals in the form of exams labeled as η. Let η denote the signal with PDF given by $f_i(\eta \mid \mu)$.

Assumption 1 *The ratio of densities $\frac{f_i(\eta|\mu=1)}{f_i(\eta|\mu=0)}$ satisfies the Monotone Likelihood Ratio Property (MLRP). That is, for any $\eta_1 > \eta_0$, $\frac{f_i(\eta|\mu=1)}{f_i(\eta|\mu=0)}$ is well defined (full support) and $\frac{f_i(\eta_1|\mu=1)}{f_i(\eta_1|\mu=0)} \geq \frac{f_i(\eta_0|\mu=1)}{f_i(\eta_0|\mu=0)}$.*

The assumption states that high ability students are prone to receiving better signals than low ability students.

The evolution of credits is a function of current signal η and amount of accumulated credits s,

$$s' = s + \tilde{\Omega}(\eta, s)$$

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[7]The assumption of no borrowing constraints is consistent with several empirical studies. See, for example, Cameron and Taber (2004) and Stinebrickner and Stinebrickner (2008).

with

$$\tilde{\Omega}(\eta, s) = \begin{cases} \Omega(\eta) & \text{if } s < T^i, \\ 0 & \text{if } s \geq T^i. \end{cases} \tag{1}$$

This equation states that, while the amount of current credits is less than the necessary amount for graduation, accumulation of credits is only a function of the received signal η, which is positively correlated with a student's ability level through Assumption 1. Moreover, it is assumed that $\Omega(\eta_1) \geq \Omega(\eta_2)$ for any $\eta_1 > \eta_2$. This assumption guarantees that, conditional on a level of previously accumulated credits, a student receiving a high grade accumulates at least as many credits as a student receiving a low grade.

Students are allowed to transfer across different types of schools and can carry with them part of the credits earned in the current institution. Let θ^i denote the operator that maps credits s in institution i to credits s in institution j, $j \neq i$. Formally, $\theta^i(s) : \mathbb{R}^+ \times I \to [0, T^j]$, where it is assumed that $\theta^i(s)$ is nondecreasing in credits s.

A high school graduate, endowed with her initial belief p_0 and initial asset level a_0, chooses her consumption stream $\{c_t : t \geq 0\}$ and whether to enroll, drop out, or transfer in A or C, in order to maximize her time-separable expected discounted lifetime utility derived from consumption,

$$\mathbb{E}\left\{\sum_0^\infty \left(\frac{1}{1+r}\right)^t \left(\frac{e^{-\gamma c_t} - 1}{-\gamma}\right)\bigg| \mathcal{F}_0\right\},$$

where $\mathcal{F}_0 = \{p_0, a_0\}$ and γ is the coefficient of Constant Absolute Risk Aversion (CARA), and where I assume that $c_t \geq 0$ for all t. The consumption profile being bounded below is a standard technical assumption. The utility function specification choice is a subtle one. A known result is that with exponential utility functions the ordering of risky projects is independent of financial wealth; this simplifies the model solution considerably as the optimal policy of students will be independent of their current wealth. Another way wealth can affect the optimal policy of students is by introducing credit constraints. In my model students do not face a credit constraint motivated by the evidence in Heckman et al. (1998), Cameron and Heckman (2001), Keane and Wolpin (2001), Cameron and Taber (2004), Foley et al. (2009), and Nielsen et al. (2010) which find no (or little) effect of credit constraints in shaping postsecondary enrollment and Stinebrickner and Stinebrickner (2008), which finds no effect of credit constraints in explaining drop out behavior.[8] In the end, the combination of exponential utility function and no credit constraints simplifies the model considerably.

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[8] Furthermore, Belley and Lochner (2007) and Lochner and Monge-Naranjo (2011) find little evidence of credit constraints affecting enrollment in NLS79 and strong evidence in NLS97, which they use to suggest a tightening of the credit constraint. My data set, constructed from NLS72, is prior to the NLS79 so my assumption of no credit constraints is also consistent with these papers.

Let $V_i(a, s, p)$ denote the value for a student currently enrolled in institution type i with asset level a, amount of credits accumulated s, and belief p. Also, let $W(a; h(GS, i, \mu)) = -\frac{1+r}{\gamma r} e^{-\gamma(ra+h(GS,i,\mu))} + \frac{1+r}{\gamma r}$ denote the value for a worker with asset level a and wage profile $h(GS, i, \mu)$.[9] Finally, let $\Lambda(a_0, p_0)$ denote the value for a high school graduate with asset level a_0 and belief p_0. This value equals,

$$\Lambda(a_0, p_0) = \max \left(W(a_0; h^w), V_A(a_0, 0, p_0), V_C(a_0, 0, p_0) \right) \ ,$$

as the agent chooses whether to join the workforce or pursue higher education (either in academic two-year colleges or four-year colleges) by comparing the value of each alternative.

Let $R_i(p)$ denote return to enrollment at institution i relative to joining the workforce,

$$R_i(p) \equiv \frac{\Delta_i(p)}{\frac{1+r}{r} h^w} \ ,$$

where $\Delta_i(p)$, which solves $V_i(a - \Delta_i(p), 0, p) = W(a; h^w)$, describes the payoff of the education option and $\frac{1+r}{r} h^w$ its price (i.e., the opportunity cost of becoming a student).

3.1 The problem of a student

The student's beliefs are updated by the stream of information that arrives through the signal η. Let $p' = b(\eta; p)$ denote the posterior that depends on the prior p and the signal η. For a given institution i, Bayes' rule is

$$b(\eta; p) = \frac{1}{1 + \frac{f_i(\eta|\mu=0)}{f_i(\eta|\mu=1)} \frac{1-p}{p}} \ .$$

The evaluation of expectations about future income flows depends on the likelihood of the signals. Any new signal can be produced by either $f_i(\eta|\mu = 1)$ or $f_i(\eta|\mu = 0)$ so that expectations about the governing CDF have to be accounted for. Define

$$H_i(\eta, p) \equiv p F_i(\eta|\mu = 1) + (1 - p) F_i(\eta|\mu = 0) \ ,$$

as the CDF that accounts for this uncertainty.

The problem faced by a student in institution type i can be written as

$$V_i(a, s, p) = \max_{c \geq 0, a'} \frac{e^{-\gamma c} - 1}{-\gamma} + \frac{1}{1+r} \int_\eta \tilde{V}_i(a', s', p') H_i(d\eta, p) \ , \tag{2}$$

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[9]The value function for a worker is the solution to the problem $W(a; h) = \max_{c,a'} \frac{e^{-\gamma c} - 1}{-\gamma} + \frac{1}{1+r} W(a'; h)$ where $a' = (1 + r)a + h - c$.
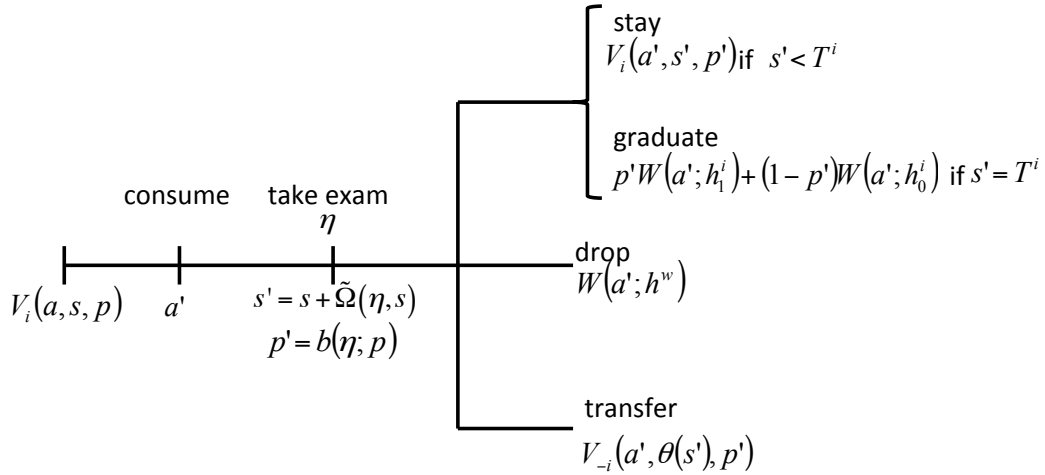
with

$$\begin{cases} a' = (1+r)a - \tau^i - c \ , \\ s' = s + \tilde{\Omega}(\eta, s) \ , \\ p' = b(\eta; p) \ . \end{cases}$$

In any given period, a student who accumulated s' credits faces alternatives. If $s' < T^i$, she can decide to stay in the current institution, transfer, or drop out. If $s' = T^i$, the graduation option is available and thus the options are reduced to graduation or transfer as dropping out is dominated. Let $\mathbb{I} = 1$ if $s' < T^i$ and $= 0$ otherwise. $\tilde{V}_i(a', s', p')$ is equal to

$$\max \left\{ W(a'; h^w), \mathbb{I} V_i(a', s', p') + (1 - \mathbb{I})[p' W(a'; h^i(1)) + (1 - p') W(a'; h^i(0))], V_{-i}(a', \theta^i(s'), p') \right\} \ . \quad (3)$$

The timing of the problem, presented in Figure 1, is as follows. For a given institutional choice i at the beginning of a period, a student chooses her consumption and level of assets for the next period given her expectations about future income streams. Next, she receives the signal η, producing Bayesian updating of her belief, $p' = b(\eta, p)$, and the amount of credits accumulated for next period s'. When the new period begins, the student chooses whether to drop out or remain a student and whether to transfer to another institution.

Figure 1: Timeline

At the beginning of the period, $V_i(a, s, p)$ denotes the value of enrollment at institution type i with wealth level a, accumulated credits s and prior p. The student chooses her consumption level producing the accumulated wealth for next period a'. Later, she takes an exam, receiving grade η. This produces a re-evaluation of beliefs $p' = b(\eta; p)$ and accumulation of credits $s' = s + \tilde{\Omega}(\eta, s)$. The student then chooses between staying in institution type i (or graduating if enough credits where accumulated), transferring or dropping out.

The solution to the problem of the student is

$$V_i(a, s, p) = -\frac{1+r}{\gamma r} e^{-\gamma(ra + v_i(s,p))} + \frac{1+r}{\gamma r} \qquad (4)$$

with

$$-\gamma v_i(s, p) = \tfrac{1}{1+r} \, \ln\left[\int_\eta - \max\left\{ \begin{array}{c} -e^{-\gamma(h^w - r\tau^i)}, -e^{-\gamma(v_{-i}(\theta^i(s'), p') - r\tau^i)}, \\ -\left(\mathbb{I}\left(p' e^{-\gamma(h^i(1) - r\tau^i)} + (1-p') e^{-\gamma(h^i(0) - r\tau^i)} \right) + (1-\mathbb{I}) e^{-\gamma(v_i(s', p') - r\tau^i)} \right) \end{array} \right\} H_i(d\eta, p) \right]$$

$$(5)$$

being the utility flow of being enrolled in institution type i, with accumulated credits s and current belief p. Also, to ease on notation, $p' = b(\eta, p)$ and $s' = s + \Omega(\eta)$. The details of the derivation can be found in Appendix C.

Notice that the solution to the value function $V_i(a, s, p)$ is such that differences in the income level simply shift up and down the value for the student. This follows from the absence of financial constraints and the assumption of Constant Absolute Risk Aversion. A similar argument implies that students find it optimal to consume the flow value of her wealth, ra, which results from a standard permanent income argument. The term $e^{-\gamma v_i(s,p)}$ accounts for the discounted expected value of all possible states the student can face tomorrow net of the current flow cost of enrollment, $r\tau^i$. This can be seen in equation (5), which shows that the flow value today of being enrolled in institution type i with credits s and belief p, $v_i(s, p)$, equals the certainty equivalent value of all possible flow values faced tomorrow by the student net of the enrollment cost.

I now impose some restrictions on the model parameters to analyze a particular pattern of education: Students with high beliefs enroll in four-year colleges, those with average beliefs enroll in academic two-year colleges, while those with low beliefs join the workforce directly. Later on, on the quantitative exploration of the model, none of these restrictions are imposed. The next assumption is useful to describe the enrollment pattern.

Assumption 2 *The primitives of the model are such that (i) $v_C(0,0) \leq v_A(0,0) \leq h^w$, and (ii) $v_C(0,1) \geq v_A(0,1) \geq h^w$.*

The assumption states that high school graduates with low ability to accumulate human capital are better off joining the workforce and, in the eventuality of enrollment, they are better off in academic two-year colleges than in four-year colleges. The opposite idea applies for high ability agents. They are better off pursuing higher education, and the best enrollment choices for them are four-year colleges. Satisfying the assumption depends crucially on four objects. First, it depends on the wages expected to be received upon graduation. High wages helps satisfy (ii) but makes (i) harder to be satisfied. Second, it also depends on the

wage differential between high skilled and low skilled agents upon graduation. Given high expected wages upon graduation, high dispersion in wages as a function of skill implies high incentives for high ability agents to enroll and low incentives for low skill agents. Third, it depends on the learning technology. If students would learn faster in academic two-year colleges than in four-year colleges even students with high beliefs would find it optimal to enroll first in an academic two-year college. Fourth, it also depends on the tuition cost.

Assumption 2 has an interesting interpretation. Under this assumption, the existence of academic two-year colleges in this model is driven by the learning mechanism and the option value attached to it. If students were to be certain about their skill level, no one would enroll in an academic two-year college. An implication of the assumption is that high school graduates with low beliefs join the workforce, those with average beliefs enroll in an academic two-year college, and those with high beliefs enroll in a four-year college. As students accumulate credits and their beliefs are updated, some of them will find it optimal to drop out, others will find it optimal to transfer, and others will find it optimal to stay in their current institution type until graduation. Because of the higher returns upon graduation, students with high beliefs will naturally sort into four-year colleges eventually. Students with low beliefs who accumulated few credits will not find it optimal to remain in school and will drop out. Characterizing the optimal dropout and transfer policy depends on the specifics of the model and its parameterization, and thus I will discuss this later on once the model is calibrated to its empirical counterpart. Nevertheless, notice that the closer to graduation a student is, i.e., the higher the amount of accumulated credits, the lower should be her incentives to dropout or transfer. This happens because the terminal payoff is closer and so its importance increases in terms of the continuation value of the student. This means the dropout and transfer bands should widen out as credits accumulate, making students less willing to dropout and transfer as they advance in their educational careers.

4   Calibration

The model explores the dynamic interaction of academic two-year colleges and four-year colleges. The evidence obtained from NLS-72 shows that vocational school (excluded from the model analysis) can be merged with the workforce as little interaction occurs between vocational school and other types of institutions (see Table 1) and the sorting in initial enrollment, presented in Table 11 and Table 12, places vocational school below academic two-year colleges. Version B in all of the tables accounts for the case where work and vocational school are merged. Further, as in the model, column 2 on the tables accounts for the cases where increases in wages only occur upon graduation.

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The operator that maps credits s in institution i to credits that remain after transferring $\theta^i(s)$ is simplified to be of the multiplicative form,

$$\theta^i(s) = \begin{cases} \theta^i s & \text{if } \theta^i s < T^{-i} \text{ ,} \\ T^{-i} & \text{if } \theta^i s \geq T^{-i} \text{ .} \end{cases}$$

Unfortunately, information on credits is not available in the data set, complicating the calibration of $\theta^i(s)$. The procedure followed in the paper for the calibration of this function is that one of direct imputation, following some observations on the timing of transferred students and their time to graduate. Students who initially enroll in an academic two-year college and transfer later on to a four-year college do so, on average, after the completion of the first year of education (see Figure 9).[10] Then, θ^A is chosen to be $\frac{1}{2}$. The evidence for students who transfer from four-year colleges to academic two-year colleges is less revealing as the fraction of students who transfer is very low (see Table 1). Figure 9 shows that students transfer during their first year of education. Among academic two-year college graduates, those that started their educational career at four-year colleges spend more time in school prior to graduation (4.5 years vs. 3.84 years). Then, $\theta^C = 0$.

The signal η plays two different roles in the model. First, it updates beliefs p as the signal conveys information regarding the likelihood of the ability level of the student. Under this definition, the signal η accounts for grades in exams, subjects, problem sets, overall experience as a student, etc. The second role of the signal η is to generate accumulation of credits through the function $\Omega_i(\eta)$, which suggests that the signal is closely tied to grades in subjects. To simplify the model, think that the signal η is the mean of the grades in a quarter obtained by a student. The set of possible values of η is simply the set of possible grades. For simplicity, assume three possible grades: $\{F, N, E\}$. That is, a student can fail, get a neutral grade, or excel in a particular exam. Let $q_\mu^i(\eta)$ denote the probability of each event. Further, assume that $q_1^A(F) = q_0^C(E) = 0$. That is, high ability students never fail an exam at academic two-year colleges and low-ability students never excel at four-year colleges. The function $\Omega(\eta)$, that maps grade η into credits s is chosen to be as follows:

$$\Omega(\eta) = \begin{cases} 0 & \text{if } \eta = F \text{ ,} \\ 1 & \text{if } \eta = N \text{ ,} \\ 1 & \text{if } \eta = E \text{ ,} \end{cases}$$

so that only students who fail in an exam do not accumulate credits. Overall, the assumptions

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[10]Students transfer before obtaining a degree or completing the course-work at academic two-year colleges. Only 12.5% of students who transfer from academic two-year colleges to four-year colleges in the NLS-72 sample hold a degree and usually transfer around one year later than those not holding a degree.

on the different signals and accumulation of credits imply that, independently of the college type, the grade E is weakly better than the grade N, which is also weakly better than the grade F.

The time period is chosen to be a quarter, therefore $T^A = 8$ and $T^C = 16$ (a student needs to accumulate T^i quarters of accumulated credits at institution i to graduate). The risk-free interest rate r is set to be 0.45 percent which implies a yearly interest rate of 1.8 percent and a yearly discount factor of 0.982. All the monetary values in the model are measured in logs and further standarized by the wage of agents with no degrees h^w, so that $h^w = 1$.[11] Academic two-year colleges are located in every city and town while four-year colleges are scarce. This way, the cost of education includes housing for four-year colleges and does not include housing for academic two-year colleges because students there can live with their parents. The standarized cost of education is then $\tau^A = 0.1152$ for academic two-year colleges and $\tau^C = 0.3205$ for four-year colleges (see Table 15). The risk aversion parameter, γ, is chosen to be equal to 8.[12]

The remaining parameters, $q_\mu^i(\eta)$ and $h^i(\mu)$ are calibrated using a two-stage method. In the first stage an Ordered Probit regression on the initial enrollment choice is used to produce estimates of the initial belief p_0. In the second stage, conditional on the distribution of beliefs obtained in the first stage, average enrollment, transition choices and wage outcomes are constructed and then matched to a set of moments through the means of a calibration exercise.

Let

$$p_0 = \left(1 + e^{-(X'\beta+\varepsilon)}\right)^{-1}, \ \varepsilon \sim N(0,1) \tag{6}$$

where X is a vector that includes all the observable characteristics of high school graduates that are correlated with the ability level of the agent and β is the vector of factor loadings, identified by an Ordered Probit regression for the initial choice of agents. The estimates for β are presented in Table 11. It is assumed that the value of the random variable ε is observed by the agent but unobserved by the econometrician. Still, by observing the initial enrollment choice of the agent, the econometrician can compute bounds to the value of ε consistent with the enrollment choice of the agent. This is useful in computing the distribution of beliefs, which are needed to calibrate the model through indirect inference, that are consistent with the vector of observable characteristics X and initial enrollment choice of each agent.

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[11]The mean wage in 1985, in 1984 dollars, for agents with no degrees was \$17740.63.

[12] The risk aversion parameter γ is hard to identify and the literature didn't spend much time estimating risk aversion parameters using CARA utility functions (for natural reasons). There is a whole string of literature in asset pricing that argues that the CRRA risk aversion parameter σ lies between 4 and 10. Using the definition of relative risk aversion it is possible to relate σ and γ, $\gamma c = \sigma$. Here, the consumption level c has a lower bound given by $ra + h^w \geq 1$ so $\gamma < 10$.

I now turn to explain how I compute the initial distribution of beliefs. Students with high beliefs join four-year colleges, with average beliefs join academic two-year colleges, and with low beliefs join the workforce. The thresholds are those of the optimal policy considered above for $s = 0$ as agents who graduate from high school didn't acquire any credits yet. Further, monotonicity of the belief p_0 as a function of $X'\beta + \varepsilon$ implies that β can be estimated by an Ordered Probit regression on the initial choice (Table 11 - Version B) and then bounds for ε can be computed using $X'\hat{\beta}$ and the enrollment choice of the agent.[13] Notice that the particular functional form chosen for the belief p_0 is irrelevant for the estimation of β and the bounds for the unobserved variable ε, as the only requirement is that the function mapping characteristics to the space of beliefs, i.e., $[0, 1]$ is monotonic and, therefore, invertible.

Conditional indirect inference is used to calibrate the remaining ten parameters (the six learning parameters and the four wages). The calibration process attempts to match the average educational histories of the individuals in the sample, each of them characterized by an educational history, a long-run wage, and a distribution of possible initial beliefs p_0 that were produced using an Ordered Probit regression as previously discussed. Because for each individual there is a distribution of possible initial beliefs which, ulteriorly, define the individual's educational history, I compute the average educational history for each individual, which allows me then to compare it with the unique educational history observed for this individual in the data. In the end, the calibration procedure computes the enrollment choice, the long-run wage, and the probability of dropout, graduation, and transfer for each agent. It then computes averages across students, and contrast these averages with their empirical counterpart. Because the educational histories greatly differ across students (see Section 2), and in the sample I have 3,462 individuals, the model is over-identified.

Before presenting and discussing the estimation results, it is important to point out that neither Assumption 1 nor Assumption 2 are imposed in the estimation process. However, both Assumptions are satisfied by the calibrated model.

Table 3 shows the wage differential to graduation at both academic two- and four-year colleges relative to joining the workforce without holding a degree. These wages in the data are measured in 1985, around eight years after the average graduation time, so that the estimated wages account for part of the premium due to experience in the labor market. For high-ability graduates, the differential is almost three times larger in four-year colleges than in academic two-year colleges, suggesting that students with high expectations currently enrolled in academic two-year colleges should be eager to transfer to four-year colleges. As students can transfer credits, many enroll in academic two-year colleges as individuals are

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[13]Therefore, for each individual on sample I have a distribution of possible beliefs p_0.

risk-averse and therefore care about the high volatility of wages.

Table 3: Calibrated wage differentials

		estimate
Academic two-year colleges		
low ability	$\frac{h^A(0)-h^w}{h^w}$	0.03
high ability	$\frac{h^A(1)-h^w}{h^w}$	0.08
Four-year colleges		
low ability	$\frac{h^C(0)-h^w}{h^w}$	0.04
high ability	$\frac{h^C(1)-h^w}{h^w}$	0.22

Table 4 shows the estimated learning parameters. Trivially, by assumption, failing an exam at academic two-year colleges signals low ability while excelling at four-year colleges signals high ability, as the probability of failing an exam at academic two-year colleges was set to be zero for high-ability students and the probability of excelling in an exam at four-year colleges was set to zero for low-ability students. The likelihood of obtaining a neutral or an excellent at academic two-year colleges is higher for high-ability students and thus receiving these signals improves the expectations about the innate ability. A similar idea happens for a neutral grade at four-year colleges while a fail lowers the expectations. Overall, notice that exams at four-year colleges produce more information than academic two-year colleges: Academic two-year colleges are easy, and thus provide information in the left tail, while four-year colleges are hard and thus provide information in both the left and right tails.

Table 5 aggregates the educational histories implied by the model and compares them with the data counterpart. The model does a good job fitting most of these aggregate moments but underestimates the number of dropouts in academic two-year colleges.

Table 4: Calibrated learning parameters

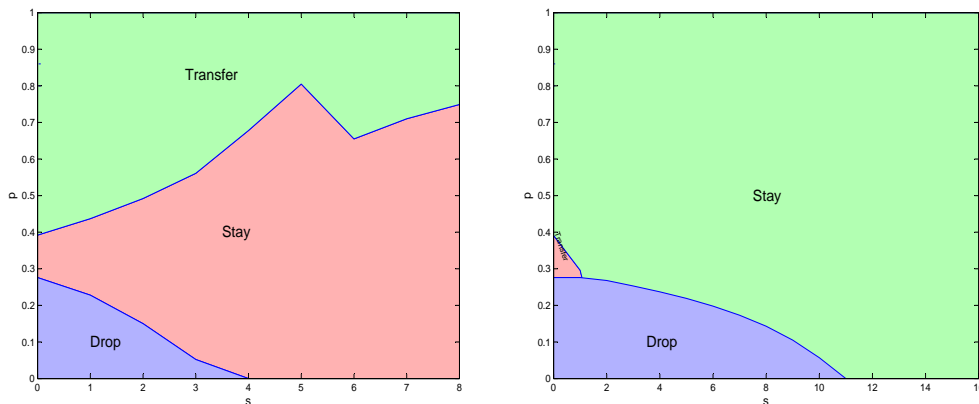|	Academic two-year colleges		Four-year colleges	
	low ability $\mu = 0$	high ability $\mu = 1$	low ability $\mu = 0$	high ability $\mu = 1$
Fail	0.23	0[*]	0.2275	0.01
Neutral	0.695	0.87	0.7725	0.81
Excel	0.075	0.13	0[*]	0.18

[*] by assumption.

The evolution of thresholds as a function of accumulated credits are presented in Figure 2. In a broad way, as previously discussed, the inaction region in both academic two- and four-year colleges increases with credits. The nonmonotonicity of the inaction region is a result of signals being discrete, which implies that the amount of credits s can only take a finite amount of values.

Table 5: Average value of educational histories in data and model

	Data	Model
% of High-school graduates who		
join Workforce	59.4	56.9
enroll in A	15.2	15.69
% of those initially enrolled in A that		
drop at A (1st spell)	63.2	48.63
transfer from A to C (1st spell)	32.2	43.79
% of those initially enrolled in C that		
drop at C (1st spell)	40.9	42.1
graduate at C (1st spell)	57.1	57.79
Mean Wage Differential for		
graduates from A	0.045	0.045
graduates from C	0.212	0.148
Moments to Discipline Priors		
FOCs from Ordered Probit (for β)		YES
distribution of X		YES

Figure 2: Regions at academic two-year and four-year colleges

Left panel: academic two-year colleges. Right panel: four-year colleges. The dropout and transfer thresholds at both academic two- and four-year colleges are as a function of amount of accumulated credits s.

To evaluate the fit of the model in Appendix D I compute the distribution of dropouts, transferred students, and graduation rates for both academic two- and four-year colleges implied by the parameterized model and contrast it with its empirical counterpart. I find that the transition patterns implied by the model resemble those in the data.

5 Risk aversion and Option Value

High dropout and transfer rates are features commonly associated with risk and thus the availability of transfer and dropout options should be highly valued by agents as they provide lower bounds to the risk of the investment. In terms of risk, keeping the primitives unaltered, the model is solved again letting γ tend to zero. Comparing the baseline model (i.e., $\gamma = 8$) with the risk-neutral case provides insights regarding the interaction of risk with the optimal policy and returns in this economy. A similar strategy is followed to evaluate the size of the option value. Still keeping the primitives unaltered, the model is solved two more times. The first time discards the transfer option, and the second time eliminates both the transfer and dropout options. The value-added of each option is then evaluated using a decomposition of returns.
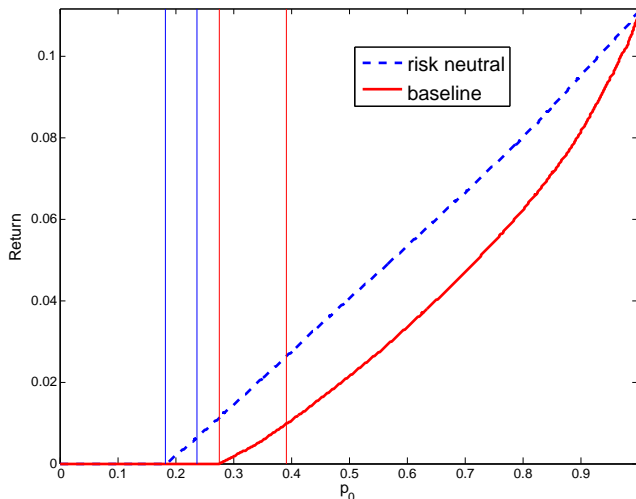
5.1 Risk aversion

For a high school graduate with any given belief p_0, Figure 3 presents the returns to enrollment for the baseline model, where $\gamma = 8$, and the model under risk neutrality, where $\gamma \to 0$. When risk aversion decreases, the enrollment thresholds shift to the left as the risk implied by education is discounted less heavily by agents. The fact that the shift to the left is stronger in the threshold between four-year colleges and academic two-year colleges than for the one between academic two-year colleges and work is not casual: Enrollment at four-year colleges is more risky than enrollment at academic two-year colleges (simple comparison of the ratio of wages). It follows that a decrease in risk aversion has a stronger effect on four-year colleges than on academic two-year colleges. Figure 3 also shows that risk aversion hinders the returns to education in an important way, and the effect is stronger the more uncertain the belief is.

Table 6 presents the mean return for the cross-section of agents that initially enroll at either academic two-year colleges and four-year colleges for both the baseline and the risk-neutral models using the estimated distribution of priors for the NLS-72 data. Lower risk aversion implies higher returns unambiguously for every prior p_0 but decreases measured returns in academic two-year colleges through the compositional change in the enrollment pattern.

Table 6: Risk aversion and average measured returns

	ac. two-year c.	four-year c.
Baseline; $\gamma = 8$	0.4 %	3.1 %
Risk neutral; $\gamma \to 0$	0.3 %	3.4 %

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Figure 3: Risk aversion and individual returns

The vertical lines define the indifference belief for enrollment between work and academic two-year colleges and between academic two-year colleges and four-year colleges.

Lower risk aversion not only implies higher returns for every prior but also affects enrollment decisions (this can be seen in Figure 3 where the vertical dotted lines denote the enrollment thresholds). Table 7 computes the distribution of initial enrollment for both cases. As expected, under risk neutrality total enrollment increases by nearly 40 percent and enrollment in four-year colleges -where risk matters the most as the wedge in wages and cost of education are higher and time until graduation longer- by around 81 percent. Finally, the mass of students still enrolling in academic two-year colleges under risk neutrality highlights the importance of the learning channel as a feature of academic two-year colleges. Risk affects the enrollment distribution both at the extensive and intensive level. At the extensive level, lower risk aversion increases total enrollment. At the intensive level, lower risk aversion affects the composition of enrollment as risk, tuition and wages upon graduation differ across types of institutions.

Table 7: Risk aversion and enrollment

|	workforce	ac. two-year c.	four-year c.
Baseline; $\gamma = 8$	56.9%	15.7%	27.4%
Risk neutral; $\gamma \to 0$	39.8%	10.4%	49.8%

5.2 How much option value?

The model is solved again to evaluate the size of the option value, this time reducing the amount of options. First, the transfer option is discarded and therefore the only available

alternative after the initial enrollment choice is to drop out. Second, the dropout option is discarded, thus no action, other than consumption decisions, is possible during tenure as student. Let $R_i^{E+D+T}(p_0)$, $R_i^{E+D}(p_0)$, and $R_i^E(p_0)$ denote the value of enrollment at institution i: with both options available to the agent; with only the dropout option available; and with no dropout or transfer options available. The following decomposition of returns applies,

$$1 = \frac{R_i^{E+D+T}(p_0) - R_i^{E+D}(p_0)}{R_i^{E+D+T}(p_0)} + \frac{R_i^{E+D}(p_0) - R_i^E(p_0)}{R_i^{E+D+T}(p_0)} + \frac{R_i^E(p_0)}{R_i^{E+D+T}(p_0)}$$

The first term in the right hand side is the value-added to total returns $R_i^{E+D+T}(p_0)$ by the transfer option, the second term provides the value-added by the dropout option, and the third term accounts for the value accrued through enrollment. Figure 4 shows that returns at four-year colleges are explained by the dropout option and by simply having the enrollment choice, in accordance with high graduation and dropout rates observed for four-year college students at NLS-72. Also, Figure 4 accounts for the importance of the transfer option in explaining returns to academic two-year college enrollment. As the figure shows, the value-added by the enrollment option that accounts for the simple human capital accumulation story has zero share of the returns.

Figure 4: Decomposition of Returns
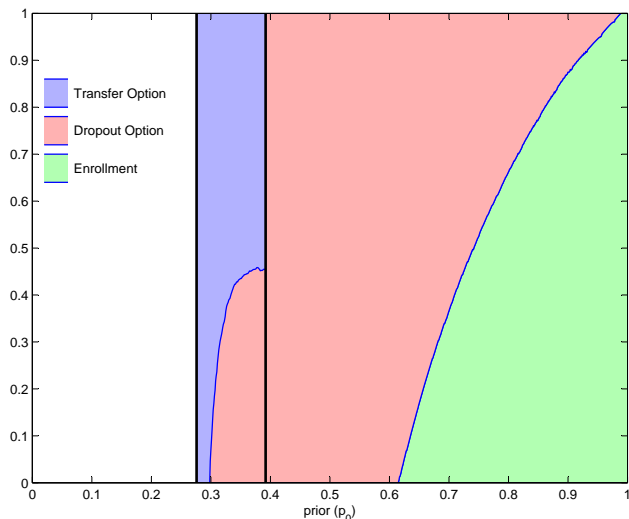
The vertical lines define the indifference belief for enrollment between work and academic two-year colleges and between academic two-year colleges and four-year colleges.

Table 8 produces the same decomposition, but this time for the mean return of the population distribution of beliefs p_0. The transfer option is very valuable in academic two-

year colleges, accounting for 69 percent of total value. The dropout option is valuable in both types of institutions but more so in four-year colleges.

Table 8: Average value added of each option

	ac. two-year c.	four-year c.
Enrollment option	0%	13%
Dropout option	31%	87%
Transfer option	69%	0%

6 How substitutable are colleges?

The main purpose of enrollment in academic two-year colleges is to learn in a less demanding and cheaper environment and eventually transfer to four-year colleges with a fraction of the already accumulated credits. Still, agents have the choice to enroll at four-year colleges. Given that a set of agents prefer to enroll in academic two-year colleges, the question that arises is how much value academic two-year colleges provide to these agents relative to initial enrollment at four-year colleges. That is, how close of a substitute are these institutions?

To explore the degree of substitutability across college types, in Table 9 I explore the effect on the enrollment distribution of changing the tuition at either academic two- or four-year colleges. To aid in the comparison, the first column reports statistics for the baseline economy. The second column of the table shows the effect of decreasing tuition in academic two-year colleges by 15 percent, while the third column shows the effect of decreasing tuition at four-year colleges, also by 15 percent. As the comparison of these and the baseline case show, even though total enrollment responds little to the changes in tuition, the distribution of enrollment changes considerably. When academic two-year colleges are cheaper, a large fraction of students that in the baseline economy enroll in four-year colleges now enroll in an academic two-year college. Because these students have high beliefs relative to the other students enrolling in an academic two-year college, the dropout rate at these colleges decreases while the transfer rate to four-year colleges increases, both in a considerable way. Furthermore, because those students that remain in a four-year college after the tuition change have high beliefs, they are less likely to drop out and more likely to graduate. Finally, notice that an opposite pattern is observed when four-year colleges are subsidized. This occurs because the pool of students enrolling in a four-year college have, on average, lower beliefs and therefore are less likely to graduate and more likely to drop out. That is, even though the decrease in the attendance cost at four-year colleges implies an increase in college participation, graduation rates at four-year colleges fall because of the compositional change in the beliefs of enrollees.

Table 9: The effect of tuition on the enrollment pattern

	baseline	$\tau_A * 0.85$	$\tau_C * 0.85$	$\tau^A \to \tau^C$
% of High-school graduates that				
join Workforce	56.9	56.1	55.9	59.7
enroll in A	15.7	26.6	8.1	0.15
enroll in C	27.4	17.3	36	40.15
% of those initially enrolled in A that				
drop at A (1st spell)	48.6	41.6	46.1	100
transfer from A to C (1st spell)	43.8	53.5	42.8	0
% of those initially enrolled in C that				
drop at C (1st spell)	42.1	34.8	46.6	53.6
graduate at C (1st spell)	57.8	65	53.3	46.4

The last column of Table 9 explores the effect of increasing tuition in academic two-year colleges up to the level at four-year colleges. The comparison of the results of this experiment with those of the baseline case are useful in underpinning the importance of academic two-year colleges. Students enroll in the latter because (i) academic two-year colleges are cheaper than four-year colleges, and (ii) the learning technology available at academic two-year colleges is better suited to marginal students as it reveals low ability at a faster pace (see Table 4). Because in this experiment (i) is shut down, the comparison of the experiment with the baseline case identifies the importance of (ii) on explaining enrollment at academic two-year colleges. Surprisingly, the table reveals that if tuition differences were to be eliminated, total participation in postsecondary education would decrease marginally, with the fraction of individuals joining the workforce directly after high school graduation increasing from 56.9 percent up to 59.7 percent. Also, after the tuition increase, most of the students enrolling in an academic two-year college now enroll in a four-year college. This shows that, once we control for tuition differences across colleges, the learning technology at academic two-year colleges is not sufficiently good so as to maintain a large enrollment. Away from a very small fraction of students who still enroll in an academic two-year college after the tuition change, those who do not enroll in a four-year college now join the workforce directly after high school graduation. These are the marginal students, those with low beliefs and therefore with low expectations of graduation. Because of their marginal status, they were also the most likely dropouts prior to the tuition hike, so their expected payoff of college education was very small. As a result, their welfare loss due to the change in tuition is minimal. Overall, this analysis shows that the main reason high school graduates find it optimal to enroll in an academic two-year college is because these colleges are cheap. The learning technology they provide, although better suited for discovering low ability, does not

provide enough value by itself to motivate students with low beliefs to enroll.

Given the apparent high degree of substitutability across college types, a natural question is whether academic two-year colleges are an important source of value for students. With this in mind, Figure 5 evaluates how the elimination of academic two-year colleges affects the return to enrollment for an agent with belief p_0. For students who enroll in four-year colleges in the baseline model, the availability of academic two-year colleges provides no value since the transfer option has no value for them. For students who enroll in academic two-year colleges in the baseline model, what happens when academic two-year colleges are eliminated is: (i) most students simply enroll in four-year colleges and the difference in value is very small, and (ii) the rest join the workforce.

Figure 5: The value added of academic two-year colleges
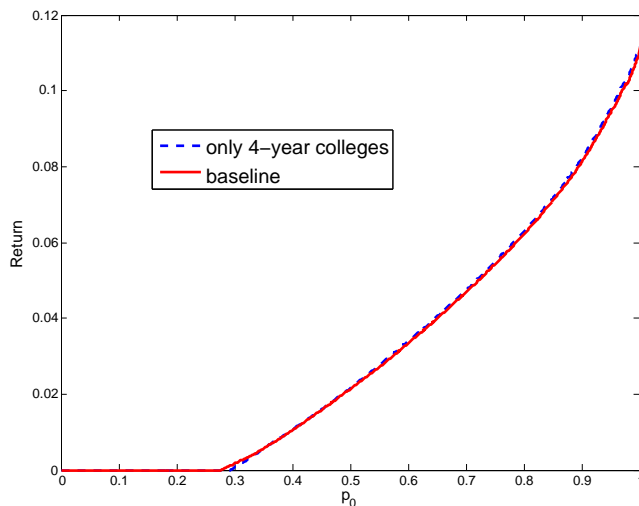
In order to characterize the welfare effect of academic two-year colleges, an analysis of population aggregates is needed. Specifically, it is important to evaluate how participation in postsecondary education and welfare change due to the availability of academic two-year colleges. Recall that $R_i(p)$ is a monotonous transformation of $v_i(0, p)$ that in turn is the certainty equivalent of enrollment at institution i abstracting from the wealth level a. As a result, $R_i(p)$ is a good measure of utility and therefore a comparison of returns provides a good approximation for welfare losses or gains. Table 10 compares total participation and measured returns for the baseline model where academic two-year colleges are available and the only enrollment options are four-year colleges. The availability of academic two-year colleges increases participation by nearly 7 percent and generates a drop in measured returns of around 6 percent due to the compositional change in total participation.

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Table 10: The effect of academic two-year colleges on enrollment and returns

	Total participation	Return
Academic two-year c. and four-year c.	43.11%	2.12%
Four-year c.	40.3%	2.25%

7 The value of the slow arrival of information

In the model students sort across college types because they have different beliefs regarding their ability level. While enrolled in a college, the arrival of information generates a reevaluation of beliefs and motivates dropout and transfer behavior. Notice that the process of reevaluation of beliefs is a slow process, something that follows from the inability of students to take more than one exam per period and signals not being perfectly revealing. The slow arrival of information hinders the returns to postsecondary education. It follows that a natural experiment is to measure how much returns are hindered by this slow-moving process. I investigate this by comparing the return to the availability of postsecondary education in the baseline economy with that one of a counterfactual economy where high school graduates obtain a fully revealing signal about their ability level prior to making the decision of college enrollment. The comparison of returns in these two economies allows me to gauge by how much the slow moving process of information acquisition hinders the value of education.

For concreteness, I now describe the counterfactual economy. Consider a high school graduate with belief p_0. With probability $1 - p_0$ the individual is of low-ability and therefore, given that in the calibration low ability students gain nothing from postsecondary participation, she finds it optimal to join the workforce. However, with probability p_0 she is of high ability and finds it optimal to enroll in a four-year college and remain enrolled until graduation. I refer to this economy as a "revealed-type" economy.

Figure 6 compares the returns to enrollment in both economies. As the figure shows, the fact that information arrives slowly does not affect the return of students with low beliefs (i.e., $p_0 = 0$) and high beliefs (i.e., $p_0 = 1$) as individuals with these levels of beliefs are fully informed about their types. As the initial belief departs from the limits of the support the two returns start to differ, with the return in the revealed-type economy always dominating. This occurs because enrollment is costly as the student has to forgo her work earnings in addition to paying tuition, and the only return to enrollment happens after graduation. Therefore, a signal that fully reveals the student's type reduces considerably the expected cost of education, as only students who are certain to graduate would pursue higher education. Moreover, notice that the largest differences in returns occur when $p_0 \approx 0.3$, the belief at which, in the baseline economy, individuals are indifferent between joining the workforce and

enrolling in an academic two-year college (this can be seen in the left panel of Figure 2). The excess return of the revealed-type economy is the largest here follows as the gains of education occur after graduating in a four-year college, something that a student enrolling in an academic two-year college with the lowest belief is very unlikely to attain.

Taking the difference between the returns for the two economies provides a measure of the excess return of the revealed-type economy with respect to the baseline economy for each initial belief p_0. Using the estimated distribution of beliefs for the NLS-72 data, I compute the average excess return. I find that a signal fully revealing the type of high school graduates increases returns quarterly by 1.5 percent, which is a large excess return given that the quarterly return in the baseline economy is 0.9 percent. The annualized excess of returns is 6.3 percent. The large gains in returns that follow from the availability of fully revealing signals show that the persistence of uncertainty that follow from the slow arrival of information hinders in a considerable way the returns to education. An obvious policy implication of this result is to provide better screening prior to enrollment. For example, by having an admission examination.

Figure 6: Value added by the availability of academic two-year colleges
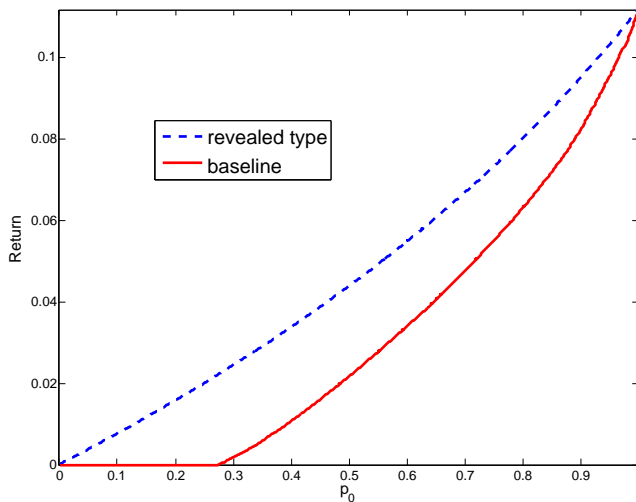
8   Concluding remarks

This paper proposes a simple, highly tractable, model of postsecondary education that incorporates together the work decision and academic two- and four-year colleges. In the model, students are allowed to drop out and transfer to more/less rewarding institutions as credits

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are accumulated and expectations are adjusted. The decision is not a trivial one. Higher rewards imply higher costs of education and more risk.

The parameterized version of the model is consistent with the data in terms of transitions, sorting, and returns. Therefore, it sheds some light on the role played by academic two-year colleges. These institutions act as a stepping stone toward more rewarding, and demanding, environments, namely, four-year colleges.

In line with the literature of investment under uncertainty, the drop out and transfer options, and the availability of academic two-year colleges, have an attached option value. A novel feature of this paper is that it presents structural estimates of the different option values. The drop out option explains a large share of ex-ante returns on both academic two- and four-year colleges. In fact, very few students would pursue higher education if dropping out is not allowed. The transfer option is not valued at four-year colleges but explains nearly 70 percent of ex-ante returns at academic two-year colleges, consistent with the idea of a stepping stone. Interestingly, academic two-year colleges as a whole are not highly valued, even though there are four students enrolling in this institutions per ten enrolling in a four-year college. Their value comes mostly by a modest increase in participation from the marginal students. All the other, would simply enroll in a four-year college given that they are still allowed to drop out.

An interesting extension would be to introduce credit constraints into the model and evaluate the importance of the stepping stone argument with respect to credit constraints on explaining the high enrollment rate observed in academic two-year colleges. Although an interesting exercise, it would require a complete reevaluation of the calibration exercise, as the whole point of the exercise would be to do a horse race between the two alternative mechanisms. As a result, external identification of the parameters would be needed, or at least finding some data that helps distinguishing the two stories. Another worthwhile extension would be to allow for different types of institutions within a college type; for example, different four-year colleges. Through the lens of my model, if different colleges charge different tuition and have different learning technologies, students would also sort across the different institutions, even within a college type. This extension would be particular suitable to relate to the results in Hoxby and Avery (2012), which finds that students with high grades in high school but from poor families do not apply to selective universities. Because the income of the student's father is positively correlated with the student's initial belief, I conjecture that a model like mine could explain the documented empirical regularity.

Appendix

A Initial enrollment choice

Ordered returns to enrollment together with the evidence presented in Table 2, suggest that the initial enrollment choice is ordered as follows: work, vocational school, academic two-year colleges, and four-year colleges. Table 11 in Appendix A presents the results of an Ordered Probit regression of the initial enrollment choice on a vector X of observable measures of ability. Let β denote the vector of factor loadings. Relative to Kane and Rouse (1999), the analysis is extended here to consider vocational school and academic two-year colleges as separate institutions. The reference column in Table 11 is Version A (Version B pools vocational school and work together).

Table 11: Ordered Probit Regression

Variable		Version A	Version B
Male		0.183	0.234
		(0.041)	(0.044)
Black		0.358	0.365
		(0.069)	(0.073)
Socio. Status:	Low	-0.901	-0.9
		(0.088)	(0.093)
	Medium	-0.600	-0.609
		(0.072)	(0.074)
Education of Father:	<HS	-0.366	-0.381
		(0.072)	(0.076)
	HS graduate	-0.120	-0.115
		(0.065)	(0.068)
	four-year graduate	0.324	0.352
		(0.084)	(0.086)
Rank		-1.317	-1.41
		(0.073)	(0.078)
Cut 1		-1.133	-0.893
		(0.078)	(0.079)
Cut 2		-0.877	-0.368
		(0.071)	(0.078)
Cut 3		-0.354	-
		(0.076)	-
# of observations		3462	3462

Ordered Probit estimation of the initial enrollment choice on observable measures of ability. Version B merges vocational school with work. Source: NLS-72.

The value $X'\beta$ is a composite measure of ability, consistently estimated by $X'\hat{\beta}$. To evaluate a measure of the degree of sorting in initial enrollment, Table 12 produces the mean and standard deviation (in the cross-section) of $X'\hat{\beta}$ across the different alternatives. See Version A in the first row of the table. The measure of ability $X'\beta$ is unit-less as it is just an ordinal representation of ability measures. Start with high school graduates who join the workforce -labeled as work in the table- and move upwards across enrollment options. The mean value for the measure of ability increases monotonically with the enrollment options.

Table 12: Measure of Ability $X'\hat{\beta}$

Version	work	vocational school	academic two-year college	four-year college
A	-1.380	-1.249	-0.993	-0.631
	(0.548)	(0.531)	(0.583)	(0.627)
B		-1.426	-1.013	-0.633
		(0.572)	(0.609)	(0.654)

Constructed from Ordered Probit Estimation (see Table 11). In version B work and vocational school are collapsed together. Source: NLS-72.

B Wage profiles

The typical Mincer regression evaluates the effect of educational histories on lifetime earnings by estimating a wage regression on years of education and work experience. There is significant ongoing literature that accounts for nonlinearities in years of education (see, for example, Grubb (1993), Heckman et al. (2008) and Kane and Rouse (1995)). The typical example in favor of nonlinearities is the graduation premium. This literature has treated years of education (or amount of credits earned) in different types of institutions as perfect substitutes. Instead, it is now assumed that different educational histories affect lifetime earnings in different ways. Further, as has been already discussed in the literature, this analysis breaks the additive form (in the log version in the Mincer regression) of years of education and experience by estimating a growth equation.

Table 13: Mincer Regression

	description	Version A	Version B	
			(1)	(2)
ϖ_C^D	drop at four-year C.	0.24	0.22	-
		(0.042)	(0.041)	
ϖ_A^D	drop at Ac. two-year C.	0.09	0.077	-
		(0.045)	(0.044)	
ϖ_V^D	drop at Voc. school	0.072	-	-
		(0.046)		
ϖ_C^G	graduation at four-year C.	0.304	0.286	0.2122
		(0.04)	(0.04)	(0.037)
ϖ_A^G	graduation at Ac. two-year C.	0.015	0.0149	0.0445
		(0.1)	(0.104)	(0.104)
ϖ_V^G	graduation at Voc. school	0.284	-	-
		(0.15)		
		Prob > F	Prob > F	
	Test: $\varpi_A^G = \varpi_A^D$	0.5698	0.5735	-
	Test: $\varpi_C^G = \varpi_V^G$	0.9092	-	-

Dependent Variable: log of wages in 1985. Independent Variables: graduation and dropout dummies for each type of institution and controls for ability measures. Version B collapse together vocational schools and workforce. Source: NLS-72.

Table 13 presents the results of the extended mincer regression, accounting for the different types of education and graduation premium. See Version A. Graduation in both vocational schools and four-year

colleges is associated with higher wages relative to dropping out. The same idea does not apply to academic two-year colleges as the return to becoming a dropout is higher than the return from graduation. The low number of students who graduate at academic two-year colleges raises questions about the significance of the difference. The significance test presented in Table 13 shows that there is not enough evidence to reject that the return to graduation is similar (or even higher) to the return to dropping out.

C   The value for a student

In this section I discuss how to derive the value function for a student. The sketch of the proof is the following. I start by guessing (i) that consumption is strictly greater than zero, and (ii) a value function that is separable in the sense that the value function can be expressed as the combination of two different value functions: one depending on wealth level a, and the other one depending on the other states $\{i, s, p\}$; I refer to the latter as $v_i(s, p)$. This guess is natural given the assumption of exponential utility function. I then go on to construct $v_i(s, p)$. Finally, I check that consumption is in fact strictly above zero confirming the guess.

I now construct the value function that solves the student's problem. Conjecture that consumption is strictly positive so that the lower bound for consumption is not binding. The first order condition reads,

$$e^{-\gamma c} = \frac{1}{1+r} \frac{d \int_\eta \tilde{V}_i(a', s', p') H_i(d\eta, p)}{da'},$$

substituting it back into equation (2) provides the maximized value function,

$$V_i(a, s, p) = \frac{1}{-\gamma(1+r)} \frac{d \int_\eta \tilde{V}_i(a', s', p') H(d\eta, p)}{da'} + \frac{1}{1+r} \int_\eta \tilde{V}_i(a', s', p') H(d\eta, p) + \frac{1}{\gamma} \ .$$

Conjecture that $\int_\eta \tilde{V}_i(a', s', p') H(d\eta, p) = -\frac{1+r}{\gamma r} e^{-\gamma(ra' + \tilde{v}_i(s, p))} + \frac{1+r}{\gamma r}$. Under the conjecture the evolution of the asset level reduces to $a' = a - \frac{\tau^i}{1+r} - \frac{\tilde{v}_i(s, p)}{1+r}$. Then, substituting into the maximized value function and using that $v_i(s, p) = \frac{\tilde{v}_i(s, p) - r\tau^i}{1+r}$, I obtain

$$V_i(a, s, p) = -\frac{1+r}{\gamma r} e^{-\gamma(ra + v_i(s, p))} + \frac{1+r}{\gamma r} \ ,$$

where $v_i(s, p)$ solves the recursive equation

$$-\gamma v_i(s, p) = (1+r)^{-1} \ \ln \left[\int_\eta - \max \left\{ \begin{matrix} -e^{-\gamma(h^w - r\tau^i)}, -e^{-\gamma(v_{-i}(\theta^i(s'), p') - r\tau^i)}, \\ -\left(\mathbb{I}\left(p' e^{-\gamma(h^i(1) - r\tau^i)} + (1 - p') e^{-\gamma(h^i(0) - r\tau^i)} \right) + (1 - \mathbb{I}) e^{-\gamma(v_i(s', p') - r\tau^i)} \right) \end{matrix} \right\} H_i(d\eta, p) \right] \ ,$$

for any i, with $s' = s + \Omega(\eta)$ and $p' = b(\eta, p)$.

Finally, recall that I constructed a solution to the value functions under the guess that $c > 0$. It follows immediately that the guess for consumption is verified and that $V_i(a, s, p) = -\frac{1+r}{\gamma r} e^{-\gamma(ra + v_i(s, p))} + \frac{1+r}{\gamma r}$, with $v_i(s, p)$ solving the system of recursive equations described above, solves the student's problem.

D Fit of Model

The prior p_0 is positively correlated with the measure of ability $X'\beta$ obtained from the Ordered Probit regression (Table 11 and Table 12) so that the enrollment pattern generated by the model fits the empirical distribution obtained from NLS-72.

The model also has predictions regarding the dropout, transfer, and graduation behavior of students. In particular, conditional on the initial enrollment choice, the model produces probabilities of different educational patterns as a function of the initial prior p_0, as shown in Figure 7. The initial prior p_0 affects the decision making of the student and the dynamic pattern in two different ways. First, it affects the likelihood of different educational histories as the distance to different threshold values changes with the prior. Second, the value of the prior is related to the likelihood of different signals as $p_0 = \Pr[\mu = 1]$. In fact, the estimation of the model attempted to match the empirical marginal density of dropouts, transfers and graduation at both academic two- and four-year colleges.

Figure 7 has three different regions (the straight vertical lines separate the different regions). The first region, given for low values of the prior p_0, is for agents who join the workforce directly. The second region, the middle one, is for agents who enroll in academic two-year colleges (average values for the prior) and the third region, the top one, is for agents who enroll in four-year colleges. Conditional on the initial enrollment choice, Figure 7 presents the likelihood of each of the three possible events (i.e., drop, transfer or graduation) in the first spell of education for a student with a given initial belief p_0. The likelihood of graduation and dropping out in academic two-year colleges are decreasing functions of the initial belief p_0, while the likelihood of transferring to four-year colleges is increasing. For students who initially enroll in four-year colleges, the likelihood of graduation increases with the initial belief, while the likelihood of dropping out decreases with the prior. Another interesting aspect observed in Figure 7 is that students who transfer from four-year colleges to two-year colleges have above average beliefs (relative to students who enroll in four-year colleges).[14]

To evaluate the predictions of the model regarding the transition probabilities, Table 14 shows students by behavior (i.e., dropout, transfer, graduation) in the first spell of education and size of the measure of ability $X'\hat{\beta}$. For students initially enrolled in academic two-year colleges, the pattern observed for dropout and transfer probabilities is similar to the model's predictions. A similar thing happens for the dropout and graduation probabilities for students initially enrolled in four-year colleges. Graduation probability in academic two-year colleges and transfer probability in four-year colleges are less revealing due to the low number of students included in these cells. Still, the evidence in these two cases does not conflict with the model's predictions.

An alternative and more involved way of evaluating the results on Figure 7 would be, conditional on the initial enrollment choice of agents, to do a nonparametric estimation of the densities of each of the different educational histories. Weighting these densities accordingly, it is possible to produce the empirical counterpart of Figure 7. First, the density associated with a given history is weighted by its share on initial enrollment in a given institution. For a given measure of ability $X'\hat{\beta}$, now it is possible to compute the proportion of agents who eventually end their first spell of education either by becoming dropouts, transferring, or by graduation. Figure 8 presents the results. Notice how the patterns in Figure 7 (model) are very similar to those in Figure 8 (data).

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[14]The shape of the figure is robust to different calibrations of the model. In particular, calibrations that match more closely dropout and transfers (in detriment of enrollment moments).

Figure 7: Probability of Dropout, Transfer, and Graduation
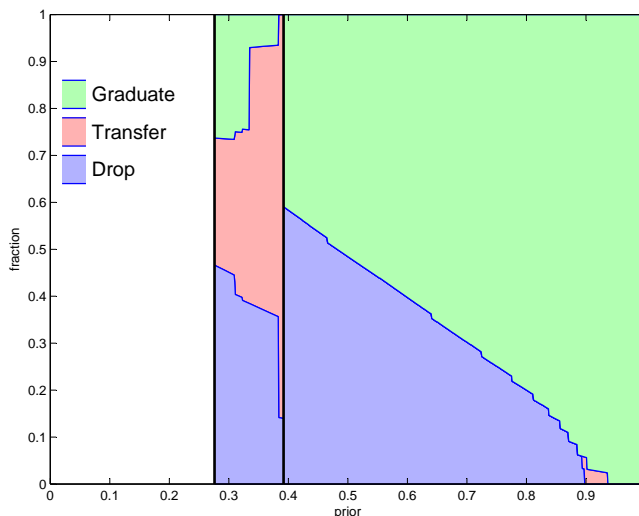
The probabilities are computed according to the initial enrollment choice of an individual with initial prior p_0. The plot has three regions (from left to right). The first region is empty as individuals with low priors join the workforce. The second region is for individuals enrolling in academic two-year colleges, while the third region is for individuals who enroll in four-year colleges.

Table 14: Proportion of agents initially enrolled in i with particular history

	Ac. two-year C.			four-year C.		
	D	T	G	D	T	G
Low $X'\hat{\beta}$	77.3%	17.6%	5.1%	51.7%	1.7%	46.6%
Med $X'\hat{\beta}$	66.1%	29.4%	4.5%	45.4%	2.4%	52.2%
High $X'\hat{\beta}$	44.9%	50%	5.1%	35.6%	2.4%	62%
# of obs.	332	192	26	392	47	474

These proportions are conditional on the initial enrollment status of the student. D: Drop, T: Transfer, G: Graduate. $X'\hat{\beta}$: estimator of the observable measures of ability. Recall that $p_0 = \frac{1}{1+e^{-(X'\beta+\varepsilon)}}$ and that a student enrolls in four-year college if her prior is high enough and in an academic two-year college if it is average (so that she compares her initial prior with thresholds). Then, the enrollment problem can be estimated by using an Ordered Probit regression from where β is estimated.

E   Other selected tables and figures

Table 15: Average Cost of Education

|	Tuition	Tuition + R&B
vocational school	3803.84	5904.74
academic two-year college	1811.36	2729.65
four-year college	3420.73	5038.8

R&B: Room and Board. Missing Values were imputed by running a cost regression and imputing missing values through observables. The values are measured in 1984 dollars. Source: NLS-72.

Figure 8: Empirical density of dropout, transfer, and graduation
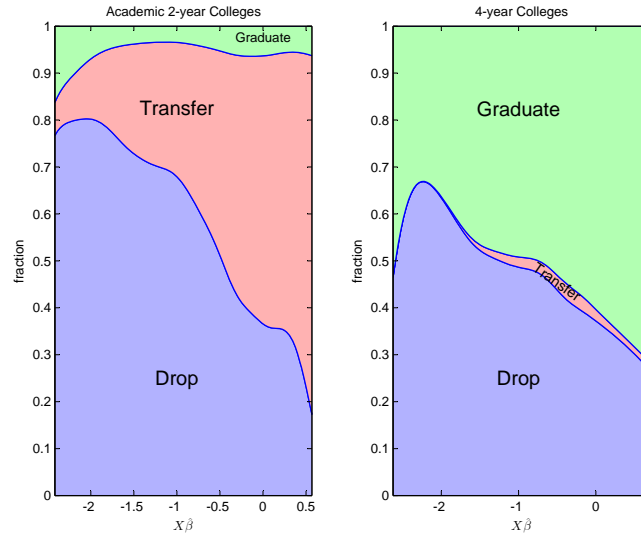
NLS-72 cohort. As a function of ability measure $X'\hat{\beta}$ and conditional on initial enrollment choice. First, a conditional (on the initial enrollment choice) nonparametric estimation of each event was performed. Next, each density was weighted by their share in initial enrollment. Finally, for every level of $X'\hat{\beta}$, the proportion of each event was constructed.

Figure 9: Transitions in ac. two-year and four-year colleges
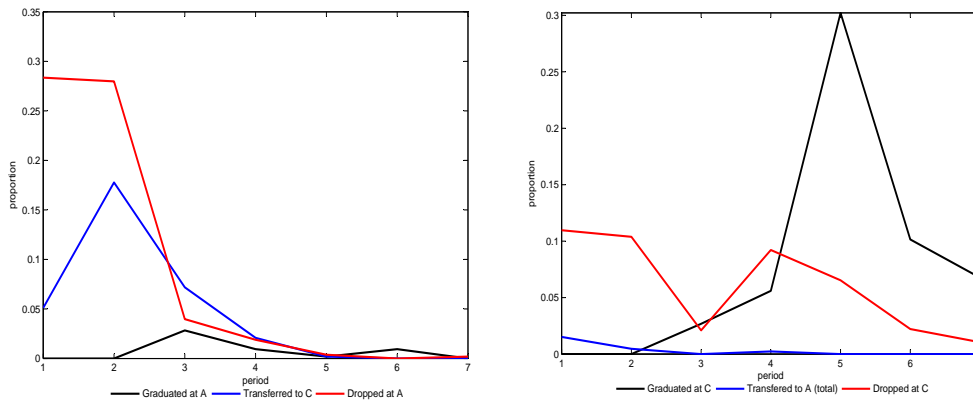
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