Learning About Consumer Uncertainty from Qualitative Surveys: As Uncertain As Ever

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Introduction

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  - involves lags, i.e. published with at least a one-month lag, and subject to 3-month and 1-year revisions
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  - Michigan Survey of Consumers indices of consumer sentiment, Institute of Supply Management index of manufacturing production, National PMIs (CES ifo Business Climate Index, etc.)
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  - Federal Reserve Banks regional indices - Atlanta, Dallas, Kansas City, New York, Philadelphia, Richmond
Introduction

- Quantitative Survey Data
  - Properties of inflation expectations - Coibion and Gorodnichenko (2012), Coibion, Gorodnichenko and Kumar (2015), ...
  - Disagreement among forecasters - D’Amico and Orphanides (2008), Sill (2012), ...

- Qualitative Survey Data
  - Forecasting - Nardo (2003), Pesaran and Weale (2006), ...
  - Assessing the state of economic activity - Bram and Ludvigson (1998), Bachmann and Elstner (2013), Bachmann, Elstner and Sims (2013), ...
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- Summarizing qualitative survey data in the form of a diffusion index:

\[ \mu \left( \frac{n^u}{n} - \frac{n^d}{n} \right) + \kappa = \mu D + \kappa \]

ISM: \( \mu = 1/2, \kappa = 1/2 \), Richmond: \( \mu = 1, \kappa = 0 \)
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- Diffusion indices summarize the direction of change in a set of disaggregated series: the breadth of change
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- The BLS gives us employment, and hence employment growth, by sector from which we can construct aggregate employment growth.
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The BLS also publishes a diffusion index of employment in its monthly statistical release, covering roughly 260 sectors (4-digit NAICS).
Actual change and the breadth of change

$$\Delta x_t = \frac{1}{n} \sum_{i=1}^{n_t^u} \Delta x_{i,t}^u - \frac{1}{n} \sum_{i=1}^{n_t^d} \Delta x_{i,t}^d,$$

where $\Delta x_{i,t}^u = \Delta x_{i,t}$ if $\Delta x_{i,t} \geq 0$, and $\Delta x_{i,t}^d = -\Delta x_{i,t}$ if $\Delta x_{i,t} < 0$.
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\[ \Delta x_t = \frac{n_t^u}{n} \mu_t^u - \frac{n_t^d}{n} \mu_t^d \]
Actual change and the breadth of change

- Define $\mu^u = T^{-1} \sum_{t=1}^{T} \mu^u_t$, $\varphi^u = T^{-1} \sum_{t=1}^{T} n^u_t / n$, 

Semantic data analysis: 

- Define $\mu^u$ and $\varphi^u$ to quantify the actual change and the breadth of change.
Actual change and the breadth of change

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- Then ...

$$
\Delta x_t^u = \left( \frac{n_t^u}{n} - \varphi^u \right) \mu^u + \varphi^u (\mu_t^u - \mu^u) + \left( \frac{n_t^u}{n} - \varphi^u \right) (\mu_t^u - \mu^u),
$$
Actual change and the breadth of change

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  $$\Delta x_t^u = \left(\frac{n^u_t}{n} - \phi^u\right) \mu^u + \phi^u (\mu^u_t - \mu^u) + \left(\frac{n^u_t}{n} - \phi^u\right) (\mu^u_t - \mu^u),$$

- Similarly, let $\mu^d = T^{-1} \sum_{t=1}^{T} \mu^d_t$, $\phi^d = T^{-1} \sum_{t=1}^{T} n^d_t / n$,
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- Similarly, let $\mu^d = T^{-1} \sum_{t=1}^{T} \mu^d_t$, $\varphi^d = T^{-1} \sum_{t=1}^{T} n^d_t / n$,

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$$\Delta x^d_t = \left( \frac{n^d_t}{n} - \varphi^d \right) \mu^d + \varphi^d (\mu^d_t - \mu^d) + \left( \frac{n^d_t}{n} - \varphi^d \right) (\mu^d_t - \mu^d),$$
Actual change and the breadth of change

- Decomposing an expansion/contraction,

\[
\Delta x_t \approx \varphi^u (\mu^u_t - \mu^u) - \varphi^d (\mu^d_t - \mu^d)
\]

Change in “how much” or intensive margin

+ \[\mu^u D_t,\]

Change in “how many” or extensive margin
Decomposing an expansion/contraction,

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Change in “how many” or extensive margin

Overall growth arises from:

- the difference between how fast “up” sectors grew and how badly “down” sectors declined,

- the difference between the proportion of sectors that expanded versus those that declined, the breadth of the expansion
Decomposition of BLS $\Delta x_t^u$ (demeaned; mean $= \varphi^u \cdot \mu^u = 3.8424\%$)

Decomposition of BLS $\Delta x_t^d$ (demeaned; mean $= -\varphi^d \cdot \mu^d = -3.2229\%$)
Decomposition of BLS $\Delta x_t$

- **Ext. margin**: $D_t \mu^u$
- **Int. margin**: $\varphi^u (\mu_t^u - \mu^u) - \varphi^d (\mu_t^d - \mu^d)$
- **Interaction**: $\epsilon \varphi_t^d + (\varphi_t^u - \varphi^u) (\mu_t^u - \mu^u) - (\varphi_t^d - \varphi^d) (\mu_t^d - \mu^d)$
Actual change and the breadth of change

- Using plant-level data from Chile and the U.S., we show that investment spikes are highly pro-cyclical, so much so that changes in the number of establishments undergoing investment spikes (the extensive margin) account for the bulk of variation in aggregate investment.

Gourio and Kashyap (2007)
Measuring the breadth of change using qualitative surveys

A sample of \( n \) survey participants, drawn randomly from a population at a point in time, is surveyed – e.g. overall business conditions?
Measuring the breadth of change using qualitative surveys

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- Possible answers: $\mathcal{A} = \{1, 2, \ldots, r\}$. Answers are indexed by $a \in \mathcal{A}$, e.g. $a \in \mathcal{A} = \{u, d, s\}$
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- Number of respondents associated with answer \( a \in \mathcal{A} \), is \( n^a \), where \( \sum_{a=1}^{r} n^a = n \).
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- Number of respondents associated with answer $a \in \mathcal{A}$, is $n^a$, where $\sum_{a=1}^{r} n^a = n$.

- Answers of type $a$ are assigned a value of $\omega^a \in [\underline{\omega}, \overline{\omega}]$, e.g. $\omega^u = 1$, $\omega^s = 0$, and $\omega^d = -1$. 

Measuring the breadth of change using qualitative surveys

- The answers are summarized in a diffusion index

\[ \hat{D} = \sum_{a=1}^{r} \omega^a \frac{n^a}{n}. \]
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- The answers are summarized in a diffusion index

\[ \hat{D} = \sum_{a=1}^{r} \omega^n a \frac{n^a}{n}. \]

- Then, we have that \( \omega \leq \hat{D} \leq \bar{\omega} \).

- An expansion in, say spending no durable goods, if \( \hat{D} > \frac{1}{r} \sum_{a=1}^{r} \omega^n a \), and no change or contraction otherwise.
Measuring the breadth of change using qualitative surveys

- $p^a$ is the probability that a participant’s answer is $a \in \mathcal{A} = \{1, 2, \ldots, r\}$, with $\sum_{a=1}^r p^a = 1$
Measuring the breadth of change using qualitative surveys

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- Survey process: $n$ random participant draws, each leading to a success for exactly one of $r$ types of responses, each answer type having success probability $p^a$
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$$f(n^1, \ldots, n^r; n, p^1, \ldots, p^r) = \frac{n!}{\prod_{a=1}^{r} n^a!} \prod_{a=1}^{r} (p^a)^{n^a}$$
p^a \text{ is the probability that a participant's answer is } a \in \mathcal{A} = \{1, 2, \ldots, r\}, \text{ with } \sum_{a=1}^{r} p^a = 1

Survey process: \( n \) random participant draws, each leading to a success for exactly one of \( r \) types of responses, each answer type having success probability \( p^a \)

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\]

\( E(n^a) = np^a, \ Var(n^a) = np^a(1 - p^a), \ Cov(n^a, n^{a'}) = -np^a p^{a'} \) for answers of types \( a \in \mathcal{A} \) and \( a' \in \mathcal{A} \)
Measuring the breadth of change using qualitative surveys

\[ \hat{p}^a = \frac{n^a}{n}, \]  

the proportion of answers of type \( a \in A \),

\[ \hat{p}^a = \frac{1}{n} \sum_{i=1}^{n} x_i^a, \]

where \( x_i^a \) takes on the value 1 when survey participant \( i \) answers \( a \), zero otherwise.
Measuring the breadth of change using qualitative surveys

\[ \hat{p}^a = \frac{na}{n}, \text{ the proportion of answers of type } a \in A, \]

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where \( x_i^a \) takes on the value 1 when survey participant \( i \) answers \( a \), zero otherwise

\( \hat{p}^a \) then has the interpretation of a sample Bernoulli mean
The multivariate Central Limit Theorem immediately gives

\[
\sqrt{n} \left( \begin{array}{c}
\hat{p}^1 - p^1 \\
\hat{p}^2 - p^2 \\
\vdots \\
\hat{p}^r - p^r - 1 \\
\end{array} \right) \overset{D}{\rightarrow} \mathcal{N} \left( 0, \begin{array}{cccc}
p^1(1-p^1) & -p^1 p^2 & \cdots \\
-p^2 p^1 & p^2(1-p^2) & \cdots \\
\vdots & \vdots & \ddots \\
-p^{r-1} p^1 & -p^{r-1} p^2 & \cdots \\
\end{array} \right)
\]
Uncertainty in the measure of direction of change

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0 \\
0 \\
\vdots \\
0
\end{pmatrix}, 
\begin{pmatrix}
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\vdots & \vdots & \ddots \\
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\end{pmatrix}
\]

- \( \hat{D} \), is a linear combination of sample Bernoulli means, \( \sum_{a=1}^{r} \omega^a \hat{p}^a \)
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\end{pmatrix} \xrightarrow{D} N \begin{pmatrix} 
0 \\
0 \\
\vdots \\
0 
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\]

- \(\hat{D}\), is a linear combination of sample Bernoulli means, \(\sum_{a=1}^{r} \omega^a \hat{p}^a\)

- Then

\[
\sqrt{n} \left( \hat{D} - D \right) \sim a N \left( 0, \left( \sum_{a=1}^{r} (\omega^a)^2 p^a \right) - D^2 \right),
\]

where \(D = E(\hat{D}) = \sum_{a=1}^{r} \omega^a p^a\).
Uncertainty in the survey-measured direction of change

- Richmond indices: $A = \{u, d, s\}$, and $\omega^u = 1, \omega^s = 0, \omega^d = -1 \Rightarrow \hat{D} = (\hat{p}^u - \hat{p}^d)$ and

$$\sqrt{n}(\hat{D} - D) \xrightarrow{D} \mathcal{N}\left(0, (1 - p^s) - D^2\right).$$
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$$\sqrt{n} \left( \hat{D} - D \right) \xrightarrow{D} \mathcal{N} \left( 0, (1 - p^s) - D^2 \right).$$

- Uncertainty in the measured direction of change...
  - decreases with the square root of the sample size
  - decreases with the diffusion index itself, $D$
  - decreases with the degree of polarization, $1 - p^s$
Distinguishing between different categories of participants

\[
\hat{D} = \sum_{j=1}^{J} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n} = \hat{D} = \sum_{j=1}^{J} \frac{n_j}{n} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j} \hat{D}_j
\]

where \( \sum_{j=1}^{J} n_j^a = n^a \), and \( \sum_{j=1}^{J} n_j = n \)
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- One may choose to rescale the weights, say by \( \gamma_j \),

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\]

\[
\sqrt{n} \left( \hat{D} - D \right) \sim^a N \left( 0, \left( \sum_{a=1}^{r} \sum_{j=1}^{J} (\omega^a \gamma_j)^2 p_j^a \right) - D^2 \right)
\]
The Distribution of Composite Indices

- $n$ survey participants responding to questions concerning $k$ economic conditions - e.g. household conditions, overall business conditions, spending on big ticket items.
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- Answers $a_k$, confined to a set $A_k$, each comprising $r$ possible types of responses, $\{1, 2, \ldots, r\}$.
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- $n$ survey participants responding to questions concerning $k$ economic conditions - e.g. household conditions, overall business conditions, spending on big ticket items.

- Answers $a_k$, confined to a set $\mathcal{A}_k$, each comprising $r$ possible types of responses, $\{1, 2, ..., r\}$.

- Participants’ answers across all components, $k = 1, ..., \bar{k}$, are collected in a $k$-tuple $a = (a_1, ..., a_k)$ that lives in the set $\mathcal{A} = \prod_{k=1}^{\bar{k}} \mathcal{A}_k$. 
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- Consider an example with 3 components, each comprising 3 possible responses, $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$
  $= \{u, d, s\} \times \{u, d, s\} \times \{u, d, s\} = \{uus, uud, uus, duu, ddu, dsu, suu, sdu, ssu, ...\}$ has 27 elements, in general $r^\bar{k}$. 
The Distribution of Composite Indices

- The Composite Index

\[ \hat{D} = \sum_{k=1}^{\overline{k}} \delta_k \hat{D}_k, \]

where \(0 < \delta_k < 1\).
The Distribution of Composite Indices

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where \( 0 < \delta_k < 1 \).

\[ \hat{D}_k = \sum_{a=1}^{r} \omega^a \sum_{a_k \in A \setminus A_k} \frac{n(a_k=a,a_{-k})}{n} = \sum_{a=1}^{r} \omega^a \frac{n_k^a}{n}, \]
The Distribution of Composite Indices

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where \( 0 < \delta_k < 1. \)

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Uncertainty in \( \hat{D} \) will need to take account of pairwise covariances between individual indices, say \( D_k \) and \( D_\ell \) - i.e. how participants’ answers compare/comove across different categories.
The Distribution of Composite Indices

- $n_{k\ell}^{aa'}$ - the number of survey participants answering $a_k = a$ and $a_{\ell} = a'$ for the components $k$ and $\ell$,

$$n_{k\ell}^{aa'} = \sum_{a_{-\{k,\ell\}} \in A \setminus A_k \times A_\ell} n(a_k = a, a_{\ell} = a', a_{-\{k,\ell\}}), \; k \neq \ell,$$
The Distribution of Composite Indices

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$$n_{k\ell}^{aa'} = \sum_{a_{\{k,\ell\}} \in A \setminus A_k \times A_\ell} n^{(a_k=a, a_\ell=a', a_{\{k,\ell\}})} \quad k \neq \ell,$$

- The number of participants answering a given response $a$ for component $k$ satisfies $n_k^a = \sum_{a' \in A_\ell} n_{k\ell}^{aa'}$
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The number of participants answering a given response \( a \) for component \( k \) satisfies \( n_k^a = \sum_{a' \in \mathcal{A}_\ell} n_{k\ell}^{aa'} \)

Let \( \hat{p}_k^a = n_k^a / n \), and \( \hat{p}_{k\ell}^{aa'} = n_{k\ell}^{aa'} / n \),
The Distribution of Composite Indices

- $n_{k\ell}^{aa'}$ - the number of survey participants answering $a_k = a$ and $a_\ell = a'$ for the components $k$ and $\ell$,

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- Let $\hat{p}_k^a = n_k^a / n$, and $\hat{p}_{k\ell}^{aa'} = n_{k\ell}^{aa'} / n$,

- Individual indices are given by

$$\hat{D}_k = \sum_{a=1}^{r} \omega^a \hat{p}_k^a = \sum_{a=1}^{r} \omega^a \sum_{a' \in A_\ell} \hat{p}_{k\ell}^{aa'},$$
The Distribution of Composite Indices

- \( p_{k\ell}^{aa'} \) - the joint probability of observing \( a_k = a \) and \( a_\ell = a' \) for the components \( k \) and \( \ell \),

\[
p_{k\ell}^{aa'} = \sum_{a_{-\{k,\ell\}} \in \mathcal{A}\setminus \mathcal{A}_k \times \mathcal{A}_\ell} p(a_{k}=a,a_\ell=a',a_{-\{k,\ell\}}), \ k \neq \ell,
\]

where \( (a_k = a, a_\ell = a', a_{-\{k,\ell\}}) \) distinguishes between answers for component \( k \), component \( \ell \), and all other components, \(-\{k,\ell\} \).
The Distribution of Composite Indices

- $p^{aa'}_{k\ell}$ - the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components $k$ and $\ell$,

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where $(a_k = a, a_\ell = a', a_{-\{k,\ell\}})$ distinguishes between answers for component $k$, component $\ell$, and all other components, $-\{k,\ell\}$

- $p_{k\ell}$ - the vector comprising all pairwise joint probabilities, $p^{aa'}_{k\ell}$ for given components $k$ and $\ell$, where the dimension of $p_{k\ell}$ is $r^2$. 

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- $p_{k\ell}^{aa'}$ - the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components $k$ and $\ell$, 

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where $(a_k = a, a_\ell = a', a_{-\{k, \ell\}})$ distinguishes between answers for component $k$, component $\ell$, and all other components, $-\{k, \ell\}$

- $\mathbf{p}_{k\ell}$ - the vector comprising all pairwise joint probabilities, $p_{k\ell}^{aa'}$ for given components $k$ and $\ell$, where the dimension of $\mathbf{p}_{k\ell}$ is $r^2$.

- Each element $\hat{p}_{k\ell}^{aa'}$, in the vector $\mathbf{p}_{k\ell}$, is the sample mean of a Bernoulli distribution 

$$\sqrt{n}(\mathbf{p}_{k\ell} - \mathbf{p}_{k\ell}) \xrightarrow{D} \mathcal{N}(0, \Sigma \mathbf{p}_{k\ell}).$$
The Distribution of Composite Indices

$$\sqrt{n}(\hat{D} - D) \sim a N \left( 0, \sum_{k=1}^{\bar{k}} \delta_k^2 \text{Var}(\hat{D}_k) + 2 \sum_{1 \leq k < \ell \leq \bar{k}} \delta_k \delta_\ell \text{Cov}(\hat{D}_k, \hat{D}_\ell) \right),$$
The Distribution of Composite Indices

\[ \sqrt{n}(\hat{D} - D) \sim^a N\left(0, \sum_{k=1}^{\bar{k}} \delta_k^2 \text{Var}(\hat{D}_k) + 2 \sum_{1 \leq k < \ell \leq \bar{k}} \delta_k \delta_\ell \text{Cov}(\hat{D}_k, \hat{D}_\ell)\right), \]

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\[ \text{Cov} (\hat{D}_k, \hat{D}_{\ell}) = \frac{1}{n} \left\{ \sum_{(a,a') \in A_k \times A_\ell} \omega^a \omega^{a'} \left[ p_{k\ell}^{aa'} - \sum_{(b,b') \in A_k \times A_\ell} p_{k\ell}^{ab} p_{k\ell}^{b'a'} \right] \right\}. \]
An Application: The Michigan Survey of Consumers

- Monthly survey of Consumers (about 500 interviews) conducted by the Survey Research Center, University of Michigan

\[
\text{ICC} = D_1 + D_5^2.6424
\]
An Application: The Michigan Survey of Consumers

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- Index of Current Conditions

  - $D_1$: Would you say that you (and your family living there) are better off or worse off financially than you were a year ago?

  - $D_5$: Generally speaking, do you think now is a good or bad time for people to buy major household items?

  - $ICC = \frac{D_1 + D_5}{2.6424}$
An Application: The Michigan Survey of Consumers

Index of Consumer Expectations

- \( D_2 \): Do you think that a year from now, you will be better off financially, or worse off, or just about the same as now?

- \( D_3 \): In the country as a whole – do you think that during the next twelve months we’ll have good times financially, or bad times, or what?

- \( D_4 \): In the country as a whole, will we have continuous good times during the next five years or so, will we have periods of widespread unemployment or depression, or what?

\[ ICE = \frac{D_2 + D_3 + D_4}{4.1134} \]
An Application: The Michigan Survey of Consumers

- **Index of Consumer Expectations**
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ICE = \frac{D_2 + D_3 + D_4}{4.1134}
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- **Index of Consumer Sentiment (headline number)**

\[
ICS = \frac{D_1 + D_2 + D_3 + D_4 + D_5}{6.7558}
\]
Economic Policy Uncertainty Index (EPU)

Stock-Market Volatility Index (VIX)
Conclusions

- **Diffusion indices:**
  - are estimates of change in quasi-extensive margin - can account for bulk of variation in aggregate series of interest
  - are linear combinations of Bernoulli averages and, therefore, approximately Normally distributed
  - variance of diffusion index reflects polarization or disagreement across responses
  - in Composite Indices, uncertainty in the index also reflects coincidence in polarization across individual indices

In Michigan Survey of Consumers:
- steady rise in uncertainty around consumer sentiment starting in the late 1990s
- uncertainty around consumer sentiment, six years removed from the Great Recession, at its highest level since 1978
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