Bubbly Recessions

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Motivation: Japanese post-bubble recession
Motivation: U.S. post bubble recession
This paper

- Asks: can collapse of bubbles precipitate long recessions?
- Explores interaction of
  - bursting of asset bubbles
  - frictional labor market (sticky wages)
  - zero lower bound
- Methodology:
  - Expansionary rational bubble: à-la Hirano Yanagawa (RES 2017)
  - Sticky wage: à-la Schmitt-Grohe Uribe (JPE 2016)
  - Simple model → analytical solution
Main findings/contributions

1. Collapse of bubbles → “overshooting” & protracted recessions
   ▶ “Leaning against bubbles” policies can help

2. Collapse of large bubbles → liquidity trap, which exacerbates unemployment & recession

Figure: K & L before, during & after a bubble episode
Related literature

- Rational bubbles (esp. with infinite-lived agents)
  - Miao Wang (2011), Hirano Yanagawa (2017), Hirano et al. (2016), ...
  - Samuelson (1958), Diamond (1965), Tirole (1985), ...
- Rational bubbles & unemployment:
- Rational bubbles & sticky prices
- New Keynesian models
  - Krugman (1998), Eggertsson Krugman (2012), Schmitt-Grohe Uribe (2016), ...
Outline

1. Model
2. Equilibrium dynamics
3. Policy discussion
4. Liquidity trap (preliminary)
Model
Firms

- Single perishable good
- Firms, entrepreneurs, workers
- Competitive firms:

\[
y_t = k_t^{\alpha} l_t^{1-\alpha} \\
w_t = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha \\
q_t = \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1}
\]
Workers & Entrepreneurs

- Identical preferences:
  \[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln c_t^i \right) \]

- Workers: unit measure, are “hand-to-mouth”:
  \[ c_t^w = w_t l_t + T_t \]

- Entrepreneurs: unit measure, provide capital
Entrepreneurs

- Idiosyncratic productivity $a^j_t \in \{a^H, a^L\}$, w. prob. $h, 1-h$
- Entrepreneurs accumulate capital (after knowing their type):
  \[ k^j_{t+1} = a^j_t i^j_t \geq 0 \]
- H-type want to borrow from L-type, but face credit constraint (à-la Kiyotaki Moore):
  \[ R_{t+1} d^j_t \leq \theta q_{t+1} k^j_{t+1} \] (CC)
  - $R_{t+1}$: interest rate between $t$ & $t + 1$
  - $q_{t+1} k^j_{t+1}$: collateral value at $t + 1$
  - $\theta \in [0, 1]$: pledgeability
  - Throughout, assume $\theta$ small so (CC) binds for H-type
**Bubble asset**

- Besides trading in credit market, entrepreneurs can trade bubble asset (after knowing their type)
- Bubble asset:
  - Tradable durable asset in fixed unit supply
  - Pays no dividend
  - Risky (Weil, 1987):
    
    $\tilde{p}_t^b = \begin{cases} 
    p_t^b & \text{w. prob. } \rho \\
    0 & \text{w. prob. } 1 - \rho 
    \end{cases}$

    Once collapsed, bubble will not re-emerge
- In equilibrium, bubble serves as savings instrument: L-type buy bubble and sell it when become H-type
Tax & Entrepreneur’s Problem

• Taking prices, productivity shock, tax $\tau$ as given:

\[
\max_{\{c_t^j, i_t^j, b_t^j, d_t^j\}_{t=0}^{\infty}} E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln c_t^j \right) \quad \text{s.t.}
\]

\[
c_t^j + i_t^j + R_t d_{t-1}^j + (1 + \tau) \tilde{p}_t b_t^j = q_t a_t^j i_{t-1}^j + d_t^j + \tilde{p}_t b_{t-1}^j \quad \text{(BC)}
\]

\[
R_{t+1} d_t^j \leq \theta q_{t+1} a_t^j i_t^j \quad \text{(CC)}
\]

\[
i_t^j, b_t^j \geq 0
\]

• Macroprudential policy: speculation tax $\tau$. Budget balance: $\tau \tilde{p}_t b_t = T_t$
Labor market

- (Exogenous) downward nominal wage rigidity (SGU 2016)

\[ P_{t+1}w_{t+1} \geq \gamma_n P_tw_t, \quad \forall t \geq 0 \]

- \( \gamma_n \): degree of nominal wage rigidity
- \( P_t \): price level of consumption good

- For now, assume fixed inflation \( \frac{P_{t+1}}{P_t} \equiv \bar{\Pi} \geq 1 \). Then:

\[ w_{t+1} \geq \gamma w_t \quad \text{(DWR)} \]

\[ \gamma \equiv \frac{\gamma_n}{\bar{\Pi}} \]

- Labor market may not clear. Workers take employment from firm as given:

\[ l_t = L_t \leq 1 \]

\[ (1 - L_t)(w_t - \gamma w_{t-1}) = 0 \]
Equilibrium

Given $\tau$, $k^j_0 = K_0$, $d^j_0 = 0$, $b^j_0 = 1$, $p^b_0$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p^b_t\}$, quantities $\{i^j_t, k^j_{t+1}, c_t\}, \{l_t, c^w_t\}, \{K_{t+1}, L_t\}$ s.t.:

- Entrepreneurs & firms optimize
- Credit market clears: $hd^H_t + (1 - h)d^L_t = 0$
- Bubble market clears: $hb^H_t + (1 - h)b^L_t = 1$ if $\tilde{p}^b_t > 0$
- Goods market clears: $h(c^H_t + i^H_t) + (1 - h)(c^L_t + i^L_t) + c^w_t = K_t^\alpha L_t^{1 - \alpha}$
- Labor market conditions: DWR and

\[
l_t = L_t \leq 1
\]

\[
(1 - L_t)(w_t - \gamma w_{t-1}) = 0
\]
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
Bubble-less equilibrium \((p^b_t = 0, \forall t)\)

- Assume \(K_0\) small and \(\gamma \leq 1\), so DWR does not bind \((L_t = 1\forall t)\)
- From binding CC & credit market clearing:

\[
K_{t+1} = \left( \frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta q_t K_t
\]

\[
R_{t+1} = a^L q_{t+1}
\]

- Bubble-less steady state:

\[
K_{nb} = (\alpha \Omega)^{\frac{1}{1-\alpha}}, \quad \Omega \equiv \left( \frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta
\]

\[
R_{nb} = a^L \alpha K_{nb}^{\alpha-1}
\]
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
Bubble equilibrium \((p_t^b > 0, \forall t)\)

- **Focus:** DWR doesn’t bind when bubble persists \((L_t = 1\) if \(\tilde{p}_t^b > 0)\)
- **Bubble has two effects on capital**
  - Crowd-in: bubble sales raise entrepreneurs’ net worth
  - Crowd-out: bubble speculation reduces investment
  - Bubble is “large” if it completely crowds out L-type’s \(k\) investment

\[
K_{t+1} = \left\{ \begin{array}{ll}
\left( h \frac{a^H - a^L}{1 - \theta a^H} + a^L \right) \beta (q_t K_t + p_t^b) - a^L (1 + \tau) p_t^b & \text{if } R_t = a^L q_{t+1} \\
a^H \beta (q_t K_t + p_t^b) - a^H (1 + \tau) p_t^b & \text{if } R_t > a^L q_{t+1}
\end{array} \right.
\]

- **Bubble may raise or lower interest rate**

\[
R_{t+1} = \max \left\{ a^L, \frac{\theta a^H (1 - (1 + \tau) \phi_t)}{1 - h - (1 + \tau) \phi_t} \right\} q_{t+1}
\]

where \(\phi_t \equiv \frac{p_t^b}{\beta (q_t K_t + p_t^b)}\) denotes bubble size (relative to agg. savings)
Proposition (Bubble existence)

A bubble steady state exists iff sufficient financial friction:

$$\theta < \frac{\beta \rho (1 - h)}{1 + \tau}$$

and bubble not too risky:

$$\rho > \frac{a^L - \theta a^H}{\beta (a^L - \theta a^H) + \beta h (a^H - a^L)}$$
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
What happens when bubble bursts?

- Assume expansionary bubble \((K_b > K_{nb})\)
- Bubble collapses in \(T\) (i.e., \(\bar{p}_t^b = 0, \forall t \geq T\))
- If \(\gamma = 0\), then \(K_t\) and \(w_t\) will ↓ towards the bubble-less SS levels

**Figure**: K before, during & after bubble: \(\gamma = 0\)
Binding wage rigidity

- If $\gamma > 0$, wage may not flexibly fall to clear labor market, causing:
  - involuntary unemployment ($L_t < 1$)
  - which reduces capital return $q_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$
  - which reduces entrepreneur’s capital income & thus net worth
  - and reduces capital accumulation

- Involuntary unemployment as long as rigid wage floor > market-clearing wage
Figure: Equilibrium wage vs. market-clearing wage
Characterizing a slump

- Let $T + s^*$ be the first post-bubble period with full employment:

$$s^* \equiv \min\{s \geq 0 | L_{T+s} = 1\}$$

- Economy is in a \textit{slump} between $T$ and $T + s^* - 1$.
- How long and deep is the slump?
Proposition (Post-bubble slump)

1. Slump duration:

\[ s^* = \begin{cases} 
0 & \text{if } \gamma = 0 \\
\left[ \omega(\gamma) - 2\alpha \log_\gamma K_T \right] & \text{if } \gamma \in (0, 1) \\
\infty & \text{if } \gamma = 1 
\end{cases} \]

where \( \omega(\gamma) \equiv \frac{2\alpha}{1-\alpha} \log_\gamma (\alpha \Omega) - \frac{3-\alpha}{1-\alpha} \)

2. During the slump:

\[ w_{T+s} = \gamma^s w_T \]
\[ L_{T+s} < 1 \]

\[ K_{T+s+1} = \alpha \Omega \left( \frac{w_{T+s}}{1 - \alpha} \right)^{\frac{\alpha - 1}{\alpha}} K_{T+s} \]
Simulation

(a) Bubble/savings ratio  
(b) Real wage  
(c) Employment  
(d) Capital  
(e) Output  
(f) Real interest rate

Figure: Bubbly boom-bust (relative to bubble-less SS)
“Proof”: backward & forward induction

- After $T + s^*$: economy follows full employment bubble-less dynamics
  \[ w_{T+s} = w_{T+s}^{\text{full}} \equiv (1 - \alpha)K_{T+s}^\alpha, \quad \forall s \geq s^* \]
  \[ K_{T+s+1} = \alpha \Omega \left( \frac{w_{T+s}}{1 - \alpha} \right)^{\frac{\alpha-1}{\alpha}} K_{T+s} \]

- Between $T$ and $T + s^* - 1$: DWR binds
  \[ w_{T+s} = \gamma^s w_b, \quad \forall 0 \leq s < s^* \]
  \[ K_{T+s+1} = \alpha \Omega \left( \frac{\gamma^s w_b}{1 - \alpha} \right)^{\frac{\alpha-1}{\alpha}} K_{T+s} \]

- By definition:
  \[ s^* \equiv \min \left\{ s \geq 0 \mid w_{T+s}^{\text{full}} \geq \gamma^s w_b \right\} \]
  \[ = \min \left\{ s \geq 0 \mid K_{T+s}^\alpha \geq \gamma^s K_b^\alpha \right\} \]
  \[ = \ldots = \left\lfloor \omega - 2\alpha \log_\gamma K_b \right\rfloor \]
Policy discussion
Proposition (Worker’s expected utility in SS)

1. **Bubble-less SS**: \( W_{nb}(K) \equiv \Gamma_2 + \frac{\alpha}{1-\beta^\alpha} \log K \)

2. **Bubble SS**: 

\[
W_b = \frac{\log c_b^w + \beta (1 - \rho) W_{burst}(K_b)}{1 - \beta \rho}
\]

\( W_{burst}(K_b) \equiv \log [(1 - \alpha)(K_b)^\alpha] \)

- **contemporaneous utility**

\[
s^* - 1 + \sum_{s=1}^{s^*} \beta^s (\Gamma_1(s) - ((1 - \alpha)s - \alpha) \log K_b)
\]

- **slump utility**

\[
+ \beta^{s^*} W_{nb} \left( \gamma^{-\left(\frac{1-\sigma}{\sigma}\right) \frac{s^*(s^*+1)}{2}} \left[ \alpha \Omega \cdot (K_b)^{\alpha-1} \right]^{s^*} K_b \right)
\]

- **post-slump continuation value**

Proposition (Welfare-reducing bubble)

Assume bubble sufficiently risky:

\[ \beta (\beta - \alpha)(1 - \rho) > \alpha (1 - \beta)^2 \]

Then:

\[ W_{nb} > \lim_{\gamma \to 1^-} W_b \]
Effect of macroprudential policy

- Changing bubble tax can change how bubble affects capital accumulation
- Tradeoff: smaller economic boom vs. less severe bust
Simulation: Effects of bubble tax

(a) Bubble/savings ratio
(b) Real wage
(c) Employment
(d) Capital
(e) Output
(f) Real interest rate
Effect of changing inflation target

- Higher $\bar{\Pi}$ lowers real wage rigidity $\gamma = \frac{\gamma_n}{\Pi}$
- Hence higher $\bar{\Pi}$ would improve welfare
  - A sufficiently high $\bar{\Pi}$ would restore full employment $\forall t$
- Weaknesses:
  - Model lacks endogenous cost of inflation (e.g., via sticky prices)
  - So far model is also silent about ZLB
Simulation: Effects of changing inflation target

(g) Bubble/savings ratio

(h) Real wage

(i) Employment

(j) Capital

(k) Output

(l) Real interest rate
Summary

- Embed DWR in rational bubble model
- Find: Collapse of bubble can → persistent & inefficient slump. Warrants policy interventions
Liquidity trap (preliminary)
Collapse of large bubble & overshooting R

Proposition (Post-bubble interest rate)

Suppose economy reaches steady state with large expansionary bubble; then bubble collapses in $T$.
If $K_{lb} >$ some threshold $\bar{K}$, then post-bubble nominal interest rate is negative:

$$R_{T+1}\bar{\Pi} < 1.$$
**Intuition**

- $R$ depends on productivity of marginal investor & on MPK
- Bubble bursts $\rightarrow$ marginal k investor switches from H-type to L-type
- Bubble causes “overinvestment” in capital relative to bubble-less SS $\rightarrow$ low MPK
  - Low MPK persists as capital is pre-determined
  - Note: Slower depreciation rate (e.g., housing) $\rightarrow$ higher persistence of low MPK
  - Note: overinvestment is endogenous here due to bubble (exogenous in Rognlie Shleifer Simsek, 2017)
Introducing money holding

- To microfound ZLB, assume entrepreneurs choose cash holding:

\[
c^j_t + i^j_t + (1 + \tau)\tilde{p}^b_t b^j_t + \frac{M^j_t - M^j_{t-1}}{P_t} = q^j_t k^j_t + d^j_t - R_{t-1,t} d^j_{t-1} + \tilde{p}^b_t b^j_{t-1}
\]

(BC)

\[
\frac{M^j_t}{P_t} \geq \varepsilon
\]

(CIA)

- Cash-less limit: \(\varepsilon \rightarrow 0^+\)

- Money supply grows at exogenous rate \(\bar{\Pi} \geq 1\)
  - All seignorage transferred to workers
Inflation in equilibrium

- In equilibrium:

$$E_t \left[ u'(c_{t+1}) R_{t+1} \right] \geq E_t \left[ u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right], \forall t \geq 0$$  \hspace{1cm} (ZLB)

  ▶ “=” if CIA is not binding (i.e., entrepreneurs want to hold cash for storage; liquidity trap)

- Price level determination (via money market clearing)
  ▶ If CIA binds: $P_t = \frac{M_t}{\varepsilon}$
  ▶ If CIA does not bind (liquidity trap): Price determined by binding ZLB
Bursting bubble $\rightarrow$ liquidity trap

- Assume no liquidity trap in steady states:
  \[
  \bar{\Pi} \geq \max \left\{ \frac{1}{R_{nb}}, \frac{1}{R_b} \right\}
  \]

- If
  - large bubble ($\tau < \bar{\tau}$)
  - and investment boom ($K_b$) sufficiently large that $a^L \alpha K_{lb}^{\alpha - 1} \bar{\Pi} < 1$,

Then
  - Liquidity trap in $T + 1$,

  \[
  R_{T+1} \frac{P_{T+1}}{P_T} = 1
  \]

  (ZLB binds)
Liquidity trap → deflated price level

- Focus on parameters s.t. liquidity trap lasts for only one period (as in Krugman 1998, Eggertsson Krugman 2012)
- Then $P_{T+1}$ fixed at target $P^*_T+1 \equiv \frac{M_{T+1}}{\varepsilon}$
- Binding ZLB becomes:

$$R_{T+1} \frac{P^*_{T+1}}{P_T} = 1$$

- So $\downarrow R_{T+1}$ (due to bubble bursting) must be associated with $\downarrow P_T$ (deflated price level)
- $\downarrow P_T$ exacerbates DWR:

$$w_T \geq \frac{\gamma}{P_T/P^*_T} w_b > \frac{\gamma}{\Pi} w_b$$
Simulation: ZLB

(m) Bubble/savings ratio
(n) Real wage
(o) Employment
(p) Capital
(q) Output
(r) Real interest rate

Figure: Post-bubble liquidity trap.

U.S. data
Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in $T$
- $\rightarrow$ Overshooting $R_{T+1}$
- $\rightarrow$ Liquidity trap
- $\rightarrow$ Deflated price level
- $\rightarrow$ Exacerbated wage rigidity: $\Pi_T \downarrow \Rightarrow \frac{\gamma}{\Pi_T} \uparrow$
- $\rightarrow$ Sufficient deflation ($\frac{\gamma}{\Pi_T} > 1$) causes unemployment ($L_T < 1$)
Conclusion

Collapse of large bubbles can trigger persistent slump and liquidity trap

Figure: K & L before, during & after a bubble episode
U.S. post-bubble ZLB & deflation