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Misallocation and Financial Frictions: The Role of Long-Term Financing

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Abstract

We analyze misallocation of capital in a model where firms face different types of financial constraints. Private firms borrow subject to a collateral constraint, while public firms issue long-term bonds subject to default risk. We estimate our model using employment and financial statistics reflecting the overall distribution of firms in conjunction with firm-level data on credit spreads that we target for the set of public firms. In our model, a productive private firm is unable to grow fast if its collateral is limited. But a productive public firm can overcome its financial constraints because it faces low borrowing costs in the debt market, a relationship we also verify in the data. As a result, financial frictions for private firms disrupt investment behavior to a greater degree and generate a larger misallocation of resources relative to financial frictions for public firms.

Keywords: misallocation, financial frictions, long-duration bonds

JEL Classification Numbers: E23, E44, G32, O47

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1 Introduction

During the past decade, there has been a considerable amount of research analyzing the effect of financial frictions on misallocation of capital and aggregate total factor productivity (see for example, Buera and Shin, 2011; Khan and Thomas, 2013; Midrigan and Xu, 2014; Moll, 2014). According to this literature, firms have limited access to borrowing in capital markets and therefore have difficulty quickly reaching their optimal scale of production. The typical way these borrowing constraints are modeled is in terms of restrictions to the amount of collateralized assets. But while collateral constraints describe well the behavior of typically smaller private firms that rely on bank loans, they are less descriptive of larger firms with access to public bond and equity markets.[1] The natural question that arises is: what is the impact of financial frictions on aggregate total factor productivity (TFP) in a model that captures more realistically the financing constraints of both private and public firms?

In this paper, we measure misallocation of capital in a model with two types of firms. First, private firms issue one-period bonds subject to a collateral constraint and can access the external equity market at a relatively high cost. Second, public firms borrow by issuing long-term defaultable bonds in the debt market and can issue external equity at a relatively low cost. Firms are born as private entities and can transition to public status by paying a one-time fixed cost. We integrate our life-cycle model with private and public firms into an otherwise standard model of firm dynamics with idiosyncratic productivity shocks, capital adjustment costs, and firm entry and exit.

We find that financial frictions have a larger impact on private firms relative to public firms. Although standard intuition suggests that private and public firms differ in size, we argue that our results hinge on another key difference between the two types of firms. A productive private firm is unable to grow fast if its collateral is limited. But a productive public firm (even a small one) can borrow cheaply in the bond market because lenders take into account the high stream of expected profits. As a consequence, the severity of financial frictions depends on the nature of financial constraints.

We estimate our model using statistics reflecting the overall U.S. distribution of firms (both young and mature) over leverage and employment in conjunction with a broader set of financial moments that characterize the behavior of larger public firms. These financial moments are constructed using micro-level data on bond issuances and credit spreads from Thomson Reuters Bond Security Master Data and financial data from Compustat.

Our model closely replicates two important features of the credit market: first, the sizable

[1] Gertler and Gilchrist (1994) construct real and financial data on manufacturing firms and document that for small firms the vast majority of financing is obtained from banks, while larger firms rely more heavily on paper markets.

dispersion in the cross-sectional distribution of credit spreads; second, the negative relationship between credit spreads and firm productivity. The necessary model element to replicate these empirical patterns is long-term financing. With long-term bonds, credit spreads are determined based on the whole sequence of default probabilities until the bond matures. The large heterogeneity in outcomes over long horizons generates a large dispersion in default probabilities across firms and, hence, credit spreads. In contrast, with one-period bonds, credit spreads are priced only based on the next period's probability of default. This results in a distribution that is too concentrated around the risk-free rate.

In our main quantitative experiment, we eliminate financial frictions and analyze the change in macroeconomic aggregates overall, as well as separately, for private and public firms. When we shut down financial frictions, capital increases overall by 33%. However, private firms increase their capital by 55% while public firms only by 7%. More importantly, when we eliminate financial frictions only for public firms, TFP increases (due to a more efficient allocation of resources) by 1.9%. In contrast, when we eliminate financial frictions only for private firms, TFP increases by 3.5%. Therefore, financial frictions are more severe for private firms that face collateral constraints relative to public firms that can issue long-term bonds.

Our paper's primary contribution is to analyze the impact of financial frictions on misallocation in a model that captures more realistically the financing constraints of both private and public firms.[2] Moll (2014) analyzes the role of persistence in productivity shocks and self-financing in a model where all firms are constrained by means of their collateralized assets. Midrigan and Xu (2014) analyze an economy where financial frictions disrupt the transition from a relative unproductive, traditional sector into a modern, productive sector as well as generate dispersion of capital within the productive sector. Both sectors face the same type of financial constraint in the form of a collateral constraint. In our model, firms start their life cycle with a collateral constraint but have the choice to use the public debt market as a source of financing. We show that the effect of financial frictions is modest among public firms since they have cheaper access to capital markets. Gilchrist, Sim, and Zakrajšek (2013) calculate misallocation based on the credit-spread distribution of U.S. public firms using a static model. We also calculate misallocation using information on credit spreads, but we estimate a dynamic model with long-term bonds.

Khan and Thomas (2013) model financial frictions and capital adjustment costs jointly. In their paper, capital adjustment costs propagate financial shocks by preventing young firms from quickly reaching their optimal scale. Moreover, Khan, Thomas, and Senga (2016) study

[2]The basic idea behind the misallocation literature is that distortions at the firm level show up as a reduction in TFP at the aggregate level. This idea was first analyzed by the seminal contributions of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

the effect of financial shocks in an economy with default risk. Jo and Senga (2019) analyze the aggregate implications of targeted credit subsidies in a model with heterogeneous firms, collateral constraints, and endogenous entry and exit. A major difference with the aforementioned studies is that our paper allows for long-term bonds and shows that it is a necessary feature to match the cross-sectional distribution of credit spreads.

Another strand of the literature studies the heterogeneity in firms' financial positions and its effect on aggregate outcomes. Crouzet and Mehrotra (2018) use detailed micro data and show that it is only the very largest firms that are less sensitive to cyclical fluctuations. Jeenas (2018) studies the effect of monetary policy on investment as a function of the firms' leverage and liquidity. Similarly, Ottonello and Winberry (2019) explore the effect of monetary policy on investment as a function of firms' financial positions. They show that firms with low default risk are the most responsive to monetary shocks. Bustamante (2019) analyzes empirically and quantitatively the effect of long-term debt on investment decisions as well as the role of monetary policy. Our addition to this literature is to demonstrate that heterogeneity in financial constraints (manifesting in terms of collateral constraints or long-term default risk) is an important aspect to consider when analyzing misallocation of capital and aggregate productivity.

In addition, our work is related to several recent papers that analyze either multiple types of firms or multiple types of debt financing. Zetlin-Jones and Shourideh (2017) measure the use of external finance by private and public firms and, as a result, their response to financial shocks. In their model, both private and public firms share the same type of financial constraint (in the form of a collateral constraint). Moreover, Dyrda and Pugsley (2018) analyze the organizational form of firms and build a model with C corporations and pass-through companies. Finally, Crouzet (2018) models heterogeneous firms with a variety of choices of debt instruments: bank loans versus market debt. The emphasis in that paper is on the aggregate and cross-sectional composition of financing while in our paper it is on the cross-sectional behavior of credit spreads.

Our paper is also related to the growing literature on long-term financing. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) were the first papers to introduce long-term borrowing in a tractable model of sovereign default. Gordon and Guerron-Quintana (2018) introduce capital in a sovereign default model with long-duration bonds and calibrate the model to cross-country moments. Gomes, Jermann, and Schmid (2016) focus on the role of monetary policy and inflation in a model with long-duration bonds and investment. Crouzet (2017), Sánchez, Sapriza, and Yurdagul (2018), and Jungherr and Schott (2020) use models with an endogenous maturity choice. We use a simpler setup with bonds maturing probabilistically in order to structurally estimate our model using firm-level data.

The paper is organized as follows. Section 2 describes the model. Section 3 describes the empirical analysis, and Section 4 describes the structural estimation exercise. Section 5 reports the main results. Section 6 explores how some of the key features of our model contribute to our findings. Finally, Section 7 concludes.

2 Model

The model is populated by a continuum of firms that make investment, borrowing, and dividend issuance decisions. There are two types of firms, indexed by j: private firms ($j = R$) and public firms ($j = B$). Firms are born as private entities and endogenously transition to public status. Private and public firms differ in terms of their financing opportunities. Private firms issue secured debt, are constrained in terms of their collateralizable assets, and also have access to relatively expensive external equity. Public firms issue long-term defaultable bonds and can also attract relatively cheap external equity.[3] Therefore, our model integrates two strands of the firm-financing literature. First, models with a collateral constraint (Khan and Thomas, 2013; Midrigan and Xu, 2014; Moll, 2014), and second, models with issuance of bonds and default risk (Hennessy and Whited, 2007; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012; Khan, Thomas, and Senga, 2016; Ottonello and Winberry, 2019).

2.1 Life Cycle

Time in the model is discrete. In order to enter the production process, firms pay a one-time fixed cost c_e. Firms start the life cycle as private entities and can make the transition to public status by paying a one-time stochastic and idiosyncratic fixed cost $\chi \sim \text{Logistic}(\mu_\chi, \sigma_\chi)$. The cost is independently distributed across time and firms. Heterogeneity in the cost to access the public bond and equity markets reflects exogenous shocks that can affect the decision to split financing between private loan markets and public debt markets. Such shocks may include, for example, a continuation or a break in a relationship between a firm and a lending facility. Once firms become public, they cannot go back to being private.

Furthermore, in each period, incumbent firms may exit the economy either by an exogenous probability π or based on an endogenous decision to stop production. The incentive for a firm to stop production is to avoid paying a fixed cost of operation c_o (discussed in detail below) or to avoid paying back its debt obligations (in the case of public firms). We allow entry and exit to prevent both private and public firms from accumulating sufficient internal funds

[3] We make a sharp distinction between private firms that issue secured debt and public firms that issue defaultable debt to simplify our analysis. For a richer description of the financial constraints faced by different type of firms, see Nikolov, Schmid, and Steri (2020).

which would diminish the effect of financial frictions.

2.2 Technology and Productivity

Firms are perfectly competitive and produce a single homogeneous good. The firm's production function is $f(z,k,n) = z\left[k^{\alpha}n^{1-\alpha}\right]^{\nu}$, where z is total factor productivity, k is the capital input, and n is the labor input. Idiosyncratic productivity z follows an AR(1) process:

$$\ln z' = \rho_z \ln z + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2_{\varepsilon z})$$

where ε is an i.i.d. shock.[4] There is no aggregate uncertainty in the model.

The production function exhibits decreasing returns to scale (i.e., we assume $\nu \in (0,1)$).[5] Firms hire labor in a perfectly competitive labor market, so that profits are given by $\pi(z,k) = \max_n \{f(z,k,n) - wn\}$, where w is the wage. As mentioned, in order to produce, firms pay a fixed cost of operation c_o every period. Due to the fixed cost, low-productivity firms will choose to stop operations and exit. If the firm decides to stop production, it still produces for one final period with its existing capital stock k and, hence, pays c_o for one last time.

2.3 Bond Issuance

Private firms issue a one-period non-defaultable bond and are subject to a collateral constraint. This type of short-term, secured financing closely describes the financing patterns of smaller firms (Gertler and Gilchrist, 1994). On the other hand, public firms issue long-term defaultable bonds. For a firm of type j, next period's stock of bonds b' is given by

$$b' = (1-\theta_j)b + i_b$$

where b is this period's stock of bonds and i_b is the number of bonds issued this period. Following Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), we assume that a constant fraction θ_j of all outstanding bonds mature each period. In the case of private firms, all bonds mature next period ($\theta_R = 1$) while for public firms only a fraction of bonds mature next period ($\theta_B < 1$). Private firms also face a collateral constraint of the form,

$$b' \leq \psi k',$$

[4] The realization of next period's productivity z' is not known when the investment decision takes place. Therefore, our model departs from Buera and Shin (2011), Moll (2014), and Midrigan and Xu (2014) where firms rent current capital after observing the realization of current productivity.

[5] According to the "span of control" models of Lucas (1978) and Rosen (1982), diminishing returns to scale can be interpreted as a consequence of the diminishing returns to entrepreneurs in managing larger operations.

where ψ is the fraction of capital accepted as collateral.

Each bond pays a coupon rate c. For the $\theta_j b$ bonds that mature, the firm pays back the principal plus interest, so that the total payment today is $(1+c)\theta_j b$. For the $(1-\theta_j)b$ bonds that do not mature, the firm only pays the coupon, so that the total payment today is $c(1-\theta_j)b$. In total, the firm will make a payment of $(\theta_j + c)b$ on both maturing and non-maturing bonds.

If the firm issues $b' - (1-\theta_j)b$ bonds this period, it receives $q_j(z, k', b')\,[b' - (1-\theta_j)b]$, where $q_j(z, k', b')$ is the bond price (explained in detail below). This price depends on current productivity z, next period's capital k', and the total number of bonds b'.[6] For convenience, we assume that bonds issued in different periods are of equal seniority.

2.4 Equity Issuance

We allow both private and public firms to raise external equity. It is common for private entities to raise capital from outside sources such as family or from offering stock ownership to individual investors as in the case of some startups. To control the substitution between debt and equity, we assume that firms incur a cost whenever they raise external equity but note that the cost is proportionally higher for private firms relative to public firms. Moreover, we also assume that both private and public firms incur a similar cost when they decide to distribute dividends.[7]

In sum, for a firm of type j, the cost of choosing dividends d (or external equity if $d < 0$) is

$$\Lambda_j(d) = (1 + \gamma \mathcal{I}_{\{j=R, d<0\}})\phi_d d^2,$$

where γ is the additional cost of private firms to issue equity. Therefore, for public firms, the cost function is symmetric for payouts and equity issuance while for private firms equity issuance is more costly than payouts. A lower cost of issuing external equity is one of the benefits of being a public company.

[6] Since the default decision depends on the total number of outstanding bonds tomorrow, b', $q_j(z, k', b')$ depends on b' rather than the amount of debt issued today (i.e., $b' - (1-\theta_j)b$).

[7] Dividend payout costs capture both pecuniary and non-pecuniary costs. For example, they can be interpreted to include dividend taxes or costs associated with share repurchases. But they can also capture the preferences of managers to smooth dividends. Lintner (1956) documents the tendency of firms to smooth dividends. With a similar motivation, Hennessy and Whited (2007) assume a progressive dividend tax. Jermann and Quadrini (2012) consider a cost function that is symmetric to both equity issuance and dividend payouts. Dividend payout costs (alongside the tax preference for debt) ensure that firms find it difficult to outgrow default risk.

2.5 Capital Adjustment Costs

We assume that capital investment is subject to adjustment costs $g(k, k')$, which take the following form:

$$g(k, k') = \phi_k \left(\frac{k' - (1-\delta)k}{k} \right)^2 k.$$

The parameter ϕ_k controls the cost of adjusting investment each period. Adjustment costs are symmetric to positive or negative investment adjustments.

2.6 Taxes

We incorporate the following tax structure in the model. Investors must pay a constant tax rate, τ_i, on interest income. Meanwhile, firms pay a tax on their corporate income. The firm's taxable income, x, is assumed to be profits minus economic depreciation and interest expense:

$$x = \pi(z, k) - \delta k - cb$$

where δ is the depreciation rate of capital and cb is the interest expense.[8] As in Hennessy and Whited (2007), the corporate tax rate is assumed to be τ_c^+ when taxable income is positive, and τ_c^- when taxable income is negative. Therefore, the firm's total corporate tax bill is given by

$$T_c(x) = \begin{cases} \tau_c^+ x & \text{if } x \geq 0 \\ \tau_c^- x & \text{if } x < 0. \end{cases}$$

In order to capture loss limitations in the U.S. corporate tax code, we will assume that $\tau_c^- < \tau_c^+$.

2.7 Value Functions

Value of Private Firms The value of a private firm ($j = R$) at the start of the period is

$$\tilde{V}_R(z, k, b) = \max \left(\underbrace{V_R^c(z, k, b)}_{\text{continue}}, \underbrace{V_R^x(z, k, b)}_{\text{exit}} \right).$$

$\tilde{V}_R$ represents the private firm's value at the beginning of the period before its decision to continue or exit. If the firm chooses to exit, it receives V_R^x (see Equation (2) below). If it

[8] We deduct the coupon expenses rather than the expenses based on the actual bond price to avoid carrying the previous period's productivity as an additional state variable. See also Begenau and Salomao (2019) for a similar assumption in a one-period bond model.

continues operations, it receives V_R^c:

$$V_R^c(z,k,b) = (1-\pi)E_\chi \max\left\{V_R(z,k,b), V_B^e(z,k,b) - \chi\right\} + \pi V_R^x(z,k,b).$$

After a private firm chooses to continue, it receives the exogenous exit shock with probability π. If it does not exit, the firm receives its draw of the entry cost χ, and it then chooses between remaining a private firm or switching to public status. If it remains private, it receives $V_R(z,k,b)$ (see Equation (3) below). If it goes public, it receives $V_B^e(z,k,b)$ (described in Equation (4) below). Since private firms issue non-defaultable debt, they always repay their debt regardless of whether they continue or exit.

Value of Public Firms The value of public firms ($j = B$) at the start of the period is

$$\tilde{V}_B(z,k,b) = \max\left(\underbrace{V_B^c(z,k,b)}_{\text{continue}}, \underbrace{0}_{\text{default}}\right). \tag{1}$$

$\tilde{V}_B$ represents the the public firm's value before its decision to continue or default. In default, we assume the firm exits and receives nothing. The value of continuing is equal to

$$V_B^c(z,k,b) = (1-\pi)V_B(z,k,b) + \pi V_B^x(z,k,b).$$

After a public firm chooses to continue, it still exits exogenously with probability π. If it does not exit, it receives $V_B(z,k,b)$ (defined in Equation (3) below). Using Equation (1), we can define a productivity default threshold, $z_d'(k',b')$, such that $V_B^c(z_d', k', b') = 0$. For $z' < z_d'(k',b')$, the public firm defaults on its debt obligations next period.[9]

Value of Exit We next focus on a firm of type j that is exiting the economy, either by choice or due to the exogenous shock. We assume that the exiting firm still produces output given its stock of capital k but chooses $k' = 0$ and $b' = 0$. As a result, the firm still pays the fixed operating cost c_o for the last year of operation. The firm also buys back all un-matured debt, pays an adjustment cost to liquidate all its capital, and distributes a final dividend to shareholders.[10] The value of exit for a firm of type j is

$$V_j^x(z,k,b) = d - \Lambda_j(d) \quad \text{where} \quad d = e_j(z,k,b) - (1-\theta_j)b - g(k,0). \tag{2}$$

[9]Because default is always an option, $\tilde{V}_B(z,k,b) \geq 0$, which implies that $V_B \geq V_B^x$ and $V_B^c \geq V_B^x$. Therefore, the firm always prefers to continue and not default rather than exit and not default. As a consequence, we do not need to separately consider the option of exiting and not defaulting for public firms in Equation (1).

[10]An exiting firm buys back all un-matured debt at the risk-free price, which is normalized to 1.

where $e_j(z,k,b)$ is defined to be the firm's internal funds:

$$e_j(z,k,b) = \pi(z,k) - T_c\left(\pi(z,k) - \delta k - cb\right) - c_o + (1-\delta)k - (\theta_j + c)b.$$

Value of Operating We now consider a firm of type j that has decided to continue operations, not default, and has not received the exogenous exit shock. The firm chooses dividends d, capital k', and debt b' according to the following program:[11]

$$V_j(z,k,b) = \max_{d,k',b'} \left\{ d - \Lambda_j(d) + \beta E_{z'|z}\left[\tilde{V}_j(z',k',b')\right]\right\} \tag{3}$$

subject to

$$d + k' = e_j(z,k,b) + q_j(z,k',b')\left[b' - (1-\theta_j)b\right] - g(k,k')$$

Value of Entry into the Public Sector The value a private firm would receive upon entry into the public sector is given by the following dynamic program:

$$V^e_B(z,k,b) = \max_{d,k',b'} \left\{ d - \Lambda_B(d) + \beta E_{z'|z}\left[\tilde{V}_B(z',k',b')\right]\right\} \tag{4}$$

subject to

$$d + k' = e_R(z,k,b) + q_B(z,k',b')b' - g(k,k')$$

In the first period of being public, the firm pays back all un-matured one-period bonds and, going forward, issues new long-term debt b'.

Value of Entry into Economy Finally, the value of a potential entrant as a private firm with productivity z is given by

$$V^e_R(z) = V_R(z,0,0)$$

Following Hopenhayn (1992), we assume that the entrant draws its initial z from the cdf $G(z)$ after paying the entry cost, where $G(\cdot)$ is the invariant distribution of the productivity process. The free entry condition is then

$$\int V^e_R(z)dG(z) = c_e.$$

The free entry condition pins down the equilibrium wage. Therefore, in the benchmark model, we calibrate the entry cost so that $w = 1$ in equilibrium. We then use the labor market clearing condition to pin down the total number of firms that are operating in the economy.[12]

[11] Denote by $g^d_j(z,k,b)$, $g^k_j(z,k,b)$ and $g^b_j(z,k,b)$ the associated policy functions for d, k' and b', respectively.

[12] For simplicity, we assume there is an exogenous labor supply equal to one. This assumption only affects

2.8 Determination of Bond Price

Both private and public firm bonds are purchased by risk-neutral lenders. Private firms issue non-defaultable one-period bonds and pay a constant premium r_p over the risk-free rate r. The extra premium faced by private firms is an additional motive to make the transition to public status.[13] Since private firms face a collateral constraint, the bond price is

$$q_R(z,k',b') = \begin{cases} \frac{1+c}{1+r+r_p} & \text{if } b' \leq \psi k' \\ 0 & \text{if } b' > \psi k'. \end{cases}$$

Public firms issue defaultable long-term bonds. The price of the bond, $q_B(z,k',b')$, is set to guarantee lenders an expected pre-tax return equal to the risk-free rate:

$$q_B(z,k',b') = \frac{1}{1+r} E_{z'|z}\left[R^d(z',k',b')\mathcal{I}_{z'<z'_d(k',b')} + R^{nd}(z',k',b')\mathcal{I}_{z'\geq z'_d(k',b')}\right] \tag{5}$$

where

$$R^d(z',k',b') \equiv \frac{1}{b'}\left[\pi(z',k') - T_c\left[\pi(z',k') - \delta k'\right] + (1-\xi)(1-\delta)k' - g(k',0)\right]$$
$$R^{nd}(z',k',b') \equiv \theta_B + c + (1-\theta_B)[(1-\pi)q_B(z', g_B^k(z',k',b'), g_B^b(z',k',b')) + \pi]$$

There are two components that determine the bond price. The first component, $R^d(z',k',b')$, reflects payments lenders will receive in default states (i.e., when $z' < z'_d(k',b')$). In this case, lenders receive the firm's after-tax profits, liquidate the firm's capital, and pay a bankruptcy cost proportional to the firm's un-depreciated capital.[14] The second component, $R^{nd}(z',k',b')$, reflects payments lenders receive in non-default states (i.e., when $z' \geq z'_d(k',b')$). In this case, lenders receive a payment $(\theta_B + c)$ on all maturing and non-maturing bonds today. For the outstanding bonds that do not mature, the lender can expect to receive more payments in the future. If the firm does not exit exogenously, the value of these claims is $q_B(z',k'',b'')$, which is the price of the un-matured bonds next period.[15] If the firm is forced to exit exogenously, the firm pays off all un-matured debt at the price 1.

In one-period bond models ($\theta_B = 1$), the bond price depends only on the probability the firm will default next period. The probability of defaulting in two periods or more has no effect on today's price. In contrast, in models with long-duration bonds, today's price is affected

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the number of firms operating in the economy.

[13]This modeling assumption follows Crouzet (2017). We analyze the implications of smaller premium in Section 6 and find slightly lower aggregate productivity losses from financial frictions.

[14]We assume that there is no tax deductibility of interest in default.

[15]Notice that the price tomorrow depends not only on the realized productivity tomorrow, z', but also on $k'' = g_B^k(z',k',b')$ and $b'' = g_B^b(z',k',b')$, which are the firm's choices for capital and debt tomorrow.

by the possibility of default in each future state of the world until the bond matures. For example, if the firm is expected to issue a high level of debt next period, then the probability of default after two periods increases and next period's bond price $q_B(z', k'', b'')$ decreases. This will be reflected in the price of bonds today, $q_B(z, k', b')$. We show that this property of long-term bonds help us generate a more reasonable distribution of credit spreads.

3 Empirical Analysis

We analyze credit spreads along three dimensions: (i) the cross-section of firms, (ii) along the life cycle, and (iii) between firms of low and high total factor productivity. These relationships inform our parameter estimation in Section 4. We document the following facts:

1. The dispersion in the cross-sectional distribution of credit spreads is substantial, reflecting a wide variety of bond qualities across firms.

2. Younger firms face higher credit spreads relative to older firms.

3. Low-productivity firms face higher credit spreads relative to high-productivity firms.

3.1 Description of Datasets

We estimate our life-cycle model using two types of statistics. First, we use statistics reflecting the overall U.S. distribution of firms over leverage and employment. Second, we use financial data that reflect the behavior of mature public firms, most notably credit spreads.

For the first set of moments we rely on a combination of statistics such as (i) employment by age groups from the Business Dynamics Statistics (BDS), (ii) the literature that has documented financing patterns between small and large firms (e.g., Crouzet and Mehrotra, 2018), and (iii) aggregate statistics from the National Accounts.

The second set of moments that serves to discipline the behavior of public firms is based on two firm-level datasets: Standard and Poor's Compustat industrial files and Thomson Reuters Bond Security Master Data (henceforth, TR).[16] Compustat includes detailed income statement, balance-sheet, and cash flow data of publicly listed companies. We use annual fundamental data from 1984-2015. We impose selection criteria common in the literature.[17]

[16] Our model characterization of public firms as firms that can jointly issue long-term debt and issue relatively cheap external equity is a simplification relative to the data. In reality, many firms in the bond market are not publicly listed in the stock market. Similarly, many firms that are in the stock market have not issued a bond during our available sample period. However, modeling such rich financing patterns would complicate the model to a large degree. As a result, we model the behavior of public firms based on the joint information from Compustat and Thomson Reuters datasets.

[17] These criteria are common in studies employing Compustat data. See Bernanke, Campbell, and Whited (1990) for details.

Table 1: Summary Statistics

Variable	#Obs.	Mean	SD	Min	p(10)	p(25)	p(50)	p(75)	p(90)	Max
Compustat	(Firms)									
Leverage	133,786	0.29	0.42	0.00	0.00	0.03	0.20	0.38	0.60	3.23
Assets/Sales	129,730	2.55	7.29	0.20	0.43	0.63	0.96	1.68	3.53	61.6
Investment rate	119,991	0.16	0.22	-0.05	0.01	0.04	0.08	0.17	0.36	1.82
Thomson Reuters	(Bond deals)									
Mkt. val. of issue ($mil.)	18,369	365.5	364.6	3.5	28.3	126.6	268.6	475.1	795.8	2000.0
Maturity (years)	18,369	11.6	8.59	1	5	6	10	11	30	40
Age	18,042	61.1	44.9	1	9	22	53	95	121	375
Credit spread (p.p.)	18,369	2.32	2.16	0.05	0.60	0.90	1.55	3.00	5.37	9.59

Note: We report statistics from the cross-sectional distribution over the entire period sample. All variables are winsorised at the 1%. For both data, the different number of observations across variables is due to missing values.

First, we exclude financial firms (SIC 6000–6999) and utilities (SIC 4900–4999). Second, we drop any firm-year observations without information on assets, capital stock, debt, or equity. Third, we drop observations that violate the accounting identity of assets equal to equity plus debt (normalized by assets) by more than 10%. Fourth, we drop firms affected by the 1988 accounting change (GM, GE, Ford, Chrysler) and include only firms reporting in USD. And finally, we keep companies that appear more than 10 consecutive years in the sample. This leaves us with a total of 7,561 firms and 135,677 observations.

Our second dataset, Thomson Reuters Bond Security Master Data, provides information on primary issuances of corporate bonds between 1980-2015. There are a total of 18,480 bond deals in our data. Available information includes the name of the company issuing the bond, the market value of the issue, the issue date, the type and purpose of the bond issuance, the maturity of the bond, and the credit spread paid by the issuer (defined as the interest rate paid over a Treasury bill of similar maturity). We drop bond issuances (1) below 1 million dollars, (2) with a maturity larger than 40 years, and (3) with a credit spread less than 5 basis points. These restrictions leave us with 18,369 deals.

In addition, using an online search, we collect age information for each firm in the TR data. In particular, the age of the firm at year of issuance t is defined as t minus the founding year of the firm. We have age information for a total of 4,172 firms that issue a total of 18,042 bonds.

3.2 Summary Statistics

Table 1 reports summary statistics for Compustat and TR data, respectively. In Compustat, the mean leverage ratio is 0.29 while the mean assets/sales ratio is 2.55. We define the investment rate as investment-to-capital ratio. The mean investment rate in Compustat is 0.16, and the standard deviation is 0.22.

In our data, the median number of bond issuances by a single firm in all years is 4 while the maximum is 88. In some cases, firms issue multiple bonds of different amounts and maturities during the same time period. Moreover, firms raise funds through the bond market at a relatively high frequency. A firm issuing a bond in year t re-issues a bond with probability 45% between years $[t, t+2]$ and with probability 57% between years $[t, t+4]$. The average amount raised in a deal is around \$365 million. The distribution is highly skewed: the maximum amount issued is \$2 billion. Most of the issues in our data involve long-term financing. The average bond matures in 11.6 years. Finally, the median firm age in our sample is 53 years old.[18]

3.3 Credit Spreads, Age, and Productivity

As we demonstrate in Table 1, on average, firms pay a spread of 2.3% over a T-Bill of similar maturity. The standard deviation is 2.2%. The sizable dispersion reflects a wide variety of bond qualities in our data. Our findings are broadly close to Gilchrist and Zakrajšek (2012). The authors use secondary market prices of outstanding securities between 1973-2010 to construct bond yields. They find a credit spread mean of 2.0% with a standard deviation of 2.8%.

We estimate the relationship between the age of the firm and the credit spread associated with a bond issuance by running the following regression

$$\mathrm{S}_{ijt} = \beta_0 + \beta_1 \mathrm{Age}_{ijt} + \beta_2 X_j + \alpha_t + \varepsilon_{ijt}. \tag{6}$$

The dependent variable S_{ijt} is the log credit spread paid by firm i associated with bond j issued in year t. We regress the spread on the age of the firm at the time of bond issuance Age_{ijt} and also include time dummies α_t as well as bond characteristics X_j (the value of the issue and the maturity of the bond). According to our estimates in Table 2, an additional year of operation decreases the credit spreads by 0.49% in the specification without bond characteristics and by 0.41% in the specification with bond characteristics.

We next analyze the relationship between firm-level total factor productivity and credit

[18] The merged Compustat-TR sample (a total of 4,372 bond deals) includes on average larger firms relative to the full Compustat sample. Nonetheless, several financial statistics such as the credit spread distribution or the relationship between credit spreads and age are similar across the TR and the merged Compustat-TR sample (please see the Online Appendix).

Table 2: Credit Spreads and Age

Dependent Variable	S_{ijt} (Credit Spreads)	
Specification	(1)	(2)
Age / 100	-0.49***	-0.41***
	(0.01)	(0.01)
# Observations	18,042	18,042
R-squared	0.06	0.40
Year F.E.	No	Yes
Bond Charact.	No	Yes

Note: This table reports estimates from the regression in Equation (6). S_{ijt} is the log credit spread paid by firm i associated with bond j issued in year t. One, two, and three stars denote significance at the 10%, 5%, and 1% level, respectively.

spreads. We construct firm TFP using the following specification:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + z_{it} + \varepsilon_{it},$$

where all variables are in logs. y_{it} is value added by firm i in year t, and k_{it} and l_{it} are the capital and labor inputs, respectively, by firm i in year t. z_{it} is the TFP of firm i in year t, which is observed by the firm but unobserved by the econometrician. Finally, ε_{it} represents either shocks unobserved to both the econometrician and the firm (and hence, not influencing labor inputs) or measurement error.

We translate nominal variables such as value added or total value of capital into physical units using the GDP price deflator and the price index for private fixed investment (both available from the Bureau of Economic Analysis). To transform the capital stock into physical units, we take into account that capital stock is shaped based on investment that occurred in different time periods. Following the method by Brynjolfsson and Hitt (2003), we calculate the average capital age as accumulated depreciation divided by current depreciation.[19] As a result, we deflate capital by the investment deflator of the respective year.

There are several challenges to estimate firm-level total factor productivity. First, there is a simultaneity problem between the firm's choices and the firm's productivity. For example, if a firm hires more workers because it is more productive, then coefficient β_l might be upward biased and productivity z_{it} might be downward biased. Moreover, there is a selection problem. The most productive firms are more likely to stay in the sample for longer. To deal with these

[19] As suggested by İmrohoroğlu and Tüzel (2014), we smooth further capital age by taking a three-year moving average.

shortcomings, we employ the approach developed by Olley and Pakes (1996).[20]

We next analyze the relationship between firm-level productivity and credit spreads. In particular, we run the following regression:

$$\mathrm{S}_{ijt} = \beta_0 + \beta_1 z_{it} + \beta_2 [b'/k']_{it} + \beta_3 [\mathrm{SD.ER}]_{it} + \beta_4 X_j + \alpha_t + \eta_i + \varepsilon_{ijt}. \tag{7}$$

The dependent variable S_{ijt} is the log credit spread paid by firm i associated with bond j issued in year t. We regress the spread on firm-level TFP z_{it} and log leverage $[b'/k']_{it}$. We also include the standard deviation of yearly equity returns for firm i denoted "SD.ER." Equity volatility is estimated using data from the CRSP daily stock files during the period of our analysis. We capture time-varying common factors using a time dummy α_t. Moreover, we include a firm fixed effect η_i. Finally, we include bond characteristics X_j such as the value of the issue (in log) and the maturity of the bond.

Table 3 shows the results of our regression. All specifications use time fixed effects and bond characteristics. We first regress spreads on firm-level TFP without and with fixed effects (specifications (1) and (2)). The fixed effect differences out time-invariant cross-sectional variation in firm characteristics and turns out to be very important. Without fixed effects, a 1% increase in TFP decreases credit spreads by 0.06%. When we include firm fixed effects, a 1% increase in TFP decreases credit spreads by 0.37%.

Specifications (3) and (4) run separately credit spreads on leverage and the volatility of firm equity returns. This is motivated by the extensive literature trying to identify the determinants of credit spreads (Duffee, 1998; Collin-Dufresne, Goldstein, and Martin, 2001; Driessen, 2005; Gilchrist, Sim, and Zakrajšek, 2014). Leverage and equity volatility are identified as important determinants of the probability of default on debt obligations. A 1% increase in leverage increases credit spreads by 0.27%. This is unaffected by the inclusion of fixed effects. Moreover, if the standard deviation of equity volatility increases 1% then credit spreads increase by 1.04% without fixed effects and 0.41% with fixed effects. Specifications (5) and (6) include all regressors – namely, TFP, leverage, and equity volatility. Compared to specifications (1)-(2), the coefficient of TFP becomes less negative, which can be explained by the inclusion of the other regressors. Once more, the inclusion of fixed effects turns out to be very important.

[20] Our measure of productivity is revenue-based (revenue total factor productivity or TFPR), so it is jointly affected by true, physical productivity (TFPQ) as well as changes in prices (Foster, Haltiwanger, and Syverson, 2008). Therefore, we recognize that our productivity measure partially captures price variation. Moreover, our approach is also susceptible to an input-price bias as we deflate capital using aggregate deflators. For this case, we explicitly check the magnitude of the bias by using not aggregate but industry-specific deflators available by the NBER-CES database. The deflators are available at the six-digit NAICS level but only for a subset of industries that reduces substantially the number of our observations. When we compare our estimates for productivity and its relation to credit spreads, in the case of mean investment deflator versus industry-specific deflators, we do not find the estimates substantially changed.

Table 3: Determinants of Credit Spreads

Dependent Variable	S_{ijt} (Credit Spreads)					
Specification	(1)	(2)	(3)	(4)	(5)	(6)
z_{it} (Productivity)	-0.06***	-0.37***			-0.01	-0.33***
	(0.02)	(0.05)			(0.01)	(0.05)
$[b'/k']_{it}$ (Leverage)			0.27***	0.27***	0.28***	0.26***
			(0.03)	(0.05)	(0.03)	(0.04)
St. Deviation of			1.04***	0.41***	1.04***	0.39***
Equity Returns			(0.05)	(0.05)	(0.05)	(0.05)
# Observations	4,151	4,151	3,780	3,780	3,644	3,644
R-squared	0.38	0.47	0.57	0.48	0.57	0.54
# Firms	724	724	625	625	608	608
Firm F.E.	No	Yes	No	Yes	No	Yes

Note: This table reports estimates from the regression in Equation (7). S_{ijt} is the log credit spread paid by firm i associated with bond j issued in year t, z_{it} is TFP (in logs) of firm i in period t, $[b'/k']_{it}$ is log-leverage, and we also include the logged standard deviation of yearly equity returns for firm i as calculated from CRSP. One, two, and three stars denote significance at the 10%, 5%, and 1% level, respectively.

With fixed effects, we find that a 1% increase in TFP decreases credit spreads by 0.33%.

Our regression explains around 50% of the variation in credit spreads. The challenge to capture the determinants of credit spreads is well known in the literature as the credit spread puzzle. Driessen (2005) analyzes the determinants of credit spreads, taking into account most of the variables proposed by the literature. He finds that only two-thirds of the variation is explained by his specification. Our explanatory power is somewhat lower but within the ranges found in the literature.

4 Model Estimation

To infer the impact of financial frictions it is necessary to jointly capture the behavior of microeconomic quantities as well as credit prices. In this section, we discuss our structural estimation technique, the estimated parameters, and assess the model's fit to the data. One set of our parameters is set outside of the model and the remaining parameters are estimated using Simulated Method of Moments (SMM).

Table 4 reports the calibrated parameters. The model is computed at an annual frequency. We normalize the wage rate to 1 and set the annual risk-free rate r equal to 4%. The returns to scale parameter, ν, is set to 0.85, which is in the middle of the estimates of Burnside, Eichenbaum, and Rebelo (1995). We set the capital share parameter equal to $\alpha = 0.35$ and the tax parameters to be $\tau_i = 0.296$, $\tau_c^- = 0.2$, and $\tau_c^+ = 0.35$, which are typical values in the literature (e.g., Katagiri (2014)). As a consequence of our parameterization, the firm's discount factor is equal to $\beta = 1/[1 + r(1 - \tau_i)] = 0.972$. We set the coupon rate, c, to be equal to the risk-free rate. This is just a normalization that ensures the risk-free bond price is equal to 1.

Table 4: Calibrated Parameters

Parameter	Notation	Value	Target / Source
Risk-free rate	r	0.04	Typical in literature
Coupon	c	0.04	Normalize risk-free bond price to 1
Capital share	α	0.35	Capital-income share
Returns to scale	ν	0.85	Burnside, Eichenbaum, and Rebelo (1995)
Min. corporate tax rate	τ_c^-	0.20	Katagiri (2014)
Max. corporate tax rate	τ_c^+	0.35	Katagiri (2014)
Interest income tax rate	τ_i	0.296	Katagiri (2014)
Bond premium, private	r_p	0.015	Schwert (2020)
Bond maturity, public	θ_B	0.086	TR Bond Security Data
Depreciation rate	δ	0.08	Median investment rate
Bankruptcy cost	ξ	0.10	Hennessy and Whited (2007)
Exogenous exit rate	π	0.02	Ottonello and Winberry (2019)
Entry cost	c_e	0.18	Normalization so that equilibrium $w = 1$

We set the bond premium for private firms to $r_p = 0.015$. This is in the middle of the estimates of the premium for bank loans versus corporate bonds, as estimated by Schwert (2020).[21] The expected maturity of a bond for public firms in our model is $\sum_{t=1}^{\infty} t\theta_B(1 - \theta_B)^{t-1} = 1/\theta_B$. Since the average maturity in our data is 11.6 years, we set $\theta_B = 1/11.6 \approx 0.086$. The depreciation rate is set to $\delta = 0.08$ to match the median investment rate in our data. We set the bankruptcy cost equal to $\xi = 0.10$ based on Hennessy and Whited (2007) which suggests that the liquidation cost in case of default is 10% of the firm's capital. Finally, we set the exogenous exit rate, π, equal to 2% based on Ottonello and Winberry (2019).

4.1 Structural Estimation

We use SMM to estimate the remaining parameters by minimizing the distance between the model statistics and their empirical counterparts. We estimate a total of 9 parameters

[21] Note that data in Schwert (2020) include only firms with access to the corporate bond market. Loans to firms without access to the corporate bond market are likely to have a higher premium. Therefore, our value of $r_p = 0.015$ is likely a conservative lower bound.

represented by the vector:

$$\Theta = [\rho_z, \sigma_{\varepsilon z}, c_o, \mu_\chi, \sigma_\chi, \psi, \phi_d, \gamma, \phi_k].$$

Let $M^m(\Theta)$ denote the vector of model-generated moments and M^d their empirical counterparts. The estimator minimizes the loss function:

$$\hat{\Theta} = \arg\min_{\Theta} \left[M^d - M^m(\Theta)\right]' W \left[M^d - M^m(\Theta)\right]. \tag{8}$$

We use a total of 11 moments as part of the estimation.[22] These are the persistence of log sales, the standard deviation of the innovation to firm log sales, the average and the standard deviation of credit spreads, the elasticity of credit spreads with respect to TFP and age, average leverage for relatively smaller firms (excluding the top 10% in terms of assets), average leverage computed in Compustat, the standard deviation of investment rates in Compustat, the investment share of Compustat firms relative to aggregate investment from national accounts, and finally, the employment of firms in their first year of production (relative to the mean employment) as computed from BDS. In sum, where moments are constructed from the joint information in Compustat and TR, the model counterparts are computed using only a sample containing public firms. When the statistics reflect the overall distribution of firms, the model counterparts are drawn from a sample of both public and private firms.

To make identification more transparent, it is useful to connect each parameter with the moments that are the most informative about their values. First, to pin down the parameters of the productivity process (ρ_z, $\sigma_{\varepsilon z}$), we estimate a first-order autoregressive process in log sales, controlling for firm fixed effects, and target the persistence and the standard deviation of the residual.

A higher fixed cost of operation c_o makes default more likely and borrowing more risky. Therefore, both the average credit spread and the standard deviation of credit spreads are influenced by this parameter. Furthermore, issuing equity at a low cost provides insurance to firms against negative productivity draws. As a result, ϕ_d affects credit spreads in general. However, we find that this parameter is particularly informative for the average credit spreads paid by younger firms that decide to enter the bond market (e.g., the relationship between credit spreads and age). The relative employment size of entrants is informative for the additional equity cost for private firms, γ. The costlier external issuance is for private firms, the smaller their initial size. Using information from the BDS between 2003-2014 we find that the average number of workers for newborn firms is around 6. The economy-wide average is

[22] Our weighting matrix W reflects specific dimensions of the data we view as more important to match. See the Online Appendix for further details.

17 workers per firm. Hence, we target a level of employment for entrants equal to 35% of economy-wide average.

We set the collateral constraint, ψ, so that the average leverage of firms excluding the top 10% (in terms of assets) is equal to 0.35 (Crouzet and Mehrotra, 2018). We set the capital adjustment cost, ϕ_k, so that the standard deviation of investment rates of public firms in the model is equal to the same moment computed for Compustat firms (equal to 0.22).[23]

The cost of entry in the bond market is heterogeneous and is determined by two parameters: (μ_χ, σ_χ). These parameters jointly affect several of our targeted moments. First, they affect the share of public firms in the economy. Therefore, we target an investment share of public firms in the model equal to 30% of aggregate investment (which is the investment share of Compustat firms relative to aggregate investment). Second, these parameters also affect which kinds of firms become public. Absent any dispersion, it is only the larger, mature, and productive firms that decide to become public. By modeling dispersion in the entry cost, we allow even relatively young and unproductive firms to participate in the bond market. As a consequence, these parameters affect several moments, including (i) mean leverage for public firms (since younger firms have higher leverage ratios in the model), (ii) average credit spreads, (iii) the relationship between credit spreads and TFP, and (iv) the life cycle of credit spreads.

4.1.1 Parameter Estimates and Model Fit

Table 5 reports the estimation results. The productivity process is mildly persistent ($\rho_z = 0.71$). This estimate is fairly close to other quantitative papers analyzing firm financing and capital misallocation for the U.S. (Hennessy and Whited, 2007; Khan and Thomas, 2013). The standard deviation of the innovation to productivity is $\sigma_{\varepsilon z} = 0.22$. We find the fixed operating cost to be equal to 0.09 which amounts to 8% of average sales. We estimate that the cost for private firms to attract external equity is around 3.6 times higher than public firms ($1 + \gamma = 3.56$ and $\phi_d = 0.39$). The average cost as a fraction of total equity issued is 10.7% for private firms and 3.0% for public firms. According to our estimation, private firms can borrow up to 55% of their capital which is close to the value used in Khan and Thomas (2013).[24] Finally, the dispersion in the bond entry cost χ implies that a fraction of firms (7.7%) has a negative bond entry cost and immediately switches to public status. We found such a large dispersion necessary to rationalize very young firms in the data issuing public debt (as we discuss below, our model matches the fraction of bond issuers that are less than

[23] An alternative strategy is to use the economy-wide dispersion in investment rates as reported in Cooper and Haltiwanger (2006). However, their investment rates are constructed based on plant-level data so they are less suitable to our model estimation that is mostly based on firm-level data.

[24] Catherine, Chaney, Huang, Sraer, and Thesmar (2017) estimate a lower value around 20% because they consider collateral constraints that jointly take into account productive capital and real estate assets.

Table 5: Estimated Parameters

ρ_z	$\sigma_{\varepsilon z}$	c_o	ϕ_d	γ	ϕ_k	ψ	μ_χ	σ_χ
0.71	0.22	0.09	0.39	2.56	0.08	0.55	0.26	0.10
(0.008)	(0.004)	(0.003)	(0.014)	(0.343)	(0.003)	(0.024)	(0.107)	(0.022)

Note: We report the parameters estimated via Simulated Method of Moments (SMM). Standard errors are reported in parentheses.

10 years old).

Table 6: Model Fit

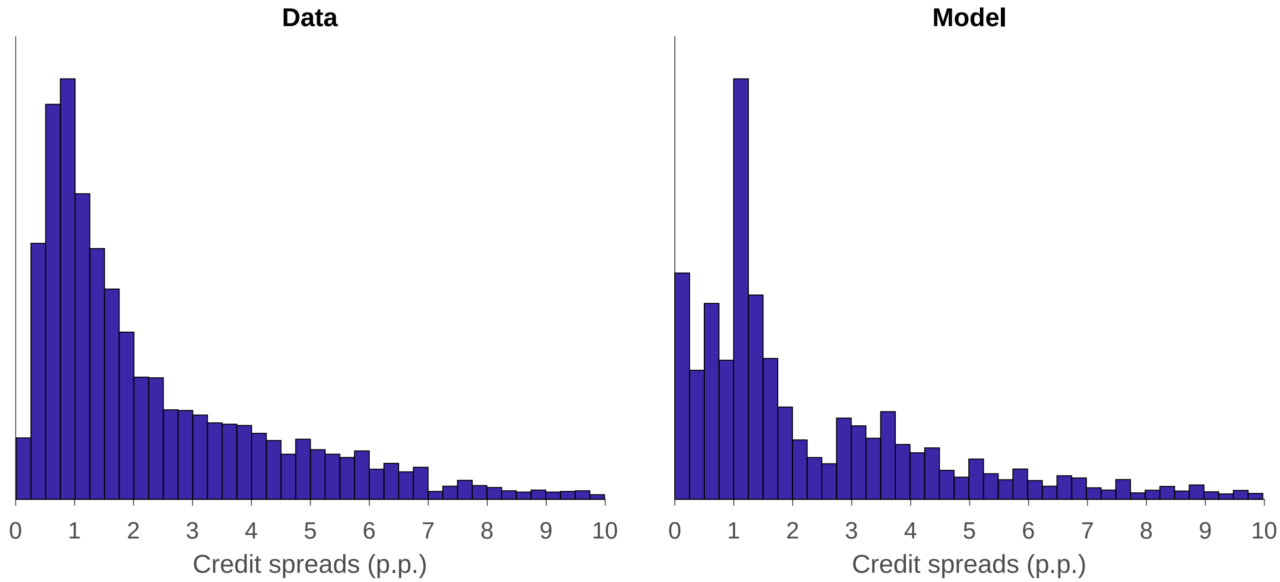
Targeted Moments	Model	Data	Untargeted Moments	Model	Data
Autocorrelation of log sales	0.76	0.75	Mean capital/sales ratio	2.92	2.55
SD of innovation to log sales	0.50	0.50	Frequency of equity issuance	0.34	0.40
Mean credit spread	2.34%	2.32%	Median credit spread	1.38%	1.55%
SD of credit spreads	2.12%	2.16%	Credit spread-Leverage elast.	0.20	0.27
Credit spread-TFP elast.	-0.35	-0.37	SD of leverage	0.20	0.42
Credit spread-Age elast.	-0.49	-0.49	Autocorrelation of inv. rates	0.24	0.17
Mean leverage [0-90] perc.	0.34	0.34	Mean default rate	3.19%	1.56%
Mean leverage, public firms	0.28	0.29	Fraction of Bond Issuers		
SD of invest. rates, public firms	0.24	0.22	$<$ Age 10	10.8%	11.0%
Investment share of public firms	0.30	0.30	$<$ Age 20	29.7%	22.5%
Rel. empl. of firms, age 0	0.34	0.35	Rel. empl. of firms, age 1-5	0.64	0.49
			Rel. empl. of firms, age 6-10	0.93	0.66
			Rel. empl. of firms, age 11-15	1.03	0.81
			Rel. empl. of firms, age 16-20	1.07	1.05

Note: The left panel reports moments that were included as part of the estimation and the right panel reports untargeted moments.

Table 6 reports the model's fit to the empirical moments. Our model replicates fairly closely the empirical behavior of credit spreads (both the level and dispersion, as well as the relationship with firm characteristics).[25] The mean credit spread in the model is 2.34% which is slightly higher than the empirical moment from Thomson Reuters. The standard deviation of credit spreads in the model is 2.12% which is slightly less than the standard deviation in

[25]In the model, we compute the credit spread for each firm as the bond's yield to maturity minus the risk-free rate r, which works out to be $(\theta_j + c)/q_j - \theta_j - r$.

Figure 1: Credit Spread Distribution: Data (Left Panel) and Model (Right Panel)



Note: A histogram of credit spreads in percentage points. Left panel shows the distribution from Thomson Reuters Bond Security Master Data based on authors' calculations. The right panel shows the distribution from the simulated data.

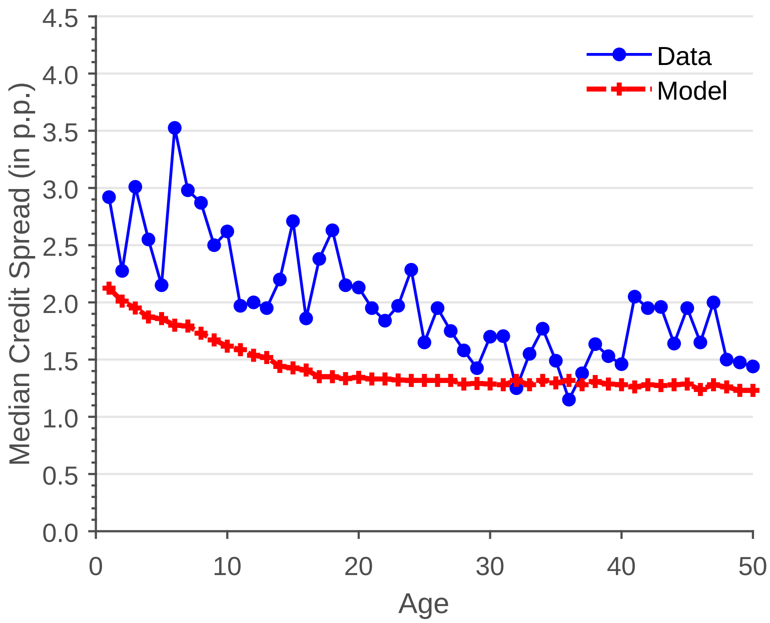
the data. The difference between the 90^{th} and the 10^{th} percentile in the data is 4.8 percentage points and in the model is 5.1 percentage points. Figure 1 shows the distribution of credit spreads in the data (left panel) and the distribution generated in the model (right panel).

There are several key model elements that allow us to generate the large dispersion of credit spreads. The necessary element for a sizable dispersion in credit spreads is long-term financing. With long-term bonds, credit spreads are determined based on the whole sequence of default probabilities until the bond matures (as we discussed in Section 2). The large heterogeneity in outcomes over long horizons generates a large dispersion in default probabilities across firms, and hence, a large dispersion in credit spreads. In contrast, with one-period bonds, credit spreads are priced only based on the next period's probability of default. This results in a distribution that is too concentrated around the risk-free rate (as we show in Section 6).

Although long-term financing is necessary to generate heterogeneity in credit spreads, it is not sufficient to match the substantial dispersion we observe in the data. Another important element that helps us match the data is the presence of young firms (i.e., including entry and exit in the model) coupled with heterogeneity in bond entry cost. To the extent that some small or unproductive firms decide to participate in the bond market (due to a relatively favorable bond entry cost draw), lenders will command a high premium resulting in a more dispersed distribution.

The model also replicates fairly well the negative relationship between credit spreads and

Figure 2: Life Cycle of Median Credit Spreads



Note: Median credit spreads in percentage points from data (Thomson Reuters) and model.

productivity. This is an important statistic to match since the extent to which productive public firms face lower borrowing costs has implications for the magnitude of misallocation. Highly productive firms are able to attract financing at a lower cost since (i) their incentive to default is lower and (ii) the lenders can recover a highly valued firm in the case of default.

The model generates a decreasing life-cycle path of credit spreads, which is a pattern we also confirm in the data.[26] Figure 2 plots the median credit spreads in the model and the data across age. In the model, relatively young firms that enter the bond market are priced high by lenders due to their small size. As they grow older and larger they manage to attract finance at a lower cost. This generates a decreasing path of credit spreads along the firm's life cycle. The median credit spread in the model is lower than in the data (1.38% versus 1.55%). As a result, the median life-cycle path of credit spreads in the model is also lower than in the data. Nonetheless, the model matches both the targeted mean credit spreads in the data as well as the targeted relationship between *mean* credit spreads (expressed in log) and age (see Table 6, left panel).

The model is also reasonably consistent with several untargeted statistics such as (i) the relationship between credit spreads and leverage, (ii) the autocorrelation of investment rates,

[26]The notion of age in both the model and the data is based on birth and not on when a firm became a public entity.

and (iii) the fraction of firms issuing external equity.[27] The model also captures closely the fraction of bond issuers that are less than the age of 10 (10.8% versus 11.0% in the data). Note that if we did not have a substantial dispersion in the bond entry cost, firms would take a longer time to enter the bond market and the fraction of young bond issuers would be small relative to the data.

Furthermore, the mean investment rate for public firms in the model is 0.13 versus 0.16 in our data (not reported in Table 6). Crouzet and Mehrotra (2018) report that firms in the [0-90] percentile (in terms of assets) have an investment rate nearly identical with Compustat firms. In our model, the investment rate of firms in the [0-90] percentile (in terms of assets) is around 2 percentage points larger than that of public firms.

However, our model evaluation also highlights some limitations. First, the model cannot match the average default rate. In the data, the expected default rate on corporate bonds is constructed using Moody's estimates on default probabilities on different classes of bonds. In our sample, for example, 20% of bond deals are rated as A1, 15% as Baa1, 12% as Baa2, and 10% as A3.[28] The average default rate is calculated by combining the distribution of bonds over categories and the default probability for each class. The average default rate of bonds turns out to be 1.56% in the data which is around half the default rate in our model.

In order for the model to generate a substantial dispersion of credit spreads consistent with the data, our estimation picked up parameter values that also imply a high default rate on long-term bonds. We found it challenging for the current model to achieve both targets simultaneously: a high credit spread dispersion with a relatively modest default rate. As a consequence of the high default rate, there are few firms in the model taking excessive amounts of leverage. Therefore, the model cannot generate either the dispersion in leverage within the public sector in the data (although it does replicate the mean leverage ratios across sectors, as mentioned).

Another model limitation is the relatively poor match with respect to the life-cycle growth of firm employment. In the model, the employment share of young firms (relative to the mean) is higher than in the data. For example, firms between the ages 1-5 have 64% of average employment in the model versus 49% in the data. Therefore, the model cannot jointly match the financial behavior of firms and the employment growth along the life cycle.

[27] We compute equity issuance following the definition of Jermann and Quadrini (2012) (i.e., sale of common stock minus the sum of purchases of common stock and distribution of cash dividends).

[28] More than half of our firms in the sample are rated by Moody's. Since every maturity has a different default probability, we choose the probability of defaulting after 12 years. This is the average maturity we observe in our sample.

5 Financial Frictions and TFP

We use our structural model to analyze the impact of financial frictions on aggregate macroeconomic variables and productivity, as well as their relative impact on private and public firms. We consider two types of experiments. First, we compare our benchmark model to an economy where we shut down financial frictions for both private and public firms. Second, we compare our benchmark model to an economy where we shut down financial frictions only for private firms or only for public firms.

Both experiments have their merit. By shutting down financial frictions for all firms we can analyze how the economy responds when both private and public firms can borrow without any constraint. By shutting down financial frictions for a single type of firm we can analyze the effect of a particular type of financial constraint on the economy. For example, if we lift financial frictions only for public firms, then we can analyze how financial constraints for public firms disrupt economic activity and generate misallocation.

Eliminating financial frictions for private firms amounts to setting the borrowing premium for private firms $r_p = 0$, the collateral constraint, ψ, equal to a very large value, and assuming that issuing equity is costless ($\phi_d = 0$ for private firms only). Eliminating financial frictions for public firms amounts to setting $\phi_d = 0$ (for public firms only) and assuming that public firms cannot default on their debt obligations.

5.1 Determinants of TFP

We start our analysis by deriving the following aggregate relationship:

$$Y = AM^{1-\nu}\left[K^{\alpha}N^{1-\alpha}\right]^{\nu}$$

where Y is aggregate output, A is TFP, M is the total mass of firms, K is aggregate capital and N is aggregate labor.[29] The first-best level of TFP, A_{fb}, can be derived by re-allocating capital and labor across firms to maximize total output subject to the constraints that aggregate capital is equal to K and aggregate labor is equal to N. At the first-best, the marginal product of capital (MPK) is equated across all firms. We define TFP loss as the percentage difference between A_{fb} and A:

$$\text{TFP loss} = \frac{A_{fb}}{A} - 1. \tag{9}$$

To build intuition about the determinants of TFP, we assume that (z, MPK) are jointly log normally distributed across firms. Under this assumption, TFP can be approximated by the

[29] According to this definition, TFP does not depend on the scale of the economy via M and can be computed as the Solow residual using average output, capital and labor per firm (i.e., $A = (Y/M)/[(K/M)^{\alpha}(N/M)^{1-\alpha}]^{\nu}$).

following expression (see the Online Appendix for details):

$$\log A \approx \underbrace{E[\log z] + \frac{1}{2}\frac{1}{1-\nu}\mathrm{Var}(\log z)}_{\text{composition}} - \underbrace{\frac{\alpha\nu(1-(1-\alpha)\nu)}{1-\nu}\mathrm{Var}(\log MPK)}_{\text{TFP loss}}. \tag{10}$$

TFP depends on three terms. First, it depends positively on the average productivity across firms. Second, TFP increases with the dispersion in productivity across firms. Since firm entry and exit are endogenous, these terms are affected by financial frictions. Third, TFP depends on the dispersion of MPK across firms, where a higher dispersion in MPK reduces TFP.

5.2 The Effect of Financial Frictions

We analyze the response of the economy when we shut down financial frictions for both private and public firms (steady-state comparison). Note that in an economy without financial frictions, the policy functions for private and public firms are identical. This happens because both entities pay the risk-free rate when they issue debt and cannot default on their debt obligations. Therefore, the only difference between private and public firms is the average age of each entity since firms are born as private. Also note that when we eliminate financial frictions, aggregate productivity is not equal to the first-best since there is still an effect from taxes and capital adjustment costs.

Table 7 reports macroeconomic variables as well as TFP losses for the benchmark model and the economy without financial frictions. We report the changes for all firms as well as separately for private and public firms. Financial constraints have a large effect on output and capital. Lifting financial frictions increases capital per firm by 33% and output per firm by 10%. The larger demand for capital translates into an increase in the equilibrium wage by 23%. The large increase in the wage decreases the demand for labor, especially for public firms. There is an increase in the number of firms producing in the economy (by 11%). Without financial frictions, the share of public-firm investment decreases by 22 percentage points (from 30% to 8%). Since both private and public firms have access to financing at the risk-free rate, the majority of firms have no incentive to switch to public status. TFP increases by 7% if we eliminate financial frictions while TFP loss (the percentage difference between first-best TFP and TFP) decreases by 3.5%.

As we have shown in Equation (10), lifting financial frictions affects TFP through two channels. First, it affects the productivity distribution of firms that operate in equilibrium (a selection effect). The left panel of Figure 3 plots the distribution of firm-level productivity in the benchmark model and the economy without financial frictions. When we eliminate

Table 7: Effect of Financial Frictions

	Benchmark Model	Model w/o Financial Frictions	Difference
All Firms			
Wage	1.00	1.23	22.5%
Capital per firm	1.93	2.57	32.9%
Labor per firm	0.60	0.54	-10.1%
Output per firm	1.09	1.20	10.1%
# firms	1.00	1.11	11.3%
Investment share of public firms	0.30	0.08	-74.1%
V(MPK)	0.41	0.33	-19.7%
TFP	1.18	1.27	7.3%
TFP loss	18.36%	14.81%	-3.5%
Private Firms			
Capital per firm	1.66	2.57	54.7%
Labor per firm	0.55	0.53	-2.2%
Output per firm	0.99	1.18	19.8%
Public Firms			
Capital per firm	2.40	2.57	7.0%
Labor per firm	0.70	0.55	-20.2%
Output per firm	1.26	1.23	-2.3%

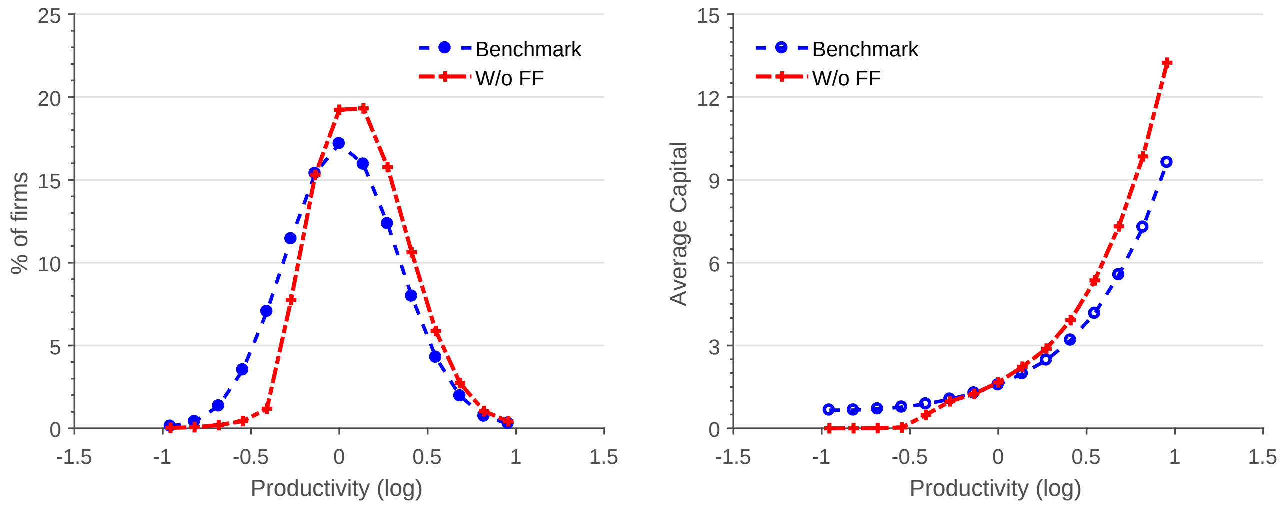
Note: Aggregate macroeconomic variables in the Benchmark and the model without financial frictions. TFP loss is computed according to Equation (9).

financial frictions, the share of firms in the bottom of the distribution decreases substantially. Due to the higher wage, relatively unproductive firms are more likely to exit. As a result, the average productivity increases when there are no financial frictions.

Second, lifting financial frictions decreases the dispersion of the marginal product of capital across firms, which leads to TFP gains. The right panel of Figure 3 plots the average capital stock (k') in the benchmark model and in the economy without financial frictions, across productivity ($\log z$). In both economies, more productive firms tend to have more capital. When financial frictions are eliminated, the average capital of high-productivity firms increases (therefore, reducing their MPK) while low-productivity firms (who are more likely to have a low MPK) exit the economy further reducing the cross-sectional dispersion in economy-wide MPK.

Table 7 also shows the effect of financial frictions for private and public firms, respectively. As mentioned, other than some differences due to age, private and public firms behave similarly

Figure 3: Productivity Distribution and Average Capital: Benchmark and Economy w/o FF



Note: The left panel plots the distribution of firm-level productivity (in log) and the right panel plots the average capital by firm-level productivity, in the Benchmark and the economy without financial frictions, respectively.

in the economy without financial frictions. As a result, the macroeconomic aggregates for private and public firms are nearly identical in the model without financial frictions. Private firms benefit more from lifting financial frictions relative to public firms. Private firms increase their capital by 55% while for public firms the increase is 7%. The labor input decreases for both private and public firms because the wage is higher. Given the small increase in capital and the large decrease in labor, output decreases for public firms.

When we lift financial frictions, the composition of firms in the economy changes. For example, many firms that would have become public now stay private. Moreover, the exit rate in the economy without financial frictions is higher due to the increase in the wage. As a result, there are more young firms in this economy relative to the benchmark.

We next analyze the effect of financial frictions keeping the composition of firms constant. In particular, to analyze the economy without financial frictions, we use the stationary distribution over productivity, age, capital, debt, and firm status from the *benchmark* economy but compute next period's capital, employment, and debt based on the policy functions derived in an economy *without* financial frictions. Therefore, in this experiment, the initial firm characteristics are the same across the two economies but the endogenous choices are different. In addition, we keep the wage equal to the value in the benchmark economy.

Table 8 shows the results from lifting financial frictions while controlling for firm characteristics. Private firms increase their capital by 42% while for public firms by 14%. This suggests that only a small part of the large increase in capital for private firms is due to compositional changes and the main driver is the behavioral response of investment when collateral con-

Table 8: Effect of Financial Frictions without Composition Effects

	Benchmark Model	Model w/o Financial Frictions	Difference
All Firms			
Capital per firm	1.93	2.50	29.1%
Labor per firm	0.60	0.73	21.8%
Output per firm	1.09	1.32	21.8%
V(MPK)	0.41	0.39	-6.3%
TFP	1.18	1.20	1.3%
TFP loss	18.36%	16.88%	-1.5%
Private Firms			
Capital per firm	1.66	2.35	41.8%
Labor per firm	0.55	0.70	28.9%
Output per firm	0.99	1.27	28.9%
Public Firms			
Capital per firm	2.40	2.74	13.9%
Labor per firm	0.70	0.78	12.3%
Output per firm	1.26	1.41	12.3%

Note: Effect of financial frictions without composition effects for private and public firms, respectively. In this experiment, we keep the composition of firms constant by analyzing the stationary distribution of firms in the benchmark economy, but compute each firm's optimal choices for capital and labor using the policy functions from the economy without financial frictions. We also keep the wage equal to the benchmark value.

straints are eliminated. In addition, labor (as well as output) increases for private firms by 29% and for public firms by 12% because capital increases while the wage remains constant.

5.3 Separating Financial Frictions by Firm Type

In our second experiment, we compare our benchmark model to an economy where we shut down financial frictions only for private firms or only for public firms. In the first case, the new economy is populated only by private firms since they can borrow without constraint while public firms still borrow subject to default risk. As a result, this economy is approximately similar to the economy where we shut down financial frictions for both types of firms as in both economies nearly all firms remain private. Table 9 shows that when we shut down financial frictions for private firms only, capital increases by 34%, output by 10%, and TFP loss due to misallocation decreases by 3.5%. Indeed, these changes coincide with the results in Table 7.

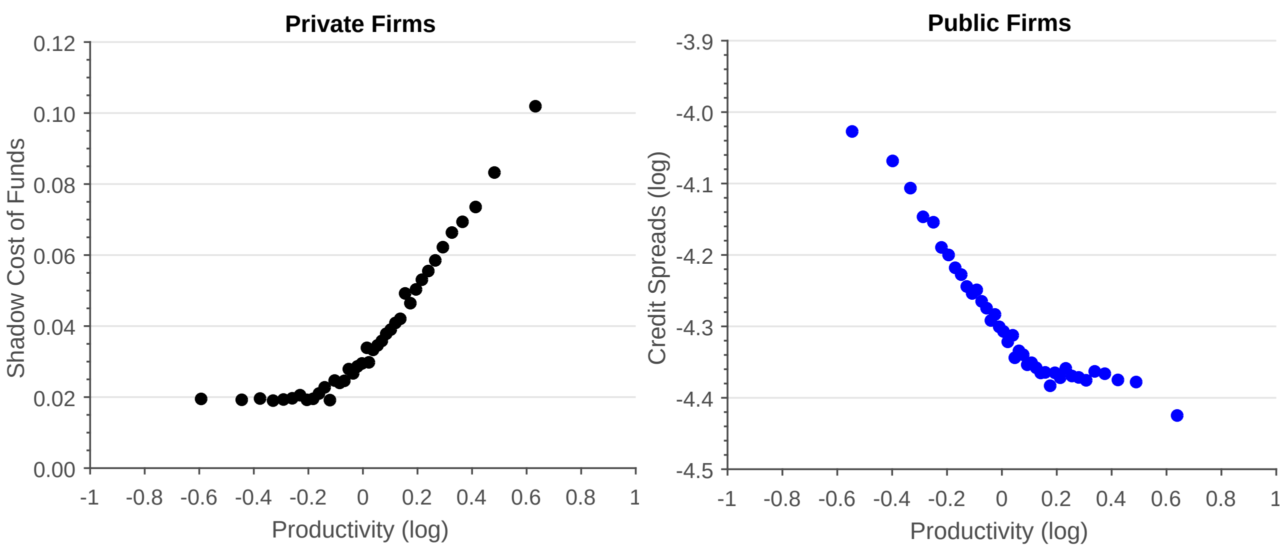
We next shut down financial frictions only for public firms. In this case, there is a larger incentive for private firms to switch to public status but since it is costly, not all private firms

Table 9: Effect of Shutting Down Financial Frictions by Firm Type

	Benchmark Model	No FF Private Firms	Diff.	No FF Public Firms	Diff.
Wage	1.00	1.22	21.6%	1.11	11.2%
Capital per firm	1.93	2.60	34.3%	2.32	19.9%
Output per firm	1.09	1.21	10.1%	1.10	1.2%
# firms	1.00	1.09	8.8%	1.10	9.9%
Inv. share of pub. firms	0.30	0.02	-92.4%	0.92	206.9%
V(MPK)	0.41	0.33	-19.5%	0.39	-6.1%
TFP	1.18	1.27	7.3%	1.20	1.0%
TFP loss	18.36%	14.83%	-3.5%	16.43%	-1.9%

Note: Effect of shutting down financial frictions only for private and only for public firms, respectively.

Figure 4: Borrowing Costs and Productivity: Private (Left Panel) and Public (Right Panel)



Note: The left panel shows the model-generated shadow cost of funds for private firms and the right panel shows the model-generated credit spread (in logs) for public firms.

will make the transition. The increase in capital and output is smaller than when we shut down financial frictions only for private firms. In addition, TFP gains from a more efficient allocation of resources are 1.9%, a little more than half of the gains when we shut down financial frictions for only private firms. This shows that misallocation arising from the public sector is modest relative to misallocation arising from the private sector.

5.4 Inspecting the Mechanism

In both experiments, financial frictions affect private firms more relative to public firms. This result hinges crucially on the different way private and public firms borrow in the capital markets (e.g., the type of financial constraint). First, external equity is costlier for private firms. Therefore, lifting financial frictions has a larger impact on their investment. Second, private firms borrow using a collateral constraint. As a result, private firms with high productivity are unable to grow fast if they do not have sufficient collateral. In contrast, productive public firms (even the smaller units) can borrow at a relatively low cost since lenders anticipate the high expected stream of profits. This can be seen in Figure 4 where we plot (using simulated data) borrowing costs for private and public firms against productivity.[30] The left panel of Figure 4 shows a positive relationship between the shadow cost of funds and productivity. This is because productive firms seek more financing to support their investment so they are more likely to hit their borrowing constraint. The right panel of Figure 4 shows the negative relationship between credit spreads and productivity for public firms which we estimated based on the joint information from Thomson Reuters and Compustat.

Nonetheless, private firms could be affected more by financial constraints simply because they are on average smaller and younger. Here, we distinguish between these competing firm characteristics. In particular, using simulated data, we regress log capital of firms on log productivity and dummy variables indicating whether the firm is young (less than or equal to 10 years) and whether it is private. If firm type is not an important factor, we should expect both private and public firms to have the same size *conditional* on age and productivity. We present our results in Table 10. All coefficients are statistically significant at the 1% level.

The results from specifications (1)-(3) are straightforward. First, productive firms have a higher capital stock. Second, younger firms are on average smaller than older firms and private firms are smaller than public firms (since firms are born as private entities). Third, as productivity increases, the size gap between young/old and private/public firms increases. For example, given a 1% increase in productivity, an old firm's capital increases by 1.71%, while a young firm's capital increases by 1.57% (suggesting that young firms are on average more constrained).

Specification (4) analyzes firm size with respect to all firm characteristics: productivity, age, and private status. We find that conditional on age and productivity, (i) private firms are smaller than public firms and (ii) the size gap between private and public firms increases with productivity. This suggests that differences in financial constraints are an important factor for the investment of private and public firms and that these differences become more

[30] The borrowing cost of private firms is computed using the Lagrange multiplier from the firms' investment Euler equation and represents the shadow cost of funds.

Table 10: Capital and Firm Characteristics

Dependent Variable	$\log k'$			
Specification	(1)	(2)	(3)	(4)
$\log z$	1.65	1.71	1.86	1.85
Young ($\leq$10 years)		-0.62		-0.61
$\log z \times$ Young ($\leq$10 years)		-0.14		-0.04
Private			-0.22	-0.02
$\log z \times$ Private			-0.33	-0.27
Constant	0.26	0.46	0.40	0.47

Note: Regression of firm capital on productivity and dummies of being young (age $\leq$ 10) and being a private firm.

pronounced for high-productivity firms. As a result, when we eliminate financial constraints, the average size of private firms increases more (in percentage terms) relative to the average size of public firms.

Our collective findings indicate that private firms, independently of their life-cycle stage, are more impacted by financial constraints relative to public firms. While private firms with high productivity are constrained if they do not have sufficient collateral, a similarly productive public firm can borrow at a relatively lower cost.

6 Robustness

We evaluate the importance of specific model elements for matching the empirical patterns documented in Section 3 and generate misallocation of capital. In each scenario, we re-estimate the benchmark model and report the full results in the Online Appendix.

Long-Term Financing We analyze how long-duration bonds affect our results by setting $\theta_B = 1$. As mentioned in Section 4, long-term financing helps us generate a distribution of credit spreads fairly close to the data. With one-period bonds, the model generates a low mean credit spread equal to 0.06% and a low standard deviation equal to 0.38%. The one-period model also exaggerates by a large degree the elasticities between credit spreads and productivity and age, respectively.

Cheap Equity for Private Firms There are two ways public firms have better access to capital markets. First, they can issue equity at a relatively lower cost than private firms. Second, they issue long-term bonds in the debt market without facing any collateral constraint. We ask here what is the relative importance of the first versus the second model element? To this end, we re-estimate the model assuming that private firms can issue equity as cheaply as public firms (i.e., $\gamma = 0$). In this case, the model does well matching all of the moments with one exception. The employment of entrants in this economy would be large relative to the data. The average number of employees for new firms would be 50% of the economy-wide average, versus 35% in the data. In this economy, TFP losses from financial frictions drop to 2.4%.[31]

Smaller Bond Premium for Private Firms Another assumption is that the debt issuance of private firms is associated with a bond premium. We re-estimate the model assuming a smaller bond premium for private firms ($r_p = 0.75\%$). In this case, the model still does well matching the moments. However, dispersion of credit spreads drops slightly to 1.99% and the average leverage of firms, public and private, increases. In this model, TFP losses from financial frictions fall to 3.1%.

Homogeneous Bond Entry Cost An important model element is the heterogeneous and stochastic bond entry cost. We assume here that the bond entry cost is same across firms and time (i.e., $\sigma_\chi = 0$). In this case, the relationship between credit spreads and productivity is too steep and there is too much dispersion in credit spreads. Note that in the identification discussion of Section 4, we claimed that a high dispersion in σ_χ allows younger and unproductive units to enter the bond market and hence increases the elasticity of age and credit spreads. The reason this exercise generates seemingly contradictory results is that we do not keep all other parameters constant but re-estimate the model. The new estimation pushes the other parameters to values that generates an elasticity that is too steep when $\sigma_\chi = 0$.

Capital Adjustment Costs Next we analyze the role of capital adjustment costs. We re-estimate the model assuming a smaller capital adjustment cost ($\phi_k = 0.04$). Naturally, the dispersion of investment rates increases from 0.24 in the benchmark model to 0.29. Mean credit spreads fall to 2.10% and the standard deviation of credit spreads also falls to 1.88%. Nevertheless, the model still does reasonably well matching the targeted moments. However, in this environment, TFP losses from financial frictions drop slightly to 3.3% showing that there is an interaction between financial frictions and capital adjustment costs (see Khan and Thomas, 2013).

[31] When we perform an experiment where we shut down financial frictions only for each firm type, we find that results are consistent with our benchmark model. Eliminating frictions frictions only for private firms decreases TFP loss by 2.3% while eliminating only for public firms by 1.6%.

Tighter Collateral Constraint for Private Firms When we re-estimate the model assuming a tighter collateral constraint for private firms ($\psi = 0.45$), the model still does well matching most of the moments. As expected, the average leverage of firms in the bottom 90% of the asset distribution decreases and the relative employment of entrants falls. Moreover, in this environment, TFP losses from financial frictions increase to 4.0%.

Lower Exogenous Exit Rate We analyze the implications of a lower exogenous exit rate ($\pi = 1\%$). In this case, most of the exit in the economy occurs endogenously. As a result, the economy is populated by relatively more productive firms. This makes the relationship between credit spreads and TFP steeper. The relative employment of entrants falls to 28% as productive incumbent firms can expect to live longer. The share of bond-issuing firms less than 10 years old also falls to 5.7%. TFP losses from financial frictions increase to 5.1%.

No Private Firms We assume here that all new firms enter the economy as public firms. In this environment, the mean credit spread is higher than the data while the standard deviation of credit spreads is lower. In addition, the employment profile for young firms is much higher than the data (as well as our benchmark model). For example, the relative employment of entrants increases to 63% from 34% in the Benchmark. This highlights the fact that public firms can reach faster their optimal scale relative to private firms. In this model, TFP losses from financial frictions are equal to 1.9%, which is identical to the losses we find when we lift financial frictions only for public firms in our benchmark economy.

Returns to Scale We re-estimate the model assuming a lower returns to scale ($\nu = 0.75$). In this case, dispersion of credit spreads drops to 1.83% and the relationship between credit spreads and TFP becomes too steep. Returns to scale is also an important parameter for the relative employment of entrants, which increases to 47%. In this environment, TFP losses from financial frictions drop to 2.1%.

Capital Share We re-estimate the model assuming a lower capital share ($\alpha = 0.30$). In this case, dispersion of credit spreads drops to 1.87% and the relative employment of entrants increases to 42%. In this environment, TFP losses from financial frictions decrease to 2.2%.

7 Conclusion

We estimate aggregate productivity losses due to misallocation of capital in a model with both private and public firms. Private firms face borrowing constraints in the form of their collateralized assets and have access to relatively expensive external equity. Public firms issue long-term bonds in the debt market and have access to relatively cheap external equity. We integrate our framework in an otherwise standard model with idiosyncratic productivity shocks, capital adjustment costs, and firm entry and exit.

We estimate our model using Simulated Method of Moments based on a set of statistics describing the overall U.S. distribution of firms over leverage and employment in conjunction with financial statistics from Compustat and Thomson Reuters. Our empirical analysis demonstrates that there is sizable cross-sectional dispersion in credit spreads and that high-productivity firms face lower credit spreads relative to low-productivity firms.

Our structural model replicates these empirical findings. We find that a necessary model element to generate the fairly realistic behavior in credit spreads is long-term financing. With long-term bonds, the credit spreads are based on the whole sequence of default probabilities until the bond matures. As a result, the large heterogeneity in outcomes over long horizons generates a large dispersion in default probabilities and, hence, credit spreads.

We use our structural model to understand (i) the impact of financial frictions on aggregate macroeconomic variables as well as misallocation of capital and (ii) how financial frictions affect private versus public firms. Financial frictions have a large effect on macroeconomic variables. Lifting financial frictions increases capital by 33% and output by 10%. TFP loss due to misallocation of capital is equal to 3.5%. Moreover, financial frictions have a larger impact on investment and misallocation through private firms. Private firms are subject to a collateral constraint and thus cannot quickly reach their optimal scale. Because public firms — especially the productive ones — can borrow cheaply in the bond market, lifting financial frictions has a smaller impact on their behavior.

Finally, our analysis demonstrates the difficulty of jointly matching the financial behavior of firms (leverage and credit spreads) and employment growth along the life cycle. In our model, employment for young firms is higher relative to the data. We view the challenge of jointly matching the financial and employment behavior of firms across the size spectrum as a meaningful topic for future research.

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