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Estimating Matching Efficiency with Variable Search Effort[*]

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Abstract

We introduce a simple representation of endogenous search effort into the standard matching function with job-seeker heterogeneity. Using the estimated augmented matching function, we study the sources of changes in the average employment transition rate. In the standard matching function, the contribution of matching efficiency is decreasing in the matching function elasticity. In contrast, for our matching function with variable search effort and small matching elasticity, search effort is procyclical, accounting for most of the transition rate volatility; and the decline of the aggregate matching efficiency accounts for a small part of the decline in the transition rate after 2007. For a large matching elasticity, search effort is countercyclical, and large movements in matching efficiency compensate for that; and the decline in the matching efficiency accounts for a large part of the decline in the transition rate after 2007. The data on employment transition rates provide evidence for endogenous search effort but do not separately identify cyclicality of search effort and matching elasticity.

Key Words: Matching efficiency. Search effort. Matching elasticity. Aggregate matching function. **JEL Codes**: E24, J63, J64.

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1   Introduction

In Diamond-Mortensen-Pissarides matching models of the labor market, the matching function plays a role similar to the aggregate production function in macroeconomic models of the goods market.[1] The matching function is a reduced form representation of how in a frictional labor market the combination of workers who are looking for employment and vacant positions that need to be filled—the inputs to the matching function—results in new matched workers and positions—the output of the matching function. In the standard matching function, inputs are homogeneous and there is no utilization variation of inputs. During and after the 2007-09 recession, the ability of the standard matching function to account for the decline in the average employment transition rate through changes in its inputs has deteriorated. Changes in the number of new matches that cannot be accounted for by changes in inputs are attributed to the residual - matching efficiency. Similarly to the measurement of total factor productivity in the growth accounting literature, measurement of the matching efficiency depends, however, on the measurement of inputs.

There are substantial differences in the employment transition rates of different groups of job seekers (by duration of nonemployment, reason of nonemployment, etc.), and the 2007-09 recession and its aftermath are associated with large changes in the composition of the search pool toward groups with typically low employment transition rates. Motivated by these observations, we augment the standard matching function by allowing for heterogeneity in search effectiveness of different groups of job seekers, which through the lens of the matching function is a multiplicative shifter of the job seeker's rate of finding a job. In contrast to the existing literature, we not only allow for exogenous but also for endogenous changes of effectiveness over time.[2] For this augmented matching function, the average employment transition rate depends on the composition of the job seeker pool and job search effectiveness, in addition to the relative number of job seekers and vacancies, and matching efficiency.

We model search effort of an observable type of job seekers as a constant elasticity function of the aggregate matching rate, which itself is a function of the total job seeker input in the economy. We allow for differences in the search effort elasticity across types to account for cyclical changes in relative transition rates. Our model is motivated by the standard

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[1]Petrongolo and Pissarides (2001) in their survey of the matching function provide an in-depth description of the role the matching function plays in macroeconomics.

[2]Existing approaches are limited to the analysis of composition effects arising from exogenous heterogeneity. For example, Veracierto (2011), Barnichon and Figura (2016), Şahin Song, Topa, and Violante (2012), Elsby, Michaels, and Ratner (2016), Kroft, Lange, Notowidigdo and Katz (2016), and Hall and Schulhofer-Wohl (2015).

search and matching model (Pissarides, 2000); however, in the estimation we do not restrict search effort to be procyclical as implied by the standard model. For this specification of endogenous search effort we show, first, that from the data on employment transition rates the group-specific search effort elasticities and matching function elasticity are not separately identified.[3] That is, the data are consistent with low matching function elasticity with respect to vacancies (α) and procyclical search effort as well as high α and countercyclical search effort. Second, the aggregate matching efficiency is identified up to a positive scalar, $1/\alpha$. Third, the matching elasticity from the standard matching function, which ignores variable search effort, is identified but is not equal to the underlying 'true' matching elasticity.

Our model of variable search effort thus defines the observed group-specific employment transition rates as nonlinear functions of the observables (market tightness and the composition of search pool) and the unobserved state (aggregate matching efficiency and group-specific exogenous effects, i.e., group-specific matching efficiencies). We use an extended Kalman filter to estimate the parameters and infer the unobserved state. In our benchmark model, we estimate a flexible random walk process for the aggregate matching efficiency and constant group-specific exogenous effects. We then extend the framework and allow for stochastic time-varying group-specific exogenous effects.

We apply our estimation procedure to three alternative groupings of the pool of nonemployed individuals: unemployed job seekers by duration of unemployment, unemployed job seekers by reason of unemployment, and nonemployed job seekers by labor force status (unemployed and out of the labor force) and gender.[4] The groupings are motivated by an attempt to maximize the possible impact of 'unobserved quality change' on the average employment transition rate: we want to have a decomposition of the search pool such that groups have large and persistent differences in transition rates, and we want to see large compositional changes of the search pool.

First, we find that the decline of the identified aggregate matching efficiency in the Great Recession was large, even after controlling for composition effects and variable search effort. This conclusion is robust to alternative sample periods, measures of vacancies, or structural breaks in the parameters. Second, the data reject the hypothesis under which our model is isomorphic to a model with constant search effort, that is, the data provide evidence in favor of group-specific variable search effort. We find that a model with group-specific variable

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[3]Throughout the paper, we use terms "type" and "group" interchangeably.

[4]In the estimation, we focus on the nonemployed job seekers. However, the framework can be easily extended to take into account employed job seekers.

search and only one exogenous shock – aggregate matching efficiency (as opposed to a number of type-specific exogenous shocks) – is sufficient to capture both comovement and relative changes in group-specific transition rates. Little is gained from adding time-varying group-specific exogenous effects. If we instead shut down the group-specific variable search effort, as is typical in the literature, the data call for large movements in exogenous group-specific efficiencies to capture relative movements in the group-specific transition rates.

Finally, using the augmented matching function, we account for the relative contributions of labor market tightness, aggregate matching efficiency, and variable search effort to changes in the average employment transition rate contingent on the elasticity of the matching function. As we just noted, in our framework the cyclicality of search effort and the magnitude of changes in matching efficiency depend on the value of the matching elasticity. For small values of the matching elasticity, search effort is procyclical and accounts for most of the procyclical transition rate. In particular, the large decline of the average transition rate in the Great Recession is mainly attributed to reduced search effort, and the matching efficiency does not decline much. On the other hand, when the matching elasticity is large, search effort is strongly countercyclical and large changes in matching efficiency are required to maintain a procyclical transition rate. This is the opposite of what one obtains for the standard matching function approach with constant search effort for which a larger matching elasticity implies a smaller contribution coming from the aggregate matching efficiency.

Independent evidence on the cyclicality of search effort is limited and mixed. Shimer (2004), Mukoyama, Patterson, and Sahin (2014), and Faberman and Kudlyak (2014) provide evidence for countercyclical search effort, whereas DeLoach and Kurt (2013) argue for acyclical search effort, and Gomme and Lkhagvasuren (2015) argue for procyclical effort. Our reading of the literature is that it favors the presence of countercyclical search effort. The evidence for endogenous and countercyclical search effort poses a hurdle for the standard search and matching model, both in terms of its technical capacity to accommodate countercyclical search effort and in terms of its ability to generate fluctuations of the employment transition rate of empirical magnitudes.

To our knowledge, ours is the first study that allows for endogenous search effort in the matching function accounting framework. A number of papers have studied how accounting for search heterogeneity might affect estimates of matching efficiency.[5] Almost all of these

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[5]Like all of this literature, we focus on the measurement of search effectiveness for job seekers, taking the recruiting effectiveness for reported vacancies as given. In seminal work, Davis, Faberman and Haltiwanger (2013) study recruiting effectiveness for vacancies.

studies assume, as we do, that after correcting for quality differences across types, the types remain perfect substitutes in the matching function. Davis (2011) and Kroft, Lange, Notowidigdo, and Katz (2016) study heterogeneity among the unemployed defined by duration of unemployment, and they construct correction factors for search effort that recover the effective input of workers to the matching function. Sahin, Song, Topa, and Violante (2014) show how the potential misallocation of unemployed workers across disaggregated labor markets affects measured matching efficiency in the reduced form aggregate matching function. Barnichon and Figura (2015) build on these contributions and study a decomposition of the unemployed by type and market segment and allow for differences in fixed type-specific efficiencies, but no aggregate matching efficiency. They argue that relative to a homogeneous search pool model, composition effects account for most of the observed decline in matching efficiency. Veracierto (2011) extends the definition of the search pool beyond the unemployed to include those out of the labor force (OLF) and allows for time-varying relative type-specific efficiencies. Sedlacek (2014) uses techniques similar to ours to study the role of time-varying relative type-specific efficiencies when the search pool includes unemployed, OLF, and employed. For the same broadly defined search pool as in Sedlacek (2014) but more demographic detail, Hall and Schulhofer-Wohl (2016) study the role of type-specific matching efficiencies when different observed search types remain imperfect substitutes in the matching function.[6] On the theory side, the cyclicality of search effort is potentially relevant for the amplification of the volatility of vacancies and unemployment in search and matching models (for example, Costain and Reiter (2008) or Gomme and Lkhagvasuren (2015)).

The remainder of the paper is structured as follows. Section 2 describes a model of the matching function with heterogeneity and endogenous search effort and derives identification results. Section 3 describes the estimation procedure. Section 4 describes the data and basic facts that motivate variable search effort. Section 5 presents our estimation results on the identified aggregate matching efficiency with variable search effort. Section 6 presents the decomposition of the average employment transition rate conditional on pro- and countercyclical search effort. Section 7 concludes.

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[6]Similar to type-specific variable search effort in our model, Hall and Schulhofer-Wohl's imperfect substitutability across types captures cyclical changes in relative transition rates. Compared with our approach there are two conceptual differences to Hall and Schulhofer-Wohl. First, unlike our approach there is no obvious interpretation for what imperfect substitutability across types means. Second, Hall and Schulhofer-Wohl exclude the possibility of changes in aggregate matching efficiency and limit themselves to changes in type-specific matching efficiencies.

2   Matching with endogenous search effort

The aggregate search and matching function in macro-labor models describes the 'production' of hires as a function of the stocks of job seekers and vacancies, and an exogenous shift term denoting the aggregate efficiency of the matching process. The standard approach for the search and matching function assumes that the inputs are homogenous and that search effort does not vary endogenously with the state of the labor market. We augment the standard search and matching function by allowing for time-varying heterogeneity in search effectiveness across observed groups of job seekers. Search effectiveness may vary for exogenous reasons or it may change in response to the aggregate matching rate. We refer to the endogenous component of group-specific search effectiveness as search effort and introduce a simple model in which search effort is a function of the aggregate matching rate.

2.1   A model of heterogeneous search effectiveness

In this section, we describe a simple extension of the aggregate matching function approach that allows for heterogeneity in search effectiveness of job seekers.

Consider an economy with a finite number of search types, $i \in I$. Time is continuous. At any point in time, $u_{i,t}$ job seekers of type i engage in search and the types differ in their search effectiveness, $\rho_{i,t}$, to be described in more detail below. Total effective search input, $u_t^* \equiv \sum_i \rho_{i,t} u_{i,t}$, and vacancies, v_t, are inputs to a Cobb-Douglas matching function that generates hires, h_t,

$$h_t = \exp\left(\kappa_t\right) v_t^\alpha \left(u_t^*\right)^{1-\alpha}, \tag{1}$$

for a given aggregate matching efficiency, κ_t, and matching elasticity, $0 < \alpha < 1$. In the standard matching function, the search types are homogeneous, $\rho_{i,t} = 1$, and aggregate search input is simply the sum of all job seekers, $u_t \equiv \sum_i u_{i,t}$.

The aggregate matching rate per search unit, $\lambda_t \equiv h_t/u_t^*$, is

$$\lambda_t = \exp\left(\kappa_t\right) \theta_t^\alpha \bar{\rho}_t^{-\alpha}, \tag{2}$$

where $\theta_t \equiv v_t/u_t$ is the standard aggregate labor market tightness, and $\bar{\rho}_t \equiv \sum_i \omega_{i,t} \rho_{i,t}$ is average search effectiveness, with $\omega_{i,t} = u_{i,t}/u_t$ being the share of type i in the search pool.

The transition rate to employment for a type i searcher is then the product of the type-specific search effectiveness and the per search unit matching rate, $\lambda_{i,t} = \rho_{i,t} \lambda_t$. The average

aggregate transition rate, $\bar{\lambda}_t \equiv \sum_i \omega_{i,t} \lambda_{i,t}$, is

$$\bar{\lambda}_t = \exp\left(\kappa_t\right) \bar{\rho}_t^{1-\alpha} \theta_t^\alpha. \tag{3}$$

For the analysis of the average transition rate, the standard matching function approach ignores changes in average search effectiveness, whether due to changes in the composition of the search pool or due to variations in type-specific search effectiveness, and sets $\bar{\rho}_t = 1$. In other words, with heterogeneous search effectiveness, the standard approach conflates changes in matching efficiency with changes in composition and search effectiveness.

2.2 Modeling endogenous search effectiveness

Let the type-specific search effectiveness ρ_{it} be the product of a type-specific exogenous component (exogenous type-specific matching efficiency), z_{it}, and a type-specific endogenous component (search effort), s_{it},

$$\rho_{it} = s_{it} \exp\left(z_{it}\right). \tag{4}$$

We model type i's search effort as a constant elasticity function of the aggregate matching rate,

$$s_{it} = \lambda_t^{\eta_i}, \tag{5}$$

where η_i is the type-specific elasticity of search effort with respect to the aggregate matching rate.

Our approach is motivated by the basic search and matching model, e.g. Pissarides (2000) or Gomme and Lkhagvasuren (2015), for which the employment transition rate of searchers is the product of search effort and the aggregate matching rate per unit of search effort, and searchers face an increasing and convex cost from search effort. For this model, search effort increases if the expected gains from search increase, in particular, if the aggregate matching rate increases.[7] In our set-up, the search elasticity η_i reflects the endogenous response of search effort to changes in the aggregate matching rate, assuming that more effort makes a searcher more effective.

Combining equations (4) and (5) yields the following expression for type-specific search effectiveness:

$$\ln \rho_{it} = z_{it} + \eta_i \ln \lambda_t, \tag{6}$$

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[7]See Appendix A for the first order condition from the basic search and matching model with endogenous search effort.

This specification nests two special cases of time-varying type-specific search effectiveness. When z_{it} is constant, all time-variation is due to the type-specific search effort. When $\eta_i = 0$, all time-variation is due to the variation in the type-specific exogenous component.

We will call search effort procyclical if $\eta_i > 0$ and countercyclical if $\eta_i < 0$. We do not restrict search effort to be procyclical as implied by the basic search and matching model, but we impose a lower bound on the search elasticity, $\eta_i \geq -1$, such that a type's employment transition rate is always procyclical,

$$\ln \lambda_{it} = z_{it} + (1 + \eta_i) \ln \lambda_t \text{ for } i \in I. \tag{7}$$

Our model of heterogenous search effectiveness thus defines observed employment transition rates of groups as a function of the unobserved aggregate matching rate and exogenous group-specific effects, that may be fixed or time-varying. Combining equations (2) and (7) then yields the aggregate matching rate as a nonlinear function of the search pool composition, aggregate matching efficiency, and exogenous type-specific effects

$$\ln \lambda_t = \kappa_t + \alpha \ln \theta_t - \alpha \ln \sum_i \omega_{it} \exp\left(z_{it} + \eta_i \ln \lambda_t\right). \tag{8}$$

In equations (7) and (8), $y_t = \left\{\theta_t, (\lambda_{it})_{i=1}^I, (\omega_{it})_{i=1}^I\right\}$ is observable, $x_t = \left\{\kappa_t, (z_{it})_{i=1}^I\right\}$ describes the unobserved state, and $\left\{\alpha, (\eta_i)_{i=1}^I\right\}$ are parameters. Our goal is to estimate the parameters and infer the unobserved state conditional on observable variables.

2.3 Identification

We first show that, conditional on using only observations on group-specific transition rates, the parameters of our model are not uniquely identified. In particular, the sign of the search effort elasticity, that is, the cyclicality of search effort, is not identified.

Proposition 1 *Conditional on observations $\{y_t\}$, the matching elasticity and search effort elasticities are identified only up to the restriction*

$$\frac{\alpha}{1-\alpha}(1+\eta_i) = \phi_i \geq 0 \text{ for } i \in I. \tag{9}$$

and the aggregate matching efficiency and matching rate are identified up to the restriction

$$\hat{\kappa}_t \equiv \frac{\kappa_t}{\alpha} \text{ and } \ln \hat{\lambda}_t \equiv \frac{1-\alpha}{\alpha} \ln \lambda_t. \tag{10}$$

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We obtain this proposition by applying the definition of ϕ_i from (9) together with the transformation of variables $\hat{\kappa}_t$ and $\hat{\lambda}_t$, to our model for employment transition rates (7) and (8) rewriting it as follows

$$\ln \lambda_{it} = z_{it} + \phi_i \ln \widehat{\lambda}_t, \tag{11}$$

$$\ln \widehat{\lambda}_t = \widehat{\kappa}_t + \ln \theta_t - \ln \sum_i \omega_{it} \exp\left(z_{it} + \phi_i \ln \widehat{\lambda}_t \right). \tag{12}$$

From these equations one can see that given observable y_t and a solution $\left(z_{it}, \hat{\kappa}_t, \hat{\lambda}_t, \phi_i \right)$, any other solution $(z_{it}, \kappa_t, \lambda_t, \alpha, \eta_i)$ that satisfies the constraints (9) and (10) is observationally equivalent. Note that ϕ_i is effectively the identified transition elasticity in (11). It then follows from equation (9) that for each type there exists a critical value of the matching elasticity that depends on the type's identified transition elasticity such that search effort is procyclical (countercyclical) if the matching elasticity is below (above) the critical value. In other words, working with observations on group-specific transition rates only, the observations are consistent with either pro- or countercyclical search effort.

Finally note that if the identified transition elasticities are the same for all types, $\phi_i = \phi$ $\forall i \in I$, that is,

$$\frac{\alpha}{1-\alpha}\left(1 + \eta_i\right) = \phi \ \forall i \in I, \tag{13}$$

our model is equivalent to one with common search effort across types $(\eta_i = \eta)$. Then by setting $\eta = 0$ and $\alpha = 1/(1+\phi)$ in equation (9), it encompasses a special case of constant search effort. Alternatively, if the identified transition elasticities differ across types, we can argue for the presence of endogenous search effort. It is an empirical question whether the identified transition elasticities differ across types, and we address this issue in the estimation section.

We get a better understanding of the model's inability to identify the cyclicality of search effort if we ignore heterogeneity and consider the case of a representative searcher. For this case, the closed form solution of the aggregate matching rate, λ_t, and the employment transition rate, λ_{1t}, in terms of the underlying parameters, matching elasticity and search effort elasticity, is

$$\ln \lambda_t = \frac{\alpha}{1 + \alpha \eta_1}\left(\ln \theta_t + \hat{\kappa}_t\right) \text{ and }$$

$$\ln \lambda_{1,t} = \underbrace{\alpha \frac{1 + \eta_1}{1 + \alpha \eta_1}}_{\equiv \hat{\alpha}}\left(\ln \theta_t + \hat{\kappa}_t\right)$$

where $\hat{\alpha}$ is the effective matching elasticity that relates observed transition rates to market tightness.[8] Let us start with a matching function elasticity α and constant search effort, $\eta = 0$. Then a 1 percentage point increase of market tightness results in an α ppt increase of the matching rate λ_t and the transition rate λ_{1t}. Suppose that search effort is procyclical, $\eta_1 > 0$, then the higher matching rate leads to an increase in search effort, ρ_{1t}, and a decline in effective tightness, θ_t/ρ_{1t}, equation (2). Thus the matching rate increases less than α ppts. The direct effect of increased search effort on the transition rate, however, more than compensates for the decline in the matching rate, and the transition rate increases by more than α ppts. The converse holds when search effort is countercyclical, $\eta_1 < 0$. In this case, a 1 ppt increase of market tightness results in a more than α ppt increase in the matching rate because search effort declines, and the effective market tightness increases by more than 1 ppt. Again, the direct effect of reduced search effort more than compensates for the increase in the matching rate, and the transition rate increases by less than α ppts. Thus the effective matching elasticity is increasing in the matching elasticity and the search effort elasticity. Any given effective matching elasticity $\hat{\alpha}$ can then be accounted for by the same matching elasticity, $\alpha = \hat{\alpha}$, and a-cyclical search effort, $\eta_1 = 0$, or by a larger (smaller) matching elasticity, $\alpha > \hat{\alpha}$ ($\alpha < \hat{\alpha}$) with a countercyclical (procyclical) search effort, $\eta_1 < 0$ ($\eta_1 > 0$).

Even though the matching elasticity and search effort elasticities are not separately identified, for a log-linear approximation of the average employment transition rate as a function of market tightness, the coefficient on market tightness is uniquely determined.

Proposition 2 *The log-linear approximation of the average employment transition rate at a point* $\left(\hat{\lambda}_0, \hat{\kappa}_0, z_0, \omega_0\right)$ *is*

$$\Delta \ln \bar{\lambda} = \bar{\alpha}\left(\Delta \ln \theta + \Delta \hat{\kappa}\right) + (1 - \bar{\alpha})\Delta \bar{z} + \Delta \bar{\varepsilon}, \text{ with} \tag{14}$$

$$\bar{\alpha} = \frac{\bar{\phi}}{1 + \bar{\phi}} \tag{15}$$

and $\bar{\phi}$, $\Delta \bar{z}$, *and* $\Delta \bar{\varepsilon}$ *are weighted averages of type-specific elasticities, efficiencies and measurement errors.*

Proof. See Appendix. ∎

The expression for the log-linear approximation of the average employment transition rate is analogous to the standard homogeneous search and matching model, but with the

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[8]Here we ignore exogenous type-specific effects since there is only one type.

reduced form matching elasticity $\bar{\alpha}$ being a function of the weighted average of the identified transition elasticities ϕ_i. Therefore, the 'reduced form' matching function elasticity $\bar{\alpha}$ is identified but is not equal to α.[9]

Finally, we note that the overall level of the aggregate matching efficiency and the type-specific effects is not identified. Using equations (11) and (12). it is straightforward to show the following proposition.[10]

Proposition 3 *The level of the state* $x_t = \left\{ \hat{\kappa}_t, (z_{it})_{i=1}^{I} \right\}$ *is identified up to an additive shift term.*

When we estimate our model we therefore normalize one of the states.

3   Estimation

Our model for the evolution of the type-specific employment transition rates has a straightforward state-space representation, albeit with a nonlinear measurement equation. Conditional on the parameters, we are using an extended Kalman-filter approach to infer the state of the system from observations on the type transition rates. We obtain parameter estimates by maximizing the likelihood function.

Given the unobserved state $x_t = (\hat{\kappa}_t, \{z_{it}\}_i)$, equation (11) for the measured type-transition rate λ_i^m with measurement noise ε_i,

$$\ln \lambda_{it}^m = z_{it} + \phi_i \ln \hat{\lambda}_t + \varepsilon_{it} \text{ for } i = 1, .., I \tag{16}$$

with $\varepsilon_{it} \sim N(0, \Sigma_\varepsilon)$, and equation (12) for the identified matching rate define the measurement equations of the state-space model.

We use three different specifications for the evolution of the unobserved state x_t. The baseline specification has time-varying aggregate matching efficiency only, and the two other alternatives use either time-varying type-specific efficiencies only or aggregate and time-varying type-specific efficiencies jointly.

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[9]In the standard matching function approach that ignores heterogeneity and endogenous search effort, the matching function elasticity $(\bar{\alpha})$ is typically estimated from the relationship between the average transition rate and labor market tightness, i.e., $\ln \bar{\lambda}_t = \bar{\kappa}_t + \bar{\alpha} \ln \theta_t$, where $\bar{\kappa}_t$ denotes the estimated aggregate matching efficiency from the standard matching function approach.

[10]The proof is in Appendix C.

Model 1. Time-varying aggregate matching efficiency and fixed type-specific matching efficiencies,

$$\widehat{\kappa}_t = \widehat{\kappa}_{t-1} + \zeta_t, \ \zeta_t \sim N\left(0, \sigma_\zeta^2\right)$$
$$z_{it} = c_i \ \forall i \in I.$$

We use a random walk to capture any trend in the aggregate matching efficiency and also any potentially substantial drop of the matching efficiency after 2007. We allow for permanent differences across types through fixed effects, and time-varying differences across types are captured through differences in the identified transition elasticities. In view of proposition 3 we normalize $c_1 = 0$.

Model 2. Time-varying type-specific matching efficiencies and fixed aggregate matching efficiency,

$$\widehat{\kappa}_t = 0$$
$$z_{it} = z_{it-1} + \xi_{it}, \ \xi_{it} \sim N\left(0, \sigma_{\xi_i}^2\right) \ \forall i \in I.$$

Similar to Model 1, we allow for flexible random walk processes for type-specific matching efficiencies. We normalize $\widehat{\kappa}_t = 0$.

Model 3. Time-varying aggregate matching efficiency and time-varying type-specific matching efficiencies,

$$\widehat{\kappa}_t = \widehat{\kappa}_{t-1} + \zeta_t, \ \zeta_t \sim N\left(0, \sigma_\zeta^2\right),$$
$$z_{it} - c_i = \gamma_i\left(z_{it-1} - c_i\right) + \xi_{it}, \ \xi_{it} \sim N\left(0, \sigma_{\xi_i}^2\right) \ \forall i \in I.$$

In Model 3, we specify the aggregate matching efficiency as a random walk and type-specific matching efficiencies as stationary $AR(1)$-processes. We normalize $c_1 = 0$. We choose this specification for two reasons. First, anticipating our estimation results for Model 2, we see substantial comovement of type-specific efficiencies at low frequencies, which suggests a common trend. We identify this common trend with the aggregate matching efficiency. Second, with this specification the stochastic processes for $\widehat{\kappa}_t$ and $\{z_{it}\}$ are separately identified to a first order approximation.[11]

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[11] Even though in Model 3 we have only I observations on type employment transition rates, but $I + 1$

4   Data and basic facts

We now construct alternative measures of the search pool using data from the Current Population Survey (CPS) and characterize these measures from the perspective of our matching framework with heterogeneity and variable search effort.

4.1   Definition of job seeker groups

In the matching function framework, we attribute differences in transition rates across job seekers to differences in search effectiveness. Therefore, we want to decompose the pool of job seekers into observable groups that are characterized by large and persistent differences in transition rates to employment. Specifically, we consider three alternative characterizations of the pool of job seekers using the CPS.

The first two characterizations define the search pool narrowly as the unemployed, that is, those reporting to be actively engaged in search. We consider two decompositions of the unemployed, by duration and by reason of unemployment. The first classification groups the unemployed into those that report unemployment of less than 5 weeks, 5-26 weeks, or more than 26 weeks. The second classification consists of the four groups that report being unemployed because they are on temporary layoff, on permanent layoff, have quit a job, or have previously been out of the labor force.

The third characterization of the search pool includes all nonemployed, that is the unemployed and those that are out of the labor force (OLF). Although those that are OLF do not report to be actively engaged in search, in any month a substantial number of them do make the transition to employment. Thus a clear cut distinction between those that are unemployed and those that are OLF may not be appropriate for a matching framework that expressly allows for differences in search effectiveness.[12] We decompose this broader definition of the search pool into four groups of nonemployed job seekers characterized by their labor market status (unemployed or OLF) and gender (male or female).

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unobserved states at any point in time, for the linear Kalman-filter the state is identified, if the state-space system is 'observable.' See, for example, Harvey (1989). Formally, the n-dimensional state of a system is said to be observable, if the last n past observations are sufficient to determine the entire history of states for the last n periods. If the measurement equations for our model were linear, we could easily verify that the observability condition is satisfied for at least one type-specific efficiency being stationary. Since our measurement equations are nonlinear, we are using an extended Kalman-filter and no general conditions for identification are available. In our estimation we check that the observability condition is satisfied for the extended Kalman-filter along the inferred path of the state.

[12]For these reasons Veracierto (2011), Barnichon and Figura (2015), and Hall and Schulhofer-Wohl (2015) have all used a broader concept of the search pool in the matching function.

For our characterizations of the search pool we take a job seeker's membership in a group at the beginning of a period as given.[13] We are agnostic of whether the groups within each decomposition reflect some ex-ante (inherent) heterogeneity among job seekers in terms of their transition rates, or whether different transition rates are a result of being/becoming a member of a particular group.

4.2 Data sources and construction of the series

For each definition of the search pool we construct the employment transition rates and the search pool shares of the different groups of job seekers.

We construct the employment transition rates for the different groups of nonemployed job seekers using the micro data from the Current Population Survey (CPS) basic monthly files, from January 1976 to December 2015. We follow Madrian and Lefgren (1999) and Shimer (2012) and match individuals from month to month using information on race, age and sex besides individual and household's identification number. In the analysis, we weight each individual by the average of the individual's CPS sampling weights from adjacent months. The transition probability of a group is the fraction of individuals that transition between labor market states in two adjacent months.[14] We transform the month-to-month transition probabilities to monthly continuous time transition rates. For the search pool definitions that cover the unemployed we use the exit probabilities to employment (E) and OLF (I), and assume that job seekers who exit do not return to unemployment in the same month. This defines a relation between the discrete time transition probabilities p_t and the continuous time employment transition rates λ_t, which we can solve for the transition rate from unemployment to employment

$$\lambda_{UE,t} = -\frac{\log\left(1 - p_{UE,t} - p_{UI,t}\right)}{1 + p_{UI,t}/p_{UE,t}}.$$

When the exit probability to OLF is small relative to the exit probability to employment, the employment transition rate is approximately $-\log\left(1 - p_{UE,t}\right)$. For most of our samples this is not a good approximation. We therefore prefer to use information on exit rates to all states when calculating the employment transition rate. For the search pool definition that covers all nonemployed, we use the exit probabilities of unemployed (OLF) to employment

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[13] The same approach is taken in other recent work that accounts for heterogeneity in matching efficiency, for example, Barnichon and Figura (2015) or Hall and Schulhofer-Wohl (2015).

[14] In the analysis, we follow the BLS approach and treat the reported labor force status as a true status. Frazis, Robinson, Evans, and Duff (2005) describes that the main reason for why the BLS does not correct responses for a potential error is a lack of methodology or the data that would guide the correction.

and OLF (unemployment). Otherwise we proceed the same way to calculate the monthly employment transition rates.[15]

We employ two alternative aggregate vacancy series: (1) the Help Wanted Index (HWI) from the Conference Board, which is available since 1951, and (2) the Job Openings and Labor Turnover Survey (JOLTS) program of the BLS, which is available starting in January 2001.[16]

For the baseline analysis, we use data from the CPS and the HWI covering the period from January 1994 to December 2015. We start the baseline sample in 1994 because of the structural breaks in the search pool shares and transition rates associated with the 1994 CPS redesign. We use the adjustment factors from Polivka and Miller (1998) to adjust the search pool share series prior to 1994. Since no comparable adjustment factors are available for our constructed exit probabilities, we introduce an additive adjustment factor in our measurement equations for the group-specific employment transition rates prior to 1994.

All monthly series are seasonally adjusted using the Watson (1996) implementation of the X-11 procedure. Since the monthly series remain highly volatile, even after this adjustment, we estimate our models on quarterly averages of the seasonally adjusted monthly data.

4.3   Basic facts

Accounting for heterogeneity in the matching function framework matters for the measurement of matching efficiency if there are large and persistent differences in employment transition rates across different groups of job seekers and large and systematic changes in the composition of the search pool over time. We now show that this is indeed the case for our two decompositions of the pool of unemployed. In particular, in recessions the search pool composition shifts toward those with relatively low employment transition rates, that is, the average search effectiveness declines. Not accounting for this change in average effectiveness is likely to bias one's estimate of the aggregate matching efficiency downward. For our broader definition of the search pool, which includes unemployed and OLF, this kind of cyclical bias is not as pronounced.

First, consider the decomposition of the search pool of unemployed by duration of unemployment. Large differences in transition rates among the unemployed by duration are immediately apparent: short-term unemployed are three times as likely to transition to em-

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[15]See Appendix D for the derivations.

[16]Given the shift in job advertising from print media to web-based means the HWI may not be consistent over time. Barnichon (2010) corrects for structural changes in the HWI series and we use his adjusted series.

ployment than long-term unemployed (Figure 1, Panel A.1). Furthermore, the differences in the transition rates among these groups persist over time, keeping the ranking of transition rates unchanged. But even though there is substantial comovement among the transition rates, the relative transition rates do change (Figure 1, Panel A.2). For example, in the 2007-09 recession the transition rates of the medium- and long-term unemployed decline more than those of the short-term unemployed. We will attribute changes in relative transition rates to time-varying differences in search effectiveness. Finally, the composition of the pool of the unemployed by duration changes systematically and substantially over time: the share of long-term unemployed increases in recessions, e.g., following the 2007-09 recession the share of long-term unemployed more than doubled (Figure 1, Panel A.3).

The alternative decomposition of the search pool of unemployed by reason has the same features as the decomposition by duration (Figure 1.B). Relative to those who give a temporary layoff as the reason for unemployment, the unemployed on permanent layoff tend to be half as likely to find employment, they suffer a relatively larger decline of transition rates in recessions, and they make up a larger share of the search pool in recessions.[17]

For the broader definition of the search pool that includes all of the nonemployed working-age population the pattern that emerges for average search effectiveness is more ambiguous (Figure 1.C). The unemployed are relatively more effective at search than those who are OLF, their employment transition rates are about five to 10 times higher than the OLF transition rates. But the transition rates of the unemployed are also much more cyclical such that the relative transition rates of those who are OLF are increasing in recessions. At the same time, the search pool share of those who are OLF tends to decline in recessions. The cyclical bias of composition changes for this search pool definition is therefore not obvious.

5 Matching efficiency with variable search effort

In this section, we present estimates of the identified transition elasticities (ϕ_i) and the identified aggregate matching efficiency ($\widehat{\kappa}_t$). We turn to the interpretation of these results conditional on the aggregate matching elasticity α in the next section.

We start with estimates of the baseline model with type-specific variable search effort and time-varying aggregate matching efficiency only (Model 1) for the search pool decomposition by unemployment duration. For this case, we find evidence for an unprecedented decline of

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[17]The sharp jump of the transition rate in 1994 for those who report being on temporary layoff likely reflects the structural break associated with the CPS redesign.

the identified matching efficiency following the Great Recession and the presence of variable search effort. For a restricted version of the baseline model with constant search effort, that is, a model with a composition effect only, we find a smaller decline in identified matching efficiency. The results are robust with respect to unobserved heterogeneity in search effort and the use of the HWI index for vacancies.

We then compare the results from this baseline model with the two alternative models that allow for time-varying type-specific matching efficiencies (Models 2 and 3) and find that relative to these alternatives the baseline model with variable search effort captures changes in relative transition rates well. Furthermore, the identified aggregate matching efficiency in the baseline model appears to capture a common trend in the time-varying type-specific matching efficiencies in Model 2.

We then shut down the time-varying search effort channel and force the model to capture time-varying heterogeneity via type-specific exogenous shocks. We find that the restricted model requires substantial volatility in type-specific exogenous shocks to capture relative movements in type-specific transition rates.

Finally, we show that estimates of the baseline model for our two alternative search pool definitions, unemployment by reason and labor force status, yield qualitatively similar paths for the identified matching efficiency. In particular, we find comparable declines of the identified aggregate matching efficiency following the Great Recession. For ease of exposition, in this section we will frequently drop the qualifier 'identified' for the transition elasticities and aggregate matching elasticity when no confusion can arise.

5.1 Aggregate matching efficiency with variable search effort

The baseline model with endogenous search effort and aggregate matching efficiency captures well the comovement of type-specific employment transition rates and changes in relative transition rates.

5.1.1 Transition elasticities and matching efficiency in a baseline model

We begin with the baseline model with aggregate matching efficiency when heterogeneity in the search pool of unemployed workers is defined by duration of unemployment, Model 1 of Section 3. For different sub-samples and specifications, Table 1 displays the parameter estimates of the model and Figure 2 displays the smoothed posterior of the identified aggregate matching efficiency.

Identified transition elasticities   The baseline specification covers the years 1994-2015 and uses the HWI for the vacancy measure, Table 1, Column $(1a)$. The type-specific fixed effects, c_i, decline with unemployment duration, capturing the persistently lower employment transition rates of the unemployed with longer durations.[18]

The identified transition elasticities, ϕ_i, are monotonically increasing with the duration of unemployment, that is, the employment transition rates of those unemployed with longer durations are more cyclically sensitive than the transition rates of those with shorter durations.

The estimates of the type-specific transition elasticities are sufficiently precise to reject the hypothesis that they are the same. As noted in the discussion of Proposition 1, if the identified transition elasticities are the same for all types, then the model is consistent with constant search effort for a particular choice of the aggregate matching elasticity. Since the transition elasticities are significantly different from each other, we reject the hypothesis of no variable search effort.

Identified aggregate matching efficiency   For the baseline specification, the estimate of the identified aggregate matching efficiency declined dramatically following the Great Recession, and it has only partially recovered over the last years; the solid black line $(1a)$ in Figure 2. Since the aggregate matching efficiency is identified only up to a positive scalar, namely the matching elasticity, we will defer discussion of the quantitative magnitude of changes in the matching efficiency to the next section. The behavior of the identified matching efficiency is, however, informative about the magnitude of movements in the aggregate matching efficiency during the Great Recession relative to the overall sample period.

To gain more perspective on the magnitude of the decline in the matching efficiency, we re-estimate the model for the extended sample period 1976-2015. As noted in section 4.2, we allow for a structural break in the employment transition rates prior to 1994 to account for the 1994 CPS data revision. For the years after 1994, the estimated aggregate matching efficiency for the extended sample and the baseline sample follow each other closely, lines (1a) and (2a) in Figure 2. In particular the matching efficiency declines dramatically in the Great Recession, and that decline remains exceptional even for the extended sample period. The estimated transition elasticities are somewhat lower in absolute value for the

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[18]The fixed effect for short duration unemployment is normalized at one. For the Kalman-filter, we take the initial prior of the identified aggregate matching efficiency as given, and this prior becomes a parameter of the likelihood function reported in the first row of Table 1.

extended sample period, but the differences between them remain significant and the cyclical sensitivity continues to increase with the duration of unemployment, columns (1a) and (2a) in Table 1.

5.1.2 Robustness

Constant search effort We evaluate the contribution of endogenous search effort by comparing our baseline model with a restricted version of the model where the transition elasticities are the same for all types. As we have argued in the discussion of Proposition 1 in Section 2.3, imposing this restriction makes the model equivalent to one with constant search effort for a particular value of the matching elasticity.[19] The restricted model can then be reinterpreted as having fixed type-specific search efficiencies, i.e., only fixed heterogeneity similar to Barnichon and Figura (2015).

The results of estimating the restricted model are presented in Table 1, column (1b), and the aggregate matching efficiency is the black o-line (1b) in Figure 2. The restricted model has no time-varying heterogeneity, only type-specific fixed effects, and the restricted transition elasticity ϕ is an average of the unrestricted type-specific transition elasticities. A priori it is not obvious whether the restriction of constant search effort will increase or decrease estimated matching efficiency. It turns out that the estimated aggregate matching efficiency of the restricted model adjusts to compensate for the model's lack of ability to accommodate relative movements in type-specific transition rates. Specifically, to compensate for the decline in relative transition rates of long duration unemployment after 2007, the restricted model requires a somewhat higher aggregate matching efficiency than the unrestricted model with type-specific time-varying search effort, Figure 2, lines (1a) and (1b). In other words, allowing for variable search effort makes the estimated matching efficiency somewhat more volatile relative to a constant search effort restriction.

Given the estimated transition elasticity from the restricted model, we infer that search effort is constant if the matching elasticity is $\alpha = 0.38$. Thus, our estimate of the identified aggregate matching efficiency $\widehat{\kappa}_t$ from the restricted model with fixed heterogeneity, line (1b) in Figure 2, is equivalent to the scaled aggregate matching efficiency κ_t/α from the standard matching function that allows for fixed heterogeneity and no time-varying changes in relative search efficiencies with $\alpha = 0.38$.

For this same matching elasticity, we can construct the aggregate matching efficiency

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[19]In Appendix E we provide a more detailed illustration of this case.

using the standard assumption of a homogeneous search pool using the expression for the average transition rate, equation (3) with a constant match quality. The scaled matching efficiency from this homogeneous case, $\overline{\kappa}_t/\alpha$, is displayed as line (1c) in Figure 2. It is comparable to our identified matching efficiencies. As we have discussed previously in Section 4.3, for the decomposition of the unemployed search pool by duration of unemployment, the composition of the search pool shifts toward groups with relatively lower employment transition rates when unemployment is high and the average transition rate is low, that is, average search quality declines. Moving from lines (1c) to (1b) demonstrates how accounting for compositional changes alone reduces the volatility of the estimated aggregate matching efficiency, that is, line (1b) shows the estimate of the aggregate matching efficiency purged of this procyclical composition effect.

Unobserved heterogeneity If one believes that changes in the duration distribution of unemployment mainly reflect unobserved heterogeneity within the three groups of the unemployed by duration, then it is possible that the unobserved composition of our search pool groups is systematically changing over time with aggregate labor market conditions.[20] The time-varying employment transition rates of a group may therefore reflect not only the actual variation of the transition rate of the job seekers in the group, but also the unobserved compositional shifts within the group. Such time-varying unobserved heterogeneity within our search pool groups would be reflected in the time-varying endogenous as well as exogenous components of group-specific matching efficiencies.[21]

If unobserved heterogeneity is quantitatively important, the identified transition elasticities, ϕ_i, should vary systematically with the aggregate state of the economy. To check for this possibility, we allow for a structural break in the identified transition elasticities that depends on the level of the unemployment rate. In particular, the identified transition elasticities are allowed to be different for periods when the unemployment rate exceeds the average of the minimum and maximum unemployment rate. For this exercise we use the extended sample period starting in 1974 and, to allow for the possibility of low frequency changes in the unemployment rate, we define separate minima and maxima for the sub-samples 1974-1987, 1987-1997, 1997-2005, and 2005-2012. Column (2b) of Table 1 shows the estimates from this specification. The estimated breaks for the transition elasticities

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[20]For recent contributions see Hornstein (2012), Ahn and Hamilton (2015), and Alvarez, Borovickova, and Shimer (2016).

[21]See Appendix F for an illustration.

are quite small. Furthermore, allowing for these breaks does not much affect the estimates for the aggregate matching efficiency, the dashed-x red line (2*b*) in Figure 2. Below we will estimate the model with aggregate and time-varying type-specific matching efficiencies, and the latter will reflect possible time-varying within-group heterogeneity.

JOLTS vacancies Finally, we consider how an alternative measure for vacancies, namely the vacancy posting series from JOLTS, affects our estimate of matching efficiency. Since JOLTS is available only from 2001 we re-estimate our model with the HWI vacancy measure for the shorter sample period, line (3*a*) in Figure 2, and also estimate our model with the JOLTS vacancy measure, line (3*b*) in Figure 2. The decline in matching efficiency starting in 2008 is comparable for the two vacancy measures, but the JOLTS-based measure does not replicate the pre-2008 decline that is quite prominent for the HWI measure. The estimates for the identified transition elasticities for the shorter sample using either the HWI or JOLTS vacancy measure are quite close to their respective estimates for the baseline sample, but with larger standard errors, Columns (1a) and (3a), respectively, of Table 1.

5.2   Alternative models of matching efficiency

In the benchmark model the comovement of the type-specific employment transition rates is captured by the aggregate matching efficiency, and relative changes of type-specific transition rates are captured by differential responses of search effort to the aggregate transition rate while exogenous differences in type-specific efficiencies remain fixed. In this section, we present results for models that also allow for time-varying type-specific matching efficiencies, Models 2 and 3 of Section 3. A comparison across the models is informative regarding the extent to which type-specific variable search effort with a common aggregate shock (as opposed to a number of type-specific exogenous shocks) is sufficient to capture both comovement and relative changes in type-specific transition rates.

In Table 2.A and Figure 3 we display the results from estimating Models 1, 2, and 3 for the sample period from 1994-2015 when the search pool of unemployed is differentiated by unemployment duration.[22] The top panel of Figure 3 displays the smoothed posterior for the aggregate matching efficiency of Models 1 and 3, and the lower three panels display the smoothed posteriors for the type-specific matching efficiencies of Models 2 and 3.

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[22]The results for Model 1 represent specification (2a) in Table 1 and Figure 2. Hornstein and Kudlyak (2016) contains a complete listing of the results for Models 2 and 3 similar to Table 1 and Figure 2 for Model 1.

For the model with time-varying type-specific matching efficiencies and fixed aggregate matching efficiency, Model 2, we find that the transition elasticities are more similar across groups than in the baseline Model 1 with fixed type-specific effects, Table 2.A, Columns (1) and (2). This means that Model 2 captures changes in relative type-specific transition rates not so much through differences in the types' responsiveness to the aggregate transition rate, but through differential changes in type-specific matching efficiencies. For example, the matching efficiency of the long-duration unemployment group is relatively more volatile and displays a larger decline in the post-2008 period than the matching efficiencies for the other two groups, lower three panels of Figure 3. Nevertheless, there is a substantial comovement between the type-specific matching efficiencies of the three groups, which suggests the presence of a common component.

Comparing the model with time-varying aggregate and type-specific matching efficiencies, Model 3, to the baseline Model 1, we find only small differences for the estimated transition elasticities and the estimated time paths for aggregate matching efficiency, Columns (1) and (3) of Table 2.A, and the top panel of Figure 3. Relatively small movements in type-specific matching efficiencies achieve some marginal improvement over the model with an aggregate matching efficiency only, but otherwise differences in search effort elasticities across types are enough to capture changes in relative transition rates.

Comparing the estimates across the three alternative models of aggregate and type-specific matching efficiencies with variable search effort, we argue that the variation of the type-specific transition rates is well captured by movements in aggregate matching efficiency together with differential endogenous type-specific responses to movements in the aggregate transition rate. Through the lens of the aggregate matching function with heterogeneity and endogenous search effort, it means that there is little exogenous movement in the type-specific transition rates beyond what is captured by the type-specific search effort and an aggregate exogenous shock.

5.2.1 Exogenous time-varying heterogeneity with constant effort

We have just argued that with variable search effort allowing for type-specific exogenous efficiencies in addition to the aggregate matching efficiency does not affect much our estimates and yields small estimated variation in the type-specific efficiencies. Once we shut down variable search effort, however, much more variation in type-specific efficiencies is required to describe relative movements in type-specific transition rates.

Consider our Model 3 estimated under restriction (13), i.e, with constant search effort.

Given the estimate of ϕ, the restricted model is akin to the model with constant search effort and $\alpha = 0.36$. In Figure 4 we display the identified aggregate matching efficiency and the type-specific matching efficiencies for the unrestricted model as the crossed red lines and for the restricted model as the circled blue lines. The restricted model captures all relative movements in the type-specific transition rates via exogenous effects. It delivers similar aggregate matching efficiency as the unrestricted model but requires much more volatile type-specific matching efficiencies to capture all relative movements in the type-specific transition rates.

5.3 Alternative search pool definitions

We now estimate models of heterogeneous employment transition rates with variable search effort for our two alternative definitions of the search pool, unemployed by reason of unemployment and nonemployed by labor force status and gender. Estimated transition elasticities are displayed in Table 2, panels B and C, for all three model specifications of the aggregate and type-specific matching efficiencies. The smoothed posteriors of the aggregate matching efficiencies for Model 1 are displayed in Figure 5.[23] For both definitions of the search pool, we continue to find evidence in favor of different transition elasticities across types, that is, variable search effort. We also find declines of the aggregate matching efficiency in the years following the Great Recession that are comparable to what we find for the baseline search pool of unemployed by duration.

Comparing the decomposition of unemployment by reason with the previous decomposition by duration, we find that the characteristics of those who claim to have been laid off temporarily are similar to those who report a short unemployment duration: their employment transition rates are higher and less cyclically sensitive than for the other groups. For Model 1 with aggregate matching elasticity only, this is reflected in their transition elasticities being lower than for the other groups, Table 2.B, Column (1). Similar to the baseline search pool definition, the differences between estimated transition elasticities of different groups are less pronounced for Model 2 with type-specific matching efficiencies only, Table 2.B, Column (1).

The second alternative definition of the search pool includes all nonemployed and differentiates between those who are actively engaged in search and those who are not. For this

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[23]Hornstein and Kudlyak (2016) contains a complete listing of the results for all search pool definitions and model specifications.

broader definition of the search pool, the groups with the higher employment transition rates tend to be more cyclically sensitive. This is the opposite of what we see for the two previous search pool definitions, which cover only the unemployed. The differences in cyclical sensitivity of the transition rates are so large across groups that they show up as differences in estimated transition elasticities for all specifications of aggregate and type-specific efficiencies, Table 2.C.

The qualitative features of the estimated aggregate matching efficiency are very similar for the different search pool definitions, Figure 5. All of them are characterized by a decline of aggregate matching efficiency in the years following the Great Recession that is exceptional relative to the full sample. Compared with the baseline search pool of unemployed by duration, the two alternative search pool definitions indicate more volatility of the aggregate matching efficiency.

6 Aggregate matching efficiency and the cyclicality of search effort

In our framework, changes in the observed average employment transition rate can be decomposed into contributions coming from aggregate labor market tightness, average search effectiveness, and aggregate matching efficiency, equation (3),

$$\ln \bar{\lambda}_t = \alpha \ln \theta_t + (1 - \alpha) \ln \bar{\rho}_t + \kappa_t \tag{17}$$

With a heterogeneous search pool, changes in average search effectiveness in turn depend on changes in the composition of the search pool and changes of type-specific search efficiency/effort,

$$\ln \bar{\rho}_t = \ln \sum_i \omega_{it} \bar{\rho}_i + \ln \sum_i \bar{\omega}_i \rho_{it} + \varrho_t. \tag{18}$$

Our framework only allows us to infer whether search effort responds to changes in the aggregate transition rate, but not whether search effort is pro- or countercyclical. Nevertheless, as discussed in Section 2.3, we can characterize the cyclicality of search effort conditional on the aggregate matching elasticity. In particular, for a sufficiently small (large) matching elasticity, search effort for each type will be procyclical (countercyclical).

We now show how the relative contributions of the components of the average employment transition rate depend on the assumed aggregate matching elasticity for the case with

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variable search effort, time-varying aggregate matching efficiency, and no exogenous changes for type-specific efficiencies, Model 1 from Section 3. We conduct our analysis for the unemployment search pool by duration and the nonemployment search pool by labor force status for the period 1994-2015. Figure 6 illustrates the dependence of search effort elasticities on matching elasticity for this case. For $0 < \alpha < 0.28$, search effort is procyclical for all three unemployment duration groups, and for $\alpha > 0.42$, search effort is countercyclical for all three unemployment duration groups.[24]

Figure 7 displays the decomposition of the average employment transition rate for the unemployment search pool differentiated by unemployment duration. For the decomposition in the top (bottom) panel of Figure 7 we choose an aggregate matching elasticity of $\alpha = 0.2$ ($\alpha = 0.5$) such that search effort is procyclical (countercyclical) for all types. Our identified measure of aggregate matching efficiency, $\hat{\kappa}_t$, is a scaled version of the actual matching efficiency, κ, with the matching elasticity being the scale factor, $\hat{\kappa}_t = \alpha \kappa_t$. We can then rewrite the expression (3) for the average employment transition rate as

$$\ln \overline{\lambda_t} = \alpha \left(\ln \theta_t + \widehat{\kappa}_t \right) + (1 - \alpha) \ln \bar{\rho}_t,$$

and it is immediate that the contributions of market tightness and aggregate matching efficiency to changes in the average employment transition rate increase with the matching elasticity. This is unlike the standard matching function with homogeneous search where a higher matching elasticity will reduce the measured contribution of matching efficiency. In our setup the difference is made up for by movements in average search effectiveness. For low values of α, procyclical average search quality more than compensates for the small impact of market tightness, and the estimated model requires a small contribution of matching efficiency. Conversely, for high values of α, countercyclical average search quality calls for a larger contribution from matching efficiency than is suggested only by the contribution from market tightness. For the two cases we study, implied matching efficiency with countercyclical search effort is more than twice as volatile than with procyclical search effort. For example, following the Great Recession, the implied matching efficiency with $\alpha = 0.2$ declines by about 15 percent, whereas it declines by more than 30 percent for $\alpha = 0.5$.

Consider first the case with a relatively low aggregate matching elasticity and procyclical search effort, the top panel of Figure 7. We start with the average employment transition rate, the solid black line, and subtract the contribution coming from changes in market

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[24]Appendix E provides a more detailed explanation of Figure 6.

tightness, the dashed red line. This gets us the standard measure of matching efficiency from the homogeneous search model, the dashed blue line. Since the average transition rate and market tightness move together, subtracting the matching elasticity-weighted contribution of market tightness from the average search effort yields a measure of matching efficiency that becomes less volatile for higher values of α for the standard approach. For the assumed low value of the matching elasticity, measured matching efficiency for the homogeneous search model is quite volatile and declines substantially in the Great Recession. For the second step, we subtract the contribution coming from changes in average match quality holding type-specific search effort fixed, the dash-dot green line. This gets the matching efficiency after accounting for changes in the composition of the search pool as in Barnichon and Figura (2015), the dash-dot blue line. Accounting for the composition effect reduces the volatility of measured matching efficiency since the composition shifts toward low transition rate components—from short-duration to long-duration unemployment—as market tightness and the average employment transition rate decline. For the third step we take into account that for the assumed low value of matching elasticity, search effort is procyclical, that is, it declines as market tightness declines. Therefore movements in search effort amplify the decline in average search effectiveness, the solid green line, and reduce the volatility of measured matching efficiency even more, the solid blue line.

Now consider the case when the matching elasticity is relatively large, $\alpha = 0.5$, and search effort is countercyclical, the bottom panel of Figure 7. Given the larger matching elasticity, the contribution of market tightness increases, the dashed red line, and the measured matching efficiency from the homogeneous search model is less volatile, the dashed blue line. Again, accounting for changes in the composition of the search pool reduces the volatility of measured matching efficiency even more, the dash-dotted green and blue lines. For a large matching elasticity, search effort is, however, countercyclical, and more than compensates for the procyclical composition effect, the solid green line. Accounting for the large increase in search effort following the decline in market tightness then recovers the much larger decline in matching efficiency, the solid blue line, than what is implied by the standard homogeneous search model.

We now briefly discuss the role of search effort for a decomposition of the average employment transition rate when the search pool is characterized according to labor force status, Figure 8. This case is of interest, since for this definition of the search pool, composition effects increase average search efficiency when market tightness and the average employment transition rate decline: the share of unemployed with relatively high exit rates increases.

Now countercyclical search effort is reinforcing the composition effect, whereas procyclical search effort is dampening the composition effect, but in any case the impact of cyclical search effort dominates the impact of composition effects.[25]

6.1 Existing empirical evidence on the cyclicality of search effort

We show above that data on type-specific transition rates and the vacancy-unemployment ratio cannot separately identify α from search effort elasticities, η_i. To separately identify α, additional data are required. Such data might come from the evidence on the cyclical behavior of search effort.

While evidence on the cyclicality of search effort is relatively scarce, the existing studies suggest that search effort is countercyclical. Specifically, using the CPS data, Shimer (2004) finds that the number of search methods used by the unemployed increases during the 2001 recession. Using the CPS and the data from the Annual Time Use Survey (ATUS), Mukoyama, Patterson and Sahin (2014) conclude that the time spent on search is countercyclical. Faberman and Kudlyak (2014) find that the number of applications sent by a job seeker per week on an online job board is significantly higher in metropolitan areas with more slack labor markets. Using ATUS, DeLoach and Kurt (2013), however, find that search appears to be acyclical. Specifically, they argue that workers reduce their search in response to deteriorating labor market conditions, but these effects are offset by the increase in search effort due to declines in household wealth. Gomme and Lkhagvasuren (2015) argue that search effort of an individual worker is procyclical and that the measured countercyclical average search effort is due to a composition effect.

Taking as given the evidence that search effort is countercyclical implies that the matching function elasticity is closer to 1 than to 0. That is, vacancies play an important role in the production of new hires in the matching function framework and the countercyclical search effort exacerbates changes in the job seeker input. Consequently, large changes in the aggregate matching efficiency are required to describe changes in the average transition rate.

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[25]See Hornstein and Kudlyak (2016) for a complete listing of the results on the cyclicality of search effort for all search pool definitions.

7   Conclusion

Modeling search effort as a constant elasticity function of the aggregate transition rate, we find a substantial decline of the aggregate matching efficiency after 2007, even after accounting for endogenous search effort. Endogenous search effort accounts well for variation in relative transition rates of different groups of job seekers. The data are consistent with both pro- and countercyclical search effort. Without additional data, we can only make a statement about cyclicality conditional on the elasticity of the matching function. We find countercyclical effort for a wide range of the elasticity of the matching function with respect to vacancies, $(1/3, 1)$. Countercyclical effort dampens transition rate volatility, and larger volatility in aggregate matching efficiency is required to compensate for that, in contrast with the standard model that ignores endogenous search effort.

References

[1] Ahn, Hie Joo , and James Hamilton. 2015. "Heterogeneity and Unemployment Dynamics," Working Paper, University of San Diego.

[2] Alvarez, Fernando, Katarina Borovickova, and Robert Shimer. 2016. Decomposing Duration Dependence in a Stopping Time Model, mimeo.

[3] Barnichon, Regis. 2010. Building a composite help-wanted index. Economics Letters 109, 175-178.

[4] Barnichon, Regis, and Andrew Figura. 2015. Labor Market Heterogeneity and the Aggregate Matching Function. American Economic Journal: Macroeconomics, Vol. 7(4): 222-49.

[5] Baxter, Marianne, and Robert G. King. 1999. Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. Review Of Economics And Statistics. Vol. 81 (4): 575-593.

[6] Christensen, Bent Jesper, Rasmus Lentz, Dale T. Mortensen, George R. Neumann, and Axel Werwatz. 2005. On-the-job Search and the Wage Distribution. Journal of Labor Economics, Vol. 23 (1): 31-58.

[7] Costain, James S., and Reiter, Michael. 2008. Business Cycles, Unemployment Insurance, and the Calibration of Matching Models. Journal of Economic Dynamics and Control, Vol. 32(4): 1120-1155.

[8] Davis, Steven J. 2011. Comment on "Job Search, Emotional Well-Being, and Job Finding in a Period of Mass Unemployment," by Alan Krueger and Andreas Mueller, Brookings Papers on Economic Activity: 58–81.

[9] Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger. 2013. The Establishment-Level Behavior of Vacancies and Hiring. Quarterly Journal of Economics 128 (2): 581-622.

[10] DeLoach, Steve, and Mark R. Kurt. 2013. Discouraging Workers: Estimating the Impact of Macroeconomic Shocks on Search Intensity of the Unemployed. Journal of Labor Research, Vol. 34(4): 433-454.

[11] Elsby, Michael W.I., Ryan Michaels, and David Ratner. 2015. The Beveridge Curve: A Survey. Journal of Economic Literature: Vol. 53 (3): 571-630.

[12] Faberman, Jason R., and Marianna Kudlyak. 2014. The Intensity of Job Search and Search Duration. Richmond FED WP No. 14-12.

[13] Frazis, Harley J., Edwin L. Robinson, Thomas D. Evans, and Martha A. Duff. 2005. Estimating Gross Flows Consistent with Stocks in the CPS. Monthly Labor Review, Vol. 128 (9): 3-9.

[14] Fujita, Shigeru, and Giuseppe Moscarini. 2013. "Recall and Unemployment," NBER Working Paper No. 19640.

[15] Gomme, Paul, and Damba Lkhagvasuren. 2015. Worker Search Effort as an Amplification Mechanism. Journal of Monetray Economics, Vol. 75 (October): 106–122.

[16] Hall, Robert E., and Sam Schulhofer-Wohl. 2015. Measuring Matching Efficiency with Heterogeneous Jobseekers. NBER Working Paper No. 20939.

[17] Harvey, Andrew C. 1989. Forecasting, Structural Time Series Models and the Kalman Filter. Cambridge University Press.

[18] Hornstein, Andreas. 2012. Accounting for Unemployment: The Long and Short of It. FRB Richmond WP No. 12-07.

[19] Hornstein, Andreas, and Marianna Kudlyak. 2016. Estimating Matching Efficiency with Variable Search Effort: Tables and Figures. Available at
https://www.richmondfed.org/publications/research/working_papers/2016/wp_16-13r

[20] Kroft, Kory, Fabian Lange, Matthew J. Notowidigdo, and Lawrence Katz. 2015. Long-Term Unemployment and the Great Recession: The Role of Composition, Duration Dependence, and Non-Participation. Journal of Labor Economics, Vol. 34 (S1): S7-S54.

[21] Krueger, Alan, and Andreas Mueller. 2011. Job Search, Emotional Well-Being, and Job Finding in a Period of Mass Unemployment, Brookings Papers on Economic Activity, Vol. 42(1): 1-81.

[22] Lise, Jeremy. 2013. On-the-job Search and Precautionary Savings. Review of Economic Studies (80): 1086–1113.

[23] Madrian, Brigitte C., and Lars John Lefgren. 1999. A Note on Longitudinally Matching Current Population Survey (CPS) Respondents. NBER Working Paper No. t0247.

[24] Mukoyama, Toshihiko, Christina Patterson, and Aysegul Sahin. 2014. Job Search Behavior over the Business Cycle. Federal Reserve Bank of New York Staff Report No. 689.

[25] Petrongolo, Barbara, and Christopher A. Pissarides. 2001. Looking into the Black Box: A Survey of the Matching Function. Journal of Economic Literature 39, 390-431.

[26] Pissarides, Christopher A.. 2000. Equilibrium Unemployment Theory. The MIT Press.

[27] Polivka, Anne E., and Stephen M. Miller. 1998. The CPS after the Redesign: Refocusing the Economic Lens. In "Labor Statistics Measurement Issues, " John Haltiwanger, Marilyn E. Manser and Robert Topel, editors. University of Chicago Press.

[28] Şahin, Ayşegül, Joseph Song, Giorgio Topa, and Givanni L. Violante. 2014. Mismatch Unemployment. American Economic Review, Vol. 104(11): 3529-64.

[29] Sedlacek, Petr. 2016. The Aggregate Matching Function and Job Search from Employment and Out of the Labor Force. Review of Economic Dynamics, Vol. 21(7): 16-28.

[30] Shimer, Robert. 2004. Search Intensity. Mimeo, downloadable from https://sites.google.com/site/robertshimer/research/workingpapers.

[31] Shimer, Robert. 2012. Reassessing the Ins and Outs of Unemployment. Review of Economic Dynamics 15, 127-148.

[32] Veracierto, Marcelo. 2011. Worker Flows and Matching Efficiency. Federal Reserve Bank of Chicago Economic Perspectives v.35 4th Quarter, 147-169.

[33] Watson, Mark M. 1996. Comment on "Is Seasonal Adjustment a Linear or Nonlinear Data Filtering Process," Journal of Business and Economic Statistics, Vol. 14, No. 3.

Appendix

A Endogenous search effort

A simple modification of the basic matching model allows for variation of individual search effort that is related to the aggregate employment transition rate, e.g. Pissarides (2000) or recently Gomme and Lkhagvasuren (2015). Let U and W denote the value of being unemployed and employed, respectively. Then, the return on unemployment is

$$rU = b - c(s) + \rho\lambda(W - U),$$

where r is the rate of time discount and b is the flow return from unemployment. Devoting effort to search increases the rate at which the worker becomes employed but it comes at a cost, $c(s)$. Determining the optimal choice of effort is a well-defined problem if the effort cost is an increasing convex function of effort. For simplicity, assume that the cost function is of the constant elasticity variety,

$$c(s) = s_0 s^v \text{ with } \nu > 1.$$

The first order condition yields the optimal search effort as

$$s = \lambda^{1/(\nu-1)} \left[(W - U) / (s_0 \nu)\right]^{1/(\nu-1)}, \tag{19}$$

that is, search effort is a constant elasticity function of the aggregate transition rate.

For the basic matching model, search effort is an increasing function of the aggregate transition rate: as the marginal benefit from search increases, the worker will devote more effort to search, yielding procyclical search effort. We propose to estimate a reduced form expression that relates the search intensity for each type to the aggregate matching rate, and a type-specific persistent component, z_{it}. The elasticity of search effort with respect to the aggregate transition rate is η_i. We do not impose any restrictions on search effort to be pro- or countercyclical, but we do impose the restriction that the type transition rate is a nondecreasing function of the aggregate transition rate, $\eta_i \geq -1$.

B   Proof of proposition 2

In this section, we obtain the log-linear approximation of the relation between the average transition rate and the standard measure of market tightness. The first order log-linear approximation of the average transition rate is

$$\Delta \ln \bar{\lambda}_t = \Delta \ln \lambda_t + \Delta \ln \bar{\rho}_t,$$

where $\Delta \ln x_t \equiv \ln x_t - \ln x_0$.

Using the first order log-linear approximation of the average search effectiveness as a function of the aggregate transition rate

$$\Delta \ln \bar{\rho}_t = \Delta \bar{z}_t + \bar{\eta}_0 \Delta \ln \lambda_t,$$

and the first order log-linear approximation of the aggregate transition rate yields

$$\Delta \ln \lambda_t = \frac{1}{1 + \alpha \bar{\eta}_0} \left(\alpha \left(\Delta \ln \theta_t - \Delta \bar{z}_t \right) + \Delta \kappa_t \right),$$

where $\Delta \bar{z}_t = \sum_i \omega_{i0} \Delta z_{it}$, $\bar{\eta}_0 = \sum_i \omega_{i0} \eta_i$, with $\omega_{i0} = \frac{\frac{u_{i0}}{u_0} \exp(z_{i0} + \eta_i \ln \lambda_0)}{\sum_i \frac{u_{i0}}{u_0} \exp(z_{i0} + \eta_i \ln \lambda_0)}$ and $\sum_i \omega_{i0} = 1$.

We then obtain the following relation between the average transition rate and the standard measure of market tightness

$$\Delta \ln \bar{\lambda}_t = \alpha \frac{1 + \bar{\eta}_0}{1 + \alpha \bar{\eta}_0} \Delta \ln \theta_t + \left[\alpha \frac{1 + \bar{\eta}_0}{1 + \alpha \bar{\eta}_0} \frac{\Delta \kappa_t}{\alpha} + \left(1 - \alpha \frac{1 + \bar{\eta}_0}{1 + \alpha \bar{\eta}_0} \right) \Delta \bar{z}_t \right]. \tag{20}$$

Intuitively, in equation (20) the coefficient on $\ln \theta_t$ captures a direct effect of the labor market tightness on $\ln \bar{\lambda}_t$ as well as an indirect effect of the labor market tightness via search effort. That is, the matching function elasticity in the standard matching function is

$$\bar{\alpha} = \alpha \frac{1 + \bar{\eta}_0}{1 + \alpha \bar{\eta}_0} \equiv \frac{\bar{\phi}_0}{1 + \bar{\phi}_0},$$

where $\bar{\phi}_0 = \sum_i \phi_i \omega_{i0}$.

C Proof of proposition 3

Consider the following shift of the state vector $(\Delta_0, \Delta_i, \Delta)$

$$
\begin{aligned}
\widehat{\kappa}' &= \widehat{\kappa} + \Delta_0 \\
z'_i &= z_i + \Delta_i \\
\ln \widehat{\lambda}'_t &= \ln \widehat{\lambda}_t + \Delta
\end{aligned}
$$

such that

$$\Delta_i = -\phi_i \Delta \text{ and } \Delta_0 = \Delta.$$

Such a shift does not have any impact on the system (11) and (12), i.e.,

$$
\begin{aligned}
\ln \lambda_i &= z'_i + \phi_i \ln \widehat{\lambda}' \\
&= (z_i + \Delta_i) + \phi_i \left(\ln \widehat{\lambda} + \Delta \right) \\
&= z_i + \phi_i \ln \widehat{\lambda}
\end{aligned}
$$

and

$$
\begin{aligned}
\ln \widehat{\lambda}' &= \widehat{\kappa}' + \ln \theta - \ln \left[\sum_i \frac{u_i}{u} \exp \left(z'_i + \phi_i \ln \widehat{\lambda}' \right) \right] \\
\left(\ln \widehat{\lambda} + \Delta \right) &= (\widehat{\kappa} + \Delta_0) + \ln \theta - \ln \left[\sum_i \frac{u_i}{u} \exp \left(z_i + \Delta_i + \phi_i \left(\ln \widehat{\lambda} + \Delta \right) \right) \right] \\
\ln \widehat{\lambda} &= \widehat{\kappa} + \ln \theta - \ln \left[\sum_i \frac{u_i}{u} \exp \left(z_i + \phi_i \ln \widehat{\lambda} \right) \right]
\end{aligned}
$$

remain satisfied.

D Continuous time transition rates from discrete time transition probabilities

First, consider the definition of the unemployed search pool by (1) duration or (2) reason. Suppose we have n groups of unemployed. For the continuous time transition rates, only consider the exit from unemployment and ignore the possibility that an individual flows back into unemployment. Then the transition probabilities are related to the continuous

time transition rates as follows

$$
\begin{aligned}
p_{iE} &= \int_0^1 \underbrace{e^{-\lambda_{iI}\tau}}_{\text{no exit to } I \text{ by } \tau} \underbrace{\left(\lambda_{iE}e^{-\lambda_{iE}\tau}\right)}_{\text{transition to } E \text{ at } \tau} d\tau = \lambda_{iE}\int_0^1 e^{-(\lambda_{iI}+\lambda_{iE})\tau}d\tau \\
&= \frac{\lambda_{iE}}{\lambda_{iE}+\lambda_{iI}}\left[1-e^{-(\lambda_{iE}+\lambda_{iI})}\right] \\
p_{iI} &= \frac{\lambda_{iI}}{\lambda_{iE}+\lambda_{iI}}\left[1-e^{-(\lambda_{iE}+\lambda_{iI})}\right].
\end{aligned}
$$

Solve for λ

$$
\begin{aligned}
\frac{p_{iE}}{p_{iI}} &= \frac{\lambda_{iE}}{\lambda_{iI}} \\
p_{iE}+p_{iI} &= 1-e^{-(\lambda_{iE}+\lambda_{iI})} \\
\log\left(1-p_{iE}-p_{iI}\right) &= -\left(\lambda_{iE}+\lambda_{iI}\right) \\
&= -\left(1+\frac{p_{iE}}{p_{iI}}\right)\lambda_{iI}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\lambda_{iI} &= -\frac{\log\left(1-p_{iE}-p_{iI}\right)}{\left(1+\frac{p_{iE}}{p_{iI}}\right)} \\
\lambda_{iE} &= -\frac{\log\left(1-p_{iE}-p_{iI}\right)}{\left(1+\frac{p_{iI}}{p_{iE}}\right)} \\
&= -p_{iE}\frac{\log\left(1-p_{iE}-p_{iI}\right)}{\left(p_{iE}+p_{iI}\right)}
\end{aligned}
$$

and for p_{iI} small relative to p_{iE} we have

$$
\lambda_{iE} \approx -\log\left(1-p_{iE}\right)
$$

Suppose instead that the pool includes OLF and unemployed, then the above equation becomes

$$
\lambda_{iE} \approx -\log\left(1-\left(p_{iE}+p_{iE}\right)\right)\frac{p_{iE}}{p_{iE}+p_{iE}}.
$$

E   Matching elasticity and cyclicality of search effort

For estimated identified transition elasticities ϕ_i, we can solve the expression for the transition elasticities, equation (11), for the search effort elasticities η_i conditional on the matching elasticity α,

$$\eta_i = \phi_i \frac{1-\alpha}{\alpha} - 1.$$

Without any additional data, the model can accommodate procyclical (countercyclical) search effort with a low (high) matching elasticity,

$$\eta_i \gtreqless 0 \text{ for } \alpha \lesseqgtr \frac{\phi_i}{1+\phi_i}.$$

For the special case that the identified transition elasticities are the same for all types, $\phi_i = \phi$, the implied effort elasticities are also the same, $\eta_i = \eta$. Since the effort elasticities are not identified, constant search effort, $\eta = 0$, is then a special case. In other words, our model with variable search effort is observationally equivalent to a model with constant search effort.

For given estimates of the identified transition rates, we can compute the range of α for which the model requires pro(counter)cyclical search effort. Figure A.1 below illustrates the dependence of search effort elasticities on matching elasticity using the estimates from Model 1 for the three duration-contingent groups of the unemployed. For $0 < \alpha < 0.28$, search effort is procyclical for all three unemployment duration groups, and for $\alpha > 0.42$, search effort is countercyclical for all three unemployment duration groups.

Since the estimated ϕ_i increase with the groups' unemployment duration, the search effort elasticity also increases with groups' unemployment duration. In particular, for $\alpha < 0.28$, when the search effort of all three groups is procyclical, search effort of long-term unemployed increases more when the aggregate transition rate increases and declines more when the aggregate transition rate declines as compared to the search effort of medium-term unemployed or short-term unemployed. For $\alpha > 0.42$, when the search effort of all three groups is countercyclical, search effort of long-term unemployed declines less when the aggregate transition rate increases and increases less when the aggregate transition rate declines as compared to the search effort of medium-term unemployed or short-term unemployed.

F   Unobserved heterogeneity within groups

In the analysis so far, we assume that job seekers within specified groups are homogenous. However, it is possible that job seekers within the groupings are heterogeneous and the composition of a group with respect to this heterogeneity is time-varying. In such a case, the time-varying transition rate of a group might reflect not only the actual variation of the transition rate of the job seekers in the group but also the compositional shifts within the group.

To see this formally, suppose that job seekers within each group i are not homogeneous. Let $\left(z_{ijt}, \eta_{ij}\right)$ represent the unobserved heterogeneity within group i and (ω_{ijt}) the distribution over types within the group, $\sum_j \omega_{ijt} = 1$. The transition rates for the different types j are

$$\ln \lambda_{ijt} = z_{ijt} + \left(1 + \eta_{ij}\right) \ln \lambda_t.$$

We observe only the 'average' transition rate for group i. To keep things simple, suppose we observe the geometric mean of the unobserved transition rates

$$
\begin{aligned}
\ln \lambda_{it} &= \sum_j \omega_{ijt} \ln \lambda_{ijt} + \varepsilon_{it} \\
&= \sum_j \omega_{ijt} \left[z_{ijt} + \left(1 + \eta_{ij}\right) \ln \lambda_t \right] + \varepsilon_{it} \\
&= z_{it} + (1 + \eta_i) \ln \lambda_t + \varepsilon_{it},
\end{aligned}
$$

where $z_{it} = \sum_j \omega_{ijt} z_{ijt}$ and $\eta_i = \sum_j \omega_{ijt} \eta_{ij}$.

That is, not only does unobserved heterogeneity show up in the estimated type-specific matching efficiency, it also shows up in our estimated search effort elasticities. Without further disaggregation of our search groups, we could check if the search effort elasticities vary systematically with the cycle.

G   Time-varying heterogeneity with constant search effort and no aggregate matching efficiency

We now consider the effect of imposing constant search effort for Model 2 with no aggregate matching efficiency and forcing the model to capture time-varying relative heterogeneity via type-specific exogenous shocks in a model. For the duration-contingent search pool definition,

the type-specific matching efficiencies for the unrestricted version of Model 2 are represented by the solid red lines in Figure A.1. When restriction (13) is imposed, the type-specific matching efficiencies are represented by the dashed blue lines in Figure A.1. There is little difference between the type-specific matching efficiencies from the restricted and unrestricted model. This is so because in the unrestricted model, there is no aggregate shock, type-specific variable search effort does not differ much across types ($\phi_i \simeq \phi \ \forall i \in I$), and the type-specific exogenous shocks essentially capture all movements in type-specific transition rates. Given the estimate of ϕ, the restricted model is akin to the model with constant search effort and $\alpha = 0.38$.

Importantly, the volatility of the type-specific exogenous shocks might differ significantly from the unrestricted case if the restricted model is estimated under an assumption for a value of α that differs from the one implied by the estimate of ϕ. Specifically, when the search effort is constant, the identification issue for α does not arise, it is then typical to use $\alpha = 0.5$ or another value from the range of admissible values. Setting $\eta_i = 0$ and $\alpha = \widetilde{\alpha}$ in our model amounts to the following restriction

$$\phi_i = \widetilde{\phi} \ \forall i \in I,$$

where $\widetilde{\phi} = \frac{\widetilde{\alpha}}{1-\widetilde{\alpha}} \left(1 + \widetilde{\eta}\right)$ and $\widetilde{\eta} = 0$. For $\widetilde{\eta} = 0$ and $\widetilde{\alpha} = 0.5$, $\widetilde{\phi} = 1$. The estimated type-specific matching efficiencies from Model 2 estimated under $\widetilde{\phi} = 1$ are represented by the dotted green lines in Figure A.1. The estimated transition rate elasticities with respect to the aggregate transition rate are set to 1 for all types, which substantially exceeds the estimated sensitivity to the aggregate transition rate in the unrestricted model or even the model with restriction $\phi_i = \phi \ \forall i \in I$, where the estimate of ϕ is 0.557 with the standard error of 0.049. Consequently, in this restricted model, the estimated type-specific matching efficiencies compensate for the imposed degree of comovement under $\widetilde{\phi} = 1$ and differ substantially from the unrestricted model.
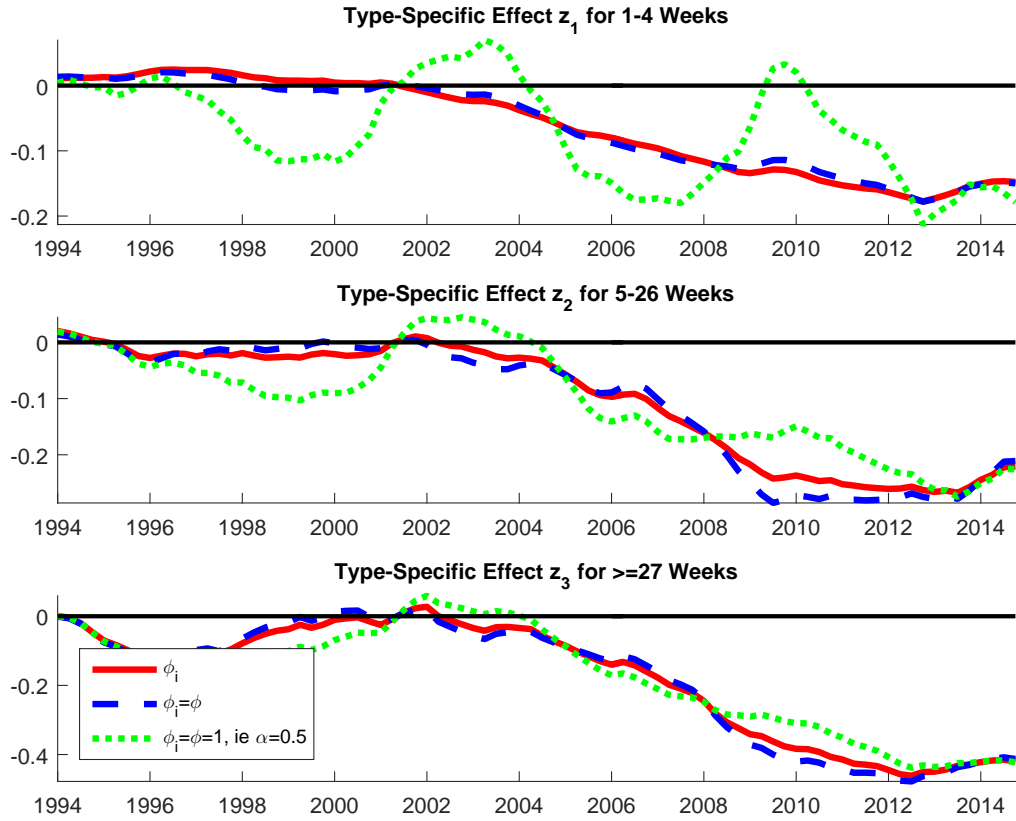
A.1 Variable vs Constant Search Effort for Model 2

Note: Smoothed posterior estimates of the type-specific matching efficiencies from Model 2 defined in the text for search pool defined as unemployed by duration. The solid red line denotes the unrestricted model, the dashed blue line denotes the restricted model with equalized identified transition elasticities, and the dotted green line denotes the model with identified transition elasticities restricted to one. The specification follows column (1a) in Table 1. All efficiencies are normalized to zero in 2001Q1.

Figures and Tables

Figure 1: TRANSITION RATES TO EMPLOYMENT AND SEARCH POOL COMPOSITION
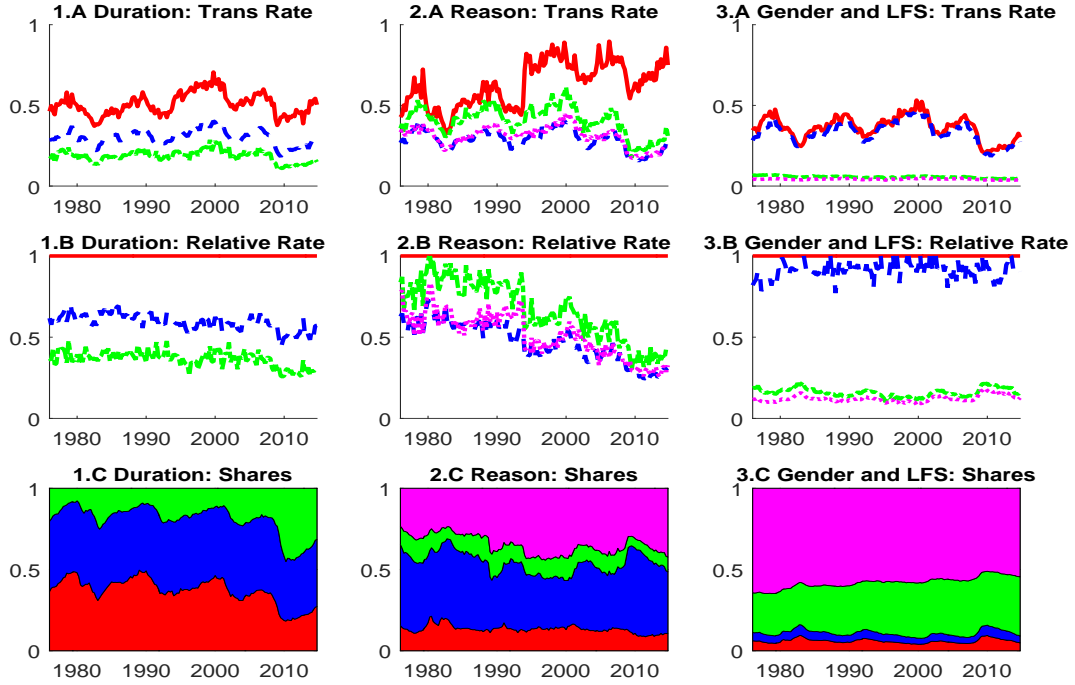
Note: The columns represent the three different search pool definitions and are characterized by the employment transition rates (first row), the relative employment transition rates (second row), and the search pool shares (third row). For the search pool defined as unemployed by duration, red denotes 1-4 weeks, blue denotes 5-26 weeks, and green denotes more than 26 weeks of unemployment. For the search pool defined by reason of unemployment, red denotes temporary layoff, blue denotes permanent layoff, green denotes job leavers, and purple denotes new entrants or re-entrants. For the search pool defined by LFS and gender, red denotes male unemployed, blue denotes female unemployed, green denotes male OLF, and purple denotes female OLF.

Figure 2: IDENTIFIED AGGREGATE MATCHING EFFICIENCY FROM THE MODEL WITH AGGREGATE MATCHING EFFICIENCY AND VARIABLE SEARCH EFFORT, UNEMPLOYMENT BY DURATION

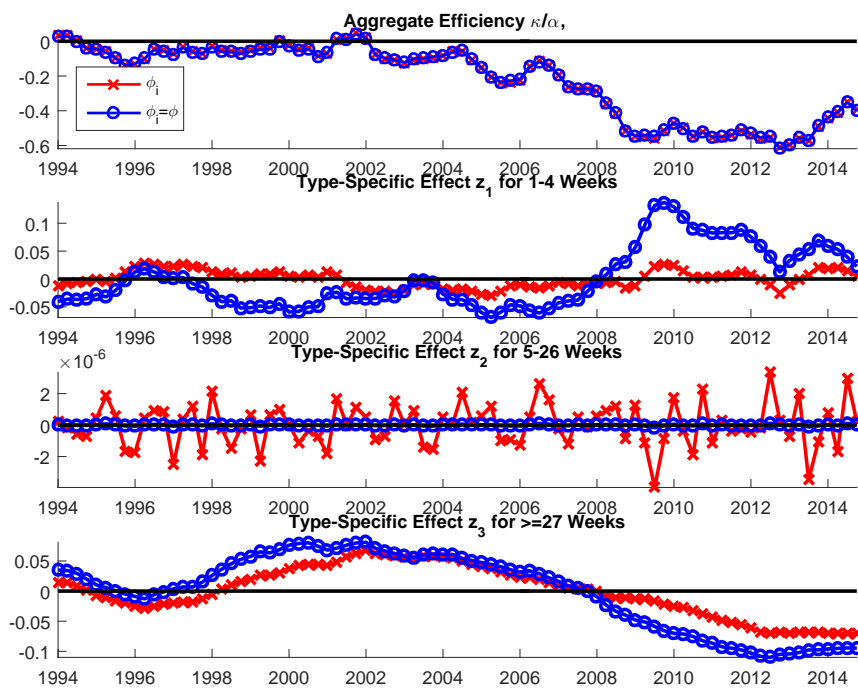**Aggregate Matching Efficiency, κ/α**
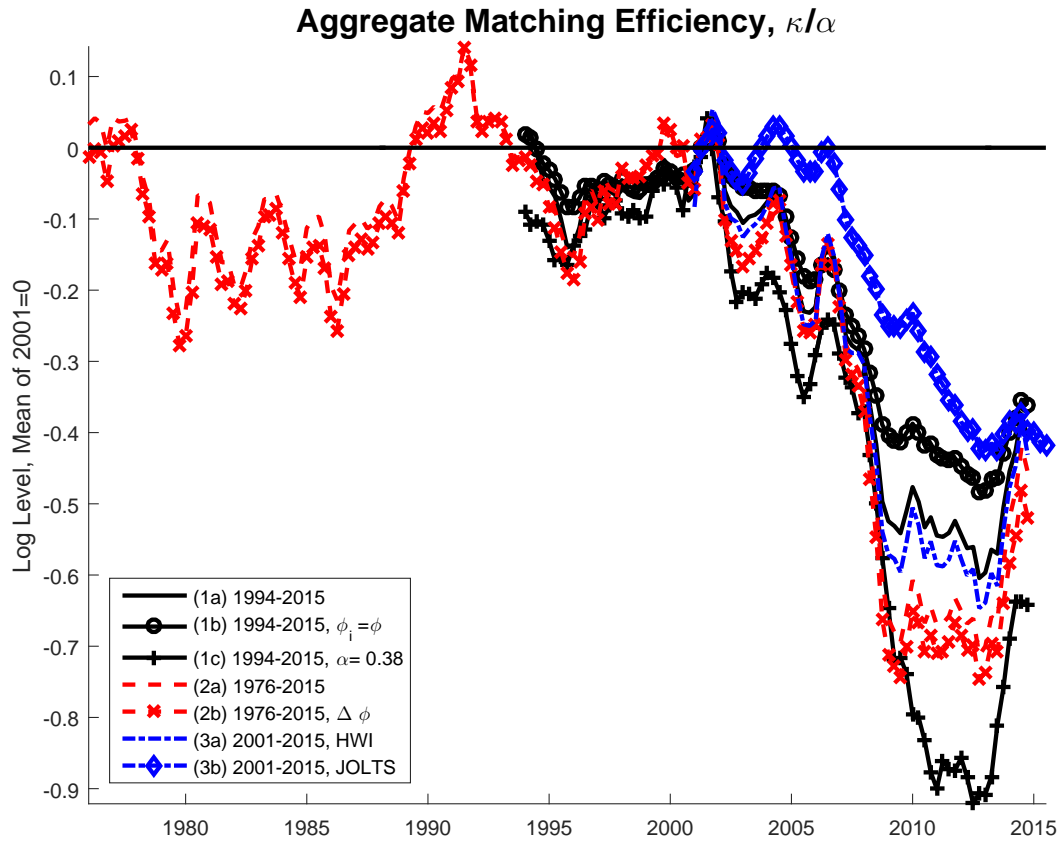
Note: Lines (1a)-(3b) show smoothed posterior estimate of $\widehat{\kappa}_t$ from a model with a random walk in aggregate matching efficiency (Model 1 in the text). Lines (1a), (1b), (2a)-(3b) show the estimates from the respective columns in Table 1, see note to Table 1 for details. Line (1c) shows the constructed $\widehat{\kappa}_t$ for the standard matching function without heterogeneity and $\alpha = 0.38$; see Section 5.3.3 for details. All efficiencies are normalized to zero in 2001Q1.

Figure 3: AGGREGATE AND TYPE-SPECIFIC MATCHING EFFICIENCIES FROM ALTERNA-
TIVE MODELS OF STOCHASTIC STATE, UNEMPLOYMENT BY DURATION
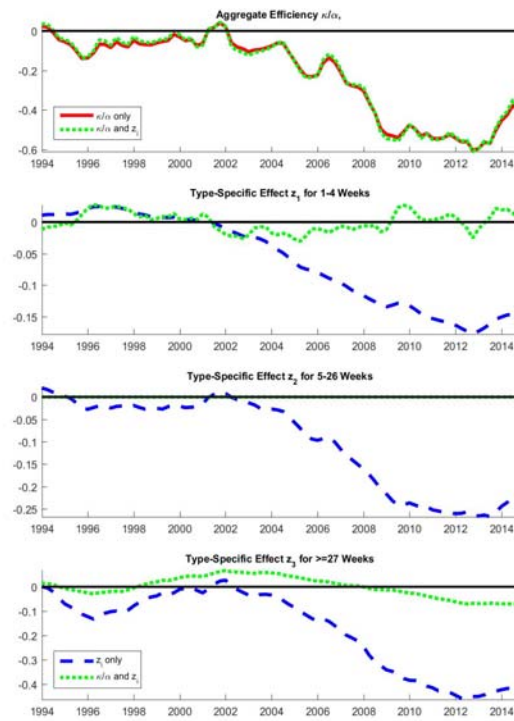
Note: Smoothed posterior estimates of the aggregate and type-specific matching efficiencies
from Models 1-3 defined in the text for search pool defined as unemployed by duration. The
specification follows column (1a) in Table 1. All efficiencies are normalized to zero in 2001Q1.

Figure 4: VARIABLE VS CONSTANT SEARCH EFFORT FOR MODEL 3 WITH DURATION CONTINGENT SEARCH POOL

Note: Smoothed posterior estimates of the aggregate and type-specific matching efficiencies from Model 3 defined in the text for search pool defined as unemployed by duration. The red crossed line imposes no restrictions on identified transition elasticities, and the blue circled line imposes equality for the identified transition elasticities. All efficiencies are normalized to zero in 2001Q1.

Figure 5: AGGREGATE MATCHING EFFICIENCIES FROM ALTERNATIVE SEARCH POOL DEFINITIONS
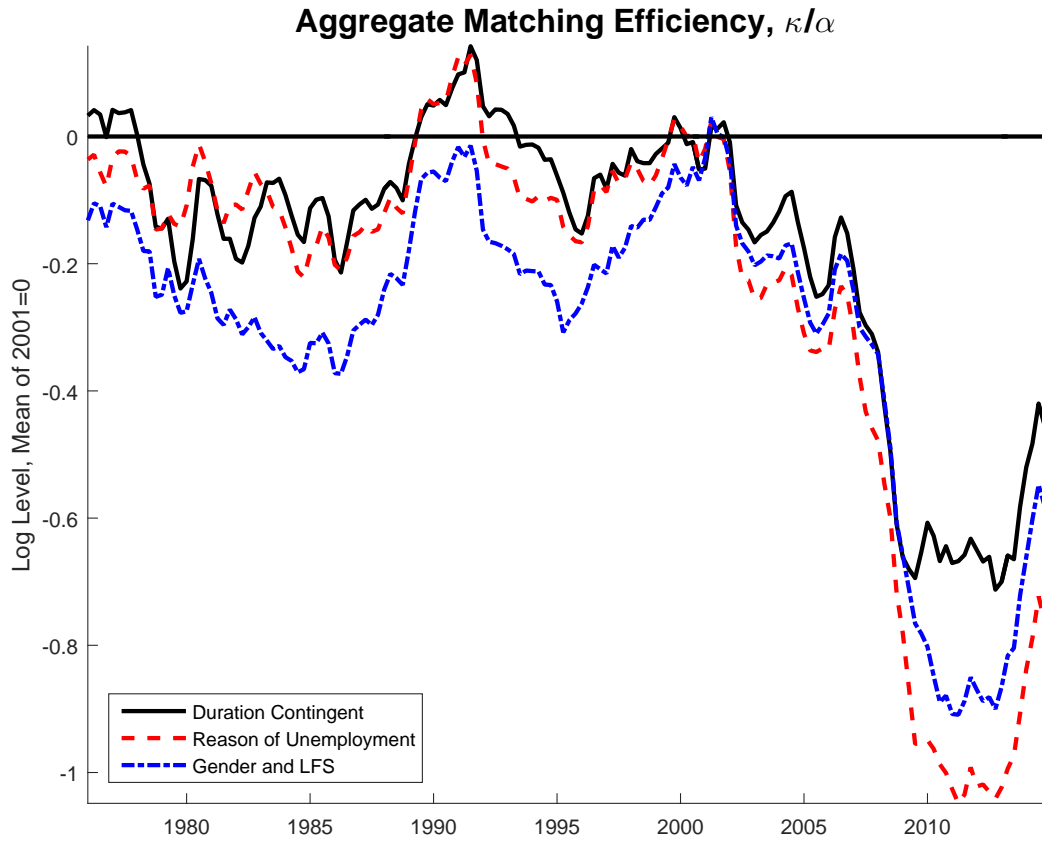
Note: Smoothed posterior estimates of the aggregate matching efficiency from a model with a random walk in aggregate efficiency (Model 1 in the text) for search pool defined as (1) unemployed by duration, (2) unemployed by reason, or (3) LFS by gender. The specification follows column (2a) Table 1. All efficiencies are normalized to zero in 1994Q1.

Figure 6: SEARCH EFFORT ELASTICITY AND MATCHING ELASTICITY
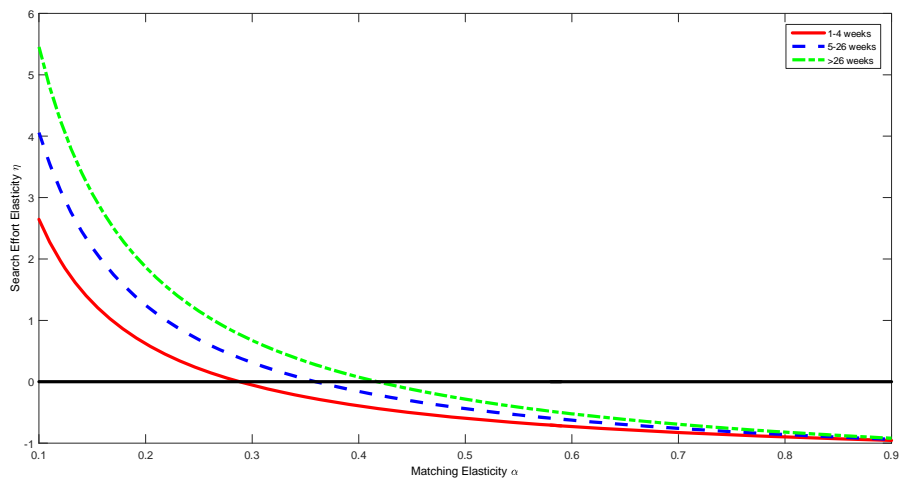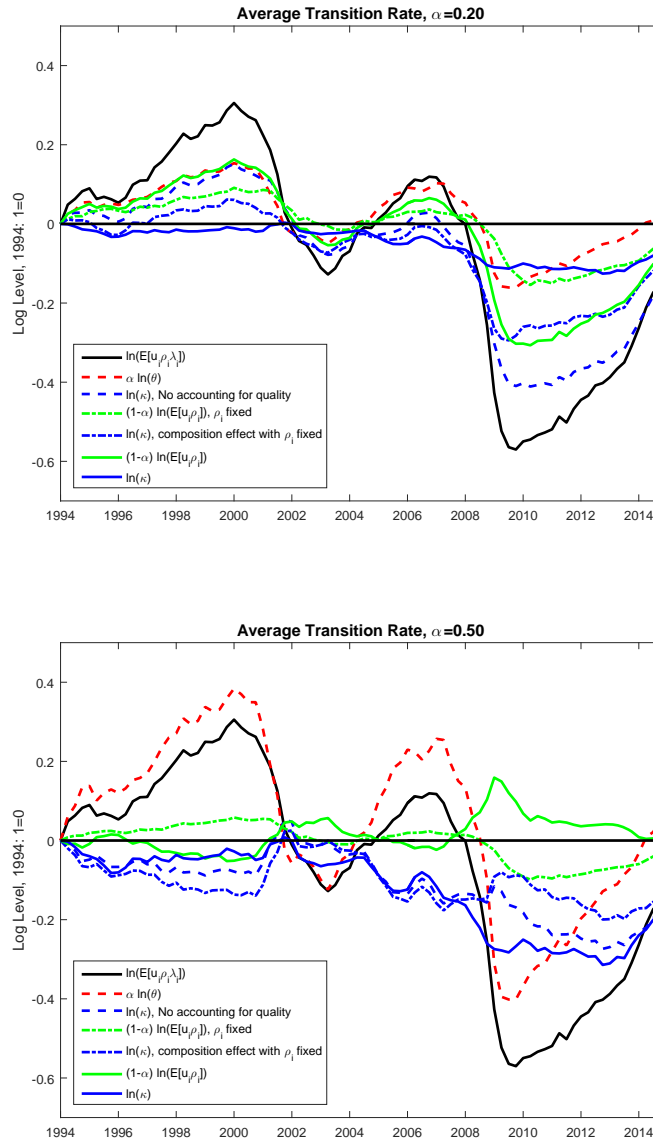
Figure 7: Decomposition of the Average Transition Rate, Unemployment by Duration

Note: Components of the average employment transition rate from a model with a random walk in aggregate efficiency (Model 1 in the text) for a search pool defined by duration of unemployment. The specification follows column (1a) Table 1. All variables are normalized to zero in 1994Q1.
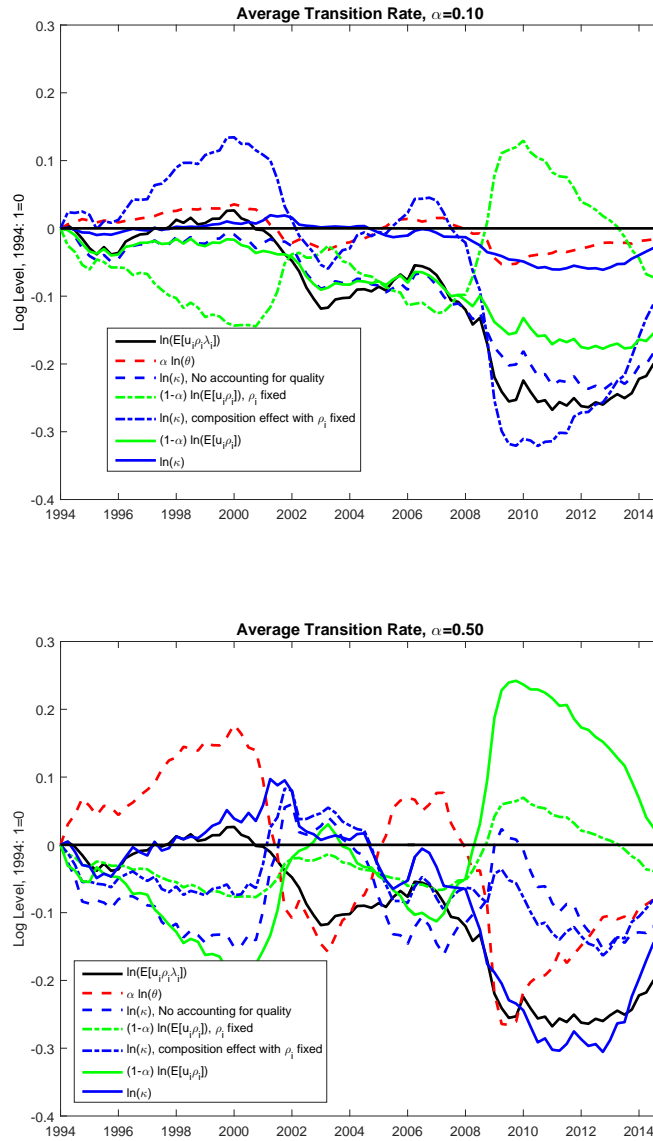
Figure 8: Decomposition of the Average Transition Rate, LFS by Gender

Note: Components of the average employment transition rate from a model with a random walk in aggregate efficiency (Model 1 in the text) for a search pool defined by labor force status and gender. The specification follows column (1a) Table 1. All variables are normalized to zero in 1994Q1.

Table 1: ESTIMATES FROM THE MODEL WITH AGGREGATE MATCHING EFFICIENCY AND VARIABLE SEARCH EFFORT, UNEMPLOYMENT BY DURATION

	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
$E[\hat{\kappa}_1]$	-2.401	-1.914	-2.548	-2.838	-2.749	-2.881
	(1.416)	(0.809)	(0.912)	(0.218)	(0.938)	(6.912)
c_2	-0.325	-0.572	-0.349	-0.314	-0.216	-0.212
	(0.245)	(0.008)	(0.051)	(0.039)	(0.417)	(0.415)
c_3	-0.587	-1.076	-0.682	-0.451	-0.423	-0.461
	(0.091)	(0.014)	(0.058)	(0.171)	(0.421)	(0.839)
ϕ_1	0.405	0.611	0.358	0.303	0.355	0.377
	(0.021)	(0.073)	(0.024)	(0.036)	(0.107)	(0.111)
ϕ_2	0.562	-	0.483	0.429	0.543	0.579
	(0.139)	-	(0.047)	(0.072)	(0.067)	(0.070)
ϕ_3	0.717	-	0.579	0.628	0.702	0.730
	(0.081)	-	(0.029)	(0.087)	(0.092)	(0.584)
$\sigma_{\varepsilon 1}$	0.048	0.065	0.045	0.045	0.049	0.050
	(0.009)	(0.006)	(0.003)	(0.003)	(0.006)	(0.006)
$\sigma_{\varepsilon 2}$	0.038	0.042	0.036	0.036	0.039	0.038
	(0.006)	(0.004)	(0.003)	(0.003)	(0.005)	(0.005)
$\sigma_{\varepsilon 3}$	0.088	0.110	0.086	0.079	0.085	0.091
	(0.017)	(0.009)	(0.005)	(0.006)	(0.010)	(0.009)
σ_{κ}	0.054	0.036	0.060	0.063	0.069	0.038
	(0.017)	(0.012)	(0.012)	(0.013)	(0.019)	(0.014)
λ_{adj1}	-	-	0.156	0.154	-	-
	-	-	(0.025)	(0.027)	-	-
λ_{adj2}	-	-	0.069	0.066	-	-
	-	-	(0.019)	(0.031)	-	-
λ_{adj3}	-	-	0.055	0.050	-	-
	-	-	(0.031)	(0.050)	-	-
$\phi_{hU,1}$	-	-	-	0.017	-	-
	-	-	-	(0.009)	-	-
$\phi_{hU,2}$	-	-	-	0.015	-	-
	-	-	-	(0.012)	-	-
$\phi_{hU,3}$	-	-	-	-0.025	-	-
	-	-	-	(0.021)	-	-

Note: Col. (1a) - (3a) display parameter estimates of a model with a random walk for aggregate matching efficiency using the HWI as a measure of vacancies. Col. (1a) displays estimates for the sample period 1994-2015, and col. (1b) imposes the restriction that ϕ_i is the same for all types. Col. (2a) displays estimates for the sample period 1976-2015 with adjustment of the search pool data prior to the 1994 CPS revision based on Polivka and Miller (1998) and a fixed estimated structural change in measured transition rates λ_i, and col. (2b) introduces an unemployment rate contingent structural break in the parameter ϕ_i as explained in the text. Col. (3a) re-estimates the model from col. (1a) for the sub-sample 2001-2015 with HWI for vacancies, and col. (3b) does the same with vacancies from JOLTS.

Table 2: IDENTIFIED MATCHING ELASTICITIES

	(1)	(2)	(3)
A. Unemployment by Duration			
1-4 weeks	0.381	0.405	0.368
	(0.037)	(0.036)	(0.174)
5-26 weeks	0.506	0.538	0.488
	(0.037)	(0.075)	(0.294)
27 weeks and above	0.604	0.523	0.521
	(0.038)	(0.108)	(0.138)
B. Unemployment by Reason			
Laid off temporarily	0.116	0.587	0.498
	(0.018)	(0.061)	(0.074)
Laid off permanently	0.563	0.719	0.634
	(0.053)	(0.103)	(0.069)
Left job	0.571	0.668	0.584
	(0.056)	(0.069)	(0.062)
LF entrant	0.516	0.461	0.451
	(0.049)	(0.070)	(0.051)
C. Unemployment and OLF by Reason			
Unemployed, male	0.686	0.731	0.588
	(0.058)	(0.064)	(0.082)
Unemployed, female	0.686	0.588	0.589
	(0.059)	(0.058)	(0.079)
OLF, male	0.261	0.218	0.195
	(0.028)	(0.043)	(0.041)
OLF, female	0.215	0.193	0.189
	(0.022)	(0.042)	(0.033)

Note: The columns display the estimated identified matching elasticities ϕ of a model with (1) a random walk for aggregate matching efficiency only, (2) random walks for type-specific matching efficiencies, and (3) a random walk for aggregate matching efficiency and stationary AR(1)'s for type-specific matching efficiencies. All models are estimated for specification (2a) in Table 1, that is, the 1976-2015 sample period with appropriate adjustment for the 1994 CPS revision. See note to Table 1 for details.