Accounting for Unemployment: The Long and Short of It

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Positive correlation between unemployment rate and share of long-term unemployed (more than 26 weeks)
Issues related to long-term unemployment

- Usually think of the US labor market as being characterized by high turnover, in particular, short unemployment durations, but the current share of LTU is exceptional.

- LTU may be related to enactment of EUB programs
  - Benefit period is extended from 26 weeks to up 99 weeks

- LTU for individuals may have long-term effects for aggregate unemployment rate
  - Scarring, hysteresis

- LTU may reflect structural change
  - ST and LT unemployed are distinct groups
Interesting policy questions

The views expressed in this presentation are my own and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.
A basic model for unemployment accounting

- Unemployed are homogeneous: inflows and outflows
- Variations in outflow rates account for most of unemployment volatility, Shimer (2007).
- If unemployment is mainly driven by outflows, then unemployment and measures of long-term unemployment are positively correlated.
- The model does not match the duration distribution, it understates long-term unemployment.
A simple model of long-term unemployment

- Unemployed are heterogeneous: short-term (ST) and long-term (LT) unemployed defined by relative exit rates
  - Ex-ante h: unemployed differ at time of entry
    - structural change
  - Ex-post h: make transition from ST to LT over time
    - hysteresis
  - Darby et al (1985)

- Match readily available date on the duration distribution

- Source of unemployment volatility
  - Exit rate volatility more important than entry rate volatility
  - Volatility of LTU (exit and entry rate) more important than STU volatility
Related Literature


- Negative duration dependence
  - Multiplicative Proportional Hazards: unemployment exit rate is the product of: duration effect x time effect x fixed individual effects (observed and unobserved)
  - Identification of unobserved heterogeneity and duration effects
Outline (1): Measurement

- Review model with homogeneous unemployment
  - Emphasis is on entry rates to unemployment (from E or OLF) and exit rates from unemployment (to E or OLF)

- Model with heterogeneous unemployment
  - Recover transition rates from duration distributions by nonlinear least squares
  - Framework is useful not just for aggregate unemployment but also for demographic groups, industries, occupations

- Measurement problems
  - Reported labor market state
  - Reported job search durations
Outline (2): What does it mean?

- Welfare costs of business cycles
  - Accounting for LT unemployment can amplify volatility of present value of income by a factor of 10

- Volatility of unemployment exit rate, Shimer (2005)
  - Share of LT unemployed is counter-cyclical
  - ‘Quality’ of unemployment pool is pro-cyclical
Homogeneous unemployment

- Law of motion for unemployment $u$ in continuous time
  \[ \dot{u}(t) = f(t) - \lambda(t) u(t) \]
- Steady state for fixed inflow, $f$, and exit rate, $\lambda$
  \[ u = f/\lambda \]
- Statistics of LT unemployment
  - Average duration of unemployment, $D = 1/\lambda$
  - Fraction who have been unemployed for at least $T$
    \[ \omega(T) = \int_T^{\infty} fe^{-\lambda s} ds / u = \exp(-\lambda T) \]
- If $u$ is mainly driven by $\lambda$ then $\text{cov}(u, D) > 0$ and $\text{cov}(u, \omega) > 0$. 
 Contributions of entry and exit rates

- Entry rate, normalized: $\sigma \equiv f/n$

- Average monthly entry and exit probability, 1950-2010
  - $\bar{\lambda} = 0.44$ and $\bar{\sigma} = 0.033$

Table 1. Accounting for Unemployment

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Entry $\sigma$</td>
<td>0.16</td>
<td>0.13</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>Exit $\lambda$</td>
<td>0.82</td>
<td>0.87</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td>Residual</td>
<td>0.02</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Measuring Transition Rates

Variance Decomposition
Duration distribution of unemployment

- Measured duration distribution, $u_{t,j}^m$: unemployed for less than 5 weeks ($j = 1$), between 5 and 14 weeks ($j = 2$), between 15 and 26 weeks ($j = 3$), and more than 26 weeks ($j = 4$)

- Implied duration distribution of model with homogeneous unemployment
  - Use unemployment entry and transition equations to construct monthly unemployment vintages
    \[ u_{t,1} = u_{t,1}^m \text{ and } u_{t,i} = \left(1 - \lambda_t\right) u_{t-1,i-1} \text{ for } t \geq 2 \]
  - Time aggregate $u_{t,i}$ to get $\hat{u}_{t,j}^m$
Actual and implied duration distribution

- Match ST unemployment (< 1 month) by construction
- Understate LT unemployment (> 6 months)
Negative duration dependence

- Increase the duration that defines ST unemployment

- As cut-off duration for ST unemployment increases the implied exit probability from unemployment declines
Model of heterogeneous unemployment

- Short-term and long-term unemployment: $\lambda^1(t) > \lambda^2(t)$

\[
\dot{u}^1(t) = f^1(t) - \lambda^1(t) u^1(t) - \gamma^1(t) u^1(t)
\]
\[
\dot{u}^2(t) = f^2(t) - \lambda^2(t) u^2(t) + \gamma^1(t) u^1(t)
\]

- Ex-ante heterogeneity: $f^1(t), f^2(t) > 0$
  - Structural change
- Ex-post heterogeneity: $\gamma^1(t) > 0$
  - Pure duration effect, scarring

- Recovering entry and exit rates
  - From 2-type CT to aggregate duration distributions

\[
x = \{f, \lambda, \gamma\} \rightarrow \{u^1_i, u^2_i\} \rightarrow \{u^m_j\}
\]

- Nonlinear least squares
The exit probability of **STU** is about four times the exit probability of **LTU**
Most of the inflow to unemployment is **STU**, about six times that of **LTU**.

Declining trend for STU inflows since 1980s.

**Transitions** from STU to LTU are relatively infrequent and volatile.
Despite the small LTU share in unemployment inflows, LTU makes up close to half of total unemployment because of its low exit rate.
In recessions exit rates from unemployment decline, and they decline more for LTU.
Sample Averages, 1950-2010

Homogeneous

Exit  \bar{\lambda} = 0.45
Entry \bar{\sigma} = 0.035

Heterogeneous

Exit  \bar{\lambda}^1 = 0.65  \bar{\lambda}^2 = 0.15
Entry \bar{\sigma}^1 = 6\bar{\sigma}^2  u^2/u = 0.45

\bar{\gamma}^1 = 0.015
Better fit of the duration distribution
Computations

"Identification"

- Solve the restricted models first, i.e., ex-ante or ex-post heterogeneity only, and use the solutions as starting values for the hybrid model
- Both converge to same solution

Estimate current transition rates from their implications for future distributions

- Required Data
- Report estimates up to 2010q4
### Table 1. Accounting for Unemployment

<table>
<thead>
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<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>A. Homogeneous Unemployment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.17</td>
<td>0.14</td>
<td>0.12</td>
<td>0.11</td>
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<tr>
<td>$\lambda$</td>
<td>0.80</td>
<td>0.84</td>
<td>0.86</td>
<td>0.89</td>
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<tr>
<td>Residual</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>B. Heterogeneous Unemployment</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\sigma^1$</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.33</td>
<td>0.32</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>$\lambda^1$</td>
<td>0.24</td>
<td>0.24</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>$\lambda^2$</td>
<td>0.34</td>
<td>0.39</td>
<td>0.42</td>
<td>0.48</td>
</tr>
<tr>
<td>$\gamma^1$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Residual</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Contributions of entry and exit rates (2)

- Exit rates account for 70% and entry rates account for 30%

- LTU transition rates account for 80% and STU transition rates account for 20%
  - Entry (exit) rates for types are positively correlated, about 0.7

- Similar results for demographic subgroups with exceptions
  - ST exit rate unimportant for males older than 45 years and for some industries (DUR, NDR, LHO) and occupations (CE, PROD)
### Selected Male Age Groups, 1976-2009

<table>
<thead>
<tr>
<th>Age</th>
<th>25-34</th>
<th>35-44</th>
<th>45-54</th>
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</thead>
<tbody>
<tr>
<td>$u$</td>
<td>5.9</td>
<td>4.4</td>
<td>4.0</td>
</tr>
<tr>
<td>$D$</td>
<td>17.1</td>
<td>19.8</td>
<td>22.6</td>
</tr>
</tbody>
</table>

#### A. Aggregate Statistics

#### B. Transition Probabilities

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(1)</th>
<th>(2)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^1$</td>
<td>0.026</td>
<td>0.10</td>
<td>0.017</td>
<td>0.08</td>
<td>0.013</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.005</td>
<td>0.18</td>
<td>0.003</td>
<td>0.33</td>
<td>0.004</td>
<td>0.37</td>
</tr>
<tr>
<td>$\lambda^1$</td>
<td>0.580</td>
<td>0.23</td>
<td>0.565</td>
<td>0.17</td>
<td>0.588</td>
<td>0.04</td>
</tr>
<tr>
<td>$\lambda^2$</td>
<td>0.150</td>
<td>0.34</td>
<td>0.130</td>
<td>0.26</td>
<td>0.132</td>
<td>0.34</td>
</tr>
<tr>
<td>$\gamma^1$</td>
<td>0.004</td>
<td>0.00</td>
<td>0.010</td>
<td>0.00</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>Res</td>
<td>0.15</td>
<td>0.16</td>
<td>0.18</td>
<td></td>
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</tr>
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</table>
Increased entry and reduced exit by LTU account for a large share of the increase in unemployment.
Measurement issues

- Classification problems
  - Number of reported inflows into U (from E and OLF) exceed number of reported durations with less than 5 weeks.
    - Elsby et al (2011)
  - Could be a classification problem
    - Poterba and Summers (1986)

- Reported durations
  - Less than half of unemployed correctly report increased duration from month to month.
    - Poterba and Summers (1984)
  - After 1994 CPS redesign the incremental duration increase for ongoing unemployment spells is measured correctly, but still a potential problem for initial reported duration of inflows into U.
What measured unemployment duration is

- Rotation structure of CPS sample
  - Households are in the sample for four consecutive months
  - Have three potential changes to the labor market status

- Unemployment duration is the reported duration of job search when unemployed (U)
  - On-the-job search even when employed (E)
  - Interrupted job search if temporarily out of the labor force (OLF)
Previous month’s employment status was either E or OLF. Based on matched household reports for consecutive months.
Duration by inflow into unemployment

Reported duration of job search for currently unemployed households that were U, E, or OLF in the previous month.
Possible correction

- Assume that all previously employed (inflow from E) have duration < 5 weeks, that is, move a constant fraction $\alpha_{i}^{E}$ of the unemployed that report a duration of more than 5 weeks, $i > 1$, to the unemployed with less than 5 weeks of unemployment, $i = 1$.

- Assume that all inflows from OLF that report more than 5 weeks of unemployment duration in the current month were unemployed in the previous month. Thus increase unemployment in the groups with more than 5 weeks duration by the OLF share $\alpha_{i}^{OLF}$
  
  - for current durations of 5-14 and 15-25 weeks assign 1/3 to the previous group and 2/3 to the current group
  - for 26-52 weeks assign 1/6 to previous group and 5/6 to current group
  - for > 52 weeks assign all to > 52 weeks
Duration distribution, corrected

A. Unemployed for < 5 weeks

B. Unemployed for 5-14 weeks

C. Unemployed for 15-26 weeks

D. Unemployed for 27+ weeks
Reported search durations

For consecutive periods of unemployment the reported increase in unemployment duration may over- or understate the actual increase, Poterba and Summers (1984)

<table>
<thead>
<tr>
<th></th>
<th>(1) Average</th>
<th>(2) ≤20 weeks</th>
<th>(3) ≥ 20 weeks</th>
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</thead>
<tbody>
<tr>
<td>&lt; 0 weeks</td>
<td>14</td>
<td>8</td>
<td>26</td>
</tr>
<tr>
<td>0-2 weeks</td>
<td>17</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>3-5 weeks</td>
<td>32</td>
<td>36</td>
<td>25</td>
</tr>
<tr>
<td>6-9 weeks</td>
<td>16</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>≥10 weeks</td>
<td>21</td>
<td>21</td>
<td>17</td>
</tr>
</tbody>
</table>

(1) is reported increase of duration; (2) is the shares for the whole sample that reported a change in duration as in (1); (3) resp (4) same as (2) but for those that reported less than 20 weeks resp. more than 20 weeks in previous month.
Reported duration: measurement error model

- Random walk for reported duration
  - $\alpha_d$ is the probability for a reported change $d = -1, 0, 1, 2, 3$
  - $f(r|s)$ probability for report $r$ conditional on actual duration $s$

- Since 1994 for continuing unemployed workers in the rotation sample $\alpha_1 = 1$
  - The conditional probability $f$ applies only for incoming rotation groups
  - Modified conditional probability for reports $g(r|s)$

- Example: $\alpha_{-1} = 0.1$, $\alpha_0 = 0.2$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$, $\alpha_3 = 0.1$
### Probability distribution for reported durations

<table>
<thead>
<tr>
<th>r</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>12</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>13</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>22</td>
<td>12</td>
<td>7</td>
<td>4</td>
<td>0</td>
<td>85</td>
<td>13</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>27</td>
<td>19</td>
<td>12</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>67</td>
<td>18</td>
<td>9</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>21</td>
<td>22</td>
<td>17</td>
<td>12</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>47</td>
<td>22</td>
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<tr>
<td>5</td>
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<td>19</td>
<td>19</td>
<td>15</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>15</td>
<td>27</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>4</td>
<td>13</td>
<td>17</td>
<td>17</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>19</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>&gt;6</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>25</td>
<td>43</td>
<td>95</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>19</td>
<td>41</td>
<td>96</td>
</tr>
</tbody>
</table>

Probability (in percent) that a household that has been unemployed for $s$ months reports $r$ months duration. $F$ is pre-94 and $G$ is post-94.
Accounting for errors in duration reports

- Pre-94: the model fit of duration distribution deteriorates significantly for the first two cells (unemployed for less than one month and for 2-3 months)
  - The estimation procedure systematically overstates the share of unemployed with duration less than 5 weeks, and understates the share of unemployed with duration 5-14 weeks.

- Post-94: the model fit deteriorates somewhat, but qualitative features on the relative contributions of entry/exit rates and STU/LTU are not affected
  - Relative to reported durations the model predicts fewer unemployed with less than 5 weeks duration and more unemployed with 5-14 weeks duration
  - Model without reporting error seems to do better on matching unconditional exit rates from unemployment
Unemployment exit rate contingent on duration, without (solid) and with (dashed) measurement error.

What does it all mean?

- **Welfare costs of unemployment**
  - Homogeneous: costs are small since mostly STU
  - Heterogeneous: substantial share of LTU increases costs
What does it all mean?

- **Welfare costs of unemployment**
  - Homogeneous: costs are small since mostly STU
  - Heterogeneous: substantial share of LTU increases costs

- **Volatility of unemployment in matching models**
  - Homogeneous: not much unemployment volatility from productivity shocks
    - Periods of high unemployment are good times to post vacancies.
    - Solution: small surplus and ‘rigid’ wages.
  - Heterogeneous: pro-cyclical ‘quality’ of unemployment pool.
    - Pro-cyclical relative exit rate of LTU generates counter-cyclical share of LTU.
Income losses from unemployment

- Effect of unemployment exit and entry rates on expected present value of income for a fixed wage $w = 1$. 
Income losses from unemployment

- Effect of unemployment exit and entry rates on expected present value of income for a fixed wage $w = 1$.

- Capital value of employment

\[
\begin{align*}
    rW_t &= w + \sigma_{1,t}(U_{1,t} - W_t) + \sigma_{2,t}(U_{2,t} - W_t) + \theta(\bar{W} - W_t) \\
    r\bar{W} &= w + \bar{\sigma}_1(\bar{U}_1 - \bar{W}) + \bar{\sigma}_2(\bar{U}_2 - \bar{W}) + \theta(W_t - \bar{W})
\end{align*}
\]

- All employed are the same, but upon job loss some become STU and others LTU

- Conditional on current transition rates which revert to sample mean transition rates at rate $\theta$.

- Analogous expressions for capital values of STU and LTU
Income losses from LTU are large

Medium-term deviation from sample average, $1/\theta = 3$ years.
Exit rates from unemployment are an important driver of unemployment, Shimer (2007).

Labor productivity fluctuations do not generate much unemployment volatility in standard versions of the DMP matching model.

Alternative versions of the DMP model with small match surplus and 'rigid' wages generate significant unemployment volatility, e.g. Hall (2005), Hagedorn and Manovskii (2008).

Study the role of pro-cyclical unobserved 'quality' for unemployment volatility.
Business cycle statistics, 1950-2009

- Quarterly averages of monthly data
- Levels detrended with HP-filter
- Correlations with unemployment rate, $\text{Corr}(u_t, x_{t+s})$

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St Dev</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^1$</td>
<td>0.034</td>
<td>0.002</td>
<td>0.37</td>
<td>0.43</td>
<td>0.39</td>
<td>0.21</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.005</td>
<td>0.001</td>
<td>0.59</td>
<td>0.67</td>
<td>0.63</td>
<td>0.49</td>
<td>0.35</td>
</tr>
<tr>
<td>$\lambda^1$</td>
<td>1.024</td>
<td>0.106</td>
<td>-0.50</td>
<td>-0.61</td>
<td>-0.67</td>
<td>-0.55</td>
<td>-0.42</td>
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<tr>
<td>$\lambda^2$</td>
<td>0.199</td>
<td>0.040</td>
<td>-0.62</td>
<td>-0.66</td>
<td>-0.61</td>
<td>-0.42</td>
<td>-0.26</td>
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<tr>
<td>$\gamma$</td>
<td>0.003</td>
<td>0.004</td>
<td>0.05</td>
<td>0.11</td>
<td>0.17</td>
<td>0.10</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\lambda^2/\lambda^1$</td>
<td>0.194</td>
<td>0.028</td>
<td>-0.53</td>
<td>-0.52</td>
<td>-0.43</td>
<td>-0.24</td>
<td>-0.10</td>
</tr>
<tr>
<td>$f^2/f$</td>
<td>0.170</td>
<td>0.035</td>
<td>0.44</td>
<td>0.49</td>
<td>0.45</td>
<td>0.38</td>
<td>0.30</td>
</tr>
<tr>
<td>$u^2/u$</td>
<td>0.420</td>
<td>0.057</td>
<td>0.44</td>
<td>0.60</td>
<td>0.69</td>
<td>0.71</td>
<td>0.65</td>
</tr>
</tbody>
</table>
A simple matching model with heterogeneity

- Two types, measure \( \phi \) of each, with \( \phi^1 + \phi^2 = 1 \)
A simple matching model with heterogeneity

- Two types, measure $\phi^i$ of each, with $\phi^1 + \phi^2 = 1$

- Both types match at the same rate, $\lambda_W$, but ...

- ... type 1 has a higher probability of being a productive match, $\psi^1 \equiv 1 > \psi^2$, and ...

- ... the two types separate at different rates, $\sigma^i$. 
A simple matching model with heterogeneity

- Two types, measure $\phi^i$ of each, with $\phi^1 + \phi^2 = 1$

- Both types match at the same rate, $\lambda_W$, but ...

- ... type 1 has a higher probability of being a productive match, $\psi^1 \equiv 1 > \psi^2$, and ...

- ... the two types separate at different rates, $\sigma^i$.

- Both types search in the same pool, ...

- ... thus vacancies cannot control who they meet, and $\omega^i$ is the probability of a match with type $i$. 
Steady state of the model (1)

Capital value equations

\[ rW^i = w^i - \sigma^i (W^i - U^i) \]
\[ rU^i = b^i + \lambda_W \psi^i (W^i - U^i) \]
\[ rJ^i = p^i - w^i - \sigma^i (J^i - V) \]
\[ rV = -c + \lambda_F \sum \omega^i \gamma^i (J^i - V) \]

Nash surplus sharing

\[ S^i = W^i + J^i - U^i - V \]
\[ W^i - U^i = \beta S^i \text{ and } J^i - V = (1 - \beta) S^i \]

Free entry condition: \( V = 0 \)
Steady state of the model (2)

Unemployment

\[ u^i = \frac{\sigma^i}{\left(\sigma^i + \lambda_W \psi^i\right)} \]
\[ u = \sum_j \phi^j u^j \]
\[ \omega^i = \frac{\phi^j u^i}{\sum_j \phi^j u^j} \]

Matching rates

\[ m = Av^{1-\alpha} u^\alpha \]
\[ \lambda_W = \frac{m}{u} \]
\[ \lambda_F = \frac{m}{v} \]
Homogeneous steady state

\[ r = 0.05/12, \ \beta = 0.72, \ p = 1, \ b = 0.4, \]
\[ \alpha = 0.72, \ \lambda_W = 0.45, \ u = 0.07 \]

Heterogeneous steady state

\[ \psi^2 = 1/4 \]
\[ \frac{\sigma^2 (1 - u^2) \phi^2}{\sigma^1 (1 - u^1) \phi^1} = 1/6 \]
\[ u^2/u = 0.4 \]
Calibration

- Homogeneous steady state
  
  \[ r = 0.05/12, \quad \beta = 0.72, \quad p = 1, \quad b = 0.4, \]
  \[ \alpha = 0.72, \quad \lambda_W = 0.45, \quad u = 0.07 \]

- Heterogeneous steady state
  
  \[
  \psi^2 = \frac{1}{4} \\
  \frac{\sigma^2 \left(1 - u^2\right) \phi^2}{\sigma^1 \left(1 - u^1\right) \phi^1} = \frac{1}{6} \\
  u^2/u = 0.4 \\
  p_1 = p_2 \\
  b_1 = b_2
  \]
Steady state elasticities

- Experiment 1: increase $p$ by one percent.
- Experiment 2: Exp 1 and increase $\psi^2$ by 14 percent.
Steady state elasticities

- Experiment 1: increase \( p \) by one percent.
- Experiment 2: Exp 1 and increase \( \psi^2 \) by 14 percent.

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Problem: both $\nu$ and $u$ decline
Steady state elasticities

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- Problem: both $\nu$ and $u$ decline

- Alternative calibration for $p_i$ and $b_i$
  - Relative productivity: $p_1 = 1 \geq p_2$
  - Type 1 match more attractive for vacancy: $S_1 \geq S_2$
  - Relative flow value of unemployment: $z = \frac{\sum \phi_i u_i b_i}{\sum \phi_i (1-u_i)p_i}$
The simple model of homogeneous unemployment does not account for the duration distribution of unemployment.

An accounting framework with unobserved heterogeneity does capture the duration distribution.

Even in good times there is a significant group in US labor markets for which unemployment is of much longer duration than the simple model suggests.

This group of LTU seems to account for most of unemployment volatility.

Most of the increase of unemployment following the 2007-09 recession is attributable to LTU. Mismatch?
Measurement issues seem manageable.

With LTU income losses from unemployment can be an order of magnitude larger than expected based on the simple model.
Measuring inflow and outflow rates

- Observations on
  - \( u_t^m \): total unemployment at the end of month \( t \)
  - \( u_{t,1}^m \): the number of unemployed at the end of month \( t \) who have been unemployed for less than 5 weeks

- Assume that the instantaneous inflow and outflow rates are constant during the month, e.g., \( f(s) = f_t \) for \( s \in (t-1, t] \)

\[
  u_t^m = \left( 1 - \bar{\lambda}_t \right) u_{t-1}^m + u_{t,1}^m
\]

- Measured inflow and outflow rates

\[
  1 - \bar{\lambda}_t = e^{-\lambda_t}
\]

\[
  u_{t,1}^m = \int_{0}^{1} f_t e^{-\lambda_t s} ds = f_t \left( 1 - e^{-\lambda_t} \right) / \lambda_t
\]
Accounting for the contributions of entry and exit

- Sequence of entry and exit rates $x = \{ f, \lambda \}$ that determine unemployment rate $u = G(x)$

- Define trends for transition rates using a band pass filter, $x^T$, and the trend unemployment rate $u^T \equiv G(x^T)$

- Define the contribution of the $i$-th transition rate to trend deviations of the unemployment rate $du^T = u - G(x^T)$ as

$$du_i^T = G(x_i, x^T_{-i}) - G(x^T_i, x^T_{-i})$$

- Define residual as

$$r^T = du^T - \sum_i du_i^T$$
Required data for current transition rates

- Inflow share of LTU in 2007 with data up to the end of 2008, 2009, or 2010
- Stable after 2009: need between one and two years of future distributions