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Learning About Consumer Uncertainty from Qualitative Surveys: As Uncertain As Ever*

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Abstract

We study diffusion indices constructed from qualitative surveys to provide real-time assessments of various aspects of economic activity. In particular, we highlight the role of diffusion indices as estimates of change in a quasi extensive margin, and characterize their distribution, focusing on the uncertainty implied by both sampling and the polarization of participants' responses. Because qualitative tendency surveys generally cover multiple questions around a topic, a key aspect of this uncertainty concerns the coincidence of responses, or the degree to which polarization comoves, across individual questions. We illustrate these results using micro data on individual responses underlying different composite indices published by the Michigan Survey of Consumers. We find a secular rise in consumer uncertainty starting around 2000, following a decade-long decline, and higher agreement among respondents in prior periods. Six years after the Great Recession, uncertainty arising from the polarization of responses in the Michigan Survey stands today at its highest level since 1978, coinciding with the weakest recovery in U.S. post-war history. The formulas we derive allow for simple computations of approximate confidence intervals, thus affording a more complete real-time assessment of economic conditions using qualitative surveys.

JEL classification: C18; C46; C83; D80; E32; E66

Keywords: Economic Uncertainty; Qualitative Data; Diffusion Index

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1 Introduction

Quantitative information regarding the state of U.S. economic activity is mostly compiled and published by various statistical agencies such as the Bureau of Labor Statistics (BLS), the Bureau of Economic Analysis (BEA), or the Federal Reserve Board. An important aspect of such data is that releases often involve a one-month lag, and are further subject to later revisions, typically at the three-month and one-year mark.[1] In part as an attempt to provide information somewhat closer to real time, or to collect information simply not compiled by official statistical agencies, a growing number of institutions and government agencies produce diffusion indices constructed from qualitative survey data. For example, the Michigan Survey of Consumers (MSC) provides several diffusion indices of consumer sentiment based on monthly nationwide surveys; the Institute for Supply Management publishes different monthly diffusion indices regarding various aspects of manufacturing, such as production, shipments or new orders; several regional Federal Reserve Banks including Atlanta, Dallas, Kansas City, New York, Philadelphia, and Richmond produce diffusion indices meant to capture the direction of change, in real time, of different facets of economic activity, such as inventories, capital expenditures, or wages, at a more regional level. In the latter cases, the information is not otherwise compiled or readily available. Moreover, by sharing a common methodology, diffusion indices allow for comparable benchmarks not just across U.S. regions but across countries, as in the case of the manufacturing Purchasing Managers Index (PMI) for France or Germany for instance.

Diffusion indices, throughout the paper, are defined in terms of the proportions of a set of disaggregated series or survey responses moving in different directions, thus defining a notion of optimism and pessimism, and provide a measure of the breadth of change in the corresponding aggregate series. This traditional interpretation is discussed in early work by Moore (1983), or the Federal Reserve Bulletin (1991) among others, and is distinct from the interpretation of diffusion indices constructed by principal components analysis in Stock and Watson (2002).

To this point, a large part of the literature on survey-based diffusion indices has focused on their ability to forecast economic activity using various approaches, as surveyed for example in Nardo (2003).[2] Diffusion indices have also been used to explore different properties of expectations, given assumptions on the underlying data, as surveyed in Pesaran and Weale (2006).[3] More recently, this second strand of research has been able to take advantage of the micro data on individual responses underlying diffusion indices, when directly available, as in Bachmann and Elstner (2013) in the case of the IFO Business Climate Survey.[4] Our paper departs somewhat from this literature

[1] See Croushore (2011) for a comprehensive survey of real-time data analysis, and the caveats that arise from the distinction between real-time and revised data. See also Runkle (1998), Croushore and Stark (2001), and Fernald and Wang (2005).

[2] See, for example, Kennedy (1994), Smith and McAleer (1995), Bram and Ludvigson (1998), and Bachmann and Sims (2012).

[3] See also Ivaldi (1992), Jeong and Maddala (1996), and Claveria, Pons, and Suriñach (2006).

[4] A challenge remains, however, in that for many published diffusion indices, individual survey responses are either

in that it studies the properties of diffusion indices as estimates of the breadth of change in an aggregate series of interest. In particular, we highlight the role of the diffusion index as an estimate of change in a quasi extensive margin, in a way closely related to the work of Gourio and Kashyap (2007), and characterize the distribution of a general diffusion index, potentially averaging different individual indices, focusing on the uncertainty implied by both sampling and the polarization of participants' responses.

While the use of survey data has traditionally emphasized quantitative surveys, for example with respect to properties of inflation expectations in the presence of nominal rigidities (Coibion and Gorodnichenko, 2012; Coibion, Gorodnichenko, and Kumar, 2015), or to measure disagreement among forecasters as an indication of uncertainty (Boero, Smith, and Wallis, 2008; Bomberger, 1996; D'Amico and Orphanides, 2008; Rich and Tracy, 2010; Sill, 2012), the use of qualitative surveys is relatively more recent.[5] Specifically, we build on Bachmann, Elstner, and Sims (2013) who use qualitative survey data capturing notions of optimism and pessimism, through the proportion of positive and negative responses to particular questions, to construct an empirical proxy for time-varying business level uncertainty. We show, by way of a central limit theorem argument that all diffusion indices, including composite indices, are asymptotically normal in large samples, and that the uncertainty proxy in Bachmann, Elstner, and Sims (2013) is the variance of a particular albeit widely used individual diffusion index (scaled by the square root of the sample size). As such, this proxy can also be used to provide confidence intervals, taking into account sampling uncertainty, for the corresponding diffusion index.

In consumer and business tendency surveys, diffusion indices are almost always published as composite indices, combining multiple individual indices corresponding to different questions around a given topic. Thus, we emphasize that a general notion of uncertainty based on multiple questions reflects not only the polarization of answers, or degree of disagreement, with respect to an individual survey question but also the extent to which agreement or disagreement coincides across individual questions. In particular, given individual response level data, we describe how the pairwise coincidence of answers across survey questions, or lack thereof, maps into the covariance between individual diffusion indices making up the composite index. In deriving the distribution of composite indices, and their associated uncertainty, we show that given data on individual responses, one need only keep track of pairwise proportions of answer types across questions, for example the proportion of responses indicating optimism to one question and pessimism to another.

We illustrate these results using micro data on individual responses used to construct different composite diffusion indices around sentiment published by the Michigan Survey of Consumers (MSC). We find a steadily rising trend, starting around 2000, in consumer uncertainty that contrasts with gradually declining uncertainty over the previous decade, and higher agreement in prior

not systematically recorded or not publicly available.

[5]See Barsky and Sims (2012), and Bachmann and Sims (2012), for applications using consumer sentiment indices constructed by the Michigan Survey of Consumers.

periods. Furthermore, following this steady increase, uncertainty arising from the polarization of responses in the Michigan Survey currently stands, six years removed from the Great Recession, at its highest level since 1978. The secular rise in uncertainty that we estimate among consumers can also be seen to a degree with respect to policy in the index of Economic Policy Uncertainty (EPU) calculated by Baker, Bloom, and Davis (2013). However, while uncertainty around consumer sentiment tends to be countercyclical as with the EPU, the period starting around 2000 is somewhat unique in that unlike the EPU, which declined sharply from its post-war period peak two years after the Great Recession, uncertainty among consumers never substantively declined after the 2007-09 recession. This continued increase in uncertainty among consumers coincides with the weakest post-war expansion on record.

2 Diffusion Indices: Measuring Changes in a Quasi-Extensive Margin of Economic Activity

Diffusion indices, such as the Index of Consumer Sentiment produced by the MSC, receive widespread coverage in part because they have been empirically found, over time, to correlate well with economic activity (Bram and Ludvigson, 1998). This section uses the diffusion index of employment, produced by the Bureau of Labor Statistics (BLS), to underscore two key points regarding diffusion indices. First, scaled appropriately, diffusion indices can be thought of as capturing a quasi-extensive margin of change in economic activity or, in this case, employment. Second, this quasi-extensive margin accounts for a large portion of the variation in aggregate employment, although interestingly to different degrees in expansions and recessions.

Beginning in 1991, the BLS began publishing a diffusion index of employment in its monthly statistical release, covering roughly 264 sectors corresponding to the 4-digit level of the North American Industrial Classification System (NAICS), separate from its release of measured employment. Overall employment growth, which reflects a weighted average of sectoral employment growth, gives us one measure of employment performance. Overall employment growth, however, does not provide a sense of how the change is shared across sectors; for instance a given aggregate growth rate may be consistent with all sectors doing equally well or, instead, a relatively few sectors growing rapidly with most others simply muddling through. In contrast, the BLS employment diffusion index summarizes the direction of change in a set of disaggregated sectors over a given time period, thus providing a measure of the breadth of the change in employment. We now explore and describe the relationship between these two measures of economic performance.

The diffusion index is most conventionally reported as

$$\mu D + \kappa \tag{1}$$

where D is the proportion of a set of disaggregated series that increased over a given time period less the proportion that decreased over the same period, $D = n^u/n - n^d/n$, where n is the total number of series or categories, such as sectors, and n^u and n^d are the number of categories that experienced an increase and decline in activity respectively; μ and κ are constants. In the case of the BLS employment diffusion index, $\mu = 1$ and $\kappa = 0$. Thus, if an individual series increases over the span of the diffusion index, it receives a value of 1; if it declines, it receives a value of -1; and if it is unchanged, it receives a value of zero. The diffusion index is then calculated by summing these values for each of the components and dividing the result by the number of series included in the diffusion index (in this case multiplied by 100). A value of the index above zero is then interpreted as an expansion in employment and vice versa for values less than zero; a value of 100 would be indicative of an expansion in all sectors.[6]

Consider employment in a given sector i and denote its monthly annualized growth rate by $\Delta x_{i,t} = 1200 \times \ln(x_{i,t}/x_{i,t-1})$. Because we are mainly concerned with issues related to the assessment of various economic conditions in real time, the key objective for many of the surveys considered below, our focus in this paper will be on monthly data. Let n represent the number of sectors covered by the employment index and denote overall employment growth at t by Δx_t. Then, we have that $\Delta x_t \approx \frac{1}{n}\sum_{i=1}^{n} \Delta x_{i,t}$ or

$$\Delta x_t = \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^u} \Delta x_{i,t}^u}_{\Delta x_t^u} - \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^d} \Delta x_{i,t}^d}_{\Delta x_t^d}, \tag{2}$$

where $\Delta x_{i,t}^u = \Delta x_{i,t}$ if $\Delta x_{i,t} \geq 0$, and $\Delta x_{i,t}^d = -\Delta x_{i,t}$ if $\Delta x_{i,t} < 0$.[7] Simply put, equation (2) distinguishes between those sectors that contribute positively to overall employment growth (up sectors), which sum to n_t^u and contribute Δx_t^u, and those that take away from overall growth (down sectors), which sum to n_t^d and contribute negatively Δx_t^d.

Let μ_t^u represent the cross sectional mean growth rate, at a point in time, of the sectors that add to overall growth in employment, $\mu_t^u = \frac{1}{n_t^u}\sum_{i=1}^{n_t^u} \Delta x_{i,t}^u$, and similarly let $\mu_t^d = \frac{1}{n_t^d}\sum_{i=1}^{n_t^d} \Delta x_{i,t}^d$ for those sectors where employment declined. Then, equation (2) can alternatively be written as

$$\Delta x_t = \frac{n_t^u}{n}\mu_t^u - \frac{n_t^d}{n}\mu_t^d. \tag{3}$$

Define $\mu^u = T^{-1}\sum_{t=1}^{T} \mu_t^u$ and $\mu^d = T^{-1}\sum_{t=1}^{T} \mu_t^d$ as the time averages, or long-run cross-sectional means, of sectors contributing and subtracting from overall employment growth respectively; we assume that the data is stationary with corresponding population conditional means $\overline{\mu}^u = E_{i,t}(\Delta x_t^i | \Delta x_t^i \geq 0)$ and $\overline{\mu}^d = E_{i,t}(\Delta x_t^i | \Delta x_t^i < 0)$. We may then express positive contributions

[6] Note that the information conveyed by the index is invariant to an affine transformation. For example, the Federal Reserve Board defines the diffusion index of Industrial Production as the proportion of sectors where production increased plus half the sectors where production was unchanged, in which case $\mu = \kappa = 1/2$ in equation (1).

[7] We define overall growth in employment using uniform weights in this case. Foerster, Sarte, and Watson (2011) show, in the context of Industrial Production, that the choice of weights, either uniform, constant mean shares, or time-varying shares, is somewhat unimportant in aggregating growth rates across sectors.

to Δx_t as

$$\Delta x_t^u = \left(\frac{n_t^u}{n} - \varphi^u\right)\mu^u + \varphi^u\left(\mu_t^u - \mu^u\right) + \left(\frac{n_t^u}{n} - \varphi^u\right)\left(\mu_t^u - \mu^u\right) + \varphi^u\mu^u, \tag{4}$$

where $\varphi^u = T^{-1}\sum_{t=1}^{T} n_t^u/n$ is the long-run average of proportion of sectors that raise overall growth.[8] In other words, at a point in time, a large increase in overall employment growth via Δx_t^u may arise because the proportion of expanding sectors is higher than usual given their average contribution, $\left(\frac{n_t^u}{n} - \varphi^u\right)\mu^u > 0$ akin to a rising extensive margin; the cross sectional average of those positive contributions is higher than usual given the typical proportion of expanding sectors, $\varphi^u\left(\mu_t^u - \mu^u\right) > 0$ akin to a rising intensive margin; or both to the extent that both are true, $\left(\frac{n_t^u}{n} - \varphi^u\right)\left(\mu_t^u - \mu^u\right) > 0$. Similarly for the negative contributions, we have that

$$\Delta x_t^d = \left(\frac{n_t^d}{n} - \varphi^d\right)\left(\mu_t^d - \mu^d\right) + \varphi^d(\mu_t^d - \mu^d) + \left(\frac{n_t^d}{n} - \varphi^d\right)\mu^d + \varphi^d\mu^d, \tag{5}$$

where $\varphi^d = T^{-1}\sum_{t=1}^{T} n_t^d/n$. Then, it follows that overall employment growth may be approximated as

$$\Delta x_t \approx \underbrace{\varphi^u\left(\mu_t^u - \mu^u\right) - \varphi^d(\mu_t^d - \mu^d)}_{\text{Change in ``how much'' or intensive margin}} + \underbrace{\mu^u D_t}_{\text{Change in ``how many'' or extensive margin}}, \tag{6}$$

where $D_t = \left(\frac{n_t^u}{n} - \frac{n_t^d}{n}\right)$ is the difference in the proportions of increasing and decreasing series defined earlier.

Put another way, by re-interpreting the scalar μ in (1), we may decompose employment growth as arising primarily from the change in an intensive margin, the difference between how fast "up" sectors grew and how badly "down" sectors declined, and the change in a quasi-extensive margin, the difference between the proportion of sectors that expanded versus those that declined or the breadth of the expansion, $\mu^u D_t$. The approximation in (6) comes about in part because the positive and negative cross-sectional growth rates μ^u and μ^d are not necessarily the same, and the fact that the difference in the way the extensive and intensive margins interact in (4) and (5) also matters in principle.[9] However, as we now illustrate, this difference in interaction terms is largely immaterial for the behavior of overall employment growth, except in the last recession.

The top panel of Figure 1 shows the decomposition of Δx_t^u (demeaned) in equation (4) for U.S. employment. Examination of the figure indicates that variations in the proportion of expanding sectors, conditional on their average contribution, closely tracks the positive contributions to overall employment growth, Δx_t^u, while the interaction term between extensive and intensive margins is

[8] Details of all derivations in the paper are given in online-only supplementary notes, Pinto, Sharp, and Sarte (2015).

[9] The derivation in (6) makes use of the fact that, without loss of generality, $\mu^u = \mu^d + \varepsilon$ for some $\varepsilon \lesseqgtr 0$. In the case of employment, μ^u and μ^d are close, 3.84 percent versus 3.22 percent respectively over the period 1991-2014, so that either one may be used to scale the diffusion index to arrive at an approximate measure of extensive margin.

mostly unimportant. Figure 1 also illustrates, in the bottom panel, the decomposition of Δx_t^d in (5). As with the top panel, the variation in the proportion of declining sectors explains a large fraction of the negative contribution to overall employment growth, although the intensive margin now also plays an unambiguous role, especially at times when Δx_t^d spikes during recessions.

Figure 2 combines these two decompositions and shows that, to a large extent, variations in overall employment growth arise from changes in the quasi-extensive margin in (6). Put another way, movements in employment tend to reflect the breadth of change (scaled appropriately) rather than large variations in cross-sectional growth rates driven by particular sectors. Table 1 summarizes these results.

Table 1. Decomposition of BLS Employment Growth Rates

Series	1990-2014
$\Delta x_t = \frac{1}{n}\sum \Delta x_{i,t}$	2.8
Components	
$D_t \mu^u$	1.8
$\varphi^u(\mu_t^u - \mu^u) - \varphi^d(\mu_t^d - \mu^d)$	0.9
$(\varphi_t^u - \varphi^u)(\mu_t^u - \mu^u) - (\varphi_t^d - \varphi^d)(\mu_t^d - \mu^d) + \varepsilon\varphi_t^d$	0.7

Note: Entries are the sample standard deviation of Δx_t and its components. Percentage points at annual rates.

This observation is reminiscent of Gourio and Kashyap (2007) who show, using plant-level data from Chile and the U.S., that "the number of establishments undergoing investment spikes (the "extensive margin") account for the bulk of variation in aggregate investment." In that sense, it is conceivable that variations in the breath of change, D_t, captured in various surveys, say regarding the business outlook of a sample of purchasing managers, would have proven a useful indicator of economic activity, even without more "concrete" data at hand. In the case of Figure 2, notable exceptions concern pronounced downturns in employment that take place during recessions. In those periods, changes in the intensive margin defined in (6) contribute as much to the downturn as the extensive margin captured by the diffusion index, and thus so does the interaction of the two margins, and reveal a potentially interesting asymmetry across expansions and downturns.

It should be noted that the notion of a "quasi extensive margin," in this context, abstracts from entry and exit since the number of sectors is held fixed in the BLS calculation of the employment diffusion index. In the many surveys that produce diffusion indices, including those of the Michigan Survey of Consumers that we study below, the sample size is also often targeted to a fixed level. In that sense, diffusion indices are then specifically concerned with the distribution of change in an existing set of disaggregated series.

3 A Formal Description of Survey-Based Diffusion Indices

In the previous section, we saw that the extensive margin captured by BLS's employment diffusion index is an important component of overall employment performance. In an attempt to assess various economic conditions in real time, a number of institutions and government agencies carry out qualitative surveys that are then similarly translated into a diffusion index. For instance, the Michigan Survey of Consumers (MSC) constructs widely used composite diffusion indices that reflect consumer sentiment regarding various current and expected economic conditions, including household financial conditions, overall business conditions, and spending on durable goods, the latter being akin to a measure of investment. Similarly, several Federal Reserve Banks construct and publicly report diffusion indices that are meant to track the breadth of economic performance in their respective regions in real time. Other diffusion indices that receive widespread attention by markets and policymakers include the monthly index of manufacturing and service conditions released by the Institute for Supply Management. Similar to the employment diffusion index described in the previous section, these surveys most often rely on trichotomous classifications whereby participants are asked whether conditions are better, worse, or unchanged relative to the previous month.

3.1 The Individual Diffusion Index and its Cross-Sectional Distribution

Consider a sample of n survey participants drawn randomly from a population at a point in time. Given the focus on timeliness underlying diffusion indices, the vast majority of indices provide monthly information. Each participant answers a survey question relating to an economic series of interest, say household financial conditions or overall business conditions, according to a predefined set of qualitative responses. We denote the set of possible answer types by $\mathcal{A}$ and consider r possible qualitative responses, $\mathcal{A} = \{1, 2, ..., r\}$. Answer types from participants are indexed by $a \in \mathcal{A}$. In the most common example, there are 3 types of responses, "up" (u), "down" (d), and "same" (s), or alternatively "better off", "worse off,", or "same." For example, a question in the Michigan Index of Consumer Sentiment (ICS) asks: "do you think that a year from now, you (and your family living there) will be better off, worse off, or just about the same as now?" In that case, we might write that $a \in \mathcal{A} = \{u, d, s\}$. A typical sample of responses might be summarized by the vector

$$(u, s, u, d, d, u, ...,). \tag{7}$$

Let n^a denote the number of respondents associated with answer $a \in \mathcal{A}$, $\sum_{a=1}^{r} n^a = n$.[10] Answers of type a are assigned a value of $\omega^a \in \mathcal{R}$ in the diffusion index, where $\underline{\omega}^a$ and $\overline{\omega}^a$ denote

[10] In principle, the number of responses of a particular type a (relative to the total number of responses) may be serially correlated over time. As will be made clear, however, our focus is on characterizing the cross-sectional distribution, and related uncertainty, that arises from the different survey responses at a point in time, so that we drop the time subscript. To the extent that the number of responses of each type are serially correlated, to reflect persistence in participants' views, so will be the measured uncertainty.

lower and upper bounds respectively associated with possible values of ω^a. In the conventional example with 3 categories, $\omega^u = 1$, $\omega^s = 0$, and $\omega^d = -1$. An alternative formulation might have $\omega^u = 1$, $\omega^s = 1/2$, and $\omega^d = 0$, as with the Board of Governors diffusion index of industrial production.

The general diffusion index statistic is then given by[11]

$$\widehat{D} = \sum_{a=1}^{r} \omega^a \frac{n^a}{n}. \tag{8}$$

By construction, the resulting index will range from $\underline{\omega}^a$ to $\overline{\omega}^a$, with numbers above $(1/r) \sum_{a=1}^{r} \omega^a$ generally being interpreted as an expansion in the condition of interest, say household spending on durable goods. Observe that different combinations of answers can result in the same index. For example, in the case comprising 3 categories, $\omega^u = 1$, $\omega^s = 0$, and $\omega^d = -1$, and 100 firms being asked about overall business conditions, a value of $D = 0$ might emerge from having 50 firms responding "up" and 50 firms responding "down," or all firms responding "no change." While both cases may be interpreted as a reading of unchanged business conditions overall, we describe below the sense in which these two cases imply a different degree of confidence in the overall reading of "no change."

Let p^a denote the probability, in a given month, that a participant's answer is $a \in \mathcal{A} = \{1, 2, ..., r\}$, with $\sum_{a=1}^{r} p^a = 1$. The survey process, according to which n participants are drawn randomly from a population, has a natural interpretation in terms of n independent trials, each of which leads to a success for exactly one of r types of responses, with each answer type having a given fixed success probability, p^a. The multinomial distribution then gives the probability, given n, of observing any particular combination of numbers of responses, $\{n^1, n^2, ..., n^r\}$, for the various categories $\{1, ..., r\}$. In particular, the probability mass function of this distribution is

$$f(n^1, ..., n^r; n, p^1, ..., p^r) = \frac{n!}{\Pi_{a=1}^{r} n^a!} \Pi_{a=1}^{r} (p^a)^{n^a}, \tag{9}$$

where the expected number of answers of type a is $E(n^a) = np^a$, with associated variance $Var(n^a) = np^a(1 - p^a)$. The covariance between the numbers of answers of types $a \in \mathcal{A}$ and $a' \in \mathcal{A}$ is given by $Cov(n^a, n^{a'}) = -np^a p^{a'}$. Observe that $E(n^a)$ and $Var(n^a)$ in this case are respectively the mean and variance of a Binomial distribution defined by the marginal distribution, for answers of type a, of the multinomial described in (9).

Let $\widehat{p}^a = n^a/n$ denote the proportion of answers of type $a \in \mathcal{A}$ observed in a given survey. We can alternatively write $\widehat{p}^a$ as

$$\widehat{p}^a = \frac{1}{n} \sum_{i=1}^{n} x_i^a, \tag{10}$$

[11] A constant is sometimes added to $\widehat{D}$, as in equation (1), to correct for sample design changes but this modification is immaterial for questions related to uncertainty.

where x_i^a is an indicator variable that takes on the value 1 when survey participant i answers a and is zero otherwise. Since $\widehat{p}^a$ then has the interpretation of a sample Bernoulli mean, the Multivariate Central Limit Theorem, combined with the fact that sums of Bernoulli random variables are binomial, immediately gives that,

$$\sqrt{n}\begin{pmatrix} \widehat{p}^1 - p^1 \\ \widehat{p}^2 - p^2 \\ \ldots \\ \widehat{p}^{r-1} - p^{r-1} \end{pmatrix} \underset{D}{\rightarrow} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \ldots \\ 0 \end{pmatrix}, \begin{pmatrix} p^1(1-p^1) & -p^1p^2 & \ldots & -p^1p^{r-1} \\ -p^2p^1 & p^2(1-p^2) & \ldots & -p^2p^{r-1} \\ \ldots & \ldots & \ldots & \ldots \\ -p^{r-1}p^1 & -p^{r-1}p^2 & \ldots & p^{r-1}(1-p^{r-1}) \end{pmatrix}\right). \quad (11)$$

Moreover, because the generic individual diffusion index, $\widehat{D}$, is a linear combination of sample Bernoulli means, $\sum_{a=1}^{r}\omega^a\widehat{p}^a$, that are asymptotically jointly normally distributed according to (11), the distribution of $\widehat{D}$ will also be asymptotically normal.[12] Thus, in large finite samples, the distribution of the diffusion index statistic is approximately normal and given by

$$\sqrt{n}\left(\widehat{D} - D\right) \sim^a \mathcal{N}\left(0, \left(\sum_{a=1}^{r}\left(\omega^a\right)^2 p^a\right) - D^2\right), \quad (12)$$

where $D = E(\widehat{D}) = \sum_{a=1}^{r}\omega^a p^a$.[13]

Given equation (12), it immediately follows that confidence intervals for $\widehat{D}$ are symmetric and can be approximately constructed as $D \pm z\sqrt{\left(\sum_{a=1}^{r}\left(\omega^a\right)^2 p^a - D^2\right)/n}$, where z is the standard score corresponding to a critical value of interest. Confidence intervals, therefore, reflect in part sampling uncertainty, which decreases with the square root of the sample size, n, and the degree of polarization among survey respondents' answers, $\left(\sum_{a=1}^{r}\left(\omega^a\right)^2 p^a\right) - D^2$. To the degree that the weights, ω^a, or the number of responses, r, determine the scale of D, the standard deviation $\sqrt{\left(\sum_{a=1}^{r}\left(\omega^a\right)^2 p^a - D^2\right)}$ then preserves the same units. The following examples provide some intuition.

3.1.1 Practical Applications and Special Cases

In many common applications, such those regarding any question making up the MSC's Index of Consumer Sentiment (ICS), we have that $\mathcal{A} = \{u, d, s\}$, and $\omega^u = 1$, $\omega^s = 0$, $\omega^d = -1$. Then, $\widehat{D} = (\widehat{p}^u - \widehat{p}^d)$ and

$$\sqrt{n}\left(\widehat{D} - D\right) \underset{D}{\rightarrow} \mathcal{N}\left(0, (1-p^s) - D^2\right). \quad (13)$$

In this example, the variance of the diffusion index is also, in large samples, the population equivalent of the uncertainty proxy estimated in Bachmann, Elstner, and Sims (2013). We make two

[12]Note that $\widehat{p}^r$ is simply defined as a residual, $\widehat{p}^r = 1 - \sum_{a=1}^{r-1}\widehat{p}^a$, and thus need not be included in equation (11).

[13]A derivation of this result is given in Appendix A1. Observe that, in principle, different functions of these probabilities might be of interest, including, for instance, measures of concentration such as that captured by the Herfindahl index, $\sum_i \left(\widehat{p}^i\right)^2$, whose distribution might be characterized following the same steps.

observations to provide intuition regarding the variance of $\widehat{D}$. First, it decreases as responses become less polarized, or alternatively split between extremes. In particular, the variance in (13) decreases with the magnitude of the diffusion index itself, $D^2 = (p^u - p^d)^2$; simply put, the only way for D^2 to be large, or more specifically to approach 1, is for all respondents to answer either "up" or "down," in which case answers coincide and there remains little room for uncertainty, $p^s = 0$ and $(1 - p^s) - D^2 = 0$. Alternatively, consider the case where where "up" and "down" responses are evenly split, $p^u = p^d$ so that $D = 0$ irrespective of p^s. Then uncertainty decreases as p^s rises (and both p^u and p^d fall while D remains unchanged); in other words, we are relatively more confident of a reading of no change, $D = 0$, when everyone reports "no change," $p^s = 1$ and all answers coincide, than when half the respondents report "up" and half the respondents report "down," $p^s = 0$ (the largest even split). Second, since $\widehat{D}$ derives from a weighted sum of means, its variance simply decreases at rate n.

As another example, consider the Federal Reserve Board's diffusion index of Industrial Production, where $A = \{u, d, s\}$, $\omega^u = 1$, $\omega^s = 1/2$, $\omega^d = 0$, and $\widehat{D} = \widehat{p}^u + (1/2)\widehat{p}^s$. In that case,

$$\sqrt{n}\left(\widehat{D} - D\right) \underset{D}{\rightarrow} \mathcal{N}\left(D, p^u + \frac{1}{4}p^s - D^2\right).$$

As before, the standard deviation of the index decreases as responses become less polarized; when either p^u or p^d is 1, $p^s = 0$ and $p^u + \frac{1}{4}p^s - D^2 = 0$.

At a more granular level, diffusion indices may, in some cases, take into account different categories or groups of respondents. For example, participants may be reporting from different sectors, $j = 1, ..., J$,

$$\widehat{D} = \sum_{j=1}^{J}\sum_{a=1}^{r}\omega^a \frac{n_j^a}{n}, \tag{14}$$

where n_j^a denotes the number of responses of type a from group j with $\sum_{j=1}^{J} n_j^a = n^a$. This last expression may be equivalently written as

$$\widehat{D} = \sum_{j=1}^{J}\frac{n_j}{n}\underbrace{\sum_{a=1}^{r}\omega^a \frac{n_j^a}{n_j}}_{\widehat{D}_j},$$

where $\sum_{j=1}^{J} n_j = n$, so that the overall diffusion index may be interpreted as a weighted sum of group-specific diffusion indices, $\widehat{D}_j$, with weights $\frac{n_j}{n}$, where n_j is the total number of responses obtained from group j. In principle, one may choose to scale those weights either up or down to emphasize particular target groups relative to the number of responses obtained in those groups, say by γ_j,[14]

[14] For instance, the weights might ultimately reflect sectoral shares in value added or gross output. In practice, membership in the ISM Business Survey Committee is in fact initially based on each industry's contribution to GDP.

$$\widehat{D} = \sum_{j=1}^{J} \gamma_j \frac{n_j}{n} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j}.$$

How then does this alternative weighting scheme affect the computation of confidence intervals? The answer depends not only on the alternative scaling, γ_j, but also the more granular information captured in different groups by way of p_j^a, the probability of observing answer type "a" in group j in the overall survey, where $\sum_{a=1}^{r} \sum_{j=1}^{J} p_j^a = 1$. In particular, letting $\widehat{p}_j^a = n_j^a/n$ denote the proportion of answers of type $a \in \mathcal{A}$ observed in sector j in the overall sample, we have that

$$\sqrt{n}\left(\widehat{D} - D\right) \sim^a \mathcal{N}\left(0, \left(\sum_{a=1}^{r} \sum_{j=1}^{J} \left(\omega^a \gamma_j\right)^2 p_j^a\right) - D^2\right), \tag{15}$$

where $D = \sum_{a=1}^{r} \omega^a \sum_{j=1}^{J} \gamma_j p_j^a$.[15]

4 Composite Indices and their Distribution: Measuring Uncertainty Using Multiple Diffusion Indices

In practice, very few, if any, diffusion indices are reported as individual indices but rather are reported as composite indices, constructed as weighted averages of individual component indices, described in the previous section, across different categories. For example, the headline Michigan Index of Consumer Sentiment combines together information from individual indices with respect to five categories: current and expected household financial conditions, current and expected overall business conditions, and spending on big ticket items or durable goods. To the extent that each series is associated with some degree of uncertainty, arising from both sampling and fundamental polarization of survey answers, the uncertainty associated with the overall composite index will in turn depend on the extent to which this polarization comoves across individual component indices.

In characterizing the approximate distribution of diffusion indices, one objective throughout the paper is to highlight the nature of the micro data needed to measure uncertainty in the survey responses, and thus how to organize and maintain incoming individual survey answers. As we will see, this involves keeping track of pairwise proportions of answer types across different questions. Intuitively, since composite indices are weighted averages of individual indices, each providing feedback on a particular question, the uncertainty surrounding the composite index will reflect not only the uncertainty associated with individual component indices but also the covariances between them. For example, survey participants responding "up" to a particular category, such as their household financial condition, may be more or less likely to also respond "up" to another category, such as spending on durable goods, and this degree of comovement may change as the state of the economy changes. Confidence intervals, therefore, must be adjusted accordingly. The next section formalizes these ideas.

[15] See appendix A2.

4.1 Composite Diffusion Indices

To describe composite diffusion indices, in a way that builds directly on the intuition presented thus far, we now consider a sample of n survey participants responding to questions concerning $\overline{k}$ economic conditions of interest. In practice, these might include financial conditions, employment conditions, etc. Answers from each participant corresponding to conditions of a particular component, k, are indexed by a_k, confined to a set $\mathcal{A}_k$, each comprising r possible types of responses, $\{1, 2, ..., r\}$. For example, $\mathcal{A}_1$ might describe whether conditions are "better off," "worse off,' or "unchanged" in a given month, with the subscript "1" identifying the category "household financial conditions," and $a_1 \in \{u, d, s\}$, and similarly the subscript "2" in $\mathcal{A}_2$ might denote "overall business conditions."

A survey participant's answers across all components, $k = 1, ..., \overline{k}$, are collected in a $k-$tuple $\mathbf{a} = (a_1, ..., a_{\overline{k}})$ that lives in the set $\mathcal{A} = \Pi_{k=1}^{\overline{k}} \mathcal{A}_k$. In our simple example with 3 components, each comprising 3 possible responses, an example of $\mathbf{a}$ might be (u, u, u), indicating that conditions are "up," meaning improving, in household financial conditions, overall business conditions, and, say, spending on durable goods. In this case, $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 = \{u, d, s\} \times \{u, d, s\} \times \{u, d, s\} = \{(u, u, s), (u, u, d), (u, u, s), (d, u, u), (d, d, u), (d, s, u), (s, u, u), (s, d, u), (s, s, u), ...\}$ has 27 elements. In general, $\mathcal{A}$ will have dimension $r^{\overline{k}}$.

It will be convenient below to distinguish between answers for a given component, k, and those for all other components, which we denote by $(a_k, \mathbf{a}_{-k})$, where $\mathbf{a}_{-k} \in \mathcal{A} \setminus \mathcal{A}_k$. We let $n^{\mathbf{a}}$ denote the number of respondents associated with answers $\mathbf{a} \in \mathcal{A}$, where $\sum_{\mathbf{a} \in \mathcal{A}} n^{\mathbf{a}} = n$. Observe that given our notation, $n^{\mathbf{a}}$ may also be expressed as $n^{(a_k, \mathbf{a}_{-k})}$. We let n_k^a denote the number of responses associated with answer $a_k \in \mathcal{A}_k$ for component k summed across all other components, $n_k^a = \sum_{\mathbf{a}_{-k} \in \mathcal{A} \setminus \mathcal{A}_k} n^{(a_k = a, \mathbf{a}_{-k})}$. For example, in the Michigan ICS, n_2^u might be the number of respondents reporting improving (u) overall business conditions (indexed by the subscript "2"), where $n_2^u = \sum_{\mathbf{a}_{-2} \in \mathcal{A} \setminus \mathcal{A}_2} n^{(u, \mathbf{a}_{-2})}$ includes all those reporting "up" on business conditions irrespective of their answers to other questions.

As before, answers of type $a_k \in \mathcal{A}_k$ for component k are assigned a value of $\omega^a \in \mathcal{R}$ independently of k. Then, an individual component diffusion index, D_k, associated with category or question k is given by

$$\widehat{D}_k = \sum_{a=1}^{r} \omega^a \sum_{\mathbf{a}_{-k} \in \mathcal{A} \setminus \mathcal{A}_k} \frac{n^{(a_k = a, \mathbf{a}_{-k})}}{n} = \sum_{a=1}^{r} \omega^a \frac{n_k^a}{n}, \tag{16}$$

and the composite index is averaged over all $\overline{k}$ components,

$$\widehat{D} = \sum_{k=1}^{\overline{k}} \delta_k \widehat{D}_k, \tag{17}$$

where $0 < \delta_k < 1$. Note that the weights, δ_k, do not necessarily sum to 1, as in the Michigan ICS which gives each of five questions an equal weight, and uses this weight to normalize the overall index to a base year.

4.2 The Distribution of Composite Diffusion Indices

Let the probability of drawing answers $\mathbf{a} = (a_1, ...a_{\overline{k}}) \in \mathcal{A}$ be denoted by $p^{\mathbf{a}}$, with $\sum_{\mathbf{a}\in\mathcal{A}} p^{\mathbf{a}} = 1$, where $p^{\mathbf{a}}$ can also be expressed as $p^{(a_k,\mathbf{a}_{-k})}$. Thus, we denote the marginal probability of drawing a given response $a \in \mathcal{A}_k$ for component k as $p_k^a = \sum_{\mathbf{a}_{-k}\in\mathcal{A}\setminus\mathcal{A}_k} p^{(a_k=a,\mathbf{a}_{-k})}$. Because the composite index (17) averages across different component indices, say D_k and D_ℓ, we will need to take into account the pairwise covariance between components. We now describe how this process in turn relies on keeping track of all pairwise joint probabilities between any two components of the composite diffusion index.

Denote the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components k and ℓ by $p_{k\ell}^{aa'}$. This probability is given by

$$p_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}}\in\mathcal{A}\setminus\mathcal{A}_k\times\mathcal{A}_\ell} p^{(a_k=a,a_\ell=a',\mathbf{a}_{-\{k,\ell\}})},\ k \neq \ell, \tag{18}$$

where the notation $(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})$ distinguishes between answers for component k, component ℓ, and all other components, $-\{k,\ell\}$. The marginal with respect to component k satisfies $p_k^a = \sum_{a'\in A_\ell} p_{k\ell}^{aa'}$. We let $\mathbf{p}_{k\ell}$ denote the vector comprising all pairwise joint probabilities, $p_{k\ell}^{aa'}$ for given components k and ℓ, where the dimension of $\mathbf{p}_{k\ell}$ is r^2. Thus, for the example with 3 components and 3 possible responses, the element p_{13}^{ud} in $\mathbf{p}_{13}$ gives the joint probability of observing "up" along dimension "1,"or improving household financial conditions, and "down" along dimension "3,"or deteriorating spending on durable goods.

Denote the number of survey participants answering $a_k = a$ and $a_\ell = a'$ for the components k and ℓ by $n_{k\ell}^{aa'}$, where similarly to equation (18),

$$n_{aa'}^{k\ell} = \sum_{\mathbf{a}_{-\{k,\ell\}}\in\mathcal{A}\setminus\mathcal{A}_k\times\mathcal{A}_\ell} n^{(a_k=a,a_\ell=a',\mathbf{a}_{-\{k,\ell\}})},\ k \neq \ell,$$

As before, the number of participants answering a given response a for component k satisfies $n_k^a = \sum_{a'\in A_\ell} n_{k\ell}^{aa'}$.

Let $\widehat{p}_k^a = n_k^a/n$ in the component index (16) and $\widehat{p}_{k\ell}^{aa'} = n_{k\ell}^{aa'}/n$. Observe that

$$\widehat{D}_k = \sum_{a=1}^{r} \omega^a \widehat{p}_k^a = \sum_{a=1}^{r} \omega^a \sum_{a'\in\mathcal{A}_\ell} \widehat{p}_{k\ell}^{aa'}, \tag{19}$$

for any component ℓ, where the components other than ℓ have already been integrated out in (18). Equation (19) effectively allows us to write each component index, D_k, in the overall index, $\widehat{D}$, in terms of joint pairwise probabilities with any other component index, D_ℓ, and, therefore, capture the pairwise comovement across these indices.

As in section 2, each element $\widehat{p}_{k\ell}^{aa'}$, which we collect in the vector $\widehat{\mathbf{p}}_{k\ell}$, may be interpreted as the sample mean of Bernoulli random variables so that

$$\sqrt{n}(\widehat{\mathbf{p}}_{k\ell} - \mathbf{p}_{k\ell}) \underset{D}{\rightarrow} \mathcal{N}(0, \Sigma_{\mathbf{p}_{k\ell}}). \tag{20}$$

As shown in Appendix A.3, the information contained in (20), constructed for all pairs of questions k and ℓ in the survey, is all that is needed to construct sampling uncertainty around a composite index that combines different individual indices.[16] A typical element of $\widehat{\mathbf{p}}_{k\ell}$, say $\widehat{p}^{ab}_{k\ell}$, is such that $E(\widehat{p}^{ab}_{k\ell}) = p^{ab}_{k\ell}$, $Var(\widehat{p}^{ab}_{k\ell}) = n^{-1}(1 - p^{ab}_{k\ell})p^{ab}_{k\ell}$, and its covariance with any other element $\widehat{p}^{b'a'}_{k\ell}$ is $Cov\left(\widehat{p}^{k\ell}_{ab}, \widehat{p}^{k\ell}_{b'a'}\right) = -n^{-1}p^{ab}_{k\ell}p^{b'a'}_{k\ell}$. In particular, we have that

$$\sqrt{n}\left(\widehat{D} - D\right) \sim^a \mathcal{N}\left(0, \sum_{k=1}^{\overline{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2\sum_{1\leq k<\ell\leq\overline{k}} \delta_k\delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)\right), \tag{21}$$

where

$$D = \sum_{k=1}^{\overline{k}} \delta_k \sum_{a=1}^{r} \omega^a p_k^a$$

$$Var\left(\widehat{D}_k\right) = \frac{1}{n}\left\{\left(\sum_{a=1}^{r} (\omega^a)^2 p_k^a\right) - (D_k)^2\right\}, \tag{22}$$

and

$$Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) = \frac{1}{n}\left\{\sum_{(a,a')\in\mathcal{A}_k\times\mathcal{A}_\ell} \omega^a\omega^{a'}\left[p^{aa'}_{k\ell} - \sum_{b,b'\in\mathcal{A}_k\times\mathcal{A}_\ell} p^{ab}_{k\ell}p^{b'a'}_{k\ell}\right]\right\}. \tag{23}$$

5 Diffusion Indices in Practice

We now illustrate what the results derived in sections 3 and 4 imply in practice. First, we revisit the BLS employment diffusion index highlighted earlier as an example of an individual diffusion index. We then consider the historical behavior of consumer uncertainty, by way of composite diffusion indices, using micro data on individual survey responses published by the Michigan Survey of Consumers (MSC). In either case, measures of uncertainty associated with the index variance tend to fall the more a given direction of change is shared across sectors, or the more answers to given survey questions agree, and rise as sectoral performance or survey responses become more polarized.

The top panel of Figure 3 reproduces the quasi-extensive margin contribution to employment growth depicted in Figure 2, along with 95 percent confidence intervals. Since the distribution in (12) is Normal, it is straightforward to construct confidence intervals for this measure, shown as the solid blue lines around μD in the top panel. Moreover, since the BLS considers 264 sectors, the sampling error associated with this quasi-extensive margin component is relatively small. The bottom panel of Figure 3 shows the standard deviation of the BLS (scaled by $\sqrt{n}$) extensive margin component of employment growth in (12), along with its Hodrick-Prescott (HP) trend and with the recessions shaded in gray. Observe that from 1990 to 2000, this measure of uncertainty, or

[16] Observe that the elements of $\mathbf{p}_{k\ell}$ sum to 1, in that $\mathbf{p}_{k\ell}$ can be used to define a marginal multinomial distribution obtained by integrating out all categories other than k and ℓ from the underlying primitive mutlinomial distribution over all categories (i.e. marginal distributions constructed from integrating out dimensions of a multinomial distribution remain multinomial).

polarization, tends to move opposite the index itself, as is typical of other conventional measures of uncertainty. As employment performance improves in the mid-1990s, the improvement also becomes more widespread across sectors and uncertainty falls; similarly, as overall employment starts declining towards the 2001 recession, it also becomes more polarized with some sectors holding up while others lose employment.

In sharp contrast, however, the Great Recession stands out in that not only does BLS employment growth reach an all time low during this period, the breadth of the decline is also unprecedented, with the degree of polarization thus also reaching an all time low. In that sense, the Great Recession was not only particularly severe along the intensive margin but also particularly widespread. One key aspect of the notion of uncertainty emphasized here, as measured by the variance of diffusion indices, is precisely that it is tied to the breadth of change; we become more or less confident in a given change as it becomes more or less widespread. Survey data, when they are purely qualitative, lack the intensive margin - for example, how strongly respondents might feel about overall business conditions. However, as we now show, we may nevertheless estimate a notion of uncertainty around sentiment based on how widely it is shared.

5.1 The Michigan Survey of Consumer Sentiment

The consumer survey carried out by the Survey Research Center at the University of Michigan publishes 3 composite diffusion indices, the Index of Current Conditions (ICC), the Index of Consumer Expectations (ICE), and the headline Index of Consumer Sentiment (ICS), from a sample of between 400 and 600 monthly interviews on average regarding household and overall conditions. Specifically, the composite indices are derived from the following 5 questions:

*Q*1) *"We are interested in how people are getting along financially these days. Would you say that you (and your family living there) are better off or worse off financially than you were a year ago, or just about the same as now?"*

*Q*2) *"Now looking ahead – do you think that a year from now you (and your family living there) will be better off financially, or worse off, or just about the same as now?"*

*Q*3) *"Now turning to business conditions in the country as a whole – do you think that during the next twelve months we'll have good times financially, or bad times, or what?"*

*Q*4) *"Looking ahead, which would you say is more likely – that in the country as a whole we'll have continuous good times during the next five years or so, or that we will have periods of widespread unemployment or depression, or what?"*

*Q*5) *"About the big things people buy for their homes – such as furniture, a refrigerator, stove, television, and things like that, generally speaking, do you think now is a good or bad time for people to buy major household items?"*

A somewhat unique feature of the MSC composite indices, relative to other published diffusion indices, concerns the individual response level data underlying the index calculations, which are made readily available through the Survey Center's online archives starting in 1978. In particular, from the archives, it is possible to obtain response-level data that allow keeping track, in each month, of a given respondent's answers to each question making up the various MSC composite indices. Given this level of detail, it is then possible to construct, in any given month, all pairwise responses, $n_{k\ell}^{aa'}$, and proportions, $\widehat{p}_{k\ell}^{aa'}$, where $k\ell$ denotes any pair of questions from the set of questions $Q1$ through $Q5$ listed above, and aa' any pair of answers, for example "better off," and "good times."

5.1.1 The Michigan Index of Current Conditions

To calculate the Index of Current Conditions (ICC), the MSC first computes the diffusion indices, or the proportion giving favorable replies less that giving unfavorable replies (plus 100) for questions $Q1$ and $Q5$ listed above, denoted D_1 and D_5 respectively. Each individual index is rounded to the nearest whole number. The ICC is then calculated as:

$$ICC = \frac{D_1 + D_5}{2.6424} + 2, \tag{24}$$

where the denominator in the above expression establishes a base period (1966), and the constant corrects for sample design changes in the $1950s$.[17] Observe that D_5 summarizes a direction of change in households' current attitudes towards the purchase of large ticket items or durable goods. As such, it is closely linked to households' attitudes towards investment. The individual index D_1 summarizes instead the direction of change in the state of households' finances, which likely has a direct bearing on their "ability and willingness to buy," which is used in part by the MSC as a working definition of consumer confidence.

The top panel of Figure 4 shows the historical behavior of the standard deviation of the ICC (times $\sqrt{n}$), as given by equation (21), along with its HP trend and the recessions shaded in gray. Beginning in the early 1990s, the uncertainty reflecting the degree of polarization in responses underlying the ICC begins a decade-long decline to its lowest point in the sample, around 1999. This decade-long decline in uncertainty corresponds to one of the strongest expansions in post-war U.S. economic history, as measured in part by consumption growth, ending with the dot-com bust and the start of the 2001 recession. Prior to the 2001 recession, the degree of polarization in the ICC begins to rise as consumers increasingly disagree on the questions related to current conditions. As in Bachmann, Elstner, and Sims (2013), and Baker, Bloom, and Davis (2013), spikes in disagreement prior to the recessions of 1991, 2001, and 2007 are clearly visible. In principle, however, disagreement about the state of current conditions would not necessarily have to rise during recessions since it may be generally recognized, and agreed upon, by consumers that the

[17] To the best of our knowledge, the micro or response level data is publicly available only since 1978.

state of the economy is poor, or more specifically in this case that it is not a good time to purchase durable goods.

The recovery from the 2001 recession was relatively subdued relative to other post-war U.S. recoveries, especially where employment is concerned, and the recovery from the 2007 recession is widely known to be the weakest on record from a number of standpoints, most notably per capita GDP growth. Interestingly, the top panel of Figure 4 indicates that throughout the 2001 and 2007 recessions, and the recovery in between, uncertainty in consumers' views regarding the state of current conditions, at least as captured by the MSC, continued to rise steadily. Moreover, the degree of polarization in answers to questions regarding current conditions remained noticeably flat in the nearly six years that followed the Great Recession, although disagreement has recently started to fall somewhat.

Remarkably, the level of uncertainty in the ICC today, almost six years removed from the most recent recession, is comparable to that which emerged during the twin recessions of the 1980s and the period immediately surrounding the 1991 recession. The standard deviation of the ICC (normalized by $\sqrt{n}$) currently stands around 20 percent higher than in 1999, at which time it began to generally rise to the level we see today. In fact, the period since 2000 is notable relative to other post-war recoveries in the sample in that uncertainty unambiguously declined during the periods of pronounced economic expansion of the late 1980s and 1990s. These observations are suggestive, as argued by Bloom (2014) and others, that the state of economic activity is intrinsically linked to the degree of uncertainty households perceive, not only at business cycle frequencies but as shown here at longer frequencies as well. As with other measures of uncertainty in the literature, uncertainty in the ICC, in spite of currently being at a near all time high 6 years into an expansion, tends to be countercyclical, falling in expansions and rising in and around recessions.

The bottom panel of Figure 4 illustrates the decomposition of uncertainty in the ICC in terms of the variances of the individual diffusion indices associated with questions $Q1$ and $Q5$ in the MSC, and the covariance between them (adding the series in the bottom panel, and dividing by 2.6424^2, gives the square of the series in the top panel). The main driver of uncertainty or variance in the ICC is the variance of the diffusion index associated with question $Q5$ in the MSC; the question related to consumers' perception of whether it is a good or bad time to purchase major household items or durable goods. The degree of polarization in answers to that question has tended to rise between 1999 and 2009, and has come down somewhat since then. However, observe that since the end of the most recent recession, the covariance between the diffusion indices D_1 and D_5 has continued to rise slightly, a trend that began after the 2001 recession, indicating rising coincidence in answers to questions $Q1$ and $Q5$ over this period. In other words, the extent to which consumers' answers to questions $Q1$ and $Q5$ have tended to locate in the extremes, good or bad rather than middle, has gradually become more consistent across questions. As described in section 4, in the case of composite diffusion indices, coincident polarization across questions increases uncertainty as captured by the variance of the composite index.

5.1.2 The Michigan Index of Consumer Expectations

The Michigan Index of Consumer Expectations (ICE) is based on diffusion indices that summarize households' perceptions of times ahead, both for themselves and the country as a whole, over different time horizons, one and five years out. In particular, the ICE is given by

$$ICE = \frac{D_2 + D_3 + D_4}{4.1134} + 2,$$

where D_2, D_3, and D_4 are the diffusion indices associated with questions $Q2$, $Q3$, and $Q4$ listed above respectively. As with any survey, the questions are potentially subject to varying interpretations by consumers, and the level of polarization in answers might then also vary considerably over time. As we now illustrate, however, uncertainty in the ICE tends to be remarkably stable throughout the 1980s and into the 1990s, before starting a steady rise that is even more pronounced with respect to expectations than that shown with respect to consumers' uncertainty around current conditions.

The top panel of Figure 5 shows the level of the ICE published by the MSC, and the bottom panel uncertainty in the ICE implied by the standard deviation in equation (21). Although more involved than in the case of the individual diffusion index, the distribution of composite indices in (21) remains Normal, so that constructing confidence intervals for the level of the ICE remains conceptually straightforward. These are shown as the solid blue lines around the MSC index in the top panel. However, as we address in more detail below, the calculations now entail the consideration of multiple pairwise objects, $\widehat{p}_{k\ell}^{ab}$, across answers and survey questions.

Observed first that the top and bottom panels of Figure 5 paint distinctly different pictures; one provides a measure of the direction of change in households' perception of times ahead, while the other is indicative of the level of polarization in those perceptions. Starting in the early 1990s, the ICE begins to climb steadily, in the top panel, as the U.S. economy enters one of its longest expansion periods and households contemporaneously grow more optimistic about the future. At the same time, as with the ICC, consensus that the next one to five years are likely to be "good times" also grows among households and the level of polarization in answers falls in the bottom panel. With the arrival of the 2001 recession, the ICE experiences a significant dip and levels off, while uncertainty among consumers about what to expect begins to increase dramatically. As the 2007 recession starts, the ICE falls further and remains at relatively subdued levels throughout the subsequent recovery period. At the same time, uncertainty about the next one to five years generally continues to increase and polarization among consumers stands today, six years into the weakest recovery of the post-war period, more than 20 percent above its 1999 level. Observe that, in contrast to the ICC, the 1991 and 2007 recessions are associated with pronounced downward spikes and a general pessimistic consensus about the future. In terms of trend, however, uncertainty about what lies ahead conveyed by consumers remains relatively stable throughout the 1980s and up to the mid-1990s. The period since 2000, in contrast, is one of striking rising uncertainty in consumers' perception of the future, indicating today a level of polarization in households' expectations unprecedented since 1978.

Figure 6 shows the decomposition of uncertainty in the Michigan ICE. The top panel of Figure 6 illustrates the behavior over time of the variances (scaled by n) associated with the individual diffusion indices, D_2, D_3, and D_5, while the bottom panel shows the behavior of the pairwise covariances. In the top panel, we see that the level of polarization in answers to question $Q2$, regarding personal finances one-year ahead, is remarkably constant throughout the entire sample. Uncertainty regarding the national outlook, however, at both the one and five year horizon, sees more variations over time. In the bottom panel, all covariances experience a generally increasing tendency starting in 2000, but this tendency is especially pronounced where the comovement between answers to questions $Q3$ and $Q4$ are concerned. Recall that questions $Q3$ and $Q4$ in the MSC differ mainly with respect to the time horizon over which households are asked about their expectations of the national outlook. Put another way, households' answers regarding the country as a whole grow more coincident between the one and five year horizon. This finding suggests that with the onset of the 2001 recession, households begin to perceive the state as more persistent, regardless of whether it is good or bad. This more coincident polarization of expectations across the one and five year horizon in turn is a key driver of the rising uncertainty in the ICE in the bottom panel of Figure 6. In this case, the covariation in the degree of disagreement across different questions of a qualitative survey plays a substantive role in the determination of overall uncertainty conveyed by the survey.

Figure 7 illustrates the more coincident polarization of responses with respect to the one and five year expectations of overall business conditions by plotting the various elements that make up the covariance between the diffusion indices associated with questions $Q3$ and $Q4$, $Cov\left(\widehat{D}_3, \widehat{D}_4\right)$. In particular, in this case, the expression in (23) reduces to

$$Cov\left(D_3, D_4\right) = \left(p_{34}^{uu} + p_{34}^{dd}\right) - \left(p_{34}^{ud} + p_{34}^{du}\right) - D_3 \times D_4. \tag{25}$$

Intuitively, the more responses to questions $Q3$ and $Q4$ in the MSC coincide, or the larger p_{34}^{uu} and p_{34}^{dd}, the more D_3 and D_4 tend to comove, while the reverse is true the more answers disagree across questions, or the larger p_{34}^{du} and p_{34}^{ud}. Furthermore, analogously to the squared term in the equation describing the variance of individual diffusion indices (22), the more one sided responses become in the same direction, or alternatively as both D_3 and D_4 approach either 1 or -1, the less room there is for the indices to comove.

In the case of questions $Q3$ and $Q4$ in the MSC, the top panel of Figure 7 shows the behavior of the various proportions making up equation (25). First note that the terms capturing the coincidence in responses, p_{34}^{uu} and p_{34}^{dd}, tend to dominate relative to the terms reflecting disagreement, p_{34}^{ud} and p_{34}^{du}, and are noticeably procyclical and countercyclical respectively. Second, in the bottom panel of Figure 7, we see that the rise in comovement between D_3 and D_4, depicted in the bottom panel of Figure 6, indeed arises from increasing coincidence, and decreasing disagreement, among responses concerning overall business conditions at the one and five year horizons. In that sense, survey responses indicate that participants increasingly see the state one year ahead as persisting into a five year horizon.

5.1.3 The Michigan Index of Consumer Sentiment

The Michigan headline Index of Consumer Sentiment (ICS) summarizes the direction of change in consumer feedback by combining both their assessment of current conditions, which drives the ICC, and their expectations, which drives the ICE,

$$ICS = \frac{D_1 + D_2 + D_3 + D_4 + D_5}{6.7558} + 2.$$

Given the historical behavior of uncertainty in the ICC and ICS, it is not surprising to see, in the bottom panel of Figure 8, uncertainty in the ICS begin to rise dramatically around 2000, and again following the 2007 recession, to reach an unprecedented level over our sample period. Uncertainty in the ICS today, as measured by the standard deviation of the ICS (times $\sqrt{n}$), is approximately 25 percent higher than throughout the 1980s and 1990s. Observe, in particular, that as consumer sentiment, depicted in the top panel of Figure 8, has steadily risen since the end of the last recession, so has the degree of polarization of responses reflected in the standard deviation of the ICS in the bottom panel. Consumers, therefore, while more confident are also more polarized in their view.

Other indices frequently used in the literature to measure uncertainty include the Chicago Board Options Exchange Market Volatility Index (VIX) and, more recently, the Economic Policy Uncertainty (EPU) index developed by Baker, Bloom, and Davis (2013).[18] Both indices are shown in Figure 9. Note that the secular behavior of ICS uncertainty, in the bottom panel of Figure 8, and that of the EPU is remarkably similar, with the EPU also reaching its peak well after the end of the Great Recession in 2012. However, the key difference between ICS uncertainty and the EPU, and indeed the VIX, concerns the current period. Both the EPU and VIX are now declining, with the decline in the VIX starting immediately after the Great Recession, whereas ICS uncertainty in Figure 8 has steadily risen throughout the post 2007 recession recovery, the weakest post-war recovery on record.

We end this section by highlighting the fact that, in characterizing the full distribution of general composite diffusion indices, the formulas derived in (21) and (23) may be used to produce confidence intervals around any diffusion index estimate. In the case of the Michigan Survey of Consumers, Table 2 summarizes the margins of error associated with 95 percent confidence intervals for its different indices averaged over different sample periods.

[18]This VIX captures the stock market's expectation of volatility over the next 30 days, and is constructed using a series of options included in the S&P 500 index. The EPU intends to capture uncertainty in economic policy and consists of three components. The first component is based on media coverage of economic policy uncertainty. The second tracks federal tax provisions that are set to expire in the next few years. And the third one captures disagreement between economic forecasters.

Table 2. CONSUMER SURVEYS: 95 PERCENT CONFIDENCE INTERVALS

Series	1978-1999	2000-2014	1978-2014
Index of Current Conditions	±4.19	±4.58	±4.34
Index of Consumer Expectations	±4.02	±4.49	±4.20
Index of Consumer Sentiment	±3.29	±3.71	±3.46

Note: Entries are time averages of $\pm1.96\sqrt{\sum_{k=1}^{\bar{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2\sum_{1\leq k<\ell\leq\bar{k}} \delta_k\delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)/n}$.

As shown in the top panels of Figures 5 and 8, with around 500 respondents, these margins are relatively tight compared to the span of the indices, although variations within those margins are not infrequently taken up or discussed as meaningful changes in direction.[19] The Institute for Supply Management, and the Philadelphia Business Outlook Survey, do not publicly report margins of errors, and indeed few if any of the most widely published diffusion indices do.

6 Concluding Remarks

In this paper, we thoroughly examine the property of diffusion indices defined as estimates of the breadth of change in an aggregate series of interest. The analysis highlights, in the first place, the relevance of diffusion indices in capturing changes in the extensive margin. We show, for instance, that the BLS's employment diffusion index explains the most part of overall employment growth.

Next, we characterize the distribution of general composite diffusion indices, defined as the weighted sum of individual indices based on responses to different individual survey questions. We show that diffusion indices are asymptotically normal, and that the uncertainty proxy in Bachmann, Elstner, and Sims (2013) is the variance of a particular albeit widely used individual diffusion index (scaled by the square root of the sample size). This proxy, which takes into account the uncertainty implied by both sampling and the polarization or disagreement of participants' responses, can be used to construct confidence intervals for the diffusion index. Our approach reveals that a general notion of uncertainty based on composite indices reflects both the degree of disagreement with respect to an individual survey question, and the degree of disagreement across individual questions. In particular, we show that only pairwise proportions of answer types across questions are relevant to derive the variance of composite indices.

Finally, we use micro data published by the Michigan Survey of Consumers to illustrate our results. We find that starting in 2000, consumer uncertainty, calculated using our approach, steadily

[19] Specialized media follows closely changes in consumer sentiment. In the wake of the government shutdown of 2013, for example, the Wall Street Journal reported that "U.S. consumers turned less optimistic about the economy in early October, according to data released Friday. The Thomson-Reuters/University of Michigan preliminary October sentiment index slipped to 75.2 from an end-September level of 77.5, according to an economist who has seen the numbers." In this case, the index decreased only 2.3 points in anticipation of the government shutdown, which statistically did not indicate a change in consumer sentiment.

increases contrasting with the gradual decline in uncertainty observed in the previous decade, and higher agreement in prior periods. Furthermore, uncertainty arising from the polarization of responses in the Michigan Survey currently stands, six years removed from the Great Recession, at its highest level since 1978. Relative to the index of Economic Policy Uncertainty (EPU), and although consumer uncertainty generally shows similar secular fluctuations throughout the period under consideration, the series tend to diverge in recent years. While the EPU declined sharply from its post-war period peak two years after the Great Recession, consumer uncertainty has tended to increase since 2009 recession, reaching the highest level on record.

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7 Appendices

A1. The distribution of individual diffusion indices, $\widehat{D} = \sum_{a=1}^{r} \omega^a \widehat{p}^a$.

Since the $\widehat{p}^a$'s are approximately normally distributed in large samples, as described by equation (11), so is their linear combination. Moreover, we have that

$$\begin{aligned} D &= E(\widehat{D}) \\ &= \sum_{a=1}^{r} \omega^a E(\widehat{p}^a) = \sum_{a=1}^{r} \omega^a p^a. \end{aligned}$$

Finally,

$$\begin{aligned} Var\left(\widehat{D}\right) &= Var\left(\sum_{a=1}^{r} \omega^a \widehat{p}^a\right) \\ &= \sum_{a=1}^{r} (\omega^a)^2 Var\left(\widehat{p}^a\right) + \sum_{a \neq a'} \omega^a \omega^{a'} Cov(\widehat{p}^a, \widehat{p}^{a'}) \\ &= \sum_{a=1}^{r} (\omega^a)^2 \frac{p^a(1-p^a)}{n} - 2 \sum_{1 \leq a < a' \leq r} \omega^a \omega^{a'} \frac{p^a p^{a'}}{n} \\ &= \frac{1}{n} \left\{ \sum_{a=1}^{r} (\omega^a)^2 p^a - \sum_{a=1}^{r} (\omega^a)^2 (p^a)^2 - 2 \sum_{1 \leq a < a' \leq r} \omega^a \omega^{a'} p^a p^{a'} \right\}, \end{aligned}$$

or

$$Var\left(\widehat{D}\right) = \frac{1}{n} \left\{ \sum_{a=1}^{r} (\omega^a)^2 p^a - D^2 \right\}, \tag{26}$$

where the last line follows from the Multinomial Theorem.

A2. The distribution of weighted individual diffusion indices, $\widehat{D} = \sum_{j=1}^{J} \gamma_j \sum_{a=1}^{r} \omega^a \widehat{p}_j^a$.

Let

$$\widehat{p}_j^a = \frac{1}{n} \sum_{i=1}^{n} x_i^a(j),$$

where $x_i^a(j)$ is an indicator variable that takes on the value 1 when survey participant i answers a and belongs to group or sector j, and is zero otherwise. The Multivariate Central Limit Theorem gives

$$\sqrt{n}\begin{pmatrix} \widehat{p}_1^1 - p_1^1 \\ \widehat{p}_1^2 - p_1^2 \\ \dots \\ \widehat{p}_J^{r-1} - p_J^{r-1} \end{pmatrix} \underset{D}{\rightarrow} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \end{pmatrix}, \begin{pmatrix} p_1^1(1-p_1^1) & -p_1^1p_1^2 & \dots & -p_1^1p_J^{r-1} \\ -p_1^2p_1^1 & p_1^2(1-p_1^2) & \dots & -p_1^2p_J^{r-1} \\ \dots & \dots & \dots & \dots \\ -p_1^{r-1}p_J^1 & -p_1^{r-1}p_J^2 & \dots & p_J^{r-1}(1-p_J^{r-1}) \end{pmatrix}\right).$$

Thus, in large finite samples, the distribution of the diffusion index statistic, $\widehat{D} = \sum_{j=1}^{J}\gamma_j\sum_{a=1}^{r}\omega^a\widehat{p}_j^a$, is approximately normal with mean $D = \sum_{j=1}^{J}\gamma_j\sum_{a=1}^{r}\omega^a E\left(\widehat{p}_j^a\right) = \sum_{j=1}^{J}\gamma_j\sum_{a=1}^{r}\omega^a p_j^a$ and variance

$$\begin{aligned}
Var(\widehat{D}) &= Var\left(\sum_{(a,j)}(\omega^a\gamma_j)\,\widehat{p}_j^a\right) \\
&= \sum_{(a,j)}(\omega^a\gamma_j)^2\,Var\left(\widehat{p}_j^a\right) + \sum_{(a,j)\neq(a'j')}\left(\omega^a\omega^{a'}\gamma_j\gamma_{j'}\right)cov\left(\widehat{p}_j^a,\widehat{p}_{j'}^{a'}\right) \\
&= \sum_{(a,j)}(\omega^a\gamma_j)^2\,\frac{p_j^a(1-p_j^a)}{n} - \sum_{(a,j)\neq(a'j')}\left(\omega^a\omega^{a'}\gamma_j\gamma_{j'}\right)\frac{p_j^a p_{j'}^{a'}}{n} \\
&= \frac{1}{n}\left\{\sum_{(a,j)}(\omega^a\gamma_j)^2\,p_j^a - \left(\sum_{(a,j)}(\omega^a\gamma_j)^2\left(p_j^a\right)^2 + \sum_{(a,j)\neq(a'j')}\left(\omega^a\omega^{a'}\gamma_j\gamma_{j'}\right)p_j^a p_{j'}^{a'}\right)\right\} \\
&= \frac{1}{n}\left\{\left(\sum_{(a,j)}(\omega^a\gamma_j)^2\,p_j^a\right) - D^2\right\}.
\end{aligned}$$

A3. The distribution of composite individual indices, $\widehat{D} = \sum_{k=1}^{\overline{k}}\delta_k\widehat{D}_k$, where $\widehat{D}_k = \sum_{a=1}^{r}\omega^a\frac{n_k^a}{n}$.

$$\begin{aligned}
D &= E(\widehat{D}) \\
&= \sum_{k=1}^{\overline{k}}\delta_k\sum_{a=1}^{r}\omega^a E\left(\widehat{p}_k^a\right) = \sum_{k=1}^{\overline{k}}\delta_k\sum_{a=1}^{r}\omega^a p_k^a.
\end{aligned}$$

$$\begin{aligned}
Var(\widehat{D}) &= Var\left(\sum_{k=1}^{\overline{k}}\delta_k\widehat{D}_k\right) \\
&= \sum_{k=1}^{\overline{k}}\delta_k^2 Var\left(\widehat{D}_k\right) + \sum_{k\neq\ell}\delta_k\delta_\ell Cov\left(\widehat{D}_k,\widehat{D}_\ell\right) \\
&= \sum_{k=1}^{\overline{k}}\delta_k^2 Var\left(\widehat{D}_k\right) + 2\sum_{1\leq k<\ell\leq\overline{k}}\delta_k\delta_\ell Cov\left(\widehat{D}_k,\widehat{D}_\ell\right). \qquad (27)
\end{aligned}$$

In equation (27), analogously to the expression in(26), we have that for each individual diffusion index D_k,

$$Var\left(\widehat{D}_k\right) = \frac{1}{n}\left\{\left(\sum_{a=1}^{r}\left(\omega^a\right)^2 p_k^a\right) - \left(D_k\right)^2\right\},$$

where $p_k^a = \sum_{a' \in A_\ell} p_{k\ell}^{aa'} = \sum_{a_{-\{k,\ell\}} \in \mathcal{A} \backslash \mathcal{A}_k \times \mathcal{A}_\ell} p^{(a_k=a, a_\ell=a', a_{-\{k,\ell\}})}, \;\; k \neq \ell$, using the notation introduced in the text.

In addition,

$$\begin{aligned}
Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) &= Cov\left(\sum_{a=1}^{r} \omega^a \widehat{p}_k^a, \sum_{a'=1}^{r} \omega_{a'} \widehat{p}_\ell^{a'}\right) \\
&= Cov\left(\sum_{a=1}^{r} \omega^a \sum_{b \in \mathcal{A}_\ell} \widehat{p}_{k\ell}^{ab}, \sum_{a'=1}^{r} \omega^{a'} \sum_{b' \in \mathcal{A}_k} \widehat{p}_{k\ell}^{b'a'}\right) \\
&= \sum_{(a,a') \in \mathcal{A}_k \times \mathcal{A}_\ell} \omega^a \omega^{a'} Cov\left(\sum_{b \in \mathcal{A}_\ell} \widehat{p}_{k\ell}^{ab}, \sum_{b' \in \mathcal{A}_k} \widehat{p}_{k\ell}^{b'a'}\right) \\
&= \sum_{(a,a') \in \mathcal{A}_k \times \mathcal{A}_\ell} \omega^a \omega^{a'} \sum_{(b',b) \in \mathcal{A}_k \times \mathcal{A}_\ell} Cov\left(\widehat{p}_{k\ell}^{ab}, \widehat{p}_{k\ell}^{b'a'}\right)
\end{aligned}$$

As explained in the text, the elements, $\widehat{p}_{k\ell}^{ab}$, of $\widehat{\mathbf{p}}_{k\ell}$ sum to 1 and define a multinomial distribution so that $\widehat{\mathbf{p}}_{k\ell}$ is approximately normally distributed in large samples, with $E(\widehat{p}_{k\ell}^{ab}) = p_{k\ell}^{ab}$, $Var(\widehat{p}_{k\ell}^{ab}) = n^{-1}(1-p_{k\ell}^{ab})p_{k\ell}^{ab}$, and the covariance between any two pairs, $\widehat{p}_{k\ell}^{ab}$ and $\widehat{p}_{k\ell}^{b'a'}$, given by $Cov\left(\widehat{p}_{ab}^{k\ell}, \widehat{p}_{b'a'}^{k\ell}\right) = -n^{-1} p_{k\ell}^{ab} p_{k\ell}^{b'a'}$. In particular,

$$Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) = \frac{1}{n}\left\{\sum_{(a,a') \in \mathcal{A}_k \times \mathcal{A}_\ell} \omega^a \omega^{a'} \left[p_{k\ell}^{aa'} - \sum_{b,b' \in \mathcal{A}_k \times \mathcal{A}_\ell} p_{k\ell}^{ab} p_{k\ell}^{b'a'}\right]\right\}.$$

Figure 1: **Decomposition of Positive and Negative Contributions to Employment Growth**

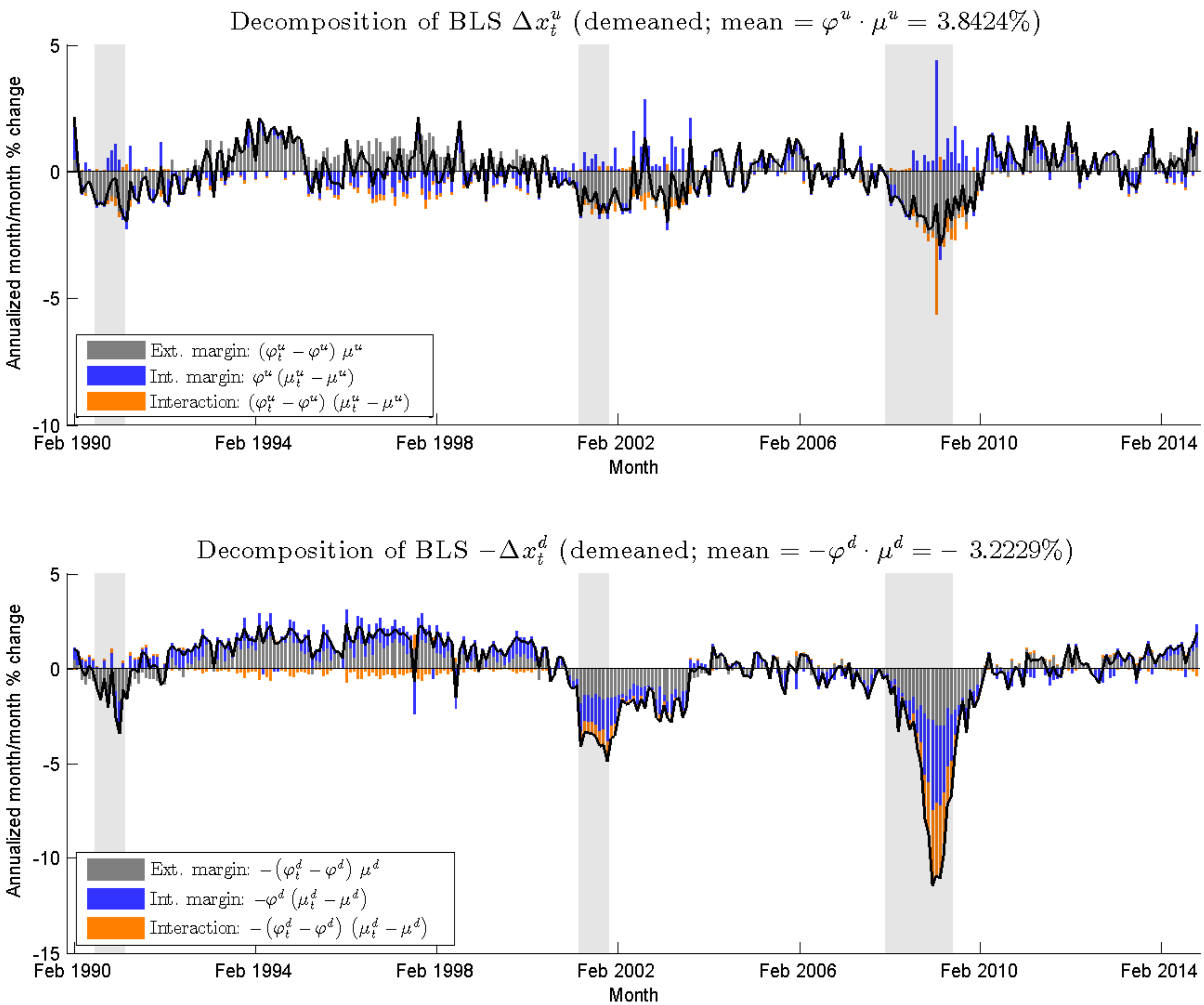


Figure 2: **Decomposition of Employment Growth**

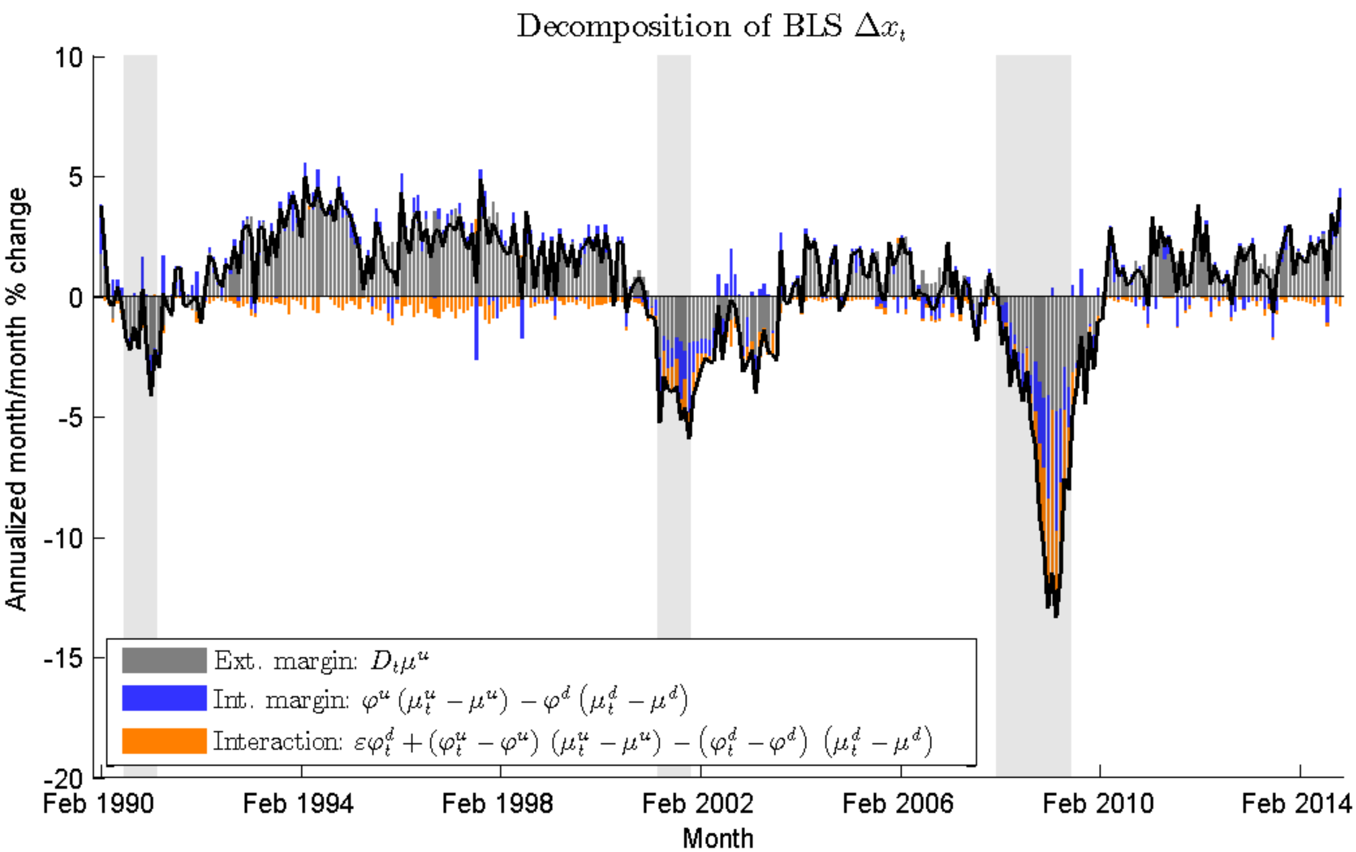


Figure 3: **Uncertainty and the Breadth of Change in Employment**

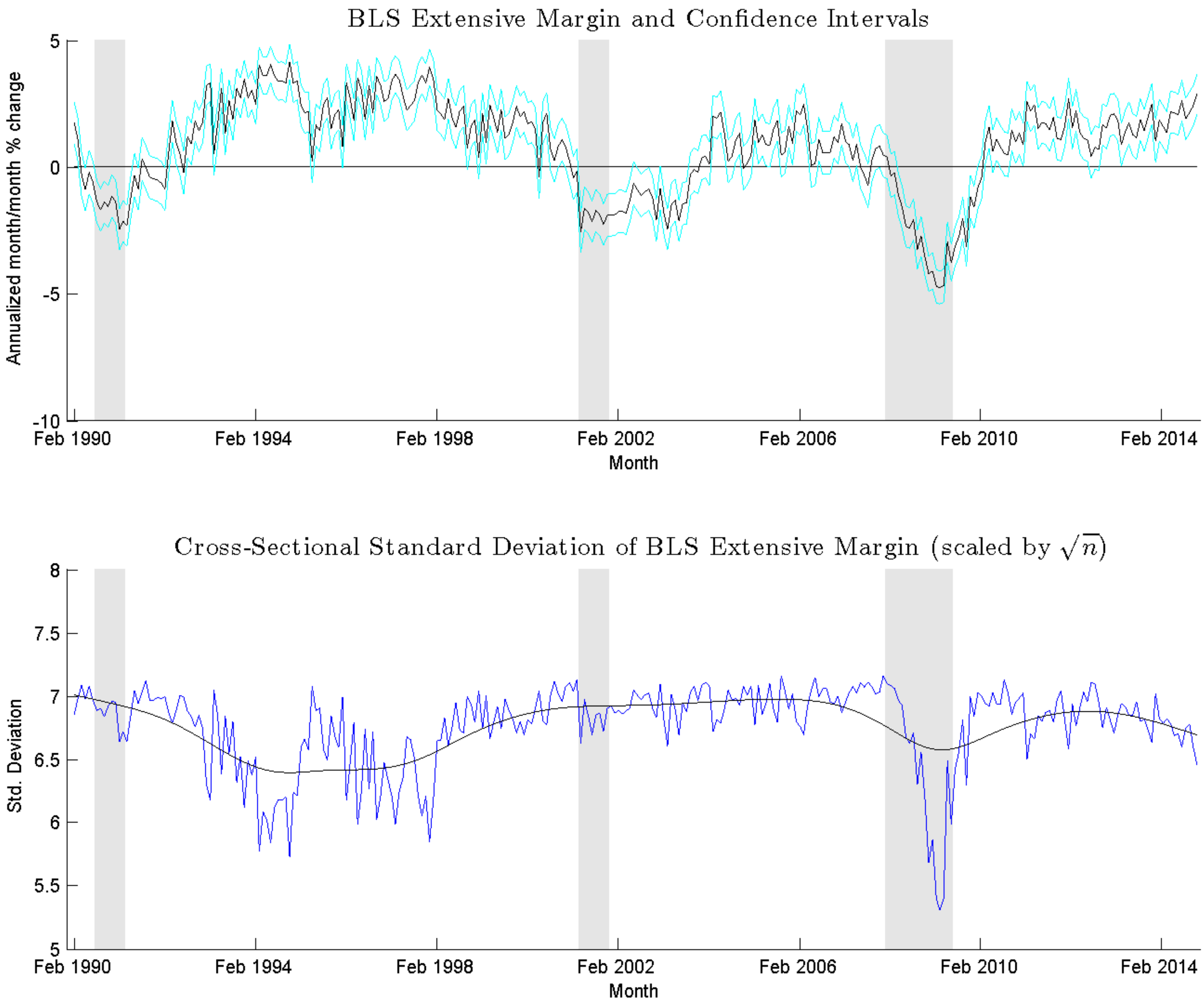
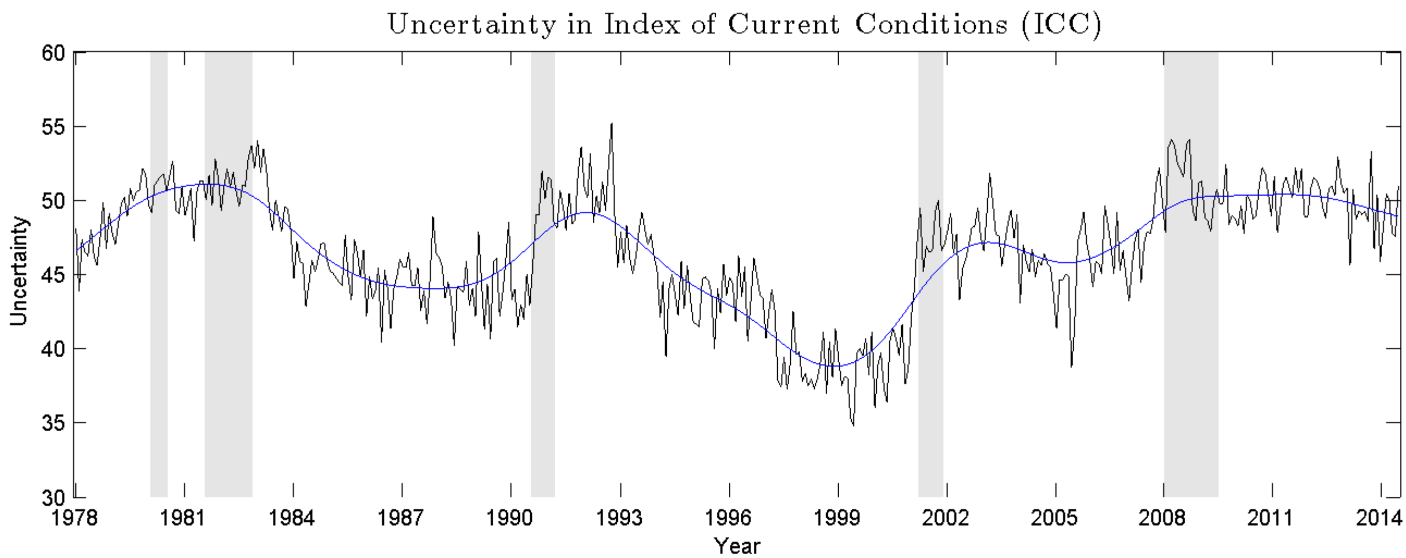


Figure 4: **Uncertainty in Consumers' Perception of Current Conditions**



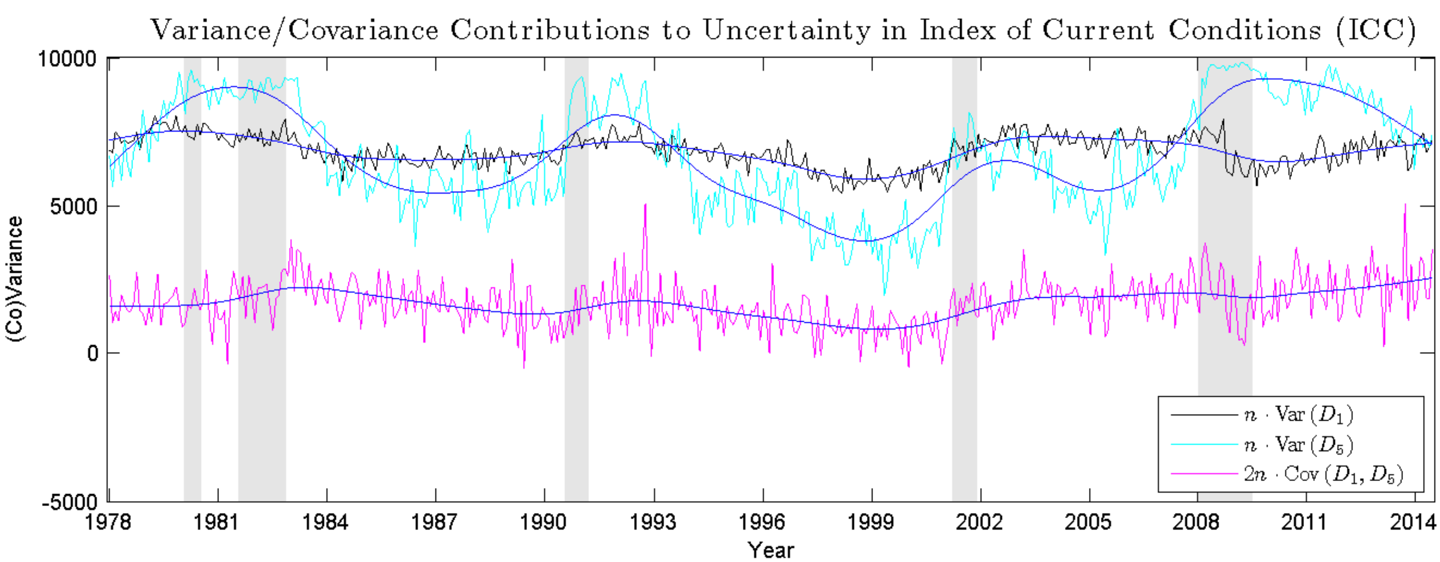


Figure 5: **Direction of Change and Uncertainty in Consumers' Expectations**

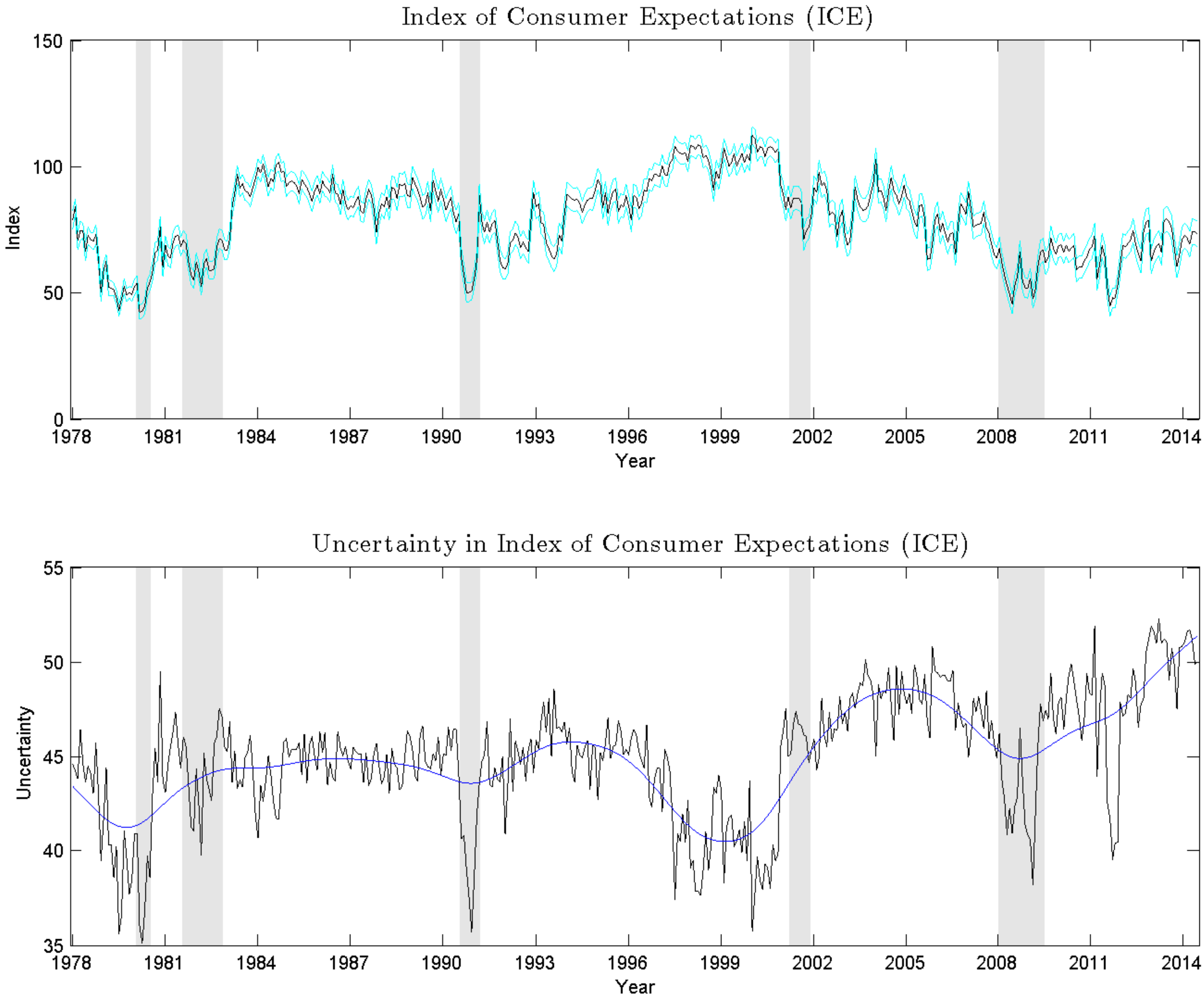
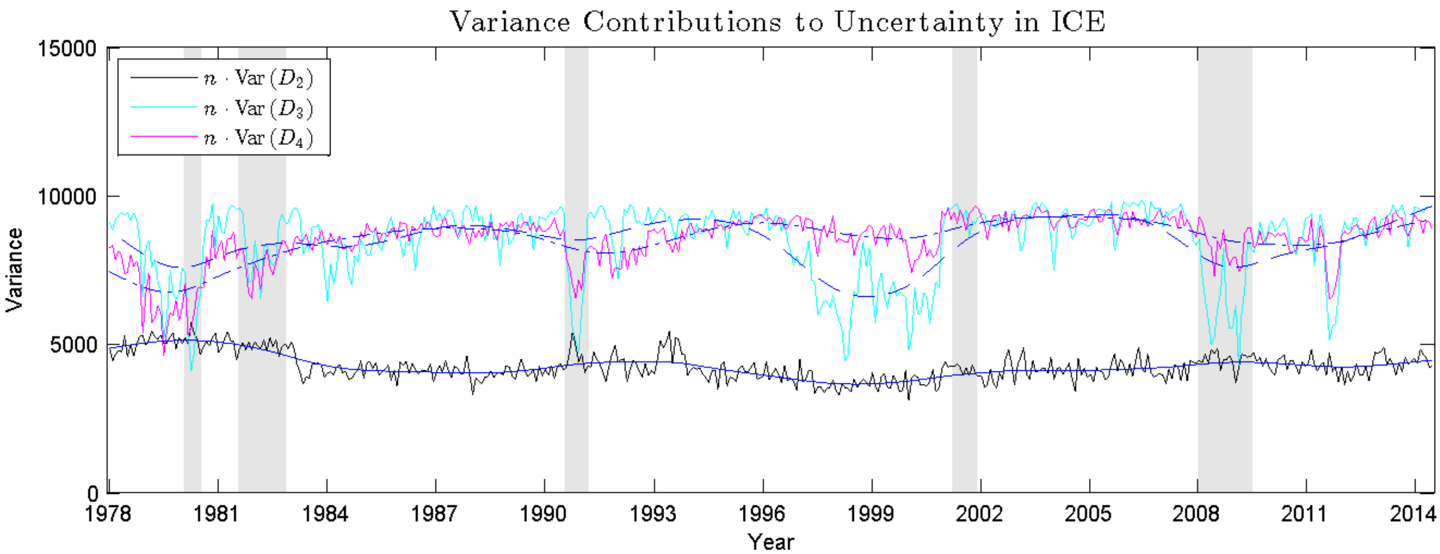


Figure 6: **Decomposition of Uncertainty in Consumers' Expectations**



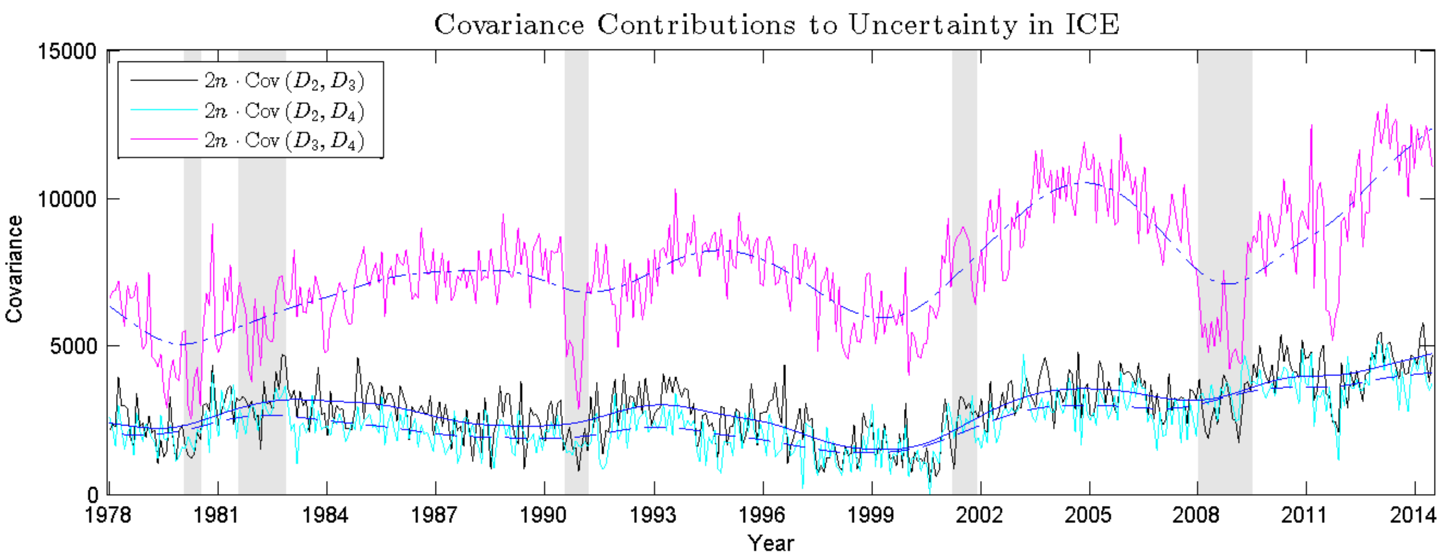
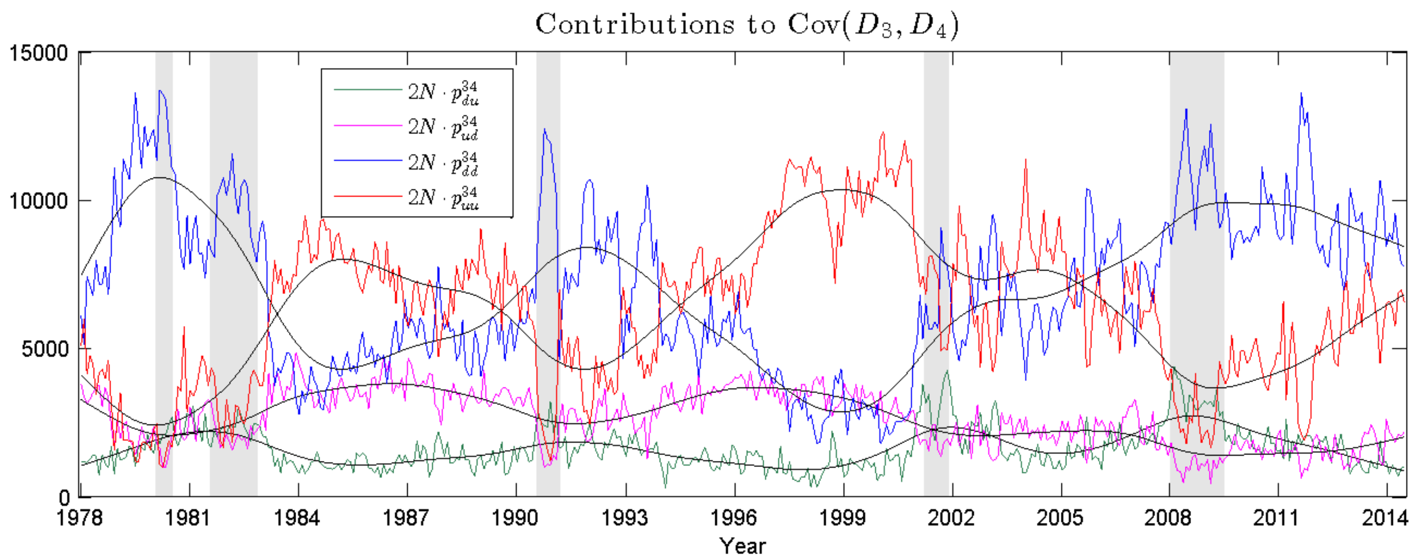


Figure 7: **Decomposition of Comovement in Answers Across the 1 and 5-Year Horizon**



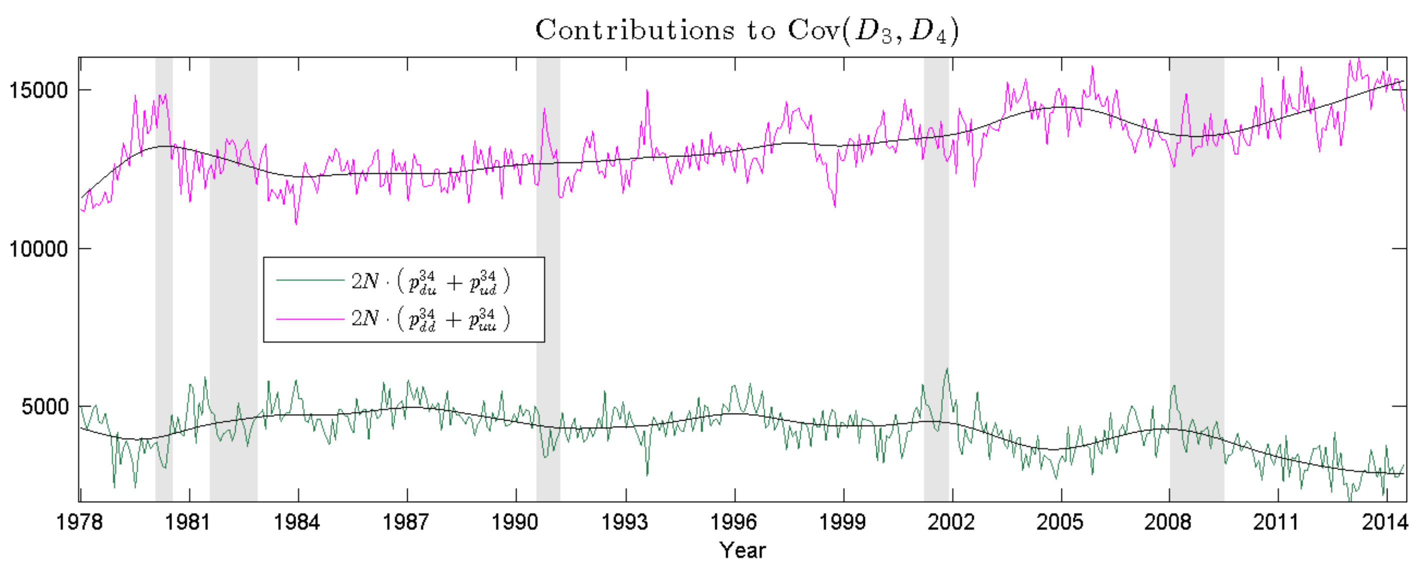


Figure 8: **Direction of Change and Uncertainty in Consumer Sentiment**

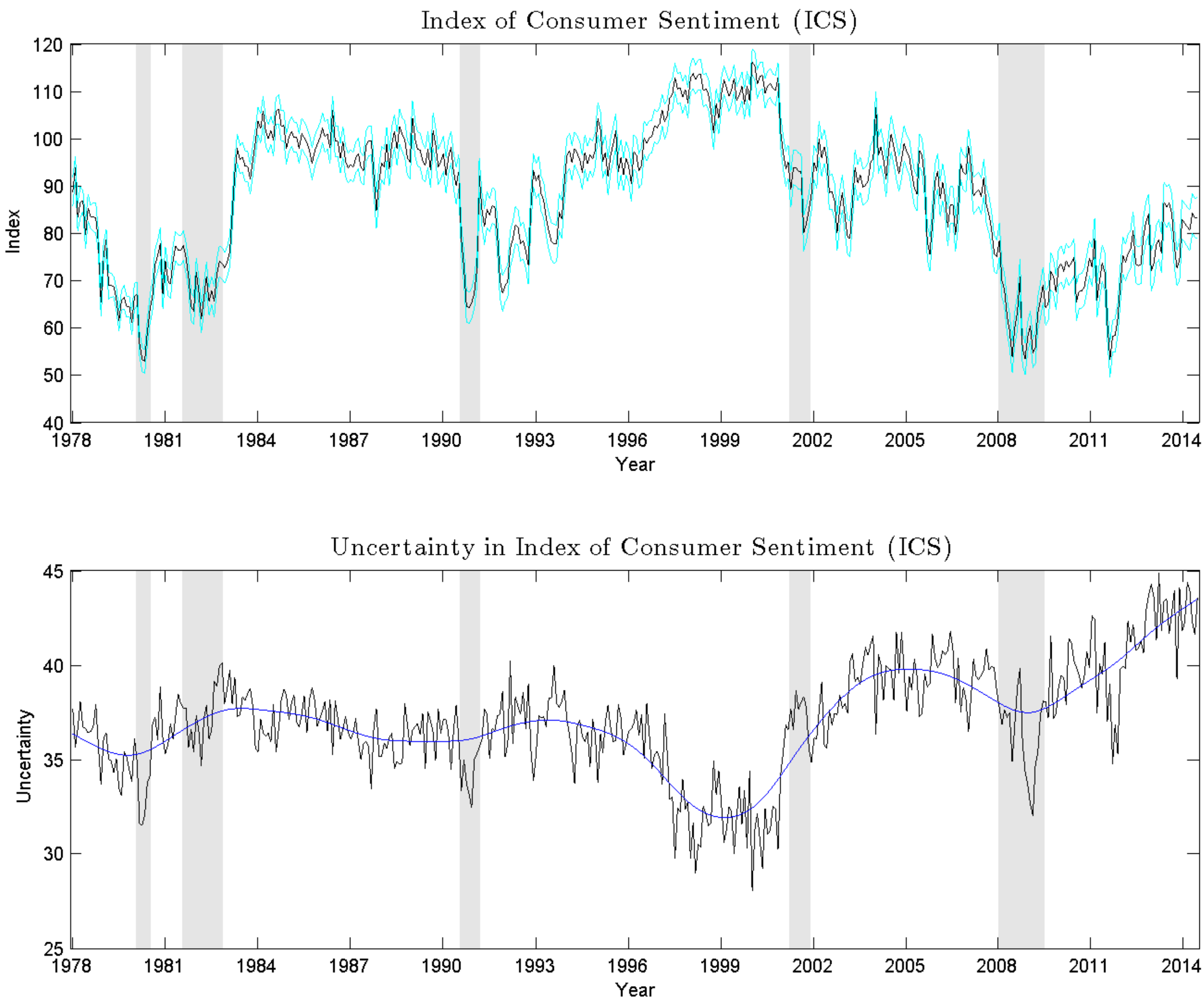


Figure 9: **Uncertainty Indicators**