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Does Redistribution Increase Output? The Centrality of Labor Supply[‡]

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Abstract

The aftermath of the recent recession has seen calls to use transfers to poorer households as a means to enhance aggregate economic activity. The goal of this paper is to study the effects of wealth redistribution from rich to poor households on consumption and output in the short run. We first demonstrate analytically how the direction and size of the output effects of such interventions depends on labor supply decisions. We then show that in a standard incomplete-markets model extended to allow for nominal rigidities and parametrized to match the U.S. wealth distribution, wealth redistribution does lead to a temporary boom in consumption but a far smaller increase in output. Our results suggest substantial value in empirical research uncovering the distribution of marginal propensities to work in the population.

Keywords: Multipliers, Redistribution, Labor supply, Idiosyncratic Risk. **JEL Codes:** D90, E21, E25, E63

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1 Introduction

The goal of this paper is to understand the driving forces behind the short-run effects on output of wealth redistributions from rich to poor. In the aftermath of the recent recession there have been numerous calls to use, as well as the actual use of, transfers to low net-wealth households as a means to increase economic activity.¹ The conventional argument for a positive effect of transfers from rich to poor on output centers on the notion that wealth-poor households have a higher marginal propensity to consume out of wealth than wealth-rich households, so that such a transfer leads to a boom in consumption. Under standard “Keynesian” intuition, where individual components of aggregate expenditure decisions can be an important determinant of output, such transfers lead to a boom in economic activity. Such a view puts household heterogeneity front and center in determining the aggregate short-run response to a change in transfers. In particular, it implies that any quantitatively plausible evaluation of the aggregate impact of transfers requires the use of a model capable of capturing observed wealth heterogeneity in considerable detail. Our analysis of such a model complements the conventional intuition by emphasizing that transfers are most likely to have stimulative impact on output to the extent that the marginal propensity to *work* of the wealth-rich is higher than that of the wealth-poor.² Our results suggest substantial value in additional empirical research aimed at uncovering the distribution of these marginal propensities to work across the population.

Figure 1, below, shows the impact of a one-time surprise wealth redistribution from rich to poor on aggregate output and consumption in the “standard incomplete-market” (SIM) or “Bewley-Aiyagari-Huggett” model, the framework most widely used to capture wealth heterogeneity and to understand interactions between inequality and the macroeconomy.³ The parameters governing household labor supply decisions are calibrated to match the joint distribution of labor supply and wealth. The remaining parameters relevant for the steady state of the model are calibrated as in Castaneda et al. (2003), so that its initial conditions match the extreme wealth concentration observed in the United States. Consistent with the conventional argument, we see that a redistribution of wealth from rich to poor agents does generate a consumption boom, and one that, furthermore, is prolonged.

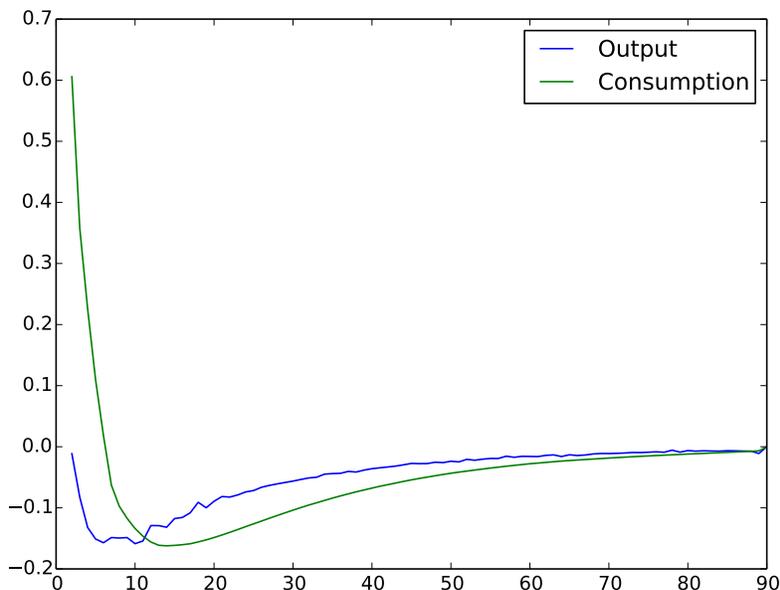
What is also clear from Figure 1, however, is that while wealth redistribution from rich to poor results in a sizeable boom in consumption, it generates a small reduction in output. Our results present a challenge to the conventional view that wealth redistribution has significant short-run

¹The most prominent redistributive policy implemented was the American Recovery and Reinvestment Act (ARRA), which contained a significant redistributive component in the form of extensions of transfer programs for the poor, such as unemployment insurance and food stamps (Oh and Reis, 2012). More generally, the idea that the post-crisis evolution of the wealth distribution may be suppressing aggregate output in the recent recovery is one that has received attention. In particular, Mian and Sufi (2014) argue that, by increasing household net worth and hence aggregate consumption, debt forgiveness may be a key for accelerating the recovery. See also Dynan (2012), and Cynamon and Fazzari (2013).

²As a quantitative matter, a nontrivial effect of wealth on labor supply is highlighted in work by Floden (2001) and Pijoan-Mas (2006).

³Examples of papers using models within the same class to study wealth redistribution policies include Heathcote (2005) and Conesa and Krueger (2006), among many others.

Figure 1: Aggregate Impact of Wealth Redistribution



stimulative power. In the paper, we will show that the response of output reflects the response of aggregate labor supply. These findings arise from a model that quantitatively captures salient household heterogeneity and thus provides a natural benchmark for analyses of redistributive policies.

Our findings are robust to numerous modifications. In particular, they are virtually indistinguishable from those arising in a model that allows for nominal rigidities. Specifically, we also extend the baseline SIM model to allow for sticky prices and a passive nominal interest rate rule consistent with the presence of a zero-lower-bound constraint on monetary policy, so that it accommodates a Keynesian channel. We find that this extension yields virtually identical results. Those results therefore highlight that, as a quantitative matter, labor supply behavior remains dominant even when changes in household expenditure decisions can shift labor demand. While our results do not rule out the possibility that an alternative calibration would overturn this result, they do suggest that under a reasonable set of assumptions, labor supply plays a central role.

The consumption boom in the model can be largely understood in terms of intertemporal decisions. First, market incompleteness and borrowing limits lead households to resort to precautionary buffer stock behavior: poor households attempt to avoid liquidity constraints by saving to reach a certain target wealth level. This implies that if they receive a transfer, they are able to achieve this target while increasing their consumption substantially. Second, those households already against their borrowing constraint will not save any portion of a marginal transfer. However, these same intertemporal considerations also push low-wealth households toward increasing their *leisure* sub-

stantially, so that, barring countervailing forces, the same incentives that lead to a consumption boom also induce a bust in labor supply and hence, in output.

The effect on output, however, depends also on intratemporal considerations. In particular, we show analytically that, all else equal, aggregate labor supply will tend to fall by less with a rich-to-poor transfer when leisure is a “luxury,” whereby rich households “spend” a greater proportion of their resources on it as compared to consumption of goods. While it may appear counter-intuitive that rich households may spend more resources on leisure than poor households, this may happen if they face much higher wages, in which case leisure for them will be more expensive. Thus, if wealthy households are also more productive, leisure could be a luxury even if they work fewer hours. Furthermore, wealthy households may spend proportionately more with leisure because of a progressive income tax system, a feature of the data that is incorporated into our quantitative analysis.

Additionally, we examine a variant of the model featuring indivisible labor. In this case, we show analytically that very rich and very poor households remain insensitive to transfers. By contrast, households with moderate levels of wealth are not: their status as donors or recipients of transfers will determine their decisions. Thus, the specific details of the redistributive policy become particularly important.

To analyze the impact of transfers on output, an important benchmark is that of a ‘neoclassical’ model in which output is produced with labor and capital purchased in competitive markets, without externalities or distortions. In this case, the labor demand function does not shift over the short run no matter what happens to expenditures. Therefore, any changes in output from redistribution must arise solely from shifts in aggregate labor supply behavior. In more general models, labor demand can also be directly affected by expenditure decisions. We show that if labor demand is shifted only by changes in total expenditures, as postulated by the usual Keynesian intuition,⁴ the model will not generate any shift in the labor demand function so long as the labor supply function is unresponsive to changes in wealth distribution. That is, shifts in the labor demand function can only matter to the extent that it amplifies or dampens the effect of labor supply responses.

We also provide an analysis of a sticky price economy. In this instance, redistribution can have aggregate effects if the monetary authority allows price distortions stemming from nominal rigidities to change with equilibrium interest rates (as would be the case if it faces a binding zero-lower-bound constraint on the interest rate). However, as a quantitative matter, we also find that given our baseline calibration labor demand still plays a very limited role. The reason is that aggregate savings at any point in time are a small fraction of the aggregate capital stock. Therefore, in the absence of adjustment costs (which we assume in our baseline), interest rates do not fluctuate much in response to a consumption boom. Hence, the pricing distortions implied by the zero-lower-bound constraint may not be very important for the effects of redistribution on output.

⁴This is, in fact, how John Maynard Keynes described it: “The amount of labour N which the entrepreneurs decide to employ depends on the sum (D) of two quantities, namely D_1 , the amount which the community is expected to spend on consumption, and D_2 , the amount which it is expected to devote to new investment.” Keynes (1936)

The results described so far rely on a calibration of the model that implies that interest rates respond very little to redistributive shock. To assess the role that interest rates can play in general equilibrium, we show quantitatively that while capital adjustment costs making interest rates more sensitive to savings do lead to a positive output impact of redistribution, they do so by a smaller amount under sticky prices. The reason is that changes in interest rates affect output primarily through their effect on labor supply, and nominal rigidities act mostly as a dampening device. Thus, even in situations where labor demand plays a significant role, understanding the determinants of labor supply remains critical.

The paper proceeds as follows. In Section 2, we present the baseline model. Section 3 presents analytical results with respect to the neutrality of aggregate outcomes to wealth redistributions. Section 4 presents the quantitative results. Section 5 concludes.

Literature Review

Our work informs a growing quantitative literature examining the short-run aggregate effects of wealth redistribution programs. Huntley and Michelangeli (2014) and Kaplan and Violante (2011) aim to explain how wealth redistribution can lead to an increase in household consumption of the size measured in empirical work (see Johnson et al., 2006; Parker et al., 2013; Jappelli and Pistaferri, 2014; and Misra and Surico, 2014), but they do not focus on the behavior of labor supply. Heathcote (2005) studies the effect of a redistributive program in a heterogeneous-agent economy, but assumes that the utility function is such that labor supply is unaffected by wealth. Heathcote's focus is, instead, on the distortionary impacts of the taxes needed to fund the transfer program. Our paper is also complementary to a rich strand in the literature that investigates the steady-state implications of redistributive programs in heterogeneous-agent economies with endogenous labor supply. Examples include Floden (2001), Pijoan-Mas (2006), Alonso-Ortiz and Rogerson (2010), and most recently, Horvath and Nolan (2013), and Zilberman and Berriel (2012).

In contrast to the work above, however, we focus entirely on the transitional dynamics of a redistribution policy that, by design, features no steady-state effects. Our paper therefore shares the emphasis on business-cycle frequencies in Oh and Reis (2012), McKay and Reis (2016), Mehrotra (2014), and Giambattista and Pennings (2014). The papers of both Mehrotra and Giambattista and Pennings analyze the effect of transfers in models with two types of agents, providing assessments of transfer multiplier given different assumptions. Oh and Reis is the most closely related, as they analyze transitional dynamics following one-off redistributive policies in a full-fledged heterogeneous-agent model. Their particular focus is on how wealth transfers ought to be targeted in order to generate output booms. By contrast, our work analyzes the impact of policies that redistribute wealth from rich to poor, as have been more typically advocated and implemented.

Lastly, our paper contributes by providing substantial analytical characterization of the effect of various policies in a model where the heterogeneity across households is potentially very rich. In existing work, analytical tractability is achieved through the use of special assumptions on income processes, preferences, or market structures (see Heathcote, Storesletten, and Violante (2014) for a recent contribution). As we will show, however, significant insight into the short-run effects of redistribution can be attained without imposing any structure on shock processes beyond what is

necessary to deliver stationary aggregate outcomes.

Our paper highlights the need for empirical research documenting the distribution of wealth effects on labor supply across the population. Unfortunately, while there is a narrow literature aimed at measuring the likely response of labor supply to plausibly exogenous changes in wealth, to our knowledge, there is essentially no work tracking how this sensitivity varies with initial household wealth. Existing work includes analysis of the effects of lottery outcomes (Kaplan, 1987; Imbens et al., 2001; Furaker and Hedenus, 2009; Cesarini et al., 2015; and Picchio et al., 2015)⁵ non-experimental data (Cheng and French, 2000; Coile and Levine, 2006) and as well as randomized control trials in less-developed economies (Banerjee, Hanna, Kreindler, and Olken, 2015). A common finding is that both intensive and extensive margin wealth elasticities, while not zero, are typically small.

2 Model

To study the short-run impact of wealth redistribution in a setting capable of capturing empirically salient heterogeneity, we introduce a single model that nests a wide variety of Bewley-Aiyagari-Huggett style environments, including extensions that allow for nominal rigidities as well as a complete markets benchmark. For notational convenience, we denote all prices in terms of units of the final consumption good, inflating them by a nominal price index whenever necessary.

2.1 Households

The economy is populated by a continuum of infinitely lived households with utility defined over consumption and leisure. Time is discrete, and given that the short-run policy decisions that our paper hopes to inform usually have annual horizons, one model period corresponds to one calendar year. In what follows we lay out a model that will provide the basis for the analytical results we derive. For the quantitative analysis, however, we will add further additional features that, for expositional ease, we will describe in detail when we introduce the parametrization of the model.

Households differ in terms of their initial wealth and labor productivity. Labor productivity for any given household changes stochastically and purely idiosyncratically. As a benchmark, we allow households to trade both risk-free bonds and Arrow securities contingent on their idiosyncratic productivity states. This benchmark nests both the full insurance case (in which case the household has access to a full set of Arrow securities at actuarially fair prices) and the standard incomplete markets framework (in which case the household only has access to a risk-free bond). While we write the problem in this general form for completeness, in the quantitative investigation we will restrict ourselves to the standard incomplete markets setup, where households are constrained to holding only riskless bonds subject to a borrowing limit \underline{b} .

⁵One interesting aspect of Cesarini et al.’s findings is that winners’ labor supply falls by much more than that of the spouse, something difficult to reconcile with a “unified household” model of labor supply.

It is also worth mentioning that lottery winnings are typically distributed over long periods of time (eg. twenty or more years in Imbens et al. (2001)), so that, given incomplete markets and borrowing limits, the estimated elasticities do not map directly into marginal propensities to work out of wealth.

Our focus throughout will be on the aggregate transitional dynamics induced by a single, one-off, and wholly unanticipated wealth redistribution. The evolution of aggregates (aggregate capital and labor) implies a path for prices (interest rates and wages) that, after the shock, are perfectly forecasted by households. We index the time since the shock by t , with $t = 0$ corresponding to the aggregate steady-state and $t = 1$ the first date after the shock. For any date $t \geq 1$, and taking $\{w_t, r_t\}_{t=0}^{\infty}$ as given, the problem of the household is completely standard and can be written recursively as:

$$\begin{aligned}
V_t(a, s) &= \max_{\{b'_{s'}\}_{s \in \mathcal{S}}, b'_f, c, l} u(c, \bar{l} - l) + \beta \sum_{s' \in \mathcal{S}} \Pr[s'|s] V_{t+1}(a'_{s'}, s') \\
s.t. &: \sum_{s' \in \mathcal{S}} q_{t,s'}(s) b'_{s'} + b'_f + c + \tau = w_t \varepsilon(s) l + (1 + r_t) a + \pi_t + x_t(a, s), \\
& a'_s = b'_s + b'_f, b'_f \geq \underline{b}, l \geq 0.
\end{aligned}$$

As usual, primed variables refer to next period values, and $\Pr[s'|s]$ is the probability of transitioning into idiosyncratic productivity state s' , given that the household is in state s . The remaining notation is as follows: $\beta < 1$ is the discount factor; c is consumption; l is labor; \bar{l} is leisure endowment; u is an increasing, concave utility function that is differentiable in both its arguments; $\varepsilon(s)$ is the labor productivity of household in idiosyncratic state; w_t is the wage per effective unit of labor; r_t is the interest rate on bonds; $q_{t,s'}(s)$ is the price of a state s' Arrow security $b'_{s'}$ when sold to a household with productivity state s ; a are initial asset holdings that, in the end of each period are equal to the amount of risk-free bonds (b_f) and realized Arrow securities b_s ; \underline{a} is an exogenous debt limit; π_t are profits that are distributed lump-sum to households; τ are lump-sum taxes (which, in the quantitative section, will be generalized to allow dependence on the household state); and $x_t(a, s)$ are transfers or, if negative, payments. In Section 2.5 we discuss in more detail our motivation for the choice of $x_t(a, s)$.

Note further that the model does not assume a one-to-one correspondence between income and wealth. Importantly, it allows for investment in financial assets. As an example, if a household that starts out being very productive but is suddenly subject to a very negative income shock and markets are incomplete, it may find itself with very low productivity and high wealth. Furthermore, in our baseline calibration, we add shocks that we interpret as capturing life-cycle aspects of household income, with households being replaced by their descendants upon their death. Descendants can be much less productive than their parents and still be quite wealthy.⁶

We denote by $\Gamma_t(a, s)$ the joint density of households with asset level a and productivity level s at the end of period t , with marginal densities $\Gamma_t(a)$ and $\Gamma(s)$. Note that since the process

⁶This life-cycle interpretation is possible in the context of an infinite horizon model under the assumption that households can bequest their wealth to their descendants, and value their welfare as their own (after time-discounting), thus being perfectly altruistic.

for s is exogenous, we can take $\Gamma(s)$ to be time invariant. We also denote the optimal choices of c , l , and a' at each date t during the transition by the policy functions $c_t(a, s)$, $l_t(a, s)$, and $a'_t(a, s)$. Letting \mathcal{S} be the (finite) set of exogenous state and \mathcal{A} the (bounded closed interval) set of wealth levels, this allows us to define aggregate consumption C_t , hours worked L_t , “effective” (i.e., productivity weighted) labor input N_t , end-of-period household wealth A_t , and aggregate net transfers X_t respectively as follows:

$$C_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} c_t(a, s) \Gamma_{t-1}(a, s) da \quad (1)$$

$$L_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} l_t(a, s) \Gamma_{t-1}(a, s) da \quad (2)$$

$$N_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} \varepsilon_t(s) l_t(a, s) \Gamma_{t-1}(a, s) da \quad (3)$$

$$A_t = \sum_{s \in \mathcal{S}} \sum_{s' \in \mathcal{S}} \int_{a \in \mathcal{A}} a'_{s',t}(a, s) \Gamma_{t-1}(a, s) \Gamma(s') da \quad (4)$$

$$X_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} x_t(a, s) \Gamma_{t-1}(a, s) da \quad (5)$$

2.2 Firms

The closest reference for the impact of redistribution in a quantitative model with heterogeneous agents and sticky prices is Oh and Reis (2012). Therefore, to maintain comparability with existing literature, we follow them closely when setting up the firm side of the model. There are two types of firms: final-goods producers and intermediate-goods producers. Final-goods producers combine the intermediate goods and capital into a single final good that can be used for consumption, investment in fixed capital, or government purchases. There is a unit mass of differentiated intermediate-goods producers indexed $i \in [0, 1]$. Each firm i is endowed with a linear technology that allows it to transform one unit of effective labor into one unit of the i -th intermediate good. Let $n_t^d(i)$ denote the demand for effective labor by firm i and $m_t(i)$ the level of production of intermediate good i . Intermediate products are imperfect substitutes in the production of the final good, and this gives their monopolistic producers some discretion over the price they charge, $p_t(i)$. However, they are subject to nominal frictions, which limit their ability to choose $p_t(i)$ in a timely manner. All intermediate-goods producing firms commit to supplying the quantity $m_t^d(i)$ that final-goods producers demand at the prevailing price, ensuring that markets for all intermediate goods clear.

The production function for final-goods producers is:

$$Y_t = F(K_{t-1}, M_t),$$

where F is a neo-classical production function, K_{t-1} is the capital stock available at the beginning of period- t , and $M_t = \left[\int_0^1 m_t^d(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ is an aggregate of different varieties of the intermediate

good. Final-goods producing firms maximize profits, behave competitively, and use the funds they borrow from households to purchase capital. Let the date t value of a final-goods firm (in terms of the final good) that enters a period with capital stock K_{t-1} be given by $\Xi_t(K_{t-1})$. This value can be described recursively as follows:

$$\Xi_t(K_{t-1}) = \max_{M_t, K_t} F(K_{t-1}, M_t) - P_t M_t - K_t + (1 - \delta) K_{t-1} + \frac{1}{1 + r_{t+1}} \Xi_{t+1}(K_t)$$

where the price index $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ denotes the per-unit price (in terms of the final good) of the cost-minimizing bundle of intermediate goods. Let δ denote the depreciation rate of capital.

Final-goods producers' choices satisfy the usual optimality conditions:

$$F_K(K_{t-1}, M_t) = r_t + \delta \tag{6}$$

$$F_M(K_{t-1}, M_t) = P_t \tag{7}$$

$$m_t^d(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} M_t. \tag{8}$$

Let Q_t be the nominal price level for the final good in the economy, so that $p_t(i) Q_t$ is the nominal price charged by firm i at time t . We assume that a fraction λ of the firms choose $p_t(i) Q_t$ at the *beginning* of the period, after shocks have been realized, whereas the remaining $1 - \lambda$ choose nominal prices that they will receive in period t at the end of the *previous* period, $t - 1$. Thus, a fraction $1 - \lambda$ set prices before they have had time to incorporate period t information. Firms distribute any profits $\pi_t(i) = (p_t(i) - w_t) m_t(i)$ to households in equal shares.⁷

To keep the model as comparable as possible to the benchmark competitive case, we assume that intermediate-goods producers receive a subsidy that is proportional to their output and are simultaneously subject to a lump-sum tax to ensure that the subsidy is revenue neutral. Given the subsidy, firms choose to set prices as close as possible to marginal cost. That is, if firm i belongs to the fraction λ of firms who set $p_t(i) Q_t$ at the beginning of period t , it sets

$$p_t(i) Q_t = w_t Q_t. \tag{9}$$

If firm i instead belongs to the fraction $1 - \lambda$ who set $p_t(i) Q_t$ at the end of $t - 1$, we have that

$$p_t(i) Q_t = E_{t-1} [w_t Q_t]. \tag{10}$$

Note that if $\lambda = 1$, the model collapses to the standard competitive flexible price environment, since in this case all firms price at marginal cost. Furthermore, in that case, M_t is equal to effective

⁷Recent work by Broer, Hansen, Krusell, and Öberg (2015) suggests that this may lead us to over-estimate the role of labor demand shifts. They argue that, under realistic assumptions about distribution of profits (namely, that they are concentrated among households who do not supply labor), the New Keynesian transmission mechanism becomes particularly weak.

labor supply N_t .

In steady state, there are no aggregate shocks, which means that all intermediate-good producing firms have had enough time to choose their prices optimally. This implies that prices coincide with what would have been set in the absence of nominal rigidities. At $t = 1$, a fraction $1 - \lambda$ find themselves forced to keep $p_1(i) Q_1 = p_0(i) Q_0$, whereas the remainder adjust, setting $p_1(i) Q_1 = w_1 Q_1$. From $t = 2$ onward, the economy proceeds as in an environment without nominal rigidities since there are no more surprises.⁸

This particular way in which we model the nominal frictions is motivated by the fact that most firms change their prices within the year, so that given the yearly calibration of the model, it is plausible to assume, as we do, that stickiness only holds for one period, with all prices flexible from the second year onward. Other forms of modeling nominal stickiness would be to allow for endogenous price changing decisions given menu costs. As discussed in a fairly extensive literature (for example, Dotsey, King and Wolman, 1999; Golosov and Lucas, 2007; Alvarez, Le Bihan and Lippi, 2016) the presence of menu costs tend to make the price stickiness channel even less potent.

2.3 Government

Finally, the government taxes and transfers resources to households. It also consumes some of the output. Government consumption confers no value on households. Its budget constraint is:

$$B_t = (1 + r_t) B_{t-1} + G_t + X_t - \tau,$$

where B_t is government debt, G_t is government consumption, and τ are the lump-sum taxes collected. Government consumption G_t is chosen to implement a particular pre-determined, nonexplosive, and perfectly forecasted path for government debt (we assume taxes are high enough relative to debt to allow such a path to be implementable). Monetary policy is set by a monetary authority, which determines the trajectory for the price level Q_t as a function of the state of the economy.⁹

2.4 Equilibrium

Equilibrium is given by sequence of prices $\{w_t, r_t, Q_t\}_{t=0}^{\infty}$, value functions $\{V_t(a, s), \Xi_t(K)\}_{t=0}^{\infty}$, choices for intermediate-goods producers $\{m_t(i), p_t(i), \pi_t(i)\}_{t=0}^{\infty} \forall i \in [0, 1]$, policy functions for households $\{c_t(a, s), l_t(a, s), a'_t(a, s)\}_{t=0}^{\infty}$, intermediate-goods input choices for final-goods producers $\{m_t^d(i)\}_{t=0}^{\infty} \forall i \in [0, 1]$, aggregate variables $\{C_t, L_t, N_t, A_t, K_t, M_t, B_t, X_t, G_t, \pi_t\}_{t=0}^{\infty}$, and the joint density of assets and idiosyncratic shocks $\{\Gamma_t(a, s)\}_{t=0}^{\infty}$ such that, given the path for transfer policies $\{x_t(a)\}_{t=0}^{\infty}$, the policy functions, value functions, and sequences of K_t and M_t correspond to the optimal choices of households and final-goods producers, the choices for intermediate-goods

⁸This approach to modeling nominal rigidities is similar to the one adopted by Oh and Reis (2012).

⁹In a wide range of sticky-price models, the government is able to implement policies of that kind even in a limiting cash-less economy, given suitable out-of-equilibrium commitments on its part. See, for example, Woodford (2011).

producers are also optimal, the government follows fiscal and monetary policy rules as described in Section 2.3 and:

1) Final-goods market clears:

$$C_t + K_t + G_t = F(K_{t-1}, M_t) + (1 - \delta) K_{t-1}$$

2) Intermediate-goods markets clear:

$$m_t(i) = m_t^d(i) \quad \forall i \in [0, 1]$$

3) Labor market clears:

$$N_t = \int_0^1 n_t^d(i) di$$

4) Capital market clears:

$$K_t + B_t = A_t$$

5) Profits from intermediate-goods producers are rebated to households:

$$\pi_t = \int_0^1 \pi_t(i) di$$

6) For any interval $[a_1, a_2] \in \mathcal{A}$, the distribution of idiosyncratic states evolves as:

$$\int_{a_1}^{a_2} \Gamma_{t+1}(a, s) da = \sum_{\tilde{s} \in \mathcal{S}} \Pr[s|\tilde{s}] \Pr[\tilde{s}] \int_{a \in \mathcal{A}'_t(a_1, a_2, \tilde{s})} \Gamma_t(a, \tilde{s}) da$$

Where $a \in \mathcal{A}'_t(a_1, a_2, \tilde{s})$ if $a'_t(a, \tilde{s}) \in [a_1, a_2]$.

2.5 Redistribution Experiment

We will focus our attention to the case where x_t are functions of a but not of s , so that we can write $x_t = x_t(a)$. We consider an experiment where at $t = 0$ the economy is in steady state, corresponding to the state approximated by the economy when $x_t(a) = 0 \quad \forall a$ for a sufficiently long time. In the subsequent period, $t = 1$, the transfer function changes from $x_0(a) = 0 \quad \forall a$ to $x_1(a) \neq 0$ for some a . From $t = 2$ onward, wealth taxes revert back to $x_0(\cdot)$, i.e., $x_t(a) = x_0(a) = 0 \quad \forall a$ for $t \geq 2$. We furthermore impose that the redistribution be revenue neutral, that is,

$$\sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} x_1(a) \Gamma_0(a, s) da = 0$$

The surprise and one-off nature of the shock is designed to isolate the role of heterogeneity on marginal propensities that lie at the heart of conventional intuition linking redistribution to short-

run aggregate outcomes. In particular, its surprise nature prevents redistribution from affecting the return to savings or labor effort directly. Thus, the presence or absence of aggregate effects of redistribution are not driven by the distortionary effects of taxes on labor or capital, but by differences in marginal propensities to consume and work out of wealth. This means that the policies we consider would have no real effect whatsoever in an economy where households can be aggregated into a single representative household. Furthermore, revenue neutrality allows us to isolate the role of wealth redistribution across households in a moment in time from the role of government debt. The latter has well-known real effects given the failure of Ricardian Equivalence in models of the class we study. Thus, in our experiments, heterogeneity in the response of households to wealth transfers is the sole underlying source of the dynamics.

We make x depend only on a in line with a broader of recent literature measuring heterogeneity in marginal propensities to consume out of wealth in order to understand the impact of redistributive policies. This includes the works cited in Footnote 1 by Mian and Sufi (2014), Dynan (2012), and Cynamon and Fazzari (2013) as well as the work by Kaplan and Violante (2014a,b). Following that literature, our purpose is to isolate one of the channels through which redistribution of wealth can have an effect. We thus avoid other channels that are also operative in many of the income based redistributive policies such as unemployment insurance, the Earned Income Tax Credit, and food stamps. The channels that we omit include altered incentives to work and save and have been discussed extensively elsewhere.

It is also worth noting that, as a practical matter, the correlation between income and wealth in the United States is 60 percent as measured by Rodriguez et al. (2002) using Survey of Consumer Finance data, so that a wealth dependent redistribution should not be too different from an income dependent one.¹⁰ Nonetheless, for completeness, in the analytical section below we discuss conditions under which income-based transfers diverge qualitatively in terms of their effects from wealth based transfers, and also provide an experiment in the quantitative section.

3 Analytics

We now proceed to present a set of analytical results that lay out various forces determining the initial impact on the equilibrium paths of an economy subjected to the one-off, unexpected redistribution experiment described in Section 2.5.

As we noted in the outset, equilibrium labor input determines aggregate output response to redistribution shocks. Specifically, to a first approximation, output is determined by a production function:¹¹

$$Y = F(K, N).$$

¹⁰This correlation is best interpreted as holding over long periods, since, as pointed out by a referee, income volatility is higher than wealth volatility and more correlated with business cycles.

¹¹This is an approximation, since it ignores potential misallocation among intermediate goods producers when there is price stickiness. We discuss this more in length in subsection 3.1.3 below

Since the capital stock is fixed immediately following a shock, we can think of the path for equilibrium output in the short-run as being determined solely by equilibrium labor input. In other words, in the short run, $Y = f(N)$. Equilibrium labor input is determined in turn by the intersection of the labor demand curve, $N^d(w, \vartheta^d)$, where ϑ^d is a vector summarizing all variables other than current period wages that may affect labor demand, and aggregate labor supply curve, $N^s(w, \vartheta^s)$, which also depends on the wage rate, w , and possibly other variables contained in the vector ϑ^s .¹² Redistribution can only have an impact on aggregate output to the extent that wealth distribution shifts, directly or indirectly, one of these curves. In what follows, we refer to redistribution as shifting or changing labor demand or supply as a short-hand to refer to effects that operate directly through ϑ^d or ϑ^s rather than equilibrium effects through wages.

In policy discussions, wealth redistribution is expected to be stimulative because it boosts consumption. This has stimulative impact under a standard Keynesian reasoning that aggregate investment is largely autonomous and that aggregate expenditures determine labor demand and, therefore, output. For what follows, it will therefore be useful to define aggregate expenditures in the context of our model, in any given period t . This is simply the sum of government consumption, household consumption, and gross fixed investment, and will be denoted E_t .

$$E_t \equiv G_t + C_t + K_t - (1 - \delta) K_{t-1}$$

Thus, E_t corresponds to the NIPAs expenditure-side definition of GDP.

We now present analytical results pertaining to each side of the labor market in turn. For labor demand we inquire specifically into how changes in redistribution can affect aggregate labor demand through changes in aggregate expenditures, in line with standard Keynesian intuition. In particular, we examine the potential role for demand externalities and sticky prices. The key result is that if household consumption only affects labor demand through its impact on aggregate expenditures, redistribution cannot have an impact on output if it does not shift labor supply as well. Redistribution will, however, have an effect on output if labor demand depends on the distribution of expenditures across goods or households. As it turns out, standard *New-Keynesian* assumptions about monetary policy rules do yield such feedback, so that the question about whether labor demand or labor supply effects dominate becomes quantitative in nature.

Afterwards we turn to labor supply, where we isolate the role of savings decisions as well as consumption/leisure trade-offs in determining the response of aggregate labor supply to redistribution. The key result is that the response of aggregate labor supply depends fundamentally on how the marginal propensity to supply labor varies with wealth. While under standard calibrations the nature of household savings decisions under incomplete markets is likely to make labor supply of wealth-poor households more sensitive to changes in wealth, this can potentially be countervailed by intratemporal allocation concerns if leisure is more wealth elastic than consumption. We finish the section with a discussion of the impact of redistribution when there are indivisibilities in la-

¹²We are grateful to an anonymous referee for helping us organize our discussion along these lines. Our exposition in the introduction to this section very closely follows his description of our framework in his referee report.

bor supply. We leave an evaluation of the relative role of the different effects for the quantitative analysis in Section 4.

3.1 Redistribution and Aggregate Labor Demand

The next subsection analyzes the benchmark “neo-classical” case, where labor demand is determined in a frictionless manner. The result that redistribution can have a short-run output impact only through labor supply will follow as one would expect, but the details of the argument help set the stage for the subsequent subsections. We then analyze the case featuring an externality through which aggregate expenditures shift labor demand, but where prices remain flexible. Finally, we consider the case with sticky prices.

3.1.1 The “neo-classical” benchmark

In the “neo-classical” benchmark there are no frictions in the production side of the model. In particular, there are no nominal rigidities. Absent nominal rigidities, i.e., when $\lambda = 1$, all intermediate-goods producers choose the same price $p_1(i) = w_1$ (recall that the subsidy keeps them from charging a markup), so that the price index at date 1 satisfies: $P_1 = \left[\int_0^1 p_1(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} = w_1$. It thus follows from the first order condition for the final-goods producer (7) that $w_1 = F_M(K_0, M_1)$, with M_1 being previously defined by $M_1 = \left[\int_0^1 m_1(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$. Also, it is the case that with price flexibility, the intermediate input composite exactly equals aggregate labor input, i.e., $M_1 = N_1$. Thus, we see that labor demand $N_1^d(w, \vartheta_1^d)$ is implicitly defined by the firms’ first-order condition for labor input:

$$w_1 = F_M(K_0, N_1^d)$$

Note that the only variable other than wages entering the determination of labor demand is K_0 , i.e., $\vartheta_1^d = K_0$. In equilibrium, $N_1^s(w, \vartheta_1^s) = N_1^d(w, K_0)$. Since K_0 is predetermined, any changes in labor input, and therefore, in production levels, can only occur through changes in determinants of labor supply *other* than wages, i.e., through changes in the elements of ϑ_1^s operating through $N_1^s(w, \vartheta_1^s)$. This means, in particular, that given the knowledge of how labor supply changes with the wealth distribution, additional knowledge of how individual consumption changes with wealth is irrelevant. This is a clear sense of the “centrality” of labor supply.

The preceding analysis has an immediate implication: if, following redistribution, labor supply remains fixed at the level it took at the steady state wage, and equilibrium is unique, redistribution will not have any effect on output or wages.¹³ To see this, notice first that we can express the condition just stated as requiring that $N_0^s(w_0, \vartheta_0^s) = N_1^s(w_0, \vartheta_1^s)$, namely that labor supply functions at $t = 1$ and $t = 0$ coincide at w_0 . Next, note that although it would be an important special case, we are *not* imposing the restriction that labor supply be independent of variables other than wages

¹³The unique equilibrium condition recognizes the possibility that under multiple equilibria the equilibrium selection mechanism might respond to the wealth distribution.

(i.e., we allow for the possibility that ϑ_t^s can be non-empty and that it can change from $t = 0$ to $t = 1$). With this in hand, notice when $w_1 = w_0$, firms' demand functions for labor remain satisfied upon impact of redistribution, because $w_1 = F_M(K_0, N_1^s(w_0, \vartheta_1^s)) = F_M(K_0, N_0^s(w_0, \vartheta_0^s)) = w_0$.

It is also the case that, in equilibrium, $r_1 = F_K(K_0, N_1^s(w, \vartheta^s)) - \delta$ and, in the frictionless case, $\pi_1 = 0$.¹⁴ Lastly, in any equilibrium aggregate expenditures equal output, so that:

$$E_1 = F(K_0, N_1), \tag{11}$$

Thus, if $N_0^s(w, \vartheta_0^s) = N_1^s(w, \vartheta_1^s)$, $E_1 = E_0$.

We collect these findings above in the following proposition:¹⁵

Proposition 1 *Consider the impact of a one-off and unexpected wealth redistribution. If there are (i) no nominal rigidities ($\lambda = 1$), and (ii) the quantity of labor supplied does not change on impact at the steady-state wage rate ($N_1^s(w_0, \vartheta_1^s) = N_0^s(w_0, \vartheta_0^s)$), then as long as the equilibrium is unique, aggregate expenditures cannot change on impact ($E_1 = E_0$), and neither can the wage rate ($w_1 = w_0$).*

This proposition provides an important benchmark, since some commonly used assumptions about preferences and/or production opportunities yield constant labor supply as an equilibrium outcome. The clearest example is of course the set of preferences proposed by Greenwood et al. (1988), whereby labor supply is solely a function of current wages (i.e., we can drop ϑ_1^s from N^s function). Another example, which we discuss in Section 3.2.3, is the case of Cobb-Douglas preferences, where aggregate investment is fixed in the short run. In that case, *equilibrium* labor supply at $t = 1$ is equal to its steady-state value because of the countervailing effects of two elements of ϑ_1^s : the wealth distribution and the interest rate.

Proposition 1 thus shows, in very stark terms, the limitations of focusing on the behavior of consumption alone in order to understand the short-run effects of redistributive policies in the class of environments without nominal rigidities. This is because it does not require any assumption about the equilibrium behavior of consumption. In particular, in a setup without nominal frictions, given an understanding of how labor supply changes, knowledge about the distribution of marginal propensities to consume out of wealth is completely uninformative about the aggregate impact of the redistribution. Also, the lack of short-run reaction of output and labor input to the redistribution is true even though the future path of equilibrium interest rates and wages can potentially change significantly.

The proposition also implies that, absent a labor supply effect, any boom in consumption engineered by a redistributive policies translates must be accompanied by a *reduction* in other

¹⁴It is also straightforward to see that, consistent with the general result that monetary policy is irrelevant in the absence of nominal rigidities, equilibrium variables are determined independently of the price level Q_t .

¹⁵The proposition is related to Proposition 2 in Mehrotra (2014). It is a slight generalization in that we do not restrict ourselves to GHH preferences, admittedly the simplest way to satisfy the condition that equilibrium labor input does not change on impact. As discussed in detail in Section 3.2.3, another case where the proposition holds is for Cobb-Douglas preferences if aggregate investment is fixed in the short run.

components of aggregate expenditures. This is an important point because it means that, holding government spending fixed—e.g., absent any compensating reductions in government expenditures— a redistributive policy that leads to a boom in consumption will also lead to a reduction in investment and, therefore, in longer-run output and consumption possibilities as the capital stock shrinks.

In the class of environments we have examined thus far, wealth redistribution cannot affect labor demand in the short run, leaving labor supply to be the sole source of output fluctuations arising from redistributions. We now extend the model to allow labor demand to respond to changes in the state of the economy in two different ways: by allowing for “aggregate demand externalities” whereby an increase in aggregate expenditures matters, and by allowing for nominal rigidities, so that firms pre-commit to adjusting supply in response to changes in aggregate expenditures, even if this results in them being “off their labor demand curve.”

3.1.2 Expenditure Externalities

We now turn to the case where expenditures carry direct implications for labor demand. For notational and conceptual clarity and convenience, we will assume for much of the exposition that ϑ_t^s is empty, so that we can write labor supply simply as $N^s(w_t)$. One leading case where this will be obtained is if preferences are of the form proposed by Greenwood et al. (1988). Thus, the calculations that follow will examine the possibility of redistribution affecting output through labor demand only.

We first focus on the case with flexible prices ($\lambda = 1$), so that $M_t = N_t$ and $P_t = w_t$. In cases where aggregate expenditures have a direct impact on labor demand, we can write the latter as:

$$N_t^d = N^d(w_t, E_t, K_{t-1}). \quad (12)$$

There are different setups that can lead to such a labor demand function. One is if demand externalities introduce a wedge between the marginal product of labor and the wage rate, as in Blanchard and Kiyotaki (1988). Another is if demand externalities affect production directly by increasing or mitigating some distortion.¹⁶

In any closed economy, we will have that, in equilibrium

$$E_t = F(N_t, K_{t-1}; E_t), \quad (13)$$

where, in order to accommodate one of the possible channels through which aggregate expenditures can affect labor demand, we introduce it as an argument into the production function. Note that here we write the equilibrium labor input N_t as an argument in the production function rather than labor demand or labor supply. In equilibrium, of course, the three are the same.

¹⁶In the “wedge” case, we have $w_t = \varphi(E_t) F_M(K_{t-1}, N_t^d)$ for some φ increasing in E_t . The equilibrium conditions are extended accordingly. In particular, profits are in this case $\pi_t = F(K_{t-1}, N_t) - w_t N_t - (r_t + \delta) K_{t-1}$, which can be different from zero.

The alternative is to have expenditures affect the production function directly, so that $Y_t = F(K_{t-1}, N_t, E_t)$. See, for example, Bai et al. (2012) for a setup where demand shocks affect both aggregate expenditures and TFP.

In equilibrium, output, expenditures, labor input, and wages are determined by the system of equations determined by (12), (13), labor supply (which under GHH preferences is given by $N_t^s = N^s(w_t)$), and by the labor market clearing condition $N_t = N_t^s = N_t^d$. At $t = 1$, given K_0 , this is a system of five equations in five unknowns, N_1 , N_1^s , N_1^d , E_1 , and w_1 . Note that all the equations are identical to the ones prevailing in steady state, so that there is an equilibrium with $N_1 = N_0$ and $E_1 = E_0$.

We can easily extend the discussion above to incorporate the case in which labor supply is a function of other variables and collect the results to get the following proposition:

Proposition 2 *Consider the impact of a one-off and unexpected wealth redistribution in the extended model described in this section. Suppose $\lambda = 1$, and the equilibrium reaction is such that aggregate labor supply does not change on impact at the initial wage ($N_1(w_0, \vartheta_1^s) = N_0(w_0, \vartheta_0^s)$). Then, if the equilibrium is unique, aggregate expenditures do not change on impact ($E_1 = E_0$) and neither does the wage rate ($w_1 = w_0$).*

Note that Proposition 2 does not preclude the possibility that demand externalities may play an amplifying or dampening role if the conditions are violated and labor supply does shift. In this framework with endogenous labor demand, labor supply is central because labor demand shifts are completely dependent on shifts in labor supply.

Proposition 2 is also interesting in that it can be used as a guide for alternative assumptions, which may lead externalities to matter even if labor supply is not shifted. For example, results could differ if externalities were only a function of aggregate consumption or of aggregate investment, or, say, of the *distribution* of consumption across households or across goods. In what follows, we show that a similar logic applies to a sticky price environment, in which labor demand is similarly distorted, and in which the distortion can be expressed as a function of the monetary policy rule.

3.1.3 Nominal Rigidities

Nominal rigidities provide a particular and, arguably, the most commonly invoked mechanism through which labor demand can depend on factors other than current wages and pre-determined capital stock. To see this, consider the generalized labor demand function obtained for the case with nominal rigidities (i.e., the labor demand equation with the labor supply function substituted in):¹⁷

$$F_M \left(K_0, \kappa(\Delta_1) N_1^d \right) = \varphi(\Delta_1) w_1, \quad (14)$$

where $\Delta_1 = \frac{w_0 Q_0}{w_1 Q_1}$ is a measure of the relative price distortion at date 1 arising from nominal

¹⁷To obtain equation (7), note that from equations (10) and (9), $p_1(i) = \frac{Q_0 w_0}{Q_1}$ for the fraction $1 - \lambda$ firms setting their prices before the shock and $p_1(i) = w_1$ for the fraction λ setting after the shock. It follows that the price index for intermediate inputs is $P_1 = \left[\lambda w_1^{1-\rho} + (1 - \lambda) \left(\frac{w_0 Q_0}{Q_1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$. Also, from manipulating the first order condition for the final goods producers (8) one can obtain the function κ .

rigidities, as measured by the relative price of intermediate inputs with sticky prices versus those with flexible prices in $t = 1$, and where

$$\kappa(\Delta_1) = \frac{\left[\lambda + (1 - \lambda)(\Delta_1)^{1-\rho}\right]^{-\frac{\rho}{1-\rho}}}{\lambda + (1 - \lambda)(\Delta_1)^{-\rho}},$$

and

$$\varphi(\Delta_1) = \left[\lambda + (1 - \lambda)(\Delta_1)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

In the last two expressions, respectively, κ denotes the distortion in labor input stemming from the difference in the prices of different goods, and φ is the ratio between wages and the price index for intermediate inputs. It is straightforward to verify that $\kappa(1) = 1$ and $\kappa'(1) = 0$. Equation (14) induces a labor demand function of the form:

$$N_1^d = N_1^d(w_1, \Delta_1, K_0).$$

New Keynesian models need to be closed by a monetary policy rule pinning down Δ_1 . It is immediate that if monetary policy aims to stabilize the distortion Δ_1 , then redistribution can only affect output on impact through its impact on labor supply. Also, if Δ_1 is only a function of aggregate expenditures, E_1 , as in conventional Keynesian intuition, we are back to the environment described in Section 3.1.2 and Proposition 2 applies.

Another instance where redistribution can only affect output through labor supply is if the monetary authority seeks price stability, setting $Q_1 = Q_0$. In that case, we have that $\Delta_1 = \frac{w_0}{w_1}$. Therefore, if real wages do not change, there is no change in the relative price distortion term Δ_1 and labor demand equals what is obtained under the neoclassical benchmark. Following analogous steps to sections 3.1.1 and 3.1.2 above, it follows that if aggregate labor supply does not change on impact ($N_1^s(w_0, \vartheta_1) = N_0(w_0, \vartheta_0)$), then neither do wages:

Proposition 3 *Consider the impact of a wealth redistribution program that is one-off and unexpected. Suppose $Q_1 = Q_0$. If the equilibrium reaction is such that aggregate labor supply does not change on impact ($N_1^s(w_0, \vartheta_1) = N_0(w_0, \vartheta_0)$) and if the equilibrium is unique, aggregate expenditures do not change on impact ($E_1 = E_0$) and neither does the wage rate ($w_1 = w_0$). In particular, the result holds if the monetary policy rule is such that Q_1 is a function of E_1 only.*

While a monetary policy that pursues price stabilization is a plausible assumption for normal times, redistribution could have a short-run impact on output in the face of constraints on the monetary authority that force it to allow changes in the price level. Of particular interest are zero-lower-bound constraints, which force the monetary authority to keep nominal interest rates constant in the face of shocks. In order to capture some of the implications of such a constraint, we now follow Krugman (1998) and Eggertsson and Krugman (2012) among others and assume that the nominal

interest rate between period 1 and 2 $(1 + r_2) \frac{Q_2}{Q_1}$ is constrained to be equal to its steady state level $1 + r_0$. Furthermore, we assume that monetary authority achieves price stability so that the price level is fixed at some level Q_0 in steady state and for $t \geq 2$. Such a monetary policy framework implies that Q_1 increases proportionately with r_2 . In such an environment redistribution can have an impact on labor demand because different wealth distributions imply different equilibrium interest rates.

Given such a constraint on monetary policy, nominal rigidities will only matter quantitatively if r_2 is significantly sensitive to changes in the wealth distribution. As we will see, under our benchmark parametrization the results are quantitatively unaffected by the presence of nominal rigidities. The reason is that the short-run impact of the redistributive policy on the capital stock and on its marginal product is very small. In Section 4, we explore this issue in more detail through a variant of the main quantitative exercise depicted in Figure 1 featuring a higher degree of concavity for capital in the production function than in the baseline Cobb-Douglas case.

3.2 Redistribution and Aggregate Labor Supply

Given the importance of aggregate labor supply movements highlighted in Section 3.1, we now examine more closely how wealth redistribution affects labor supply decisions. In particular, we examine which assumptions change aggregate labor supply, as well as the direction of the implied changes. As we will show, a key driving force is how marginal propensities to work correlate with wealth across the population. We locate conditions under which these correlations are positive, negative, or zero.

Intuitively, one can imagine recipients from the redistribution program dividing a given dollar received between increased savings, increased consumption, and increased leisure (and symmetrically for contributors). For given prices, redistribution can have an aggregate impact on labor supply if recipients and contributors to the redistribution program differ systematically on the fraction that they dedicate to increased leisure. This, in turn, is a function both of how much of the additional resources households save and of how they divide the remainder between consumption and leisure. Accordingly, for the analysis that follows, it is convenient to separate intertemporal decisions made by households over consumption and leisure from intertemporal decisions over wealth accumulation, which we do next.

3.2.1 Separating the determinants of labor supply

In order to separate the various determinants of labor supply, we start by defining z to be the sum of the household's expenditures on goods *and* leisure. Thus,

$$z \equiv c + w\varepsilon(s) (\bar{l} - l).$$

The problem of the household can be written as

$$\begin{aligned}
V_t(a, s) &= \max_{a', z} \tilde{u}(z; w_t, s) + \beta \sum_{s' \in \mathcal{S}} \Pr[s'|s] V_{t+1}(a', s') \\
s.t. &: \sum_{s' \in \mathcal{S}} q_{t,s'}(s) b'_{s'} + b'_f + z = w_t \varepsilon(s) \bar{l} + (1 + r_t) a + \omega(s) + \pi_t + x_t(a), \\
& a'_s = b'_s + b'_f, a \geq \underline{a},
\end{aligned}$$

with $\tilde{u}(\cdot)$ the value function obtained from the within-period problem

$$\begin{aligned}
\tilde{u}(z; w, s) &= \max_{c, l} u(c, \bar{l} - l) \\
s.t. &: c + w \varepsilon(s) (\bar{l} - l) = z.
\end{aligned}$$

In addition to the usual policy and value functions, the problem above also yields the policy function for z , $z_t(a, s)$. Aggregate expanded consumption expenditures therefore are given as:

$$Z_t = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} z_t(a, s) \Gamma_{t-1}(a, s) da$$

Finally, we can write policy functions for consumption and hours coming from static sub-problem as:

$$\begin{aligned}
c &= c^{static}(z, s, w) \\
l &= l^{static}(z, s, w)
\end{aligned}$$

Naturally, it is the case that $c_t(a, s) = c^{static}(z_t(a, s), s, w)$ and $l_t(a, s) = l^{static}(z_t(a, s), s, w)$. The alternative, static, policy functions are useful in that they clarify that within this setup, the choice of consumption and labor supply only depend on the aggregate state through wages w_t and through whatever its impact is on the choice of expanded consumption expenditures z .

3.2.2 No Wealth Effects on Labor Supply

One immediate implication from the decomposition above is that labor supply can only change in response to a redistribution if $l^{static}(z, s, w)$ depends on expanded consumption z . It then follows immediately from Proposition 1 that wealth redistribution upon impact keeps labor supply unchanged. The leading case where $l^{static}(z, s, w)$ does not depend on z is under preferences postulated by Greenwood et al. (1988). Under those preferences,

$$u(c, \bar{l} - l) = h(c + g(\bar{l} - l)),$$

where h is increasing and concave. At any t , the optimality condition for labor supply in the static problem satisfies

$$g'(\bar{l} - l_t^{static}(z, s; w)) = \varepsilon(s) w,$$

so labor supply depends, at the individual level, only on the current effective wage rate $\varepsilon(s) w$. Thus,

$$N_0(w_0) = N_1(w_0) = \sum_{s \in \mathcal{S}} \varepsilon(s) (\bar{l} - g'^{-1}(\varepsilon(s) w_0)) \Gamma_0^s(s),$$

where $\Gamma_0^s(s)$ is the marginal distribution of exogenous idiosyncratic productivity states s at time 0. Therefore, in this case, we have that $N_1(w) = N_0(w)$ for all w and, in particular, for $w = w_0$. The result highlights the importance of wealth effects on labor supply in determining the aggregate short-run impact of wealth redistribution.

3.2.3 Wealth Effects on Labor Supply

We now examine the effect of redistribution in the presence of wealth effects on labor supply. We start with the benchmark case of Cobb-Douglas utility within periods, and then we proceed to examine more general cases.

Cobb-Douglas Utility We will define a utility function to be “Cobb-Douglas within the period” if it is of the form

$$u(c, \bar{l} - l) = h(c^{1-\mu} (\bar{l} - l)^\mu), \quad \mu \in (0, 1),$$

where h is an increasing and strictly concave function. The utility function is, of course, fairly standard. In particular, it is widely used in the business cycle literature, as some of its particular cases are compatible with a balanced growth-path for the economy. Also, it is featured in heterogeneous agent models such as Krusell and Smith (1998).

Given Cobb-Douglas preferences, we have that expenditures on consumption and leisure are a constant fraction of z , so that:

$$\begin{aligned} c^{static}(z, s; w) &= (1 - \mu) z, \\ l^{static}(z, s; w) &= \bar{l} - \frac{\mu}{w\varepsilon(s)} z. \end{aligned}$$

Thus, for a given wage, any shock that leads a household to increase its consumption will also lead it to decrease its labor supply. More generally, any change in the environment that leads households to reduce savings (and, therefore, increase z), will also induce them to increase consumption and reduce labor supply.

This same logic translates to aggregate quantities. To see this, first integrate $w_t \varepsilon(s) (\bar{l} - l_t(a, s)) = \mu z_t(a, s)$ across households to get

$$N_t = \bar{N} - \mu \frac{Z_t}{w_t}, \quad (15)$$

$$C_t = (1 - \mu) Z_t, \quad (16)$$

where $\bar{N} \equiv \bar{l} \sum_{s \in \mathcal{S}} \Gamma_0^s(s) \varepsilon(s)$. Thus, given the wage rate, labor supply is *decreasing* in the amount of wealth allocated for current period consumption. Second, aggregating household budget constraints yields the following relationship between aggregate savings A_t and aggregate expanded consumption Z_t :

$$A_t = w_t \bar{N} + (1 + r_t) A_{t-1} - Z_t.$$

It follows that effective hours worked are increasing in aggregate savings. Hence, any change in the economic environment that leads to a reduction in aggregate savings, will, for a given wage, lead at the same time to an increase in aggregate consumption and a reduction in aggregate effective hours worked.

In particular, if prices are fully flexible and aggregate savings do not change (for example because asset supply is fixed, as in Guerrieri and Lorenzoni (2011), or adjustment costs to capital are extremely large), Proposition 1 holds, and neither aggregate labor supply, aggregate consumption, nor aggregate expenditures change. More generally, given decreasing returns to labor input, under price flexibility any redistribution that increases aggregate consumption C_t , will also reduce effective labor supply. We state the result in the following Proposition:

Proposition 4 *If $\lambda = 1$, $Z_1 = Z_0$, $u(c, \bar{l} - l) = h(c^\kappa (\bar{l} - l)^{1-\kappa})$, then $w_1 = w_0$, $N_1 = N_0$, and $C_1 = C_0$. If $Z_1 > Z_0$, then $C_1 > C_0$ and $N_1 < N_0$, with the inequalities reversed if $Z_1 < Z_0$.*

Note that with Cobb-Douglas preferences, it is *not* necessarily the case that wealthier households consume more leisure, since what is being kept constant are expenditures in leisure as a fraction of wealth. If wealthier households are also more productive (so that $\varepsilon(s)$ is higher), they can conceivably supply much more labor time than less wealthy households. The reason effective labor supply stays constant is that aggregate effective hours worked is the sum of individual labor supply weighted by productivity $\varepsilon(s)$.

Proposition 4 implies that Cobb-Douglas preferences yield a particular kind of aggregation in consumption and labor-supply decisions. A natural question then is: how stringent is the Cobb-Douglas requirement? In classical demand theory, homothetic preferences are typically sufficient to allow for aggregation of household choices, meaning that wealth redistribution does not change outcomes. The reason this is not sufficient in the current setting is because different households face effectively different wages. If the elasticity of substitution between consumption and leisure is not equal to 1, this implies that high-productivity households consume a different share of their

expanded consumption, z , than low-productivity households, so that redistribution between them will have an impact on aggregate effective labor supply.

Small Redistributions with General Preferences We now turn to the impact of redistributive policies for general preferences. While all preceding results hold for any wealth redistribution, we now focus on redistributions that transfer from the wealthy to the poor, that is, that feature an $x_1(a)$ that is monotonically decreasing in a . In general, the impact of the redistribution on effective labor supply is

$$N_1 - N_0 = \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} (\varepsilon(s) (l_1(a, s) - l_0(a, s))) \Gamma(a, s) da.$$

In the case of small redistribution policies, we will show that the various (competing) ways in which wealth redistribution can affect labor supply can be clearly separated. Restricting attention to policy functions that are differentiable with respect to all states and prices, and defining “small” redistributions to be those for which we can approximate the policy function $\phi_t \left(a + \frac{x_1(a)}{1+r_1}, s \right)$ by $\phi_t(a, s) + \frac{\partial \phi_t(a, s)}{\partial a} \frac{x_1(a)}{1+r_1}$, we have that

$$\begin{aligned} & \varepsilon(s) l^{static} \left(z_0 \left(a + \frac{x_1(a)}{1+r_1}, s \right), s; w_0 \right) - \varepsilon(s) l^{static} (z_0(a, s), s; w_0) \\ \cong & \underbrace{\frac{x_1(a)}{1+r_1}}_{(+/-)} \underbrace{\frac{\partial z_0(a, s)}{\partial a}}_{(+)} \underbrace{\frac{\partial \varepsilon(s) l^{static}(z(a, s), s; w_0)}{\partial z}}_{(-)}. \end{aligned} \quad (17)$$

with the signs in brackets indicating the likely direction of change in the corresponding terms. The expression above denotes the change in labor supply for a given household, holding prices fixed, and allowing only its wealth to change. It therefore captures the partial equilibrium component of a household’s reaction. We denote the expression in equation (17) by $\mathcal{P}_1(a, s)$

The expression (17) has three components. The first one is the wealth transfer itself, normalized by the interest rate, so that it is in the same units as beginning-of-period assets. Given revenue neutrality, $x_1(a)$ is negative for very wealthy households and positive for less wealthy ones. Together, the two last terms give the household’s marginal propensity to work out of financial wealth. The term $\frac{\partial z_0(a, s)}{\partial a}$ denotes the effect of increased wealth on “expanded consumption” z . This is, of course, inversely related to the marginal propensity to save out of wealth. It is positive if both consumption and leisure are normal goods, since an increase in wealth relaxes the intertemporal budget constraint of the household. The final component is the static wealth effect on *effective* labor supply. It is negative so long as leisure is a normal good. Overall, the partial equilibrium component is negative for households that are on the receiving side of the redistribution policy and positive for the households that are on the contributing side.

Changes in prices affect labor supply directly, through current wages w_1 , and indirectly, through changes in the function $z(a, s)$, from $z_0(a, s)$ in the initial steady state to $z_1(a, s)$ in $t = 1$. The

general equilibrium component can therefore be written as

$$\begin{aligned}
& \varepsilon(s) l^{static}(z_1(a, s), s; w_1) - \varepsilon(s) l^{static}(z_0(a, s), s; w_0) \\
\cong & \sum_{v=1}^{\infty} \left(\frac{\partial z_0(a, s)}{\partial r_v} dr_v + \frac{\partial z_0(a, s)}{\partial w_v} dw_v \right) \frac{\partial \varepsilon(s) l^{static}(z_0(a, s), s, w_1)}{\partial z} \\
& + \frac{\partial \varepsilon(s) l^{static}(z_0(a, s), s, w_1)}{\partial w_1} dw_1.
\end{aligned} \tag{18}$$

The component in parenthesis summarizes whose savings decisions are affected by the path of prices over time. Those changes multiply the static wealth effect on labor supply. Additionally, current wages exert direct influence on labor supply for any given level of savings. We denote the expression in equation (18) by $\mathcal{G}_1(a, s)$. Collecting all terms, it follows that, to a first-order approximation, the change in effective labor following a redistribution can be denoted by

$$N_1 - N_0 \cong \sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} (\mathcal{P}_1(a, s) + \mathcal{G}_1(a, s)) \Gamma(a, s) da.$$

We will from this point onward focus our analysis on the determinants of the partial equilibrium component $\sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} \mathcal{P}_1(a, s) \Gamma(a, s) da$. This ought to be informative about the overall impact of redistribution on labor input so long as the term is large relative to the general equilibrium component. Furthermore, in the quantitative analysis in Section 4 we verify that the intuitions derived from this analysis remain intact in the calibrated GE environments.

Before proceeding, we state a proposition linking the shape of the marginal propensity to work out of wealth $\frac{\partial z_0(a, s)}{\partial a} \frac{\partial \varepsilon(s) l^{static}(z(a, s), s, w_0)}{\partial z}$ to the partial equilibrium component of the change in N . Let $\Gamma_0(s|a)$ denote the proportion of households with exogenous productivity s given wealth a . Then, the following proposition holds:

Proposition 5 *Given a “small” wealth redistribution with $x_1(a)$ decreasing in a , the partial equilibrium component of its impact on aggregate effective labor supply is given by $\sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} \mathcal{P}_1(a, s) \Gamma(a, s) da$. Suppose, moreover, that $\frac{\partial z_0(a, s)}{\partial a} \geq 0$ and $\frac{\partial \varepsilon(s) l^{static}(z(a, s), s, w_0)}{\partial z} \leq 0$. This component is positive if $\sum_{s \in \mathcal{S}} \frac{\partial z_0(a, s)}{\partial a} \left| \frac{\partial \varepsilon(s) l^{static}(z(a, s), s, w_0)}{\partial z} \right| \Gamma_0(s|a)$ is increasing in a .*

The proposition states that the partial equilibrium component of a redistribution is positive if, after taking the average across productivity levels, the labor supply of high-wealth households is relatively more sensitive to their wealth. Intuitively, if this is the case, high-wealth households will increase their effective labor supply more in response to a dollar contributed than a low-wealth household will reduce theirs in response to that same dollar received. This is the analogue to the conventional intuition about how heterogeneity in marginal propensities to consume influence the aggregate consumption response to redistribution. Note that, given the policy functions z_0 and l^{static} , the partial equilibrium component does not depend on the specifics of financial markets: we can calculate it whether there are complete markets, partial insurance or no insurance cases.

We can extend Proposition 5 to a more general case where transfers are a function both of wealth and productivity. This accommodates the empirically relevant case of income based wealth transfers. In that case we have:

Proposition 6 *Let where $MPL(a, s) \equiv \frac{\partial z_0(a, s)}{\partial a} \frac{\partial \varepsilon(s)^{l^{static}}(z(a, s), s, w_0)}{\partial z}$ be the marginal propensity to work given state (a, s) . Given a “small” wealth redistribution with $x_1(a, s)$ with $\bar{x}(a) \equiv \sum_{s \in \mathcal{S}} x(a, s) \Gamma(s|a)$ decreasing in a , the partial equilibrium component of its impact on aggregate effective labor supply is given by $\sum_{s \in \mathcal{S}} \int_{a \in \mathcal{A}} \mathcal{P}_1(a, s) \Gamma(a, s) da$. Suppose, moreover, that $\frac{\partial z_0(a, s)}{\partial a} \geq 0$ and $\frac{\partial \varepsilon(s)^{l^{static}}(z(a, s), s, w_0)}{\partial z} \leq 0$. This component is positive if $\sum_{s \in \mathcal{S}} \left[\frac{\partial z_0(a, s)}{\partial a} \left| \frac{\partial \varepsilon(s)^{l^{static}}(z(a, s), s, w_0)}{\partial z} \right| \Gamma_0(s|a) + cov \left(\frac{x(a, s)}{\sum_s x(a, s) \Gamma(s|a)}, MPL(a, s) | a \right) \right]$ is increasing in a .*

Compared to Proposition 5, Proposition 6 adds two requirements. The first is that even if transfers differ across households with the same wealth, on average they decline with wealth. This will naturally hold in a setup where productivity and wealth are highly correlated and transfers decline with productivity. The second requirement is that the covariance term $cov \left(\frac{x(a, s)}{\sum_s x(a, s) \Gamma(s|a)}, MPL(a, s) | a \right)$ does not decline too quickly with a . That covariance term will be very small if the marginal propensity to work depends little on productivity or if there is a high correlation between labor productivity and wealth. Under those conditions there will be little variance in $\frac{x(a, s)}{\sum_s x(a, s) \Gamma(s|a)}$, as most of the mass will be concentrated in a specific s for given wealth. We now proceed to discuss each of the terms determining the marginal propensity to work separately.

Intertemporal Determinants of Aggregate Labor Supply The first component determining the marginal propensity to work, $\frac{\partial z_0(a, s)}{\partial a}$, denotes how wealth affects the intertemporal choices made by the household. This is because, given the budget constraint, this term is symmetric to the change in wealth accumulation decision, so that $\frac{\partial z_0(a, s)}{\partial a} = 1 + r_1 - \frac{\partial a'_0(a, s)}{\partial a}$. Therefore, $\frac{\partial z_0(a, s)}{\partial a}$ is increasing in a if the policy function for asset accumulation is concave in a .

In our quantitative work, we will focus on the incomplete markets case (without Arrow securities). The literature on incomplete market models points to a strong presumption that $a'_0(a, s)$ ought to be convex, so that $\frac{\partial z_0(a, s)}{\partial a}$ is decreasing in a . The reason, emphasized by Gourinchas and Parker (2002) is that households with liquid wealth in small amounts engage in buffer-stock savings behavior, going through a lot of effort to save to self-insure against income risk. As their liquid wealth grows, they are able to self-insure without having to consume so little and work so hard, so they increase their expenditures and reduce their hours worked very quickly. For households with a lot of liquid wealth, marginal changes in their wealth do not affect their ability to self-insure. Thus, their main reason for saving is for intertemporal smoothing, saving more out of a given dollar received. A second mechanism at work here is that wealth-poor households are closer to binding borrowing constraints, which allows the wealth transfer to immediately relax the constraint and hence boost current consumption.¹⁸ To the extent that this logic carries through to a world with two goods (consumption and leisure), this implies that households with a large amount of liquid

¹⁸We thank an anonymous referee for emphasizing the first force.

wealth will not change their consumption and labor supply much in response to a one-off, temporary change in their wealth. Thus, redistribution from wealth-rich to wealth-poor households is likely to lead to a boom in aggregate consumption and a drop in aggregate effective labor supply.

Given the consensus in the literature that $a'_0(a, s)$ is convex, Proposition 5 suggests that wealth redistributions can very plausibly lead to a boom in consumption at the same time as they lead to a drop in effective hours worked. One case where this holds is the case discussed in the beginning of this section, when preferences are Cobb-Douglas.¹⁹ In that case, the intratemporal component $\frac{\partial \varepsilon(s)l^{static}(z(a,s),s,w_0)}{\partial z} = -\frac{\mu}{w_0}$, so that it does not vary across households, and the partial equilibrium component of the change in labor supply is entirely determined by the shape of $\frac{\partial z_0(a,s)}{\partial a}$. We now turn to a more detailed discussion of which features of preferences (and also the tax-code) determine the variation $\frac{\partial \varepsilon(s)l^{static}(z(a,s),s,w_0)}{\partial z}$.

One caveat to the discussion above is that, whereas we simplify the model to allow all wealth to be liquid, in reality it has been shown both empirically (Johnson, Parker and Souleles, 2006) and theoretically (Kaplan and Violante, 2014; Huntley and Michelangeli, 2014) that it is not the total wealth THAT IS the main driver of the high MPC, but the amount of liquid wealth in portfolio. In fact, the presence of liquidity constraints is the leading explanation for the large observed consumption responses to transitory shocks. Of course, this form of wealth may well not line up cleanly with conventional measures of wealth that are broader. In order to focus as cleanly as possible on the dynamics created by wealth changes, and without conflating our analysis with issues related to liquidity, we study a setting with a single asset. In the end, however, such a restriction means that the model counterpart to wealth in the data is most closely related net worth. In terms of the quantitative results, the presence of wealthy households with high MPC's could lead to a more stimulative impact of a total wealth-based redistribution policy, to the extent that those households would be likely to be contributors to the redistribution policy.

Lastly, it is worth noting that even under complete markets, there is no guarantee that $\frac{\partial z_0(a,s)}{\partial a}$ will be constant with respect to a . For instance, if preferences over consumption and leisure do not follow the Gorman form, and if, as assumed in the quantitative section, there are non-linear taxes on capital income (as we will assume in the calibration of the model), $\frac{\partial z_0(a,s)}{\partial a}$ may vary with wealth, leading to non-trivial impacts of wealth redistribution.

Intratemporal Determinants of Aggregate Labor Supply The second component determining the marginal propensity to work, $\frac{\partial l_0^{static}(z,s;w)}{\partial z}$, denotes how changes in the amount of resources dedicated to current period expenditures, and hence to savings, affect labor supply. It, therefore, relates to the intratemporal decision that households make about how to allocate these resources between consumption and leisure. Such intratemporal considerations are important since, in typical calibrations, intratemporal preferences between consumption and leisure do not satisfy homotheticity. Notably, Pijoan-Mas (2006) has emphasized the fact that departure from homotheticity is necessary to explain the low observed cross-sectional correlation between wages and hours. A similar departure is present in the calibration used by Castaneda et al. (2003) and for

¹⁹This also holds for proportional income taxes. See Lemma 1.

the experiment in Figure 1.

We characterize the shape of $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ in terms of the share of expenditures dedicated to leisure, $\mu(z; s, w) = \frac{w\varepsilon(s)(\bar{l} - l^{static}(z,s;w_t))}{z}$. This allows us to connect the cross-sectional variation in the static component of labor supply to classical demand theory, which emphasizes the role of variation of expenditure shares with wealth. We focus on cases in which μ is twice differentiable in z and denote by $\mu_z(z; s, w)$ and $\mu_{zz}(z; s, w)$ the first and second derivatives with respect to z . We have that:

Proposition 7 *Consider the partial equilibrium component of the impact of a wealth redistribution $x_1(a)$, with x decreasing in a . Then $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ increases with z if $\mu_z(z; s, w_t) > 0$ and $\frac{\mu_{zz}(z; s, w_t)z}{\mu_z(z; s, w_t)} > -2$ and it decreases with z if $\mu_z(z; s, w_t) < 0$ and $\frac{\mu_{zz}(z; s, w_t)z}{|\mu_z(z; s, w_t)|} < 2$.*

The proposition states that redistribution increases labor supply if leisure is a luxury good in the intratemporal problem, i.e., if for a given wage rate received by the household (given by $w\varepsilon(s)$), leisure increases with z more quickly than consumption (i.e., $\mu_z(z; s, w_t) > 0$). The bounds on the curvature of the share function $\mu(z; s, w)$ ensure that the intensity with which households of different wealth levels spend on leisure does not change too quickly with their wealth. It is important to notice that the luxury nature of a good is tied to the amount spent for given prices. Thus, it is entirely consistent with leisure being a luxury good that a typical wealthy household consumes a smaller quantity of leisure than a typical poor household, so long as they face different wages (since they have different productivities). For example, for a household facing a wage of \$100 per hour worked, working one hour less implies a much greater expenditure on leisure than for a household facing a wage of \$10 per hour.

The following corollary shows the proposition applies to the commonly used case of separable utility:

Corollary 1 *Suppose the utility function is:*

$$u(c, \bar{l} - l) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{\phi(\bar{l} - l)^{1-\psi}}{1-\psi},$$

with $\sigma > 0$, $\psi > 0$, and $\phi > 0$. Then, $\frac{\partial \varepsilon(s) l_0^{static}(z,s;w)}{\partial z}$ increases with z if $\psi < \sigma$ and decreases otherwise.

Thus, wealth transfers have a stimulative effect so long as the intertemporal elasticity of substitution for leisure ($\frac{1}{\psi}$) is larger than for consumption ($\frac{1}{\sigma}$). In the intratemporal problem of the household, this implies that leisure is a luxury good and consumption is a necessity. This relation between ψ and σ is exactly the one that Pijoan-Mas (2006) has argued for.

The conditions for the propositions above appear to hinge on properties of households' underlying preferences. However, it is important to note that in a more general setting with non-linear

taxes it is possible for the tax code to induce households to behave as if leisure were a luxury good even if underlying preferences are Cobb-Douglas. More formally, the following Lemma holds:

Lemma 1 *Suppose taxes are progressive, so that $\tau''(y) > 0$ and $u(c, \bar{l} - l) = h\left(c^\kappa (\bar{l} - l)^{1-\kappa}\right)$, with h increasing and concave. Suppose furthermore that wealth effects on labor supply are mild enough that $ra + w\varepsilon(s)l_0(a, s)$ is increasing in a . Then the ratio $\frac{w\varepsilon(s)(\bar{l}-l)}{z}$ increases with z .*

Thus progressive taxation can be a further factor leading to a more positive impact of redistributive policies.

3.2.4 Indivisible Labor

We have thus far focused on cases in which labor supply can be smoothly varied. However, households might be unable to adjust labor in such a smooth fashion. Therefore, we conclude the analytical section by examining the short-run effect of wealth redistribution when labor supply is indivisible. The extension of the heterogeneous agent model for this case has been examined by Alonso-Ortiz and Rogerson (2010) among others. In particular, Alonso-Ortiz and Rogerson (2010) show that labor supply functions are characterized by a threshold rule, such that for each exogenous productivity level, there is a cutoff level of wealth, $\bar{a}(s)$, such that labor supply is zero if wealth exceeds the threshold and maximal otherwise. The proposition below characterizes the partial equilibrium response of such an economy to a wealth redistribution.

Proposition 8 *Consider a “small” redistributive policy $x_1(a)$, with x decreasing in a . Suppose labor supply decisions are characterized by a threshold $\bar{a}(s)$ so that $l(a, s) = 0$ if $a > \bar{a}(s)$ and $l(a, s) = \bar{l}$ if $a \leq \bar{a}(s)$. Then, the partial equilibrium component of the change in effective labor supply is positive if and only if*

$$\sum_{s \in \mathcal{S}} \varepsilon(s) x(\bar{a}(s)) \Gamma(\bar{a}(s), s) > 0.$$

Therefore, output increases if the workers at threshold points are, on average, net contributors to the redistribution, where the average is weighted by how much they receive and by their labor productivities. The result highlights that the direction of the impact of redistribution on output is highly sensitive to the details of the program and of the joint distribution of wealth and productivity.

4 Quantitative Analysis

Our analytical results highlight that the short-run effect of a wealth redistribution depends fundamentally on the specific factors that influence labor supply decisions, including properties of household savings behavior. Therefore, the sign and size of the overall impact can only be resolved with the use of a quantitative model. Given our focus on the short-run reaction of the economy to a one-off redistributive shock, it is essential that the initial state accurately captures observed

wealth heterogeneity in the U.S. economy. We therefore employ a baseline incomplete markets model that is specifically calibrated to match the wealth distribution in the United States along several dimensions, including the extreme concentration in its right tail. Furthermore, given our focus on the relationship between labor supply and wealth, we take the joint distribution of labor force participation and wealth as an additional target.

4.1 Parametrization

For quantitative realism, we generalize the household problem in Section 2 in several directions:

1. *Two-earner households*: Our generalized model allows for heterogeneity in the labor supply behavior across workers within a household. This is especially important in light of empirical work identifying a relatively high labor supply elasticity for the “second” earners (typically female in two-worker households, see for example Blundell, Pistaffferri, and Saporta-Eksten 2016).
2. *Indivisibility in labor supply*: We assume that primary earners can only choose to work full time or not at all, in line with the empirical literature, which has found very low elasticity of labor supply along the intensive margin for this group, while still allowing for non-trivial extensive margin decisions. The indivisibility also captures the notion that primary earners are likely to be in industries or occupations where employers are unwilling to let them work fewer hours in exchange for a pay cut.²⁰
3. *Unemployment*: We now assume that with some probability workers are subject to an “unemployment” shock that makes them completely unproductive. Furthermore, we assume that once a worker stops working (either by choice or because of an unemployment shock), he or she can only become employed again in the following period with some probability smaller than one.
4. *Means-tested social insurance*: Since the seminal work of Hubbard, Skinner and Zeldes (1994,1995), it has been known that means-tested insurance can substantially alter both savings behavior (and hence the proportion of households with little to no wealth) and labor supply.
5. *Preference heterogeneity*: The model now allows for heterogeneity in individual labor elasticities within the household. Importantly, we exploit and discipline this preference heterogeneity by calibrating the model to match the observed cross-sectional relationship between wealth and labor supply in the cross-section (see, e.g., Mustre Del-Rio, 2014).

²⁰As a means to model the extensive margin, we also experimented with non-convexity in labor productivity, as in e.g. Rogerson and Wallenius (2009) and others. Specifically, we experimented with the assumption that workers cannot reduce their hours worked at a going wage below a certain point. This captures the notion, brought to our attention by a referee, that both salaried employees and wage workers cannot easily negotiate lower hours for a marginal cut in earnings. However, when calibrating the model, we find that the closest match to available data occurs when we assume that the lower limit is equal to zero.

6. *Income Taxes and Social Security*: We assume non-linear income taxes following Gouveia and Strauss (1994). We also allow for transfers to retirees and estate taxation. The government budget constraint is suitably modified to accommodate those, with as before government expenditures serving as the margin that is adjusted to ensure a non-explosive path for government debt.

Given these new elements, we now write the household problem as:

$$\begin{aligned}
V_t(a, l; s, d) &= \max_{c, h^f, h^m} \frac{c^{1-\sigma}}{1-\sigma} + \chi_m(d) (\bar{h} - h^m) + \chi_f(d) \frac{(\bar{h} - h^f)^{1-\psi}}{1-\psi} \\
&\quad + \beta \sum_{l'} \lambda^{l'}(h^f, h^m) E[V_{t+1}(a', l'; s', d) | s, l] \\
a' + c &= a + y - \tau(y) + \omega(a, y, s) + x_t(a) + \pi_t \\
y &= r_t a + w_t \varepsilon_m(s, d, l) h^m + w_t \varepsilon_f(s, d, l) h^f \\
\omega(a, y, s) &= \max\{0, \bar{c} - a - [y - \tau(y)] - \bar{\omega}(s)\} + \bar{\omega}(s) \\
h^m &\in \{0, 1\}
\end{aligned}$$

where now $l \in \{ee, eu, ue, uu\}$ captures the employment status of the household in the end of the period (eg. $l = ee$ means both members of the household are employed, whereas $l = eu$ means only the primary earner is employed, etc.) and d captures heterogeneity in preferences and in the income process, which we take as fixed for each household. We label the primary earner by m (“males”) and secondary earners by f (“females”).

For many of the parameters we adopt the parametrization on Castaneda et al. (2003). In particular we set the discount factor to 0.924 per year, the inverse of the intertemporal elasticity of substitution in consumption σ to 1.5, and the total endowment of labor equal to 3.2, so that working males work a little less than a third of their total endowment. Also, as in that paper, we extend the parametrization of the tax function in Gouveia and Strauss (1994), so that

$$\tau(y) = \alpha_0 \left[y - (y^{-\alpha_1} + \alpha_2)^{-\frac{1}{\alpha_1}} \right] + \alpha_3 y,$$

with the parameters α_0 , α_1 , α_2 , and α_3 equal to, respectively, 0.258, 0.768, 0.491 and 0.144 and set transfers to retirees $\bar{\omega}(s)$ equal to 0.7 (about 2/5 of median income). Castaneda et al. (2003) also allow for estate taxes. We follow their calibration, by assuming that estate taxes are 16%, with exemption level equal to about 10 times median income. The function $\lambda^{l'}(h^f, h^m)$ gives the probability of a transition in the unemployment status of the household conditional on their employment decisions. Having it as a function of labor supply decision captures the notion that deciding to offer zero hours of labor may not be easily reversed. The transitions in and out of retirement are parametrized so that working life is equal to 45 years, on average, and retirement 18 years.

For the income process, we adopt a variant of the parameterizations of Castaneda et al. (2003), since more standard parameterizations, e.g. Aiyagari (1994), normally miss the concentration present in the right tail of the distribution. Two salient features of the parameterizations in Castaneda et al. (2003) are as follows: First, the authors specify a labor productivity process designed to capture life-cycle movements in income. As such, they incorporate both retirement and the arrival of new cohorts through shocks that, respectively, turns the labor productivity of households to zero and that restore it to a positive value. When relating the quantitative results below to the analytical results in Section 3, it is therefore important to keep in mind that retirees, by definition, supply zero effective hours irrespective of wealth or wages. Thus, they behave like inframarginal households in the indivisible labor case described in Section 3.2.4. Second, in order to capture the extreme concentration of wealth in the U.S. economy, Castaneda et al. (2003) employ an extremely high and brief productivity state, which is reached with very low probability. We assume that the income processes for both male and female are perfectly correlated, so that the process applies to the household as a whole. We also assume female productivity is 23 percent smaller than that of males, as a way to capture the observed gender wage gap. We depart from Castaneda et al. (2003) by allowing for ex-ante heterogeneity in the income process. In particular, we assume that in each idiosyncratic state some households have half of the productivity assumed by Castaneda et al. (2003), some of the households have the exact same productivity, and some of the households have twice the productivity. Those differences in income process are fixed for each household. Therefore the household type d indexes both the household disutility and their income process.

We choose the exponent on the component of utility depending on the leisure of secondary earners (ψ) so that the average working female would face an elasticity of labor supply equal to 0.832, the value estimated by Blundell et al. (2016) when assuming separable preferences as we do. Finally, we pick transition probabilities for the unemployment state that are independent across members of the household. Consistent with our yearly calibration, we set the probability of finding a job given that a member of the household is unemployed to 28.24 percent.²¹ We calibrate the probability of being subject to an unemployment shock to match the fraction of workers who remain unemployed for more than 52 weeks after the Great Recession. Therefore, the results can be suitably interpreted as valid for the effects of a redistributive shock in a recessionary period. The fraction of long-term unemployed among the unemployed has been about 35 percent of after 2008 (from Kroft, Lange, Notowidigdo and Katz, 2016). Given that unemployment was close to 10 percent this yields a probability of becoming unemployed of 3.5 percent for the post-recession period. This yields a probability of entering unemployment for employed individuals of 2.8%. Finally, we choose $\bar{c} = 0.3$, implying transfers close to 20% of median income.

²¹This is equal to one minus the probability of a worker who has not been employed for more than 6 months not finding a job within the year. As measured by Elsby, Hobijn, Sahin and Valletta (2011), the probability of finding a job within a month for a long-term unemployed is approximately 10 percent. While this probability falls steeply over the first six months of unemployment, it stays fairly constant at that level thereafter.

In order to keep the number of parameters manageable, we assume that for any given household, χ_f is proportional to χ_m . We allow for three different levels of χ_m (and corresponding χ_f). We pick values for χ_m , the proportionality factor between χ_m and χ_f , the overall fraction of households of each disutility type in the population, and the fraction of households with each of the income process within each disutility type to match the fraction of male and female workers in the labor force for each wealth quintile as documented by Mustre-del-Rio (2014), as well as to match the fact that working females work on average 15 percent fewer hours over the course of a year than working males (see Table 22 in the Labor Force Statistics from the Current Population Survey, produced by the Bureau of Labor Statistics).²² This procedure yielded values for χ_m equal to 0.55, 0.045, and 0.008, with 12 percent of the population with the highest value for χ_m , 38 percent in the middle value and 50 percent with the lower value. The households with high disutility of labor have half of the income of the households with medium disutility in every state. Out of those with low disutility 70 percent have the same income process as the ones in the middle, but 30 percent have twice the income for every state. Females have disutility parameters which are 45 times larger. We are able to hit the target for average hours worked by females fairly closely. We show the values for the targeted participation rates and the model equivalents in Table 1. We also checked that the new specification of labor supply is a comparable fit to wealth and earnings distribution data relative to what was obtained under Castaneda et al.’s (2003) original calibration.

Table 1: Labor Force Participation by Wealth Quintile and Gender

Wealth Quintiles	Data		Model	
	Males	Females	Males	Females
First Quintile	0.69	0.56	0.65	0.64
Second Quintile	0.89	0.64	0.92	0.46
Third Quintile	0.90	0.70	0.96	0.64
Fourth Quintile	0.92	0.60	0.94	0.50
Fifth Quintile	0.94	0.54	0.91	0.47

Since Castaneda et al. (2003) examine steady states in which nominal rigidities are irrelevant, we need to select parameters for the nominal frictions. We examine two cases: one with $\lambda = 1$, corresponding to the benchmark without nominal rigidities, and one with $\lambda < 1$. For the second case, we chose $\lambda = 0.5$, so that half of the firms change prices in a given year. Given that one period corresponds to a year, this is in line with the evidence in Bils and Klenow (2004) that firms take on average close to half a year to change their prices. We also set the elasticity of substitution between different varieties of intermediate goods to $\theta = 10$, as suggested by Woodford (2003). For tractability, we assume that the profits and losses generated by intermediate-goods producers are rebated to the government in the form of taxes.

In the baseline case, the production side of the economy is a Cobb-Douglas production function for final-goods production. As we will see, in that case there is very little quantitative short-run

²²<http://www.bls.gov/cps/cpsaat22.htm>, viewed on 5/18/2016

impact on the interest rate. To understand the effects of changes in interest rate upon equilibrium, we also investigate a variant of the model in which we augment the production function with a quadratic term $-\frac{1}{2}(K_{t-1} - K_0)^2$, so that interest rates become more sensitive to changes in the capital stock.²³

The wealth transfer at the center of our analysis is structured to move wealth, a , from richer to poorer households and takes the functional form: $x(a) \equiv \eta - \chi a$, with η and $\chi < 1$. This function gives the net change in wealth for an individual with initial wealth a after the transfer. We will consider a tax of 2 percent, i.e. set $\chi = 0.02$, and set η so that the transfer is revenue neutral.

4.2 Description of the Initial Conditions

We now describe the initial state of the economy immediately following redistribution but prior to agents having had time to react. It is this initial condition that determines the short-run aggregate impacts. The first column in Table 2 shows the average change in wealth implied by the policy for different wealth quantiles. The average change in wealth is positive for the first four quintiles. Indeed, close to 80 percent percent of the households are net recipients of wealth transfers. Consistent with the very skewed wealth distribution in the data, we find that the average transfer paid by a household in the top wealth percentile is equal to approximately 30 times that received on average by households in the bottom three quintiles.

Table 2 also collects, for different wealth quantiles, the conditional means of (1) the change in wealth, referred to above ($x_1(a)$); (2) the *net* marginal propensity to save, defined as $\frac{\partial a'(a,s)}{\partial a} - 1$, and denoted by MPS; (3) the marginal propensity to consume out of wealth, defined as $\frac{\partial c(a,s)}{\partial a}$ and denoted by MPC; and (4) the marginal propensity to supply hours defined as $\frac{\partial l(a,s)}{\partial a}$ and denoted by MPL. This “net” definition of marginal propensity to save adjusts for the fact that, absent a behavioral response, households would mechanically increase their wealth by the amount that they receive. The second column shows that the marginal propensity to save rises with wealth. It is a symptom first of low wealth pushing households to work more and consume less in order to build up precautionary balances. Thus, the shape of the policy function for savings conforms to the general finding in the literature, discussed in Section 3.2.3. If marginal propensity to save increases with wealth, then the marginal propensity to consume decreases with wealth. At the same time, the marginal propensity to work decreases with wealth, except between the first and second quintiles. This occurs because the first wealth quintile includes a large fraction of retirees and other households who do not supply labor, for whom labor supply is perfectly inelastic.

4.3 Results

We now turn to the quantitative results. The state of the economy at any given point in time is given by the joint distribution of productivity and wealth. One summary of the state is wealth

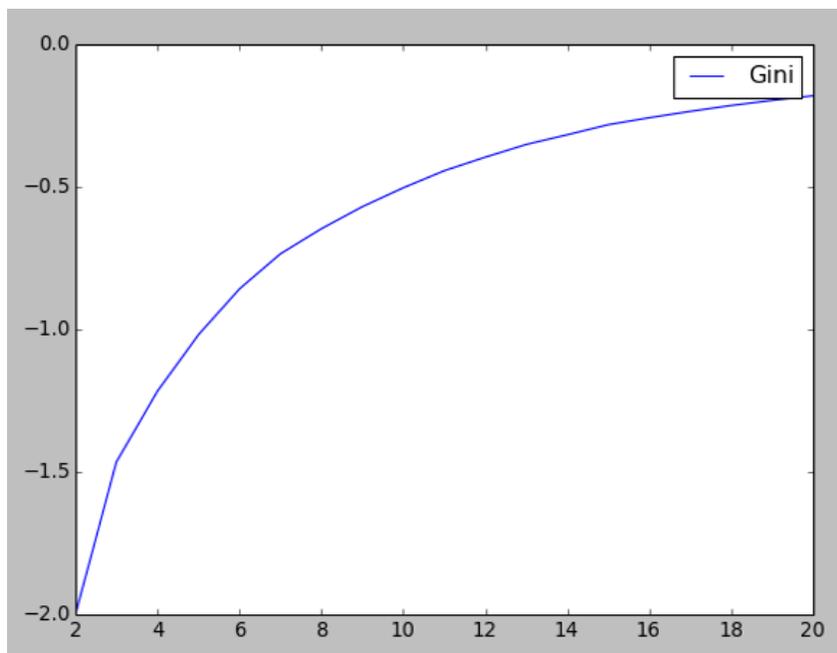
²³In that case, the production function exhibits decreasing returns to scale, so that final-goods producers earn economic profits. We assume that they are taxed lump-sum by the government and used to finance government expenditures.

Table 2: Average Wealth Transfer and Marginal Propensities, by Wealth Quintile

Wealth Quintiles	$x_1(a)$	MPS	MPC	MPL
First Quintile	0.34	-0.85	0.56	-0.08
Second Quintile	0.33	-0.20	0.13	-0.13
Third Quintile	0.28	-0.10	0.10	-0.05
Fourth Quintile	0.14	-0.09	0.08	-0.03
Fifth Quintile	-1.08	-0.07	0.07	-0.01
90-95	-0.66	-0.07	0.07	-0.01
95-99	-1.35	-0.08	0.06	-0.01
99-100	-11.11	-0.04	0.07	0.00

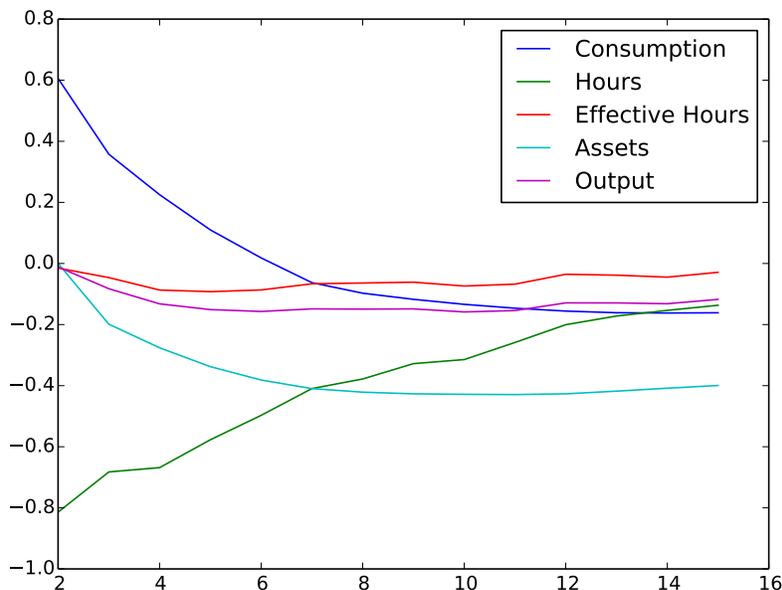
inequality. Figure 2 shows how inequality, as measured by the Gini coefficient, evolves over the transition. We see that upon impact inequality falls by 2 percent. However, as the transition continues, we see that inequality monotonically increases throughout. The driving forces behind this mean-reversion include, most directly, the recipients' consumption of much of the transfer, as well as efforts by the donors to replenish their wealth balances. The relatively slow transition (with a half life of around five years) reflects households' attempts to smooth consumption, thus keeping wealth dynamics rather slow.

Figure 2: Gini Coefficient



We now turn to the impact of the wealth redistribution program on macroeconomic aggregates. Figure 3 shows that the effect of the transfer program on aggregate output, consumption, invest-

Figure 3: Aggregate Variables



ment, and hours is bounded above by around 1.5 percent. Output falls, as noted above, but does so over the entire transition by less than one-quarter of 1 percent. In terms of dynamics, we see that the initial decline in output is followed by a slow increase over time as the economy returns to its steady-state equilibrium. If aggregate output does not change by much, its composition changes much more noticeably, with a boom in consumption and a bust in investment. There is also a large drop in hours worked but a much more muted reduction in effective hours worked, i.e., the total amount worked weighted by individual labor productivity. This compositional change therefore generates an increase in aggregate labor productivity. As implied by Proposition 1, the small drop in effective hours worked explains the small drop in output and the negative comovement between consumption and investment.

In order to understand the link between the aggregate results and the intuition developed through analytics in Section 3, it is useful to observe the average sensitivity of agents' behavior within a given wealth category to changes in their wealth. We construct two related decompositions in Tables 3 and 4. Each element in these two tables reports the contribution – via wealth effects alone – of individuals in a given wealth (Table 3) or productivity (Table 4) category to the total change in a given aggregate. More precisely, the elements of the first column of Table 3 are given by $\left[\sum_{s \in \mathcal{S}} \sum_{a \in A_k} \left(\frac{\partial a'_0(a,s)}{\partial a} - 1 \right) x(a) \Gamma(a,s) da \right] / A_0$, with A_k denoting different wealth quantiles, and the elements in the first column of Table 4 are given by $\frac{\left[\sum_{s \in S_k} \sum_{a \in \mathcal{A}} \left(\frac{\partial a'_0(a,s)}{\partial a} - 1 \right) x(a) \Gamma(a,s) da \right]}{A_0}$, where S_k are different productivity groups. Other columns in the two tables are defined similarly.

This decomposition is accurate if (i) the wealth redistribution is small relative to individual wealth holdings and, (ii) changes in the policy function induced by general equilibrium effects of prices are relatively small.

The decomposition of aggregate effects by household wealth groups is depicted in Table 3. Recall that each of these numbers gives an approximation for the total change within a given group. It is clear that in response to the transfer, low-wealth households contribute negatively to the change in the aggregate savings rate, whereas wealth-rich households contribute positively. Low-wealth households achieve lower savings both by increasing their consumption and by decreasing their labor supply.

In particular, the consumption boom is disproportionately driven by those in the first quintile, who, by themselves, generate a 1.26 percent change in total consumption from the steady state. Furthermore, lower-wealth households reduce their hours by proportionately more than high-wealth households increase theirs. However, once weighted by productivity, higher-wealth groups' aggregate effort rises substantially more than otherwise, since they are more likely to be also more productive. This explains why there is a substantial drop in aggregate hours worked at the same time that effective hours fall very little.

Table 3: Decomposition of Changes in Aggregates by Wealth Quintile

Wealth Quintiles	$S_1 - S_0$	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$
First Quintile	-0.34	1.26	-0.55	-0.19
Second Quintile	-0.08	0.29	-0.97	-0.31
Third Quintile	-0.03	0.18	-0.32	-0.11
Fourth Quintile	-0.02	0.07	-0.10	-0.08
Fifth Quintile	0.08	-0.47	0.09	0.35
90-95	0.01	-0.08	0.02	0.06
95-99	0.03	-0.11	0.04	0.16
99-100	0.03	-0.24	0.00	0.11
Total	-0.39	1.32	-1.85	-0.32

Table 4 organizes households by their idiosyncratic labor productivity. Following Castaneda et al. (2003), we apply a discretization to labor productivity along the four productivity states of working families, $s \in \{1, 2, 3, 4\}$ and among the retirees. Recall that retirees are not endowed with labor time. The table shows that more productive households are more inclined than less productive households to reduce their consumption and increase their hours worked in response to redistribution. This is consistent with productive households being wealthier and therefore more likely net contributors, whereas less productive households are less wealthy and hence more likely net recipients. We see the sharp drop in hours overall, as already noted, but also the concentration of that drop among the relatively unproductive ($s = 1$), explaining the increase in aggregate labor productivity. As for consumption, the bulk of the boom is concentrated in the more unproductive households ($s = 1$) and the retirees. The retirees feature prominently since they effectively discount

the future more heavily than other agents – they face a positive probability of being replaced by a descendant they care for and who, in expectation, will be better off than them. Furthermore, since they do not supply labor, all their dissaving takes place through increased consumption rather than through reduced labor supply. As a result, this group has relatively diminished intertemporal smoothing motives and a higher marginal propensity to consume out of wealth.²⁴

It is interesting to note that whereas retirees are a major group benefiting from the wealth transfers, their labor supply is by definition completely inelastic. Given that leisure is a normal good, this would tend to push aggregate labor supply up as compared to a program where recipients were all working age households.

Table 4: Decomposition in Changes in Aggregates by Exogenous State

exogenous state (s)	$S_1 - S_0$	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$
1	-0.23	0.25	-2.00	-0.68
2	0.01	0.01	0.12	0.09
3	0.03	-0.17	0.03	0.16
4	0.02	-0.03	0.00	0.10
Retirees	-0.21	1.26	0.00	0.00
Total	-0.39	1.32	-1.85	-0.32

Table 5 turns to the general equilibrium effects upon impact of wealth redistribution, comparing outcomes with and without nominal rigidities. The first two lines compare the results in the baseline case with standard Cobb-Douglas production function. The changes in consumption and output in both cases are very similar. This reflects the fact that the interest rate and wages change very little in the case without nominal rigidities. Hence, sticky prices play a very small role. The following two rows consider the case where we change the production function as described in Section 4.1 so that the interest rate is more sensitive to changes in savings. In that case, both models, with and without nominal rigidities, generate a boom in consumption and output, although the boom in consumption is smaller than in the benchmark case. It is notable that in the presence of nominal rigidities, output rises by less than does in its absence. Thus, rather than amplifying the output boom induced by redistributive shocks, sticky prices dampen it. The example highlights how even when labor demand shifts, the results need not conform with the usual Keynesian intuition and that labor supply plays a dominant role.

The analytical results in Section 3 imply that the aggregate impact of the redistribution shock on output is a function of preferences, the tax system, and the incentives for precautionary savings. To further demonstrate the extent to which the intuitions developed in the analytical section inform the quantitative results, we now examine alternative specifications. Table 6 shows how aggregate responses upon impact change if we alter the specification of the model. The last column provides

²⁴Note that the overall change in consumption predicted by the decomposition is about twice as large as the one implied by the model. This is in large part an artifact of the linearization and, specifically, because the marginal propensity to consume of the retirees falls very rapidly as their wealth increases.

Table 5: The Role of Nominal Rigidities

		$C_1 - C_0$	$N_1 - N_0$	$Y_1 - Y_0$
Baseline	$\lambda = 1$	0.60	-0.02	-0.01
	$\lambda = 0.5$	0.61	-0.00	-0.00
Sensitive Interest Rates	$\lambda = 1$	0.14	0.79	0.60
	$\lambda = 0.5$	0.14	0.76	0.57

a summary of the welfare impact of the policy, as defined by the factor by which one would need to multiply the consumption of a household in every date and state to make them, on average, indifferent between remaining in a steady-state economy and being subject to the redistributive shock.²⁵ The first line of the table restates the results for the benchmark model.

Rows 2 and 3 in Table 6 show how removing the progressivity of the tax code and making preferences homothetic both lead to a progressively larger drop in effective labor supply and output on impact. This is in line with the discussion in Section 3.2.3 and more specifically in Proposition 7 and Lemma 1, regarding how, respectively, non-homotheticity in preferences and a progressive tax-code lead wealthy households to choose to spend proportionately more on leisure than on consumption. Once we make preferences homothetic and taxes linear, the wealthy reduce their expenditure on leisure by less than in the baseline case. Since wages change by little on impact, this means that they also increase their labor supply by less. The increase in the curvature of the utility of leisure in Row 2 also provides some indication of the role of labor supply as an adjustment mechanism available to households in addition to consumption and savings. As it turns out, the change in the parameter has a larger impact among wealthy households. This is because many poorer households are not supplying any labor to begin with, so at the margin they are unresponsive. Furthermore, the low value of ψ relative to σ in the baseline calibration implies that wealthy households supply proportionately more labor, in line with Corollary 1, so they are more sensitive to changes in their ability to adjust labor supply. Given that wealthy households adjust to the redistributive shock by increasing their labor supply, we see that with a higher ψ labor supply change is smaller than in the baseline. Since the wealthy will not adjust their labor supply by as much, they are forced to adjust their consumption and savings by more, resulting in a smaller consumption boom and a larger drop in assets.

Rows 4 and 5 explore the role of precautionary savings. In Row 4, we increase household income risk by increasing the probability that households transition between productivity states. This extra risk leads to a slightly larger drop in hours and output, in line with the discussion on the

²⁵Specifically, we calculate λ so that $E \left[u \left(\lambda c_0(a, l, s, d), h_0^m(a, l, s, d), h_0^f(a, l, s, d) \right) \right] = E \left[V_1 \left(a + \frac{x(a)}{1+r_1}, l, s, d \right) \right]$, where, following the notation in the paper, the policy functions indexed 0 refer to steady-state values and the value function indexed 1 refers to the value after redistribution takes place. Expectations are taken over the steady-state distribution of households over the state-space.

intertemporal determinants of labor supply in Section 3.2.3.²⁶ In Row 5 we interact the reduction in risk with the same reduction in the labor supply elasticity that we explored in Row 2. This interaction allows us to evaluate the role of the labor supply elasticity in allowing households to insure through working longer hours, as in Pijoan-Mas (2006). Without this insurance mechanism, there is an even larger decrease in labor supply.

Row 6 revisits the case in which interest rates are sensitive to savings. In that case, the sensitivity of interest rates suppresses the increase in consumption and leads to an output boom. This highlights the role of intertemporal choices in keeping aggregate labor supply from rising in the baseline model. With sensitive interest rates, aggregate assets barely change in equilibrium, and intratemporal choices dominate. Row 7 examines the role of capital taxes in affecting individual household's incentives to save out of the transferred wealth. In order to do this, we set capital taxation to zero, so that income taxes only depend on labor income. We find that output rises by a little instead of falling by a little, but at the same time there is a larger increase in consumption. It is interesting to note that the net effect is a smaller drop in investment. This is in line with the intuition that capital taxation incentivizes wealth-rich households to reduce their savings in response to a tax on their wealth by more than they would otherwise.

Row 8 provides the results of an income-based redistribution. As before, households receive a once and for all unexpected tax on their wealth that is then redistributed lump-sum to all households in the economy. The difference is that the new tax rate is calculated based on the after-tax income that households would receive in $t = 1$ were redistribution to not take place. There is now a considerable output boom. The reason can be deduced from the covariance term in Proposition 6. At any wealth level there are households in which secondary earners do not participate in labor markets. Those households are likely to be at the same time income poorer and less labor elastic. This induces a negative covariance between labor supply elasticity and transfers for any wealth level. Furthermore, the covariance is likely to increase in absolute value, since high-wealth households are also more likely to be more productive, increasing the earnings disparity within wealth levels. The result suggests that income-based redistribution programs are more likely to be effective in increasing output than wealth-based programs exactly because they improve the alignment between transfers and elasticity of labor supply.

The last two rows of Table 6 examine whether particular assumptions made in Castaneda et al. (2003) matter for the results. Two natural alternatives are the quantitative environments in Floden (2001) and Alonso-Ortiz and Rogerson (2010). Rows 9 and 10 present the results for these two cases, respectively. In both cases there is a clear decline in both hours and effective hours upon impact. The environment in Alonso-Ortiz and Rogerson (2010) presumes indivisible labor. As highlighted by Proposition 8, the fact that output falls in that case suggests that given the proposed redistributive policy, recipients are more likely to be close to the margin between working

²⁶In order to do this exercise, we reduce the probability of the household remaining in states $s = 1$ to 3 once they are there. At the same time we leave the probability of transitions out of the very rare and very productive state $s = 4$ the same. Finally, we calibrate the transitions into $s = 4$ to keep the stationary distribution of productivity types the same as before. We rescale the probabilities of transitions to states other than $s = 4$ to accommodate those changes.

and not working.

Table 6: Alternative Parameterizations

	$C_1 - C_0$	$L_1 - L_0$	$N_1 - N_0$	$Y_1 - Y_0$	$A_1 - A_0$	CE Factor
1. Baseline	0.61	-0.81	-0.02	-0.01	-0.20	1.02
2. $\psi = 1$	0.55	-1.47	-0.21	-0.16	-0.21	1.02
3. Flat Tax	0.66	-1.12	-0.08	-0.06	-0.20	1.03
4. High Risk	0.83	-1.07	-0.04	-0.03	-0.20	1.03
5. High risk and $\psi=1$	0.76	-1.62	-0.20	-0.16	-0.20	1.03
6. Sensitive Interest Rate	0.14	-0.10	0.79	0.60	-0.01	1.02
7. No Capital Taxation	0.68	-0.84	0.01	0.01	-0.18	1.03
8. Income Based	0.53	-0.49	0.24	0.19	-0.13	1.02
9. Floden (PE)	0.44	-0.83	-0.34	-	-	-
10. Alonso-Ortiz and Rogerson	0.31	-6.49	-1.85	-1.19	-	-

Finally, we look at the welfare impact of the redistributive policy along different groups. We do this by calculating the factor by which one would need to multiply the consumption of a given household in every state of the world so that it is indifferent between remaining within the “no-redistribution” steady-state and being subject to the redistributive shock.²⁷ The values in Table 7 refer to the factor as applied to a household with productivity state depicted in the column (including retirement) and whose wealth equals the mid-point of the wealth quantiles depicted in the row. The factor is only smaller than one in the upper quintile, as one would expect, given that this is where households contributing to the redistribution are concentrated. The gains are most pronounced among retirees, reaching 6% of consumption if they find themselves in the bottom 20% of wealth. The gains are substantial for households in the low productivity state. There are virtually no gains or losses for those in the extremely high productivity $s = 4$ state.

5 Conclusion

The recent severe recession and slow recovery have drawn renewed attention to the possibility that wealth redistribution can stimulate output. In this paper, we have taken a step, both analytically and quantitatively, in evaluating the aggregate impact of short-term redistributive economic policy that transfers wealth from rich to poor households. We show that the conventional intuition with respect to the stimulative effect of wealth redistribution indeed holds for the behavior of aggregate consumption. However, we show that while redistributive policies can have a stimulative impact

²⁷Specifically, for different (a_0, s_0) values we calculate λ^{a_0, s_0} to satisfy

$$\begin{aligned}
 & E \left[\sum_{k=0}^{\infty} \beta^k u \left(\lambda^{a_0, s_0} c_0(a_k, s_k, l_k, d_k), h_0^m(a_k, s_k, l_k, d_k), h_0^f(a_k, s_k, l_k, d_k) \right) \middle| a_0, s_0 \right] \\
 &= E [V_1(a_0, s_0, l_0, d_0) | a_0, s_0]
 \end{aligned}$$

where a_k , s_k , l_k and d_k refer to the realized values of the state variables as of date k .

Table 7: Welfare

Wealth Quantiles	Exogenous State				Retirees
	$s = 1$	$s = 2$	$s = 3$	$s = 4$	
First Quintile	1.02	1.01	1.01	1.00	1.07
Second Quintile	1.02	1.01	1.01	1.00	1.05
Third Quintile	1.01	1.01	1.00	1.00	1.02
Fourth Quintile	1.01	1.00	1.00	1.00	1.01
Fifth Quintile	0.99	0.99	1.00	1.00	0.99
90-95	0.99	0.99	1.00	1.00	0.99
95-99	0.99	0.99	0.99	1.00	0.99
99-100	0.99	0.99	0.99	1.00	0.99

on consumption, their effect on aggregate output depends, potentially quite importantly, on the nature of household labor supply. We show analytically that in an important class of settings, redistributions will be output-neutral on impact unless they alter aggregate labor supply, and tease out conditions under which redistribution will lead to either a boom or a bust in output. We also show that this “centrality” of labor supply holds even in the presence of “aggregate demand” externalities and sticky prices. Our quantitative benchmark is a standard incomplete-markets model of consumption and labor supply that incorporates nominal rigidities and, in its quantitative version, also accurately captures the U.S. wealth distribution. In particular, we highlight the role of wealth effects on labor that, in our quantitative model, are strong enough to largely negate the output effects of the consumption boom. Our results make clear that research aimed at measuring the impact of redistributive policies on output will benefit strongly from further empirical research on how marginal propensities to work vary with wealth.

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7 Proofs of Propositions

Proofs of Propositions 1 through 3:

See text.



Proof of Proposition 4

Recall that with flexible prices, wages satisfy $w_t = F_M(N_t, K_{t-1})$ (since $M_t = N_t$) so that, holding K_0 fixed, w_1 is decreasing in N_1 . Substituting out Z_t from the expression for aggregate labor supply (15), we have that

$$\begin{aligned}
N_1 &= \bar{N} - \frac{\mu}{1-\mu} \frac{C_1}{w_1} \\
&= \bar{N} - \frac{\mu}{1-\mu} \frac{C_1}{F_M(N_1, K_0)}
\end{aligned}$$

This induces an implicit function of N_1 on C_1 with derivative given by the implicit function theorem:

$$\frac{dN_1}{dC_1} = - \frac{\frac{\mu}{1-\mu} \frac{1}{F_M(N_1, K_0)}}{1 - \frac{\mu}{1-\mu} \frac{C_1 F_{MM}(N_1, K_0)}{(F_M(N_1, K_0))^2}}$$

Thus, given that $F(M_1, K_0)$ is neo-classical (so that $F_{MM}(M_1, K_0) \leq 0$), any increase in C_1 leads to a reduction in N_1 .

Proof of Proposition 5:

Let $\mathcal{A}^- = \{a | x(a) < 0\}$, $\mathcal{A}^+ = \{a | x(a) > 0\}$. Then

$$\left| \int_{a \in \mathcal{A}^-} x(a) \Gamma(a) da \right| = \int_{a \in \mathcal{A}^+} x(a) \Gamma(a) da = M$$

For notational simplicity, let $m(a) \equiv \sum_{s \in \mathcal{S}} \frac{\partial z_0(a, s)}{\partial a} \frac{\partial \varepsilon(s) l^{static}(z(a, s), s, w_0)}{\partial z} \Gamma_0(s | a)$ denote the marginal propensity to work out of wealth averaged out across productivity types s .

Since $x(a)$ is decreasing in a , it follows that if $a \in \mathcal{A}^+$ and $a' \in \mathcal{A}^-$, then $a < a'$. Moreover, by assumption, $|m(a)|$ is increasing in a . It follows from both of these observations that, for all s , $\sup_{a \in \mathcal{A}^+} |m(a)| \leq \inf_{a \in \mathcal{A}^-} |m(a)|$. Thus,

$$\begin{aligned}
\int_{a \in \mathcal{A}^+} x(a) |m(a)| \Gamma(a) da &< M \sup_{a \in \mathcal{A}^+} |m(a)| \leq \\
&\leq M \inf_{a \in \mathcal{A}^-} |m(a)| \\
&< \left| \int_{a \in \mathcal{A}^-} x(a) |m(a)| \Gamma(a) da \right|.
\end{aligned}$$

Therefore, since $\int_{a \in \mathcal{A}^-} x(a) |m(a)| \Gamma(a) da < 0$,

$$\int_{a \in \mathcal{A}^+} x(a) |m(a)| \Gamma(a) da + \int_{a \in \mathcal{A}^-} x(a) |m(a)| \Gamma(a) da < 0,$$

Finally, if $\frac{\partial z_0(a, s)}{\partial a} \geq 0$ and $\frac{\partial \varepsilon(s) l^{static}(z(a, s), s, w_0)}{\partial z} \leq 0$, then $m(a) < 0$, so that

$$\int_{a \in \mathcal{A}} x(a) m(a) \Gamma(a) da > 0.$$

The proof for the converse case in which $|m(a)|$ is decreasing in a is analogous.

■

Proof of Proposition 7:

Let $\mu(z, s, w_t) = \frac{w_t \varepsilon(s) (\bar{l} - l_t^{static}(z, s, w_t))}{z}$, so that $l_t^{static}(z, s, w_t) = \bar{l} - \frac{\mu(z, s, w_t) z}{w_t \varepsilon(s)}$. We have that, for all t ,

$$\frac{\partial \varepsilon(s) l_0^{static}(z, s; w)}{\partial a} = -\frac{1}{w_t} [\mu_z(z_0(a, s), s, w_0) z_0(a, s) + \mu(z_0(a, s), s, w_0)],$$

Thus, $\frac{\partial \varepsilon(s) l_0^{static}(z, s; w)}{\partial a}$ increases with z if $\mu_z(z, s, w_t) z + \mu(z, s, w_t)$ to increase with z . It is increasing in z if

$$\mu_{zz}(z, s, w_0) z + 2\mu_z(z, s, w_0) > 0$$

This will be true if $\mu_z(z, s, w_0) > 0$ (leisure is a luxury) and $\frac{\mu_{zz}(z, s, w_0) z}{\mu_z(z, s, w_0)} > -2$. Conversely, $N_1 - N_0 < 0$ if $\mu_z(z, s, w_t) z + \mu(z, s, w_t)$ decreases with z . This will be the case if

$$\mu_{zz}(z, s, w_0) z + 2\mu_z(z, s, w_0) < 0$$

Which is the case if $\mu_z(z, s, w_0) < 0$ (leisure is a necessity) and $\frac{\mu_{zz}(z, s, w_0) z}{|\mu_z(z, s, w_0)|} < 2$

■

Proof of Corollary 1

In the case of separable utility, we have that

$$w \varepsilon(s) c^{-\sigma} = \chi (\bar{l} - l)^{-\psi}$$

The budget constraint is:

$$w \varepsilon(s) (\bar{l} - l) + c = z$$

Bringing the two together defines $\mu(z, s, w)$ implicitly as

$$\mu(z, s, w) + \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} \mu(z, s, w)^{\frac{\psi}{\sigma}} z^{\frac{\psi}{\sigma}-1} = 1$$

From the implicit function theorem, we have that

$$\mu_z(z, s, w) = \left(1 - \frac{\psi}{\sigma}\right) \frac{\chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} \mu(z, s, w)^{\frac{\psi}{\sigma}} z^{\frac{\psi}{\sigma}-2}}{1 + \frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w) z)^{\frac{\psi}{\sigma}-1}}$$

Thus, $\mu_z(z, s, w) > 0$ if $\psi < \sigma$ and $\mu_z(z, s, w) < 0$ otherwise. Also, taking natural logarithms on both sides and differentiating with respect to $\ln z$ yields

$$\begin{aligned} \frac{\mu_{zz}(z, s, w)z}{\mu_z(z, s, w)} &= \frac{\psi}{\sigma} \frac{\mu_z(z, s, w)z}{\mu(z, s, w)} + \frac{\psi}{\sigma_C} - 2 \\ &\quad - \frac{\frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}}{1 + \frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}} \left(\frac{\psi}{\sigma} - 1 \right) \left(\frac{\mu_z(z, s, w)z}{\mu(z, s, w)} + 1 \right) \end{aligned}$$

Note that $\frac{\frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}}{1 + \frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}} = \frac{1}{1 - \frac{\psi}{\sigma}} \frac{\psi}{\sigma} \frac{\mu_z(z, s, w)z}{\mu(z, s, w)}$, so that

$$\frac{\mu_{zz}(z, s, w)z}{\mu_z(z, s, w)} = \frac{\psi}{\sigma} \left(\frac{\mu_z(z, s, w)z}{\mu(z, s, w)} + 2 \right) \frac{\mu_z(z, s, w)z}{\mu(z, s, w)} + \frac{\psi}{\sigma} - 2$$

So that if $\mu_z(z, s, w) > 0$, $\frac{\mu_{zz}(z, s, w)z}{\mu_z(z, s, w)} > -2$. If $\mu_z(z, s, w) < 0$, we have that

$$\frac{\mu_{zz}(z, s, w)z}{|\mu_z(z, s, w)|} = \frac{\psi}{\sigma} \left(2 - \frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} \right) \frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} - \frac{\psi}{\sigma} + 2$$

So that $\frac{\mu_{zz}(z, s, w)z}{|\mu_z(z, s, w)|} < 2$ if

$$\frac{\psi}{\sigma} \left(2 - \frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} \right) \frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} < \frac{\psi}{\sigma}$$

Simplifying and rearranging,

$$\left(\frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} \right)^2 - 2 \frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} + 1 > 0$$

Note that the LHS is equal to zero if $\frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} = 1$. Also, if we take the first derivative of the LHS and set $\frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} = 1$, we see that this is a local minimum. Thus, unless $\frac{|\mu_z(z, s, w)|z}{\mu(z, s, w)} = 1$, the condition is satisfied. This would require

$$\left(1 - \frac{\sigma}{\psi} \right) \frac{\frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}}{1 + \frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}} = 1$$

Under the assumption that $\psi > \sigma$ (so that $\mu_z(z, s, w) < 0$), we have that $0 < \frac{\sigma}{\psi} < 1$, so that $1 - \frac{\sigma}{\psi} < 1$. Also, $\frac{\frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}}{1 + \frac{\psi}{\sigma} \chi^{-\frac{1}{\sigma}} w \varepsilon(s)^{\frac{1-\psi}{\sigma}} (\mu(z, s, w)z)^{\frac{\psi}{\sigma}-1}} < 1$, so that the equality cannot hold. Thus, $\frac{\mu_{zz}(z, s, w)z}{|\mu_z(z, s, w)|} < 2$.

■

Proof of Lemma 1

The intratemporal optimality condition is

$$c = \frac{\kappa}{1 - \kappa} w \varepsilon(s) (1 - \tau' (ra + w \varepsilon(s) l + \omega(s))) (\bar{l} - l)$$

Given the definition

$$\mu_0(a, s) = \frac{w\varepsilon(s)(\bar{l} - l_0(a, s))}{c_0(a, s) + w\varepsilon(s)(\bar{l} - l_0(a, s))}$$

we have that

$$\mu_0(a, s) = \frac{1}{\frac{\kappa}{1-\kappa}(1 - \tau'(ra + w\varepsilon(s)l(a, s) + \omega(s))) + 1}$$

Thus,

$$\frac{\partial \mu_0(a, s)}{\partial a} = \frac{\kappa}{1-\kappa} \frac{\tau''(ra + w\varepsilon(s)l + \omega(s))}{\left(\frac{\kappa}{1-\kappa}(1 - \tau'(ra + w\varepsilon(s)l(a, s) + \omega(s))) + 1\right)^2} \frac{\partial (ra + w\varepsilon(s)l(a, s))}{\partial a}$$

If taxes are progressive, so that $\tau''(ra + w\varepsilon(s)l + \omega(s)) > 0$, then $\frac{\partial \mu_0(a, s)}{\partial a} > 0$. ■

Proof of Proposition 8

We want to consider the impact of an infinitesimally small redistribution program. We consider a sequence of vanishingly small programs with transfers to households with wealth level a given by $vx_1(a)$, where v is a perturbation parameter. Under the indivisible labor case, effective labor supply at $t = 0$ and $t = 1$ is, in partial equilibrium, given by

$$\begin{aligned} N_0 &= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \int_{a \in A} \mathbf{1}(a \leq \bar{a}(s)) \Gamma_0(a, s) da \\ N_1 &= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \int_{a \in A} \mathbf{1}\left(a + \frac{vx_1(a)}{1+r} \leq \bar{a}(s)\right) \Gamma_0(a, s) da \end{aligned}$$

To characterize the change in effective labor supply given the program, we take the limit

$$\begin{aligned} \lim_{v \rightarrow 0} \frac{N_1 - N_0}{v} &= \lim_{v \rightarrow 0} \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \int_{a \in A} \left[\frac{\mathbf{1}(a + vx_1(a) \leq \bar{a}(s)) - \mathbf{1}(a \leq \bar{a}(s))}{v} \right] \Gamma_0(a, s) da \\ &= \lim_{v \rightarrow 0} \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \int_{a \in A} \left[\frac{\mathbf{1}(a + vx_1(a) \leq \bar{a}(s)) - \mathbf{1}(a \leq \bar{a}(s))}{x_1(a)v} \right] x_1(a) \Gamma_0(a, s) da \\ &= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \lim_{u \rightarrow 0} \frac{\int_{\bar{a}(s)-u}^{\bar{a}(s)} x_1(a) \Gamma_0(a, s) da}{u} \end{aligned}$$

where in the last line we substitute $x_1(a)v \equiv u$. Using L'Hospital's rule and Leibnitz' rule,

$$\begin{aligned}
\lim_{v \rightarrow 0} \frac{N_1 - N_0}{v} &= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \lim_{u \rightarrow 0} \frac{\partial \int_{\bar{a}(s)-u}^{\bar{a}(s)} x_1(a) \Gamma_0(a, s) da}{\frac{\partial u}{\partial u}} \\
&= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) \lim_{u \rightarrow 0} \frac{x_1(\bar{a}(s) - u) \Gamma_0(a, s) da}{1} \\
&= \bar{l} \sum_{s \in \mathcal{S}} \varepsilon(s) x_1(\bar{a}(s)) \Gamma_0(a, s) da
\end{aligned}$$

■

8 Solution Algorithm for the Computational Model

1. We solve for policy functions using value function iteration on a discrete grid. We use cubic splines to interpolate the expected value function over the grid for assets. There are 250 nonlinearly spaced gridpoints for assets. The rest of the state space consists of the 4 stochastic income states, 2 retirement states, 4 male and female work opportunity pairs, and 6 states for leisure preference and ex-ante income heterogeneity.

- (a) Households choose between available work/not work pairs. There are potentially 4 options, NN, NW, WN, WW for male/female. Since the distribution of employment opportunity is endogenous, the expected value function depends on the choice of work status.
- (b) We define $U(a'|OO, X)$ as the utility from choosing savings a' given work status OO and state X . Optimal consumption and leisure are determined by solving an intratemporal FOC, a single variable non-linear equation.
- (c) We maximize the RHS of the Bellman equation

$$U(a'|OO, X) + EV(a'|OO, X)$$

over a' , and then choose the best OO . There is potentially more than one local maximum because of the non-convexity created by \bar{c} . In order to make the maximization robust, we first evaluate the objective function at all feasible gridpoints and choose the best point on the grid. Then we use a continuous optimization routine to choose from the surrounding region.

- (d) We iterate on the Bellman equation until convergence.
2. The steady state distribution of households over the state vector is constructed by iterating on a distribution operator until it reaches a fixed point. The distribution operator maps current distributions to the one next period, accounting for asset choices from the policy

functions and the various stochastic processes in the model. In order to assure household's asset choices are near a gridpoint, we interpolate the policy functions onto a finer grid with 10,000 gridpoints for assets. We provide details upon request.

3. The steady state general equilibrium is computed by iterating on r until $r_t = r_t^*$, where r_t^* comes from the capital market clearing condition, to be described below in more detail. As for the impulse response functions, we compute them by first modifying the initial steady state wealth distribution in accordance with the redistribution policy. Next, we guess a path for interest rates. The transition policy functions are computed by applying the Bellman operator backwards from the steady state value functions, then the sequence of distributions and aggregates is computed by applying the distribution operator forwards starting with the modified initial distribution as a starting point and using the new policy functions. The GE interest rate paths are computed by iterating on the r vector until $r_t = r_t^*$ at all dates.
4. For the determination of the target interest rates $\{r_t^*\}_{t=1}^\infty$ and of the wage rates, $\{w_t\}_{t=1}^\infty$ there are two relevant cases:

(a) *Flexible Prices* ($\lambda = 1$): Under flexible prices, $r_t^* = \alpha(K_{t-1}/N_t)^{\alpha-1} - \delta$, where we use the fact that under flexible prices $M_t = N_t$ (for the sensitive interest rate case, we further subtract $\chi(K_{t-1} - K_0)$ from the expression for r_t^*). When prices are flexible ($\lambda = 1$), we can express wages as a function of the guessed interest rate as r_t , given by $(1 - \alpha) \left(\frac{r_t + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$.

(b) *Sticky Prices* ($\lambda = 2$): Sticky prices only hold for $t = 1$, so that from $t = 2$ onwards, r_t^* and w_t are determined exactly as above. For $t = 1$ we first calculate the change price level using the monetary policy condition. This is $\frac{Q_1}{Q_0} = \frac{1+r_2}{1+r_0}$. Also, using the same steps as in a., we can calculate the relative price of the intermediate input bundle M_t , which under flexible prices turns out to be identical to the wage rate, $P_t = (1 - \alpha) \left(\frac{r_t + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$. Given the pricing frictions, this price index is also given by $P_1 = \left[\lambda w_1^{1-\theta} + (1 - \lambda) \left(\frac{Q_0 w_0}{Q_1}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$. Given w_0 (calculated in steady-state) and $\frac{Q_0}{Q_1}$ calculated from the monetary policy rule, we can invert that expression to obtain w_1 . Finally, profits received by households are equal to $\pi_1 = \left(\frac{w_0}{Q_1} - w_1\right) \lambda \frac{N_0}{\lambda + (1-\lambda) \left(\frac{w_0 Q_0}{w_1 Q_1}\right)^\theta}$.
When solving for the sticky prices, we iterate on π_1 as well as on r_t .