Learning About Consumer Uncertainty from Qualitative Surveys: As Uncertain As Ever

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Federal Reserve Bank of Richmond

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  - involves lags, i.e. published with at least a one-month lag, and subject to 3-month and 1-year revisions

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  - Federal Reserve Banks regional indices - Atlanta, Dallas, Kansas City, New York, Philadelphia, Richmond

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  - Properties of inflation expectations - Coibion and Gorodnichenko (2012), Coibion, Gorodnichenko and Kumar (2015), ...
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- Qualitative Survey Data
  - Forecasting - Nardo (2003), Pesaran and Weale (2006), ...
  - Assessing the state of economic activity - Bram and Ludvigson (1998), Bachmann and Elstner (2013), Bachmann, Elstner and Sims (2013), ...

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- Summarizing qualitative survey data in the form of a diffusion index:

$$\mu\left(\frac{n^u}{n}-\frac{n^d}{n}\right)+\kappa=\mu D+\kappa$$

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- Diffusion indices summarize the direction of change in a set of disaggregated series: the breadth of change

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- The BLS also publishes a diffusion index of employment in its monthly statistical release, covering roughly 260 sectors (4-digit NAICS).

Actual change and the breadth of change

- $$\Delta x_t = \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^u} \Delta x_{i,t}^u}_{\Delta x_t^u} - \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^d} \Delta x_{i,t}^d}_{\Delta x_t^d},$$

  where $\Delta x_{i,t}^u = \Delta x_{i,t}$ if $\Delta x_{i,t} \geq 0$, and $\Delta x_{i,t}^d = -\Delta x_{i,t}$ if $\Delta x_{i,t} < 0$

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- $$\Delta x_t = \frac{n_t^u}{n}\mu_t^u - \frac{n_t^d}{n}\mu_t^d$$

Actual change and the breadth of change

- Define $\mu^u = T^{-1} \sum_{t=1}^{T} \mu_t^u$, $\varphi^u = T^{-1} \sum_{t=1}^{T} n_t^u / n$,

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- Then ...

$$\Delta x_t^u = \left(\frac{n_t^u}{n} - \varphi^u \right) \mu^u + \varphi^u \left(\mu_t^u - \mu^u \right) + \left(\frac{n_t^u}{n} - \varphi^u \right) \left(\mu_t^u - \mu^u \right),$$

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- Similarly, let $\mu^d = T^{-1}\sum_{t=1}^{T}\mu_t^d$, $\varphi^d = T^{-1}\sum_{t=1}^{T} n_t^d / n$,

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Actual change and the breadth of change

- Decomposing an expansion/contraction,

$$\Delta x_t \approx \underbrace{\varphi^u (\mu_t^u - \mu^u) - \varphi^d (\mu_t^d - \mu^d)}_{\text{Change in "how much" or intensive margin}} + \underbrace{\mu^u D_t}_{\text{Change in "how many" or extensive margin}},$$

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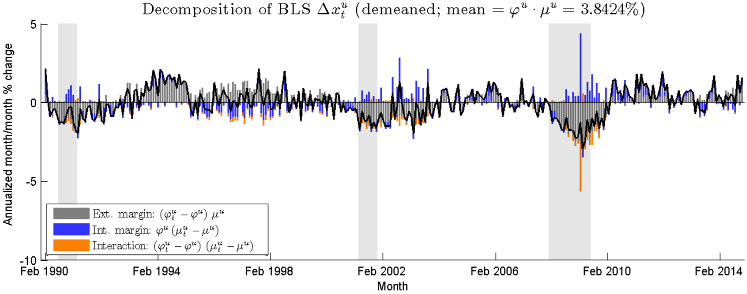
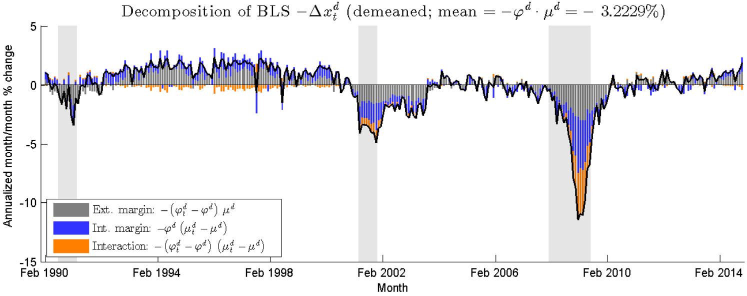
- Overall growth arises from:
  - the difference between how fast “up” sectors grew and how badly “down” sectors declined,
  - the difference between the proportion of sectors that expanded versus those that declined, the breadth of the expansion



Decomposition of BLS Δx_t^u (demeaned; mean $= \varphi^u \cdot \mu^u = 3.8424\%$)
Annualized month/month % change
5
0
-5
-10
Ext. margin: $(\varphi_t^u - \varphi^u)\ \mu^u$
Int. margin: $\varphi^u\ (\mu_t^u - \mu^u)$
Interaction: $(\varphi_t^u - \varphi^u)\ (\mu_t^u - \mu^u)$
Feb 1990
Feb 1994
Feb 1998
Feb 2002
Feb 2006
Feb 2010
Feb 2014
Month



Decomposition of BLS $-\Delta x_t^d$ (demeaned; mean $= -\varphi^d \cdot \mu^d = -\ 3.2229\%$)
Annualized month/month % change
5
0
-5
-10
-15
Ext. margin: $-(\varphi_t^d - \varphi^d)\ \mu^d$
Int. margin: $-\varphi^d\ (\mu_t^d - \mu^d)$
Interaction: $-(\varphi_t^d - \varphi^d)\ (\mu_t^d - \mu^d)$
Feb 1990
Feb 1994
Feb 1998
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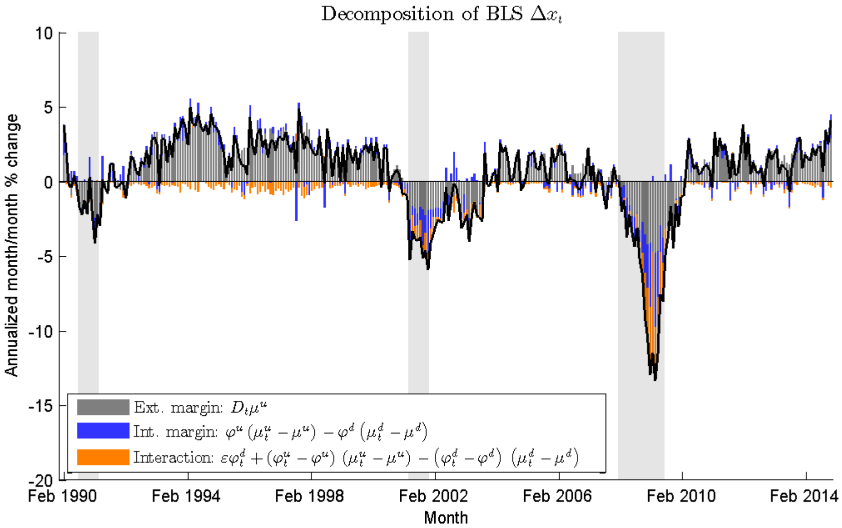
Month



Decomposition of BLS Δx_t
10
5
0
-5
-10
-15
-20
Annualized month/month % change
Ext. margin: $D_t \mu^u$
Int. margin: $\varphi^u(\mu_t^u - \mu^u) - \varphi^d(\mu_t^d - \mu^d)$
Interaction: $\varepsilon\varphi_t^d + (\varphi_t^u - \varphi^u)(\mu_t^u - \mu^u) - (\varphi_t^d - \varphi^d)(\mu_t^d - \mu^d)$
Feb 1990
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Actual change and the breadth of change

- Using plant-level data from Chile and the U.S., we show that investment spikes are highly pro-cyclical, so much so that changes in the number of establishments undergoing investment spikes (the extensive margin) account for the bulk of variation in aggregate investment.

Gourio and Kashyap (2007)

Measuring the breadth of change using qualitative surveys

- A sample of n survey participants, drawn randomly from a population at a point in time, is surveyed – e.g. overall business conditions?

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- Number of respondents associated with answer $a \in \mathcal{A}$, is n^a, where $\sum_{a=1}^{r} n^a = n$.
- Answers of type a are assigned a value of $\omega^a \in [\underline{\omega}, \overline{\omega}]$, e.g. $\omega^u = 1$, $\omega^s = 0$, and $\omega^d = -1$.

Measuring the breadth of change using qualitative surveys

- The answers are summarized in a diffusion index

$$\widehat{D} = \sum_{a=1}^{r} \omega^a \frac{n^a}{n}.$$

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- An expansion in, say spending no durable goods, if $\widehat{D} > (1/r)\sum_{a=1}^{r} \omega^a$, and no change or contraction otherwise.

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- p^a is the probability that a participant's answer is $a \in \mathcal{A} = \{1, 2, ..., r\}$, with $\sum_{a=1}^{r} p^a = 1$

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- $$f(n^1, ..., n^r; n, p^1, ..., p^r) = \frac{n!}{\Pi_{a=1}^{r} n^a!} \Pi_{a=1}^{r} (p^a)^{n^a}$$

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- $E(n^a) = np^a$, $Var(n^a) = np^a(1 - p^a)$, $Cov(n^a, n^{a'}) = -np^a p^{a'}$ for answers of types $a \in \mathcal{A}$ and $a' \in \mathcal{A}$

Measuring the breadth of change using qualitative surveys

- $\widehat{p}^a = n^a / n$, the proportion of answers of type $a \in \mathcal{A}$,

$$\widehat{p}^a = \frac{1}{n} \sum_{i=1}^{n} x_i^a,$$

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  where x_i^a takes on the value 1 when survey participant i answers a, zero otherwise

- $\widehat{p}^a$ then has the interpretation of a sample Bernoulli mean

Uncertainty in the measure of direction of change

- The multivariate Central Limit Theorem immediately gives

$$\sqrt{n}\begin{pmatrix} \widehat{p}^1 - p^1 \\ \widehat{p}^2 - p^2 \\ \dots \\ \widehat{p}^{r-1} - p^{r-1} \end{pmatrix} \longrightarrow^D \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \end{pmatrix}, \begin{pmatrix} p^1(1-p^1) & -p^1p^2 & \dots \\ -p^2p^1 & p^2(1-p^2) & \dots \\ \dots & \dots & \dots \\ -p^{r-1}p^1 & -p^{r-1}p^2 & \dots \end{pmatrix}\right)$$

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- $\widehat{D}$, is a linear combination of sample Bernoulli means, $\sum_{a=1}^{r} \omega^a \widehat{p}^a$

- Then

$$\sqrt{n}\left(\widehat{D} - D\right) \sim^a \mathcal{N}\left(0, \left(\sum_{a=1}^{r} (\omega^a)^2 p^a\right) - D^2\right),$$

where $D = E(\widehat{D}) = \sum_{a=1}^{r} \omega^a p^a$.

Uncertainty in the survey-measured direction of change

- Richmond indices: $\mathcal{A} = \{u, d, s\}$, and $\omega^u = 1$, $\omega^s = 0$, $\omega^d = -1 \Rightarrow \widehat{D} = (\widehat{p}^u - \widehat{p}^d)$ and

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- Uncertainty in the measured direction of change ...
  - decreases with the square root of the sample size
  - decreases with the diffusion index itself, D
  - decreases with the degree of polarization, $1 - p^s$

Distinguishing between different categories of participants

- $$\widehat{D} = \sum_{j=1}^{J} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n} = \widehat{D} = \sum_{j=1}^{J} \frac{n_j}{n} \underbrace{\sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j}}_{\widehat{D}_j}$$

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- One may choose to rescale the weights, say by γ_j,

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  $$\widehat{D} = \sum_{j=1}^{J} \gamma_j \frac{n_j}{n} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j}$$

- $$\sqrt{n}\left(\widehat{D} - D\right) \sim^a \mathcal{N}\left(0, \left(\sum_{a=1}^{r} \sum_{j=1}^{J} \left(\omega^a \gamma_j\right)^2 p_j^a\right) - D^2\right)$$

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- n survey participants responding to questions concerning k economic conditions - e.g. household conditions, overall business conditions, spending on big ticket items

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- Participants' answers across all components, $k = 1, ..., \overline{k}$, are collected in a $k-$tuple $\mathbf{a} = (a_1, ..., a_{\overline{k}})$ that lives in the set $\mathcal{A} = \Pi_{k=1}^{\overline{k}} \mathcal{A}_k$.

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- Consider an example with 3 components, each comprising 3 possible responses, $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$ $= \{u, d, s\} \times \{u, d, s\} \times \{u, d, s\} = \{uus, uud, uus, duu, ddu, dsu, suu, sdu, ssu, ...\}$ has 27 elements, in general $r^{\overline{k}}$.

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- Uncertainty in $\widehat{D}$ will need to take account of pairwise covariances between individual indices, say D_k and D_ℓ - i.e. how participants' answers compare/comove across different categories

The Distribution of Composite Indices

- $n_{k\ell}^{aa'}$ - the number of survey participants answering $a_k = a$ and $a_\ell = a'$ for the components k and ℓ,

$$n_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}} \in \mathcal{A} \backslash \mathcal{A}_k \times \mathcal{A}_\ell} n^{(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})}, \; k \neq \ell,$$

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- The number of participants answering a given response a for component k satisfies $n^a_k = \sum_{a' \in A_\ell} n^{aa'}_{k\ell}$
- Let $\widehat{p}^a_k = n^a_k / n$, and $\widehat{p}^{aa'}_{k\ell} = n^{aa'}_{k\ell} / n$,

The Distribution of Composite Indices

- $n_{k\ell}^{aa'}$ - the number of survey participants answering $a_k = a$ and $a_\ell = a'$ for the components k and ℓ,

$$n_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}} \in \mathcal{A} \setminus \mathcal{A}_k \times \mathcal{A}_\ell} n^{(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})}, \; k \neq \ell,$$

- The number of participants answering a given response a for component k satisfies $n_k^a = \sum_{a' \in A_\ell} n_{k\ell}^{aa'}$
- Let $\widehat{p}_k^a = n_k^a / n$, and $\widehat{p}_{k\ell}^{aa'} = n_{k\ell}^{aa'} / n$,
- Individual indices are given by

$$\widehat{D}_k = \sum_{a=1}^{r} \omega^a \widehat{p}_k^a = \sum_{a=1}^{r} \omega^a \sum_{a' \in \mathcal{A}_\ell} \widehat{p}_{k\ell}^{aa'},$$

The Distribution of Composite Indices

- $p_{k\ell}^{aa'}$ - the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components k and ℓ,

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where $(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})$ distinguishes between answers for component k, component ℓ, and all other components, $-\{k, \ell\}$

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- Each element $\widehat{p}_{k\ell}^{aa'}$, in the vector $\widehat{\mathbf{p}}_{k\ell}$, is the sample mean of a Bernoulli distribution

$$\sqrt{n}(\widehat{\mathbf{p}}_{k\ell} - \mathbf{p}_{k\ell}) \to^D \mathcal{N}(0, \Sigma_{\mathbf{p}_{k\ell}}).$$

The Distribution of Composite Indices

- $$\sqrt{n}\left(\widehat{D}-D\right) \sim^{a} \mathcal{N}\left(0, \sum_{k=1}^{\overline{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2 \sum_{1\leq k<\ell\leq \overline{k}} \delta_k \delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)\right),$$

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The Distribution of Composite Indices

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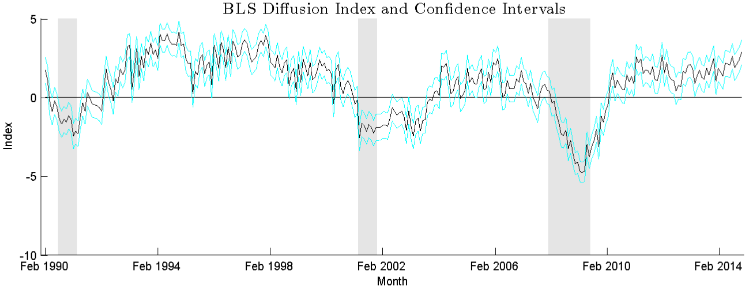
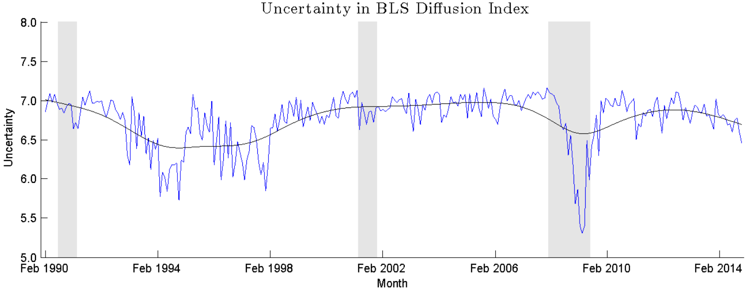
- $$Var\left(\widehat{D}_k\right) = \frac{1}{n}\left\{\left(\sum_{a=1}^{r} (\omega^a)^2 p_k^a\right) - (D_k)^2\right\},$$
- $$Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) = \frac{1}{n}\left\{\sum_{(a,a')\in\mathcal{A}_k\times\mathcal{A}_\ell} \omega^a \omega^{a'}\left[p_{k\ell}^{aa'} - \sum_{(b,b')\in\mathcal{A}_k\times\mathcal{A}_\ell} p_{k\ell}^{ab} p_{k\ell}^{b'a'}\right]\right\}.$$


BLS Diffusion Index and Confidence Intervals
5
0
-5
-10
Index
Feb 1990
Feb 1994
Feb 1998
Feb 2002
Feb 2006
Feb 2010
Feb 2014
Month


Uncertainty in BLS Diffusion Index
8.0
7.5
7.0
6.5
6.0
5.5
5.0
Uncertainty
Feb 1990
Feb 1994
Feb 1998
Feb 2002
Feb 2006
Feb 2010
Feb 2014
Month

An Application: The Michigan Survey of Consumers

- Monthly survey of Consumers (about 500 interviews) conducted by the Survey Research Center, University of Michigan

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- Index of Current Conditions
  - D_1: Would you say that you (and your family living there) are better off or worse off financially than you were a year ago?
  - D_5: Generally speaking, do you think now is a good or bad time for people to buy major household items?
  - $$ICC = \frac{D_1 + D_5}{2.6424}$$

An Application: The Michigan Survey of Consumers

- Index of Consumer Expectations

  - D_2: Do you think that a year from now, you will be better off financially, or worse off, or just about the same as now?
  - D_3: In the country as a whole – do you think that during the next twelve months we'll have good times financially, or bad times, or what?
  - D_4: In the country as a whole, will we have continuous good times during the next five years or so, will we have periods of widespread unemployment or depression, or what?
  - $$ICE = \frac{D_2 + D_3 + D_4}{4.1134}$$

An Application: The Michigan Survey of Consumers

- Index of Consumer Expectations
  - D_2: Do you think that a year from now, you will be better off financially, or worse off, or just about the same as now?
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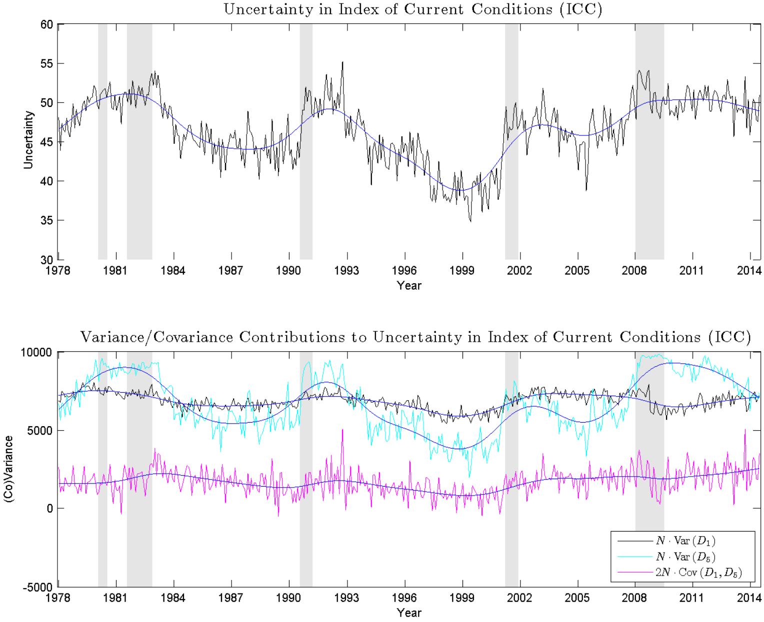
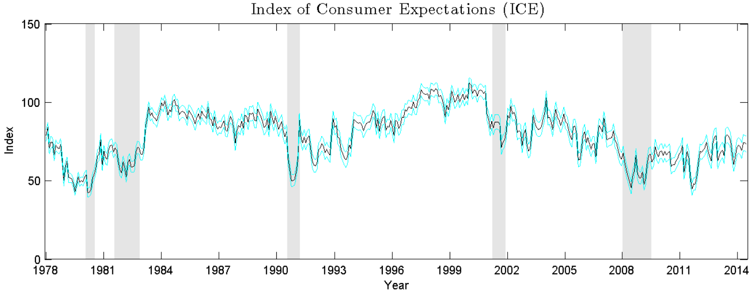
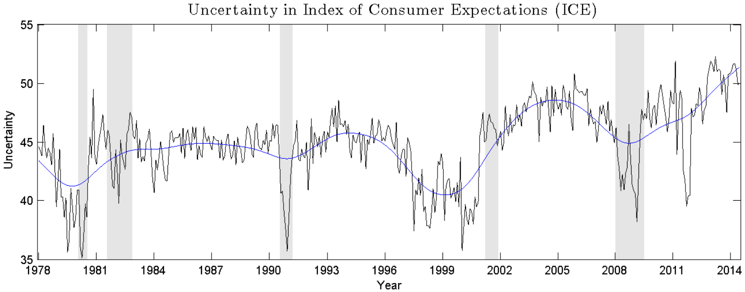
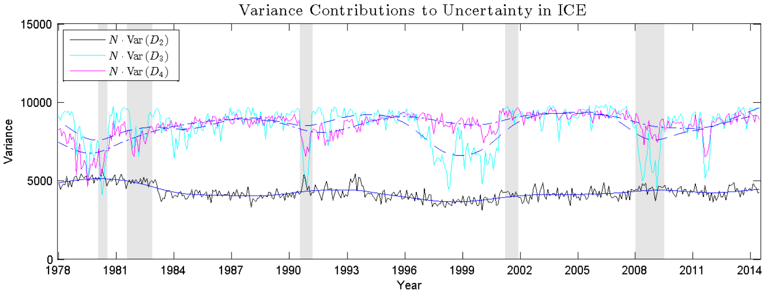
  - $$ICE = \frac{D_2 + D_3 + D_4}{4.1134}$$
- Index of Consumer Sentiment (headline number)

$$ICS = \frac{D_1 + D_2 + D_3 + D_4 + D_5}{6.7558}$$

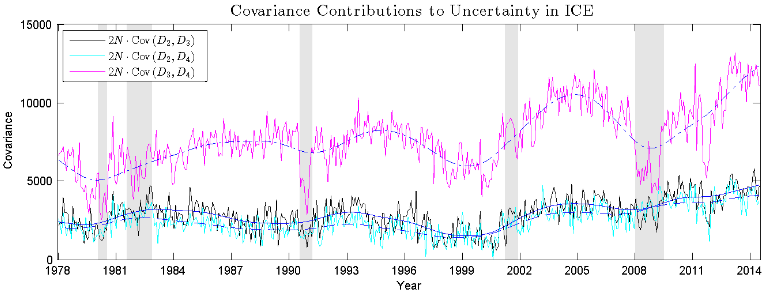

Uncertainty in Index of Current Conditions (ICC)
Uncertainty
Year
Variance/Covariance Contributions to Uncertainty in Index of Current Conditions (ICC)
(Co)Variance
Year
$N \cdot \mathrm{Var}(D_1)$
$N \cdot \mathrm{Var}(D_5)$
$2N \cdot \mathrm{Cov}(D_1, D_5)$


Index of Consumer Expectations (ICE)
150
100
50
0
Index
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year

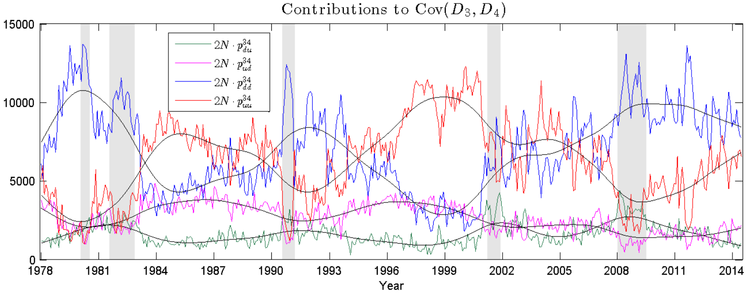

Uncertainty in Index of Consumer Expectations (ICE)
55
50
45
40
35
Uncertainty
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year

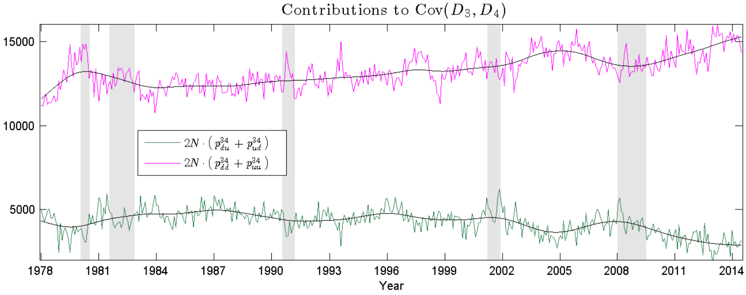


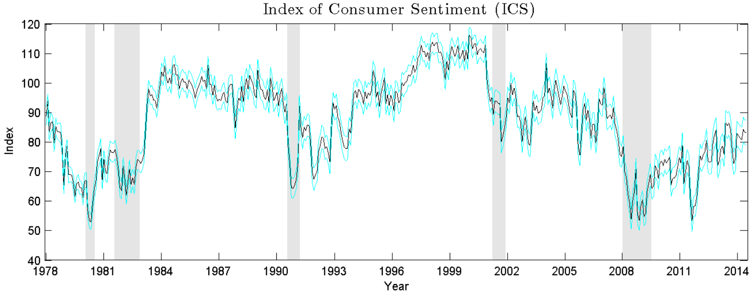
Variance Contributions to Uncertainty in ICE
$N \cdot \mathrm{Var}(D_2)$
$N \cdot \mathrm{Var}(D_3)$
$N \cdot \mathrm{Var}(D_4)$
Variance
15000
10000
5000
0
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year

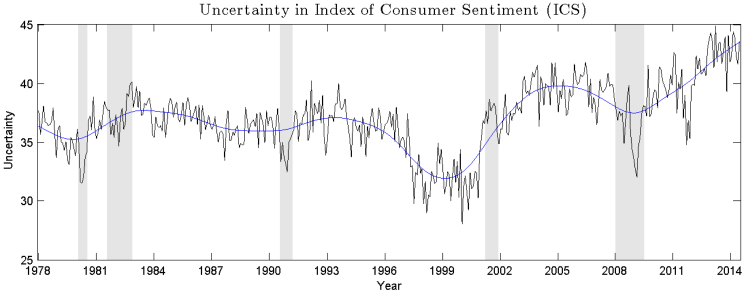


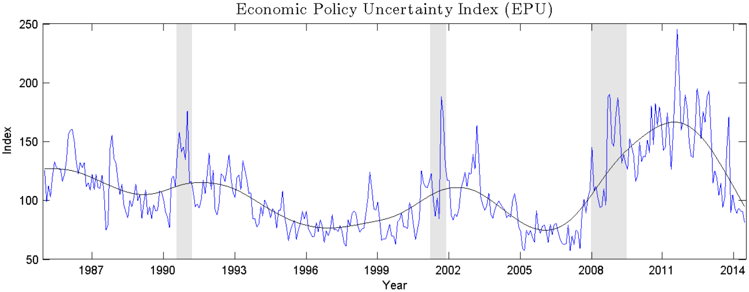
Covariance Contributions to Uncertainty in ICE
$2N \cdot \mathrm{Cov}(D_2, D_3)$
$2N \cdot \mathrm{Cov}(D_2, D_4)$
$2N \cdot \mathrm{Cov}(D_3, D_4)$
Covariance
15000
10000
5000
0
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year

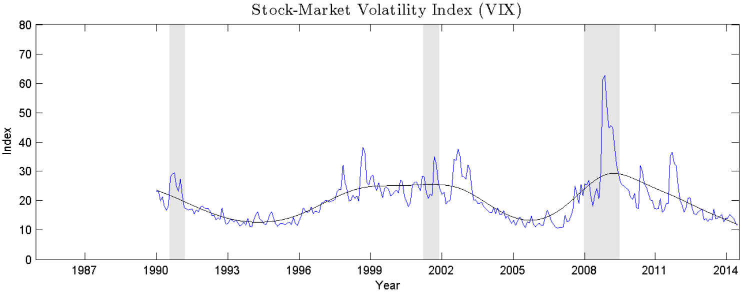

Contributions to Cov(D_3, D_4)
15000
10000
5000
0
$2N \cdot p^{34}_{du}$
$2N \cdot p^{34}_{ud}$
$2N \cdot p^{34}_{dd}$
$2N \cdot p^{34}_{uu}$
1978 1981 1984 1987 1990 1993 1996 1999 2002 2005 2008 2011 2014
Year


Contributions to Cov(D_3, D_4)
15000
10000
5000
$2N \cdot (p^{34}_{du} + p^{34}_{ud})$
$2N \cdot (p^{34}_{dd} + p^{34}_{uu})$
1978 1981 1984 1987 1990 1993 1996 1999 2002 2005 2008 2011 2014
Year


Index of Consumer Sentiment (ICS)
Index
120
110
100
90
80
70
60
50
40
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year


Uncertainty in Index of Consumer Sentiment (ICS)
Uncertainty
45
40
35
30
25
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year


Economic Policy Uncertainty Index (EPU)
250
200
150
100
50
Index
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year


Stock-Market Volatility Index (VIX)
80
70
60
50
40
30
20
10
0
Index
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Year

Conclusions

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