Aggregate Implications of Changing Sectoral Trends

Technical Appendix and Supplementary Material

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Contents

1 Statistical Model of Trend Growth

1.1 Low-Frequency Transformation of the Annual Growth Rates

As discussed in the text, we follow the methods discussed in Müller and Watson (2020) to extract low frequency trends in the growth rates of GDP, TFP, and labor input. This process is summarized in Figure A1 which shows results for the growth rate of aggregate GDP. Panel (a) shows the raw data, that is, the cyclically adjusted annual growth rates of GDP. Panel (b) plots nine regressors, a constant (in blue) and eight cosine functions, $\Psi_j(s) = \sqrt{2}\cos(js\pi)$, with $s = (t-1/2)/T$, for $j = 1, ..., 8$, where $T = 69$ years is the sample size. Note that $\Psi_j(s)$ has period $2T/j$ so that the first cosine function has a period of 138 years, the second has a period 69 years, and so forth. The last cosine function, $j = 8$, has period 17.25 years. Panel (c) shows the fitted values from the regression of the data from panel (a) onto the regressors in panel (b). The solid line is from the regression onto the constant and all $q = 8$ cosine functions; this captures periodicities longer than 17.25 years and is the trend used in the body of the paper. The figure also shows the fitted values using only the first $q = 6$ cosine functions corresponding to periods longer than 23 years. This trend was used in the robustness analysis reported in Figure 11 in the paper. Panel (d) plots the OLS regression coefficients from the regressions of the data on the cosine functions. These eight regression coefficients are denoted by $\mathbf{X}$ and summarize the variation in the low-frequency trends plotted in panel (c). The values of $\mathbf{X}$ are called the *cosine transforms* of the raw data.

1.2 Factor Model

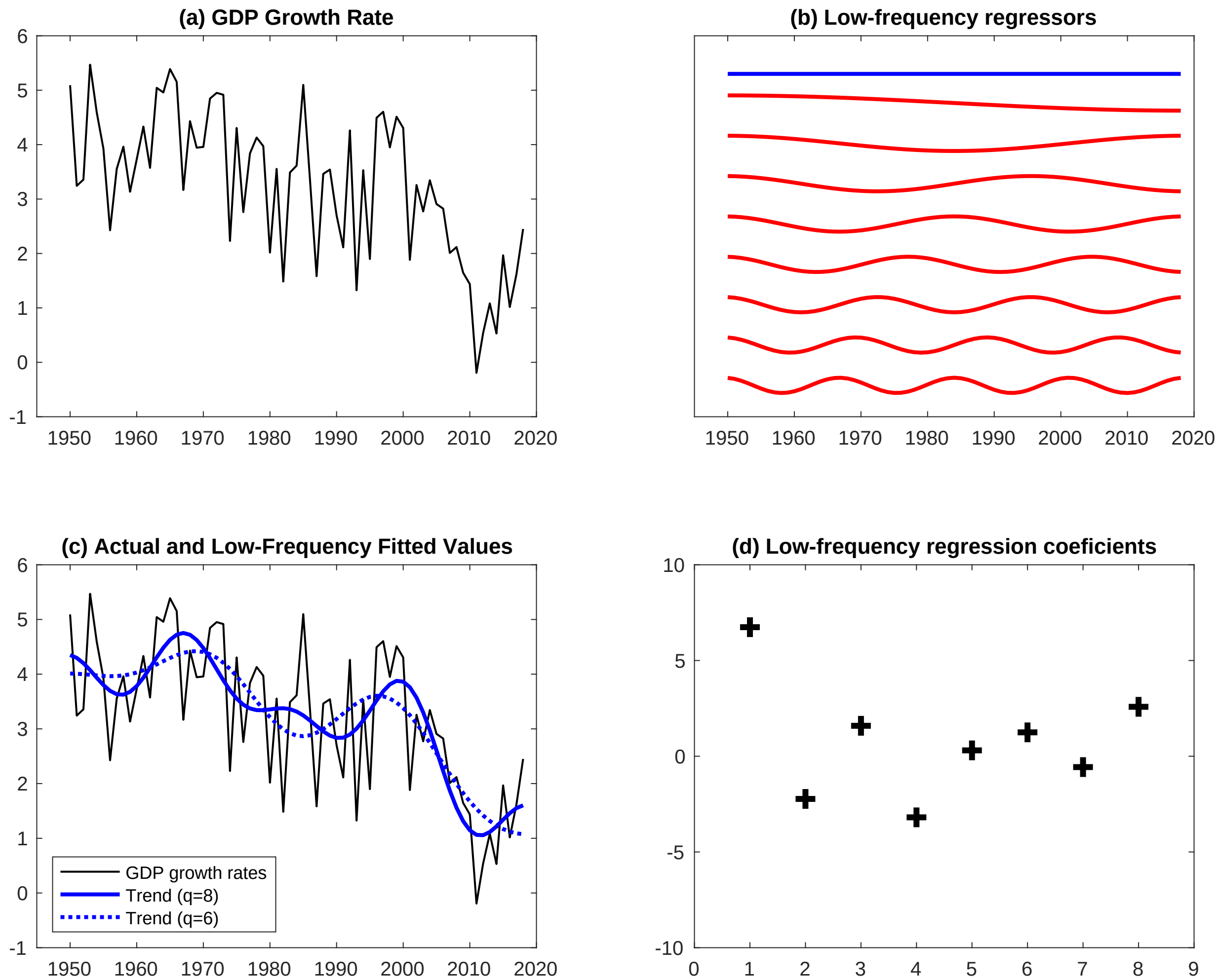
We use a low-frequency factor model. Written in terms of the growth rates of labor and TFP, the model is

$$\begin{bmatrix} \Delta \ln \ell_{i,t} \\ \Delta \ln z_{i,t} \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell & 0 \\ 0 & \lambda_i^z \end{bmatrix} \begin{bmatrix} f_t^\ell \\ f_t^z \end{bmatrix} + \begin{bmatrix} u_{i,t}^\ell \\ u_{i,t}^z \end{bmatrix}, \tag{1}$$

where $f_t = (f_t^\ell \ f_t^z)'$ are unobserved common factors, $\lambda_i = (\lambda_i^\ell \ \lambda_i^z)'$ are factor loadings, and $u_{i,t} = (u_{i,t}^\ell \ u_{it}^z)'$ are sector-specific disturbances. Written in terms of the cosine transforms $(\mathbf{X}_i^\ell, \mathbf{X}_i^z, \mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$, the factor model is:

$$\begin{bmatrix} \mathbf{X}_i^\ell \\ \mathbf{X}_i^z \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell I_q & 0 \\ 0 & \lambda_i^z I_q \end{bmatrix} \begin{bmatrix} \mathbf{F}^\ell \\ \mathbf{F}^z \end{bmatrix} + \begin{bmatrix} \mathbf{U}_i^\ell \\ \mathbf{U}_i^z \end{bmatrix}, \tag{2}$$

Figure A1: Computing low-frequency transformations



which characterizes the low-frequency variation and covariation in the data. See Müller and Watson (2020) for a detailed *Handbook* discussion related to low-frequency factor models. A key result from that analysis is that $(\mathbf{X}_i^\ell,\ \mathbf{X}_i^z, \mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$ is approximately normally distributed in large T samples, so that (2) can be estimated by standard factor analysis methods after parameterizing the various covariance matrices.

We use a 'local-level model' parameterization for each of the covariance matrices. This model parameterizes the low frequency spectrum (that is, the trend variation) by linearly combining a flat spectrum (from an $I(0)$ component) and a steeply decreasing spectrum (from an $I(1)$ component). In this model, the growth rate time series behave like the sum of independent $I(0)$ and $I(1)$ processes over the long run. The resulting covariance matrix of the cosine transforms depends on two parameters, (σ^2, γ), where σ is an overall scale parameter and γ governs the relative importance of the $I(0)$ and $I(1)$ components; larger values of σ

produce a more variable low frequency trend and larger values of γ produce a more persistent trend. Let $\mathbf{X}$ denote one of the $q \times 1$ vector of cosine transforms in $(\mathbf{F}^{\ell}, \mathbf{F}^{z}, \mathbf{U}_i^{\ell}, \mathbf{U}_i^{z})$. The resulting covariance matrix has the form:

$$\mathrm{Var}(\mathbf{X}) = \sigma_X^2 D(\gamma_X), \tag{3}$$

where the notation emphasizes that each component has its own (σ, γ) parameter. As shown in Müller and Watson (2020), the matrix $D(\gamma)$ is diagonal with $D_{jj}(\gamma) = 1 + \gamma^2(j\pi)^{-2}$.

We use this local-level parameterization to characterize the covariance matrix of each of the components in $(\mathbf{F}^{\ell}, \mathbf{F}^{z}, \{\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z}\}_{i=1}^{16})$, where each component has its own value of (σ, γ); thus, for example, $\mathrm{Var}(\mathbf{F}^{\ell}) = \sigma_{F,\ell}^2 D(\gamma_F^{\ell})$, and similarly for $\mathbf{F}^{z}$ and each of the $\mathbf{U}_i^{\ell}$ and $\mathbf{U}_i^{z}$ components. As in a standard factor model, we assume that $\mathbf{F}$ and $\mathbf{U}$ are uncorrelated, as are $\mathbf{U}_i$ and $\mathbf{U}_j$ for $i \neq j$. We allow $\mathbf{F}^{\ell}$ and $\mathbf{F}^{z}$ to be correlated by introducing a covariance parameter $\sigma_{F,\ell z}$ and letting $\mathrm{Cov}(\mathbf{F}^{\ell}, \mathbf{F}^{z}) = \sigma_{F,\ell z} D(\gamma_F^{\ell})^{1/2} D(\gamma_F^{z})^{1/2}$. We use an analogous parameterization for the covariance between each of the sectoral values of $\mathbf{U}_i^{\ell}$ and $\mathbf{U}_i^{z}$. Thus, each pair $(\mathbf{F}^{\ell}, \mathbf{F}^{z})$ or $(\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z})$ is characterized by a parameter pair, $\gamma = (\gamma^{\ell}, \gamma^{z})$, that governs persistence and a 2×2 covariance matrix, say Σ, that includes $(\sigma_\ell^2, \sigma_z^2, \sigma_{\ell z})$.

1.3 Bayes Estimation

Our ultimate goal is to decompose the various sectoral and aggregate trends into components associated with the common factors (f^l, f^z) and sector specific terms $\{u_i^{\ell}, u_i^{z}\}_{i=1}^{16}$. As shown in Figure A1, the trends in these factors are deterministic functions of $(\mathbf{F}^{\ell}, \mathbf{F}^{z}, \{\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z}\}_{i=1}^{16})$. The probability distribution of $(\mathbf{F}^{\ell}, \mathbf{F}^{z}, \{\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z}\}_{i=1}^{16})$ given $\{\mathbf{X}_i^{\ell}, \mathbf{X}_i^{z}\}_{i=1}^{16}$ can be computed using standard signal extraction formulae, given values for the various parameters of the model. The probability distribution of the parameters given $\{\mathbf{X}_i^{\ell}, \mathbf{X}_i^{z}\}_{i=1}^{16}$ can be computed using standard Bayes methods. As discussed in Müller and Watson (2020), the likelihood is Gaussian and estimation is facilitated by using standard conjugate priors for some of the parameters and multinomial priors for others.

1.3.1 Priors

There are three sets of parameters in the model:

- The factor loadings, $(\lambda^{\ell}, \lambda^{z})$,

- The covariance matrices,

$$\Sigma_F = \begin{bmatrix} \sigma^2_{F,\ell} & \sigma_{F,\ell z} \\ \sigma_{F,z\ell} & \sigma^2_{F,z} \end{bmatrix} \text{ and } \Sigma_{U,i} = \begin{bmatrix} \sigma^2_{U,i,\ell} & \sigma_{U,i,\ell z} \\ \sigma_{U,i,z\ell} & \sigma^2_{U,i,z} \end{bmatrix} \text{ for } i = 1, ..., 16,$$

- The persistence parameters, $\gamma_F = (\gamma^\ell_F, \gamma^z_F)$ and $\gamma_{U,i} = (\gamma^\ell_{U,i}, \gamma^z_{U,i})$ for $i = 1, ..., 16$.

As discussed in the text, we use the following independent priors for these parameters:

- $\lambda^\ell \sim N(\mathbf{1}, P_\ell)$, where $\mathbf{1}$ is a $q \times 1$ vector of ones and $\mathbf{P}_\ell = \eta^2(\mathbf{I}_n - s_\ell(s'_\ell s_\ell)^{-1}s'_\ell)$, where s_ℓ is the vector of sectoral labor shares. The prior for λ^z is analogous, but uses s_z, the sectoral TFP shares. The parameter η governs the tightness of the prior. The benchmark specification uses $\eta = 1$, and results using $\eta = 0.5$ and $\eta = 2.0$ are summarized in the text and Tables A2-A3.

- For each Σ matrix, we use an inverse-Wishart with $\nu = 0.01$ degrees of freedom and scale $\nu\mathbf{I}_2$. The small value for ν makes this prior nearly uninformative.

- Unlike for λ and Σ, there isn't a conjugate prior for γ. We use a prior with $\ln(\gamma) \sim U(0, \ln(500))$. This puts relatively more weight on small values of γ, i.e., small weight on the $I(1)$ component of the local-level model (consistent with a body of evidence beginning in Stock and Watson (1998)) but allows for low-frequency behavior dominated by $I(1)$ dynamics. We appoximate this prior by a 15-point equally-spaced discrete grid on $(0, \ln(500))$, with equal prior weight on each of the grid points.

1.3.2 MCMC Algorithm

Müller and Watson (2020) discusses an MCMC algorithm for a closely related model. The steps are standard and are outlined here. Let $\mathbf{X} = \{\mathbf{X}^\ell_i, \mathbf{X}^z_i\}^{16}_{i=1}$, $\mathbf{F} =(\mathbf{F}^\ell, \mathbf{F}^z)$, and $\mathbf{U} = \left\{\mathbf{U}^\ell_i, \mathbf{U}^z_i\right\}^{16}_{i=1}$. Let $\theta_1 = (\Sigma_F, \{\Sigma_{U,i}\}^{16}_{i=1})$, $\theta_2 = ((\gamma^\ell_F, \gamma^z_T), \left\{\gamma^\ell_{U,i}, \gamma^z_{U,i}\right\}^{16}_{i=1})$, $\theta_3 = (\lambda^\ell, \lambda^z)$, and $\theta = (\theta_1, \theta_2, \theta_3)$. The MCMC algorithm uses is a Gibbs algorithm with four steps:

1. Draw $\mathbf{F}$ from the distribution $\mathbf{F}|(\mathbf{X}, \theta)$. The distribution of $\mathbf{F}|(\mathbf{X}, \theta)$ is normal, and the draw uses standard multivariate normal formulae.

2. Draw θ_1 from the distribution of $\theta_1|(\mathbf{X}, \mathbf{F}, \theta_2, \theta_3)$. Given $\mathbf{F}$, $D(\gamma^\ell_F)$ and $D(\gamma^z_F)$, the draw of Σ_F is a draw from the inverse Wishart distribution. $\Sigma_{U,i}$ is drawn analogously given $\mathbf{U}^\ell_i = \mathbf{X}^\ell_i - \lambda^\ell_i\mathbf{F}^\ell$ and $\mathbf{U}^z_i = \mathbf{X}^z_i - \lambda^z_i\mathbf{F}^z$.

3. Draw θ_2 from the distribution of $\theta_2|(\mathbf{X}, \mathbf{F}, \theta_1, \theta_3)$. Given $\mathbf{F}^{\ell}$ and σ_F^{ℓ}, the likelihood for γ_F^{ℓ} can be computed at each of the grid points making up the support of γ_F^{ℓ}; the distribution of $\gamma_F^{\ell}|\mathbf{F}^{\ell}, \sigma_F^{\ell}$ is multinomial. Draws for the other γ parameters are similarly obtained.

4. Draw θ_3 from the the distribution of $\theta_3|(\mathbf{X}, \mathbf{F}, \theta_1, \theta_2)$. Note that this distribution is normal and corresponds to drawing linear regression coefficients in a regression model with a known covariance matrix for the regressions errors.

The results shown in the text were computed from 550,000 draws from this algorithm. The first 50k draws were discarded and every 200th draw of the remaining 500k draws were saved. The code was tested using the procedure outlined in Geweke (2004).

1.4 Results

Table A1 summarizes the posterior for the benchmark model ($q = 8$, $\eta = 1.0$). The entries in the table show the posterior median, 68% and 90% equal-tail credible intervals (in parentheses and brackets, respectively) for each of the model parameters. Also shown are the fraction of the variability of the trends in labor and TFP explained by the common factors, R_{ℓ}^2 and R_z^2, and the correlation of the sector-specific labor and TFP trends (labeled $corr(\ell, z)$ in the table). The final row of the table shows the parameters associated with the factors, where in this row the R^2 measures the fraction of variance in aggregate labor and TFP explained by the common factors and $corr(\ell, z)$ shows the correlation of the trends in the common factors. Tables A2-A4 show results for the alternative models presented in Figures 11 in the text.

Table A1: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 1.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	2.01	0.59	1.14	5.88	-5.35	45.80	1.00	0.21	0.02	-0.32
	(1.24,2.71)	(-0.59, 1.64)	(0.38, 2.41)	(4.46, 7.99)	(-13.22, -1.62)	(17.61, 119.16)	(0.00, 2.60)	(0.06, 0.44)	(0.00, 0.13)	(-0.52, -0.15)
	[0.51, 3.15]	[-1.30, 2.23]	[0.18, 3.57]	[3.65, 9.95]	[-24.62, -0.62]	[10.92, 310.00]	[0.00, 6.77]	[0.01, 0.60]	[0.00, 0.34]	[-0.69, -0.05]
Min	0.73	1.10	10.36	11.08	-38.63	1.61	4.20	0.01	0.01	-0.35
	(-0.17, 1.64)	(0.10, 2.09)	(7.58, 14.29)	(5.94, 15.82)	(-111.87, -3.35)	(0.00, 6.77)	(1.00, 28.40)	(0.00, 0.07)	(0.00, 0.04)	(-0.63, -0.06)
	[-0.83, 2.35]	[-0.61, 2.81]	[3.71, 18.26]	[1.05, 20.14]	[-220.92, 13.71]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.18]	[0.00, 0.10]	[-0.78, 0.11]
Utl	1.13	1.36	2.01	4.28	1.17	6.77	2.60	0.24	0.05	0.22
	(0.41, 1.82)	(0.36, 2.35)	(0.48, 3.13)	(2.08, 6.12)	(-0.33, 6.46)	(1.00, 73.88)	(1.00, 28.40)	(0.04, 0.58)	(0.00, 0.29)	(-0.06, 0.58)
	[-0.16, 2.31]	[-0.32, 3.08]	[0.16, 4.18]	[0.41, 8.08]	[-3.71, 14.71]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.78]	[0.00, 0.63]	[-0.33, 0.77]
Con	1.55	1.26	3.45	1.22	-2.05	1.61	45.80	0.33	0.02	-0.25
	(0.95, 2.08)	(0.21, 2.66)	(2.57, 4.73)	(0.30, 3.80)	(-8.83, -0.26)	(0.00, 4.20)	(10.92, 192.20)	(0.10, 0.61)	(0.00, 0.19)	(-0.55, -0.04)
	[0.42, 2.49]	[-0.50, 3.24]	[1.95, 6.09]	[0.15, 6.71]	[-19.53, 0.10]	[0.00, 17.61]	[4.20, 500.00]	[0.02, 0.77]	[0.00, 0.42]	[-0.72, 0.02]
DurG	0.40	1.31	2.15	4.80	-2.16	10.92	6.77	0.03	0.03	-0.35
	(-0.23, 1.03)	(0.44, 2.17)	(0.59, 3.72)	(1.17, 7.44)	(-12.28, -0.10)	(4.20, 73.88)	(1.00, 73.88)	(0.00, 0.18)	(0.00, 0.15)	(-0.63, -0.05)
	[-0.67, 1.45]	[-0.10, 2.72]	[0.17, 5.38]	[0.29, 9.61]	[-26.63, 0.75]	[1.00, 310.00]	[0.00, 310.00]	[0.00, 0.39]	[0.00, 0.38]	[-0.77, 0.16]
NdG	0.59	1.22	2.48	3.97	-2.24	6.77	6.77	0.06	0.04	-0.36
	(-0.20, 1.38)	(0.36, 2.13)	(0.49, 4.01)	(2.40, 5.79)	(-9.03, -0.17)	(1.00, 119.16)	(1.61, 17.61)	(0.01, 0.29)	(0.00, 0.23)	(-0.65, -0.06)
	[-0.75, 1.86]	[-0.26, 2.80]	[0.17, 5.29]	[0.62, 7.50]	[-18.98, 1.77]	[0.00, 310.00]	[0.00, 119.16]	[0.00, 0.54]	[0.00, 0.52]	[-0.80, 0.15]
WT	1.09	0.88	1.20	2.05	0.34	4.20	10.92	0.53	0.04	0.20
	(0.62, 1.49)	(0.06, 1.74)	(0.65, 1.81)	(0.37, 3.71)	(-0.06, 2.08)	(1.00, 28.40)	(1.61, 119.16)	(0.17, 0.81)	(0.00, 0.20)	(-0.06, 0.53)
	[0.22, 1.82]	[-0.55, 2.35]	[0.18, 2.44]	[0.15, 4.97]	[-0.84, 4.65]	[0.00, 119.16]	[0.00, 310.00]	[0.03, 0.90]	[0.00, 0.44]	[-0.30, 0.72]
RT	0.80	1.14	1.76	3.42	0.24	2.60	2.60	0.26	0.05	0.06
	(0.26, 1.29)	(0.17, 2.82)	(1.02, 2.51)	(0.92, 5.05)	(-1.85, 2.42)	(1.00, 17.61)	(1.00, 10.92)	(0.04, 0.60)	(0.00, 0.85)	(-0.25, 0.62)
	[-0.19, 1.65]	[-0.49, 3.48]	[0.26, 3.29]	[0.31, 6.57]	[-5.40, 5.41]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.77]	[0.00, 0.98]	[-0.51, 0.94]
TW	-0.04	0.88	2.91	3.04	0.35	4.20	2.60	0.05	0.06	0.06
	(-0.75, 0.72)	(-0.02, 1.79)	(1.91, 4.12)	(2.15, 4.26)	(-2.73, 3.67)	(1.00, 10.92)	(1.00, 10.92)	(0.00, 0.23)	(0.00, 0.28)	(-0.25, 0.36)
	[-1.23, 1.33]	[-0.62, 2.40]	[0.61, 5.29]	[0.90, 5.49]	[-7.43, 8.65]	[0.00, 73.88]	[0.00, 45.80]	[0.00, 0.43]	[0.00, 0.54]	[-0.49, 0.59]
Inf	1.34	0.77	3.81	3.05	-2.54	1.61	4.20	0.22	0.03	-0.25
	(0.69, 2.01)	(-0.18, 1.81)	(2.80, 5.31)	(0.70, 4.58)	(-9.15, -0.03)	(0.00, 6.77)	(1.00, 73.88)	(0.04, 0.51)	(0.00, 0.19)	(-0.56, -0.00)
	[0.19, 2.47]	[-0.75, 2.79]	[1.48, 7.03]	[0.20, 6.02]	[-19.33, 3.42]	[0.00, 45.80]	[0.00, 310.00]	[0.01, 0.70]	[0.00, 0.49]	[-0.74, 0.21]
F(x-H)	1.92	0.35	1.59	1.59	0.02	2.60	2.60	0.76	0.08	0.01
	(1.34, 2.48)	(-0.42, 1.34)	(1.01, 2.54)	(0.90, 2.30)	(-1.02, 1.20)	(1.00, 10.92)	(0.00, 10.92)	(0.35, 0.92)	(0.01, 0.40)	(-0.41, 0.40)
	[0.76, 2.90]	[-0.92, 2.18]	[0.47, 3.44]	[0.27, 2.97]	[-2.72, 2.89]	[0.00, 28.40]	[0.00, 73.88]	[0.08, 0.97]	[0.00, 0.80]	[-0.73, 0.62]
PBS	1.87	0.90	1.42	1.65	-1.90	6.77	10.92	0.64	0.06	-0.92
	(1.48, 2.29)	(-0.01, 1.80)	(0.52, 2.65)	(0.59, 3.20)	(-7.57, -0.31)	(1.61, 45.80)	(2.60, 45.80)	(0.31, 0.87)	(0.00, 0.39)	(-0.98, -0.67)
	[1.15, 2.63]	[-0.40, 2.42]	[0.24, 3.66]	[0.25, 4.41]	[-14.38, -0.07]	[0.00, 73.88]	[1.00, 119.16]	[0.12, 0.96]	[0.00, 0.73]	[-1.00, -0.32]
EdHe	0.59	1.36	1.42	2.23	-1.84	6.77	4.20	0.16	0.10	-0.63
	(-0.06, 1.05)	(0.24, 2.49)	(0.56, 2.18)	(0.66, 3.68)	(-5.17, -0.38)	(1.00, 45.80)	(1.00, 45.80)	(0.01, 0.56)	(0.01, 0.54)	(-0.88, -0.26)
	[-0.49, 1.40]	[-0.41, 3.09]	[0.23, 3.03]	[0.22, 4.92]	[-10.12, -0.09]	[0.00, 119.16]	[0.00, 192.20]	[0.00, 0.77]	[0.00, 0.84]	[-0.96, -0.10]
AEFS	1.19	0.37	0.53	2.04	-0.22	45.80	2.60	0.37	0.05	-0.18
	(0.69, 1.75)	(-0.39, 1.31)	(0.17, 1.39)	(1.16, 2.89)	(-1.20, 0.02)	(10.92, 119.16)	(1.00, 17.61)	(0.11, 0.67)	(0.00, 0.26)	(-0.51, 0.02)
	[0.27, 2.12]	[-0.93, 2.06]	[0.09, 2.36]	[0.28, 3.73]	[-2.64, 0.56]	[2.60, 192.20]	[0.00, 119.16]	[0.02, 0.83]	[0.00, 0.51]	[-0.75, 0.24]
OthS	0.68	0.74	3.10	0.65	-0.13	6.77	73.88	0.06	0.02	-0.07
	(-0.10, 1.48)	(-0.10, 1.63)	(0.66, 4.83)	(0.19, 2.22)	(-1.78, 0.23)	(1.00, 73.88)	(17.61, 192.20)	(0.01, 0.23)	(0.00, 0.10)	(-0.35, 0.17)
	[-0.61, 2.05]	[-0.65, 2.26]	[0.20, 6.31]	[0.11, 4.30]	[-7.00, 1.22]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.42]	[0.00, 0.26]	[-0.56, 0.45]
Hous	0.82	0.75	10.38	1.31	0.32	6.77	6.77	0.01	0.10	0.07
	(-0.13, 1.74)	(0.08, 1.50)	(1.82, 15.95)	(0.43, 2.10)	(-2.98, 5.61)	(1.00, 119.16)	(1.00, 45.80)	(0.00, 0.04)	(0.01, 0.44)	(-0.21, 0.40)
	[-0.77, 2.34]	[-0.38, 2.17]	[0.53, 20.39]	[0.15, 2.85]	[-11.58, 16.28]	[0.00, 310.00]	[0.00, 192.20]	[0.00, 0.10]	[0.00, 0.74]	[-0.44, 0.64]
Fac			0.48	0.69	-0.08	28.40	4.20	0.67	0.30	-0.29
			(0.18, 1.05)	(0.28, 1.33)	(-0.46, 0.05)	(10.92, 73.88)	(1.00, 28.40)	(0.48, 0.82)	(0.10, 0.58)	(-0.35, 0.22)
			[0.10, 1.58]	[0.13, 1.85]	[-1.17, 0.36]	[2.60, 119.16]	[0.00, 73.88]	[0.34, 0.89]	[0.03, 0.75]	[-0.25, -0.35]

Notes: For each sector, the entries are the posterior median and (68%) and [90%] credible intervals for each parameter in the model. Also shown are the fraction of variance explained by the common factor (R_ℓ^2 and R_z^2) and the correlation between between the trends associated with the u components (labeled $corr(\ell, z)$). The row labeled *Fac* shows the parameter values for the factors, the fraction of variance for aggregate labor and TFP explained by the common factors (R_ℓ^2 and R_z^2), and the correlation of factor trends (labeled $corr(\ell, z)$).

Table A2: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 0.5$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	1.38	0.84	1.19	6.06	-5.32	45.80	1.61	0.15	0.01	-0.32
	(0.90, 1.84)	(0.36, 1.35)	(0.36, 2.48)	(4.68, 8.17)	(-13.50, -1.48)	(17.61, 192.20)	(0.00, 4.20)	(0.05, 0.36)	(0.00, 0.05)	(-0.54, -0.14)
	[0.54, 2.11]	[-0.01, 1.68]	[0.17, 3.73]	[3.82, 10.37]	[-25.68, -0.57]	[10.92, 310.00]	[0.00, 6.77]	[0.02, 0.54]	[0.00, 0.13]	[-0.70, -0.04]
Min	0.91	1.04	10.34	11.06	-39.25	2.60	4.20	0.02	0.00	-0.35
	(0.45, 1.37)	(0.55, 1.49)	(7.51, 14.22)	(6.22, 15.77)	(-108.08, -3.10)	(0.00, 6.77)	(1.00, 17.61)	(0.00, 0.08)	(0.00, 0.02)	(-0.62, -0.06)
	[0.14, 1.72]	[0.18, 1.80]	[3.37, 18.06]	[1.02, 19.90]	[-209.93, 12.34]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.16]	[0.00, 0.05]	[-0.76, 0.11]
Utl	1.09	1.12	1.95	4.24	1.35	4.20	2.60	0.33	0.02	0.26
	(0.66, 1.51)	(0.61, 1.57)	(0.58, 3.01)	(2.11, 6.07)	(-0.03, 6.51)	(1.00, 73.88)	(1.00, 28.40)	(0.10, 0.64)	(0.00, 0.14)	(-0.00, 0.60)
	[0.35, 1.81]	[0.27, 1.90]	[0.19, 3.98]	[0.38, 7.82]	[-2.98, 14.31]	[0.00, 192.20]	[0.00, 192.20]	[0.02, 0.83]	[0.00, 0.34]	[-0.25, 0.79]
Con	1.15	1.02	3.61	0.98	-1.43	1.61	73.88	0.27	0.01	-0.20
	(0.78, 1.52)	(0.52, 1.45)	(2.69, 4.93)	(0.26, 3.75)	(-7.47, -0.24)	(0.00, 4.20)	(10.92, 310.00)	(0.08, 0.55)	(0.00, 0.05)	(-0.45, -0.03)
	[0.47, 1.75]	[0.19, 1.76]	[2.10, 6.32]	[0.14, 6.62]	[-17.67, 0.12]	[0.00, 10.92]	[4.20, 500.00]	[0.03, 0.70]	[0.00, 0.12]	[-0.64, 0.02]
DurG	0.78	1.09	2.31	4.61	-2.02	10.92	6.77	0.11	0.01	-0.29
	(0.41, 1.15)	(0.66, 1.50)	(0.67, 3.88)	(1.11, 7.13)	(-11.07, -0.02)	(2.60, 73.88)	(1.00, 73.88)	(0.02, 0.31)	(0.00, 0.08)	(-0.61, -0.01)
	[0.16, 1.42]	[0.35, 1.81]	[0.18, 5.46]	[0.26, 9.33]	[-25.20, 1.70]	[1.00, 310.00]	[0.00, 310.00]	[0.00, 0.51]	[0.00, 0.22]	[-0.75, 0.21]
NdG	0.89	1.06	2.78	4.09	-3.05	4.20	6.77	0.16	0.02	-0.37
	(0.48, 1.32)	(0.59, 1.52)	(0.79, 4.12)	(2.62, 5.88)	(-10.78, -0.29)	(1.00, 73.88)	(1.61, 17.61)	(0.03, 0.41)	(0.00, 0.10)	(-0.65, -0.07)
	[0.16, 1.58]	[0.29, 1.87]	[0.20, 5.44]	[0.85, 7.69]	[-20.14, 1.54]	[0.00, 310.00]	[0.00, 119.16]	[0.01, 0.60]	[0.00, 0.29]	[-0.78, 0.13]
WT	0.94	0.94	1.21	2.12	0.19	4.20	10.92	0.55	0.03	0.13
	(0.64, 1.24)	(0.48, 1.43)	(0.59, 1.83)	(0.37, 3.74)	(-0.20, 1.77)	(1.00, 28.40)	(1.61, 119.16)	(0.22, 0.80)	(0.00, 0.12)	(-0.12, 0.44)
	[0.41, 1.46]	[0.15, 1.73]	[0.17, 2.53]	[0.14, 5.03]	[-1.55, 4.45]	[0.00, 119.16]	[0.00, 310.00]	[0.07, 0.90]	[0.00, 0.32]	[-0.36, 0.65]
RT	0.84	0.99	1.75	3.81	0.00	2.60	2.60	0.39	0.03	0.00
	(0.53, 1.16)	(0.53, 1.43)	(1.14, 2.44)	(2.56, 5.27)	(-2.36, 2.23)	(1.00, 10.92)	(1.00, 10.92)	(0.14, 0.68)	(0.00, 0.13)	(-0.30, 0.29)
	[0.31, 1.40]	[0.22, 1.75]	[0.31, 3.24]	[0.54, 6.68]	[-6.20, 5.31]	[0.00, 119.16]	[0.00, 119.16]	[0.03, 0.81]	[0.00, 0.30]	[-0.54, 0.51]
TW	0.61	0.97	3.04	3.11	0.11	4.20	2.60	0.07	0.04	0.01
	(0.14, 1.06)	(0.50, 1.45)	(1.85, 4.43)	(2.24, 4.26)	(-3.84, 3.19)	(1.00, 17.61)	(1.00, 10.92)	(0.01, 0.23)	(0.00, 0.19)	(-0.31, 0.31)
	[-0.17, 1.41]	[0.19, 1.76]	[0.42, 5.87]	[1.42, 5.70]	[-10.11, 7.66]	[0.00, 119.16]	[0.00, 28.40]	[0.00, 0.42]	[0.00, 0.41]	[-0.56, 0.52]
Inf	1.14	0.96	3.78	3.11	-2.26	1.61	4.20	0.23	0.02	-0.22
	(0.72, 1.56)	(0.51, 1.44)	(2.85, 5.21)	(0.75, 4.59)	(-9.08, 0.16)	(0.00, 6.77)	(1.00, 73.88)	(0.07, 0.51)	(0.00, 0.15)	(-0.53, 0.02)
	[0.40, 1.82]	[0.19, 1.76]	[1.99, 6.88]	[0.20, 5.95]	[-18.68, 3.29]	[0.00, 28.40]	[0.00, 310.00]	[0.02, 0.68]	[0.00, 0.36]	[-0.72, 0.22]
F(x-H)	1.40	0.79	1.83	1.70	0.13	2.60	1.61	0.64	0.08	0.05
	(0.99, 1.76)	(0.38, 1.24)	(1.23, 2.84)	(1.18, 2.40)	(-1.01, 1.62)	(1.00, 10.92)	(0.00, 10.92)	(0.26, 0.87)	(0.01, 0.33)	(-0.30, 0.40)
	[0.67, 2.02]	[0.08, 1.57]	[0.85, 3.81]	[0.45, 3.12]	[-2.81, 3.77]	[0.00, 28.40]	[0.00, 45.80]	[0.08, 0.94]	[0.00, 0.58]	[-0.55, 0.62]
PBS	1.49	1.08	1.50	1.56	-1.79	6.77	10.92	0.61	0.06	-0.87
	(1.20, 1.77)	(0.61, 1.52)	(0.51, 2.84)	(0.53, 3.05)	(-6.90, -0.28)	(1.00, 28.40)	(2.60, 45.80)	(0.29, 0.86)	(0.01, 0.37)	(-0.98, -0.54)
	[0.95, 1.96]	[0.26, 1.84]	[0.22, 3.90]	[0.22, 4.42]	[-14.13, -0.07]	[0.00, 73.88]	[1.00, 119.16]	[0.12, 0.96]	[0.00, 0.68]	[-1.00, -0.23]
EdHe	0.73	1.33	1.51	2.02	-1.66	2.60	6.77	0.39	0.08	-0.58
	(0.42, 1.02)	(0.80, 1.87)	(0.87, 2.17)	(0.52, 3.39)	(-4.69, -0.34)	(1.00, 17.61)	(1.00, 73.88)	(0.10, 0.70)	(0.01, 0.50)	(-0.85, -0.21)
	[0.11, 1.29]	[0.40, 2.20]	[0.32, 2.89]	[0.19, 4.54]	[-8.49, -0.07]	[0.00, 73.88]	[0.00, 192.20]	[0.01, 0.87]	[0.00, 0.88]	[-0.95, -0.05]
AEFS	1.09	0.83	0.48	2.08	-0.19	45.80	2.60	0.43	0.04	-0.16
	(0.75, 1.45)	(0.31, 1.33)	(0.17, 1.27)	(1.25, 2.91)	(-1.12, 0.05)	(10.92, 119.16)	(1.00, 17.61)	(0.17, 0.71)	(0.00, 0.23)	(-0.48, 0.04)
	[0.49, 1.70]	[-0.02, 1.68]	[0.09, 2.13]	[0.31, 3.80]	[-2.56, 0.76]	[4.20, 192.20]	[0.00, 119.16]	[0.07, 0.83]	[0.00, 0.48]	[-0.71, 0.27]
OthS	0.88	0.91	2.99	0.60	-0.13	10.92	73.88	0.10	0.02	-0.09
	(0.42, 1.32)	(0.43, 1.42)	(0.52, 4.88)	(0.18, 2.16)	(-1.89, 0.15)	(1.61, 119.16)	(17.61, 192.20)	(0.02, 0.26)	(0.00, 0.08)	(-0.37, 0.15)
	[0.12, 1.62]	[0.10, 1.73]	[0.19, 6.50]	[0.10, 3.97]	[-7.00, 0.95]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.43]	[0.00, 0.20]	[-0.59, 0.43]
Hous	0.96	0.94	10.14	1.31	0.41	6.77	6.77	0.01	0.11	0.08
	(0.48, 1.46)	(0.50, 1.38)	(1.59, 15.88)	(0.45, 2.02)	(-2.00, 5.67)	(1.00, 119.16)	(1.00, 45.80)	(0.00, 0.04)	(0.01, 0.45)	(-0.16, 0.41)
	[0.14, 1.76]	[0.20, 1.75]	[0.52, 20.41]	[0.15, 2.63]	[-8.94, 15.43]	[0.00, 310.00]	[0.00, 119.16]	[0.00, 0.09]	[0.00, 0.74]	[-0.40, 0.66]
Fac			0.48	0.54	-0.11	28.40	4.20	0.75	0.22	-0.37
			(0.19, 1.16)	(0.20, 1.13)	(-0.60, 0.00)	(10.92, 73.88)	(1.00, 28.40)	(0.58, 0.87)	(0.04, 0.53)	(-0.35, 0.26)
			[0.11, 1.80]	[0.10, 1.70]	[-1.58, 0.18]	[4.20, 192.20]	[0.00, 73.88]	[0.46, 0.92]	[0.01, 0.72]	[-0.20, -0.29]

Notes: See Notes for Table A1.

Table A3: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 2.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	2.90	1.18	1.34	5.50	-5.61	45.80	1.61	0.17	0.05	-0.35
	(1.56, 4.11)	(-0.95, 2.69)	(0.42, 2.72)	(4.17, 7.61)	(-14.63, -1.60)	(17.61, 119.16)	(0.00, 4.20)	(0.05, 0.39)	(0.00, 0.24)	(-0.55, -0.15)
	[0.38, 4.94]	[-2.52, 3.64]	[0.19, 3.97]	[3.38, 9.81]	[-26.65, -0.60]	[10.92, 310.00]	[0.00, 10.92]	[0.01, 0.57]	[0.00, 0.48]	[-0.72, -0.05]
Min	0.18	0.97	10.51	10.99	-45.62	1.61	4.20	0.01	0.01	-0.39
	(-1.58, 1.81)	(-1.13, 3.19)	(7.81, 14.55)	(5.84, 15.76)	(-121.76, -4.81)	(0.00, 6.77)	(1.00, 17.61)	(0.00, 0.07)	(0.00, 0.06)	(-0.67, -0.08)
	[-2.80, 3.20]	[-2.67, 5.10]	[4.31, 18.58]	[0.93, 20.14]	[-229.55, 11.08]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.18]	[0.00, 0.21]	[-0.80, 0.11]
Utl	1.32	1.52	1.99	4.42	0.68	6.77	2.60	0.16	0.05	0.14
	(-0.11, 2.47)	(-0.53, 3.72)	(0.45, 3.13)	(1.72, 6.45)	(-1.01, 6.38)	(1.00, 73.88)	(1.00, 17.61)	(0.02, 0.51)	(0.00, 0.35)	(-0.17, 0.53)
	[-1.24, 3.38]	[-1.86, 5.47]	[0.16, 4.13]	[0.37, 8.50]	[-5.21, 13.89]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.73]	[0.00, 0.90]	[-0.55, 0.73]
Con	2.53	3.49	3.25	1.60	-3.86	1.61	45.80	0.43	0.13	-0.37
	(1.67, 3.39)	(0.14, 4.69)	(2.42, 4.49)	(0.47, 3.75)	(-10.93, -0.48)	(0.00, 4.20)	(10.92, 119.16)	(0.16, 0.69)	(0.01, 0.35)	(-0.70, -0.10)
	[0.56, 4.03]	[-1.60, 5.46]	[1.85, 5.76]	[0.19, 5.81]	[-19.45, -0.01]	[0.00, 10.92]	[6.77, 310.00]	[0.02, 0.83]	[0.00, 0.58]	[-0.84, -0.00]
DurG	-0.42	1.15	1.70	5.17	-2.10	17.61	4.20	0.04	0.02	-0.32
	(-1.56, 0.63)	(-0.17, 2.53)	(0.35, 3.40)	(1.71, 7.64)	(-11.00, -0.10)	(6.77, 119.16)	(1.00, 45.80)	(0.00, 0.16)	(0.00, 0.11)	(-0.61, -0.06)
	[-2.50, 1.39]	[-1.15, 3.56]	[0.13, 4.80]	[0.33, 9.75]	[-23.25, 0.45]	[1.61, 310.00]	[0.00, 310.00]	[0.00, 0.32]	[0.00, 0.26]	[-0.75, 0.10]
NdG	-0.21	1.75	1.50	3.85	-1.13	28.40	6.77	0.06	0.06	-0.32
	(-1.64, 1.30)	(0.19, 3.17)	(0.28, 3.46)	(2.26, 5.63)	(-5.97, -0.03)	(2.60, 119.16)	(1.61, 17.61)	(0.01, 0.27)	(0.00, 0.27)	(-0.63, -0.01)
	[-2.55, 2.29]	[-1.04, 4.30]	[0.13, 4.88]	[0.65, 7.39]	[-13.61, 2.50]	[1.00, 310.00]	[0.00, 119.16]	[0.00, 0.50]	[0.00, 0.53]	[-0.81, 0.20]
WT	1.62	1.49	1.30	1.88	0.73	4.20	17.61	0.56	0.08	0.30
	(0.94, 2.26)	(-0.05, 2.72)	(0.84, 1.90)	(0.40, 3.71)	(0.05, 2.78)	(1.00, 17.61)	(1.61, 119.16)	(0.18, 0.83)	(0.01, 0.28)	(0.03, 0.62)
	[0.19, 2.79]	[-1.24, 3.71]	[0.30, 2.51]	[0.16, 4.98]	[-0.37, 6.03]	[0.00, 73.88]	[0.00, 310.00]	[0.02, 0.90]	[0.00, 0.51]	[-0.18, 0.79]
RT	0.72	3.87	1.73	1.35	0.54	4.20	2.60	0.13	0.79	0.33
	(-0.32, 1.59)	(0.24, 5.23)	(0.73, 2.53)	(0.45, 4.16)	(-0.40, 2.45)	(1.00, 28.40)	(1.00, 10.92)	(0.01, 0.47)	(0.02, 0.98)	(-0.10, 0.90)
	[-1.06, 2.22]	[-1.37, 6.05]	[0.21, 3.30]	[0.19, 5.66]	[-2.90, 5.05]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.70]	[0.00, 1.00]	[-0.43, 0.98]
TW	-1.07	0.29	2.63	3.04	0.25	4.20	2.60	0.13	0.06	0.05
	(-2.15, 0.03)	(-1.22, 2.23)	(1.62, 3.74)	(2.07, 4.28)	(-2.18, 3.40)	(1.00, 17.61)	(1.00, 10.92)	(0.01, 0.42)	(0.00, 0.28)	(-0.24, 0.36)
	[-3.02, 1.05]	[-2.17, 3.42]	[0.40, 4.94]	[0.69, 5.44]	[-6.31, 8.21]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.64]	[0.00, 0.55]	[-0.47, 0.57]
Inf	1.85	-0.18	3.85	2.68	-2.37	1.61	4.20	0.18	0.06	-0.26
	(0.69, 3.01)	(-1.81, 2.31)	(2.81, 5.36)	(0.58, 4.29)	(-8.82, -0.02)	(0.00, 6.77)	(1.00, 73.88)	(0.03, 0.49)	(0.00, 0.38)	(-0.60, -0.00)
	[-0.38, 3.86]	[-2.92, 5.37]	[1.42, 7.01]	[0.19, 5.75]	[-18.15, 2.99]	[0.00, 45.80]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.81]	[-0.82, 0.22]
F(x-H)	3.26	-0.53	1.16	1.42	-0.08	2.60	2.60	0.89	0.20	-0.07
	(2.35, 4.21)	(-1.54, 1.33)	(0.55, 2.02)	(0.54, 2.08)	(-0.91, 0.89)	(1.00, 10.92)	(1.00, 17.61)	(0.60, 0.98)	(0.01, 0.71)	(-0.71, 0.46)
	[1.51, 4.93]	[-2.33, 3.19]	[0.26, 2.92]	[0.21, 2.73]	[-2.26, 2.28]	[0.00, 28.40]	[0.00, 73.88]	[0.17, 0.99]	[0.00, 0.95]	[-0.94, 0.74]
PBS	2.42	-0.05	1.36	1.57	-1.65	17.61	17.61	0.47	0.02	-0.95
	(1.81, 3.10)	(-0.73, 1.24)	(0.50, 2.74)	(0.58, 3.21)	(-8.16, -0.30)	(1.61, 45.80)	(4.20, 45.80)	(0.19, 0.75)	(0.00, 0.12)	(-0.99, -0.82)
	[1.35, 3.66]	[-1.19, 2.36]	[0.25, 3.83]	[0.27, 4.44]	[-15.10, -0.10]	[0.00, 119.16]	[1.00, 119.16]	[0.08, 0.89]	[0.00, 0.34]	[-1.00, -0.56]
EdHe	0.16	0.33	1.09	2.68	-1.47	17.61	4.20	0.06	0.03	-0.57
	(-0.69, 1.13)	(-0.81, 2.05)	(0.32, 2.03)	(0.74, 4.10)	(-5.00, -0.30)	(1.61, 73.88)	(1.00, 45.80)	(0.00, 0.36)	(0.00, 0.20)	(-0.84, -0.24)
	[-1.29, 1.66]	[-1.59, 3.40]	[0.16, 2.82]	[0.22, 5.31]	[-9.97, -0.08]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.61]	[0.00, 0.51]	[-0.92, -0.08]
AEFS	1.30	-0.16	0.55	2.05	-0.20	45.80	2.60	0.20	0.07	-0.14
	(0.39, 2.14)	(-1.31, 1.38)	(0.17, 1.57)	(1.20, 2.91)	(-1.17, 0.05)	(10.92, 119.16)	(1.00, 17.61)	(0.03, 0.51)	(0.01, 0.35)	(-0.45, 0.03)
	[-0.23, 2.87]	[-2.02, 2.63]	[0.09, 2.67]	[0.36, 3.77]	[-2.91, 0.76]	[2.60, 310.00]	[0.00, 73.88]	[0.00, 0.73]	[0.00, 0.60]	[-0.69, 0.24]
OthS	0.20	0.31	3.30	0.64	-0.12	4.20	73.88	0.04	0.02	-0.05
	(-0.98, 1.50)	(-0.86, 1.66)	(0.90, 4.93)	(0.19, 2.37)	(-1.89, 0.30)	(1.00, 73.88)	(17.61, 192.20)	(0.00, 0.18)	(0.00, 0.08)	(-0.34, 0.15)
	[-1.89, 2.51]	[-1.79, 2.66]	[0.23, 6.31]	[0.11, 4.43]	[-7.35, 1.50]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.37]	[0.00, 0.20]	[-0.55, 0.41]
Hous	0.48	0.35	10.62	1.32	0.12	4.20	10.92	0.01	0.05	0.03
	(-1.35, 2.27)	(-0.49, 1.39)	(2.21, 15.90)	(0.38, 2.15)	(-4.27, 4.45)	(1.00, 73.88)	(1.61, 73.88)	(0.00, 0.04)	(0.00, 0.27)	(-0.25, 0.36)
	[-2.61, 3.53]	[-1.14, 2.37]	[0.62, 20.75]	[0.14, 2.78]	[-13.11, 14.19]	[0.00, 310.00]	[0.00, 192.20]	[0.00, 0.12]	[0.00, 0.53]	[-0.49, 0.61]
Fac			0.44	0.71	-0.00	17.61	2.60	0.46	0.22	-0.00
			(0.17, 0.85)	(0.38, 1.06)	(-0.18, 0.14)	(6.77, 73.88)	(1.00, 10.92)	(0.28, 0.66)	(0.09, 0.44)	(-0.39, 0.14)
			[0.10, 1.21]	[0.19, 1.45]	[-0.51, 0.40]	[1.61, 119.16]	[0.00, 45.80]	[0.17, 0.80]	[0.04, 0.61]	[-0.37, -0.32]

Notes: See Notes for Table A1.

Table A4: Posterior Summary for Benchmark Model, $q = 6$ and $\eta = 1.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l, z)$
Agr	1.20	0.72	0.60	3.04	-0.56	73.88	4.20	0.18	0.08	-0.28
	(0.51, 1.87)	(0.07, 1.41)	(0.20, 1.86)	(0.48, 5.27)	(-3.66, -0.02)	(17.61, 192.20)	(1.00, 73.88)	(0.03, 0.48)	(0.01, 0.32)	(-0.60, -0.03)
	[-0.11, 2.32]	[-0.44, 1.96]	[0.11, 3.89]	[0.15, 7.39]	[-11.80, 0.23]	[10.92, 310.00]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.57]	[-0.76, 0.23]
Min	0.56	1.63	10.71	2.79	-14.47	2.60	45.80	0.02	0.04	-0.37
	(-0.32, 1.47)	(0.61, 2.58)	(7.10, 16.05)	(0.52, 10.01)	(-73.84, -2.13)	(0.00, 10.92)	(4.20, 192.20)	(0.00, 0.09)	(0.00, 0.19)	(-0.71, -0.11)
	[-0.96, 2.15]	[-0.03, 3.22]	[1.19, 21.76]	[0.26, 15.41]	[-179.11, -0.18]	[0.00, 119.16]	[1.00, 500.00]	[0.00, 0.24]	[0.00, 0.42]	[-0.85, -0.01]
Utl	1.27	2.87	2.35	2.47	-3.71	4.20	4.20	0.44	0.72	-0.79
	(0.71, 1.68)	(1.99, 3.50)	(1.14, 3.62)	(0.98, 4.06)	(-11.52, -0.39)	(1.00, 28.40)	(1.00, 28.40)	(0.12, 0.75)	(0.30, 0.91)	(-0.97, -0.24)
	[0.22, 2.00]	[1.12, 3.94]	[0.31, 5.18]	[0.25, 5.96]	[-24.64, 0.19]	[0.00, 119.16]	[0.00, 119.16]	[0.02, 0.86]	[0.05, 0.97]	[-0.99, 0.05]
Con	1.29	1.87	3.05	1.09	-1.83	1.61	45.80	0.42	0.20	-0.33
	(0.85, 1.73)	(1.05, 2.58)	(2.08, 4.61)	(0.28, 3.50)	(-8.59, -0.25)	(0.00, 6.77)	(10.92, 192.20)	(0.14, 0.71)	(0.03, 0.58)	(-0.66, -0.09)
	[0.43, 2.07]	[0.46, 3.10]	[0.92, 6.23]	[0.13, 6.45]	[-21.87, 0.02]	[0.00, 45.80]	[4.20, 310.00]	[0.03, 0.85]	[0.00, 0.79]	[-0.83, 0.01]
DurG	0.70	1.25	2.21	6.15	-3.18	10.92	4.20	0.10	0.06	-0.34
	(0.10, 1.21)	(0.49, 1.98)	(0.54, 4.19)	(1.36, 9.78)	(-19.27, -0.06)	(2.60, 73.88)	(1.00, 73.88)	(0.01, 0.40)	(0.01, 0.22)	(-0.67, -0.03)
	[-0.43, 1.57]	[-0.01, 2.51]	[0.16, 6.11]	[0.30, 13.84]	[-45.52, 1.49]	[0.00, 310.00]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.43]	[-0.82, 0.17]
NdG	1.56	2.10	4.30	1.10	-3.77	1.61	45.80	0.37	0.21	-0.33
	(0.86, 1.98)	(1.32, 2.67)	(2.94, 6.43)	(0.39, 2.53)	(-11.65, -0.78)	(0.00, 4.20)	(17.61, 119.16)	(0.11, 0.63)	(0.07, 0.46)	(-0.57, -0.13)
	[0.06, 2.29]	[0.58, 3.06]	[1.28, 8.66]	[0.19, 4.07]	[-22.17, -0.07]	[0.00, 28.40]	[6.77, 192.20]	[0.01, 0.80]	[0.01, 0.65]	[-0.78, -0.02]
WT	1.09	0.68	0.82	3.36	0.29	4.20	6.77	0.82	0.06	0.18
	(0.80, 1.38)	(-0.09, 1.48)	(0.33, 1.40)	(0.72, 5.67)	(-0.21, 2.44)	(1.00, 28.40)	(1.00, 73.88)	(0.48, 0.94)	(0.01, 0.25)	(-0.12, 0.55)
	[0.56, 1.63]	[-0.57, 2.13]	[0.13, 2.07]	[0.20, 7.99]	[-2.10, 6.02]	[0.00, 119.16]	[0.00, 310.00]	[0.19, 0.98]	[0.00, 0.49]	[-0.44, 0.74]
RT	0.68	1.63	1.83	3.40	2.49	4.20	4.20	0.28	0.27	0.48
	(0.29, 1.05)	(0.92, 2.40)	(1.01, 2.87)	(1.12, 5.43)	(0.31, 8.43)	(1.00, 28.40)	(1.00, 45.80)	(0.04, 0.63)	(0.05, 0.64)	(0.13, 0.79)
	[-0.05, 1.38]	[0.39, 2.96]	[0.27, 4.08]	[0.31, 7.82]	[-0.70, 19.24]	[0.00, 119.16]	[0.00, 192.20]	[0.00, 0.80]	[0.01, 0.85]	[-0.10, 0.90]
TW	-0.14	-0.17	2.55	0.41	-0.01	4.20	28.40	0.08	0.08	-0.02
	(-0.78, 0.62)	(-0.50, 0.34)	(0.98, 4.25)	(0.15, 1.26)	(-0.94, 0.35)	(1.00, 28.40)	(10.92, 119.16)	(0.01, 0.32)	(0.01, 0.34)	(-0.39, 0.33)
	[-1.25, 1.19]	[-0.74, 1.00]	[0.24, 6.00]	[0.08, 2.64]	[-4.34, 1.68]	[0.00, 192.20]	[2.60, 192.20]	[0.00, 0.57]	[0.00, 0.61]	[-0.66, 0.63]
Inf	1.09	0.04	4.26	3.51	-5.17	1.61	4.20	0.22	0.03	-0.38
	(0.49, 1.68)	(-0.73, 0.94)	(3.11, 6.25)	(0.72, 5.71)	(-17.51, -0.48)	(0.00, 6.77)	(1.00, 73.88)	(0.03, 0.53)	(0.00, 0.18)	(-0.68, -0.06)
	[0.01, 2.12]	[-1.25, 1.61]	[2.00, 8.37]	[0.23, 8.06]	[-37.44, 1.86]	[0.00, 17.61]	[0.00, 192.20]	[0.00, 0.74]	[0.00, 0.42]	[-0.84, 0.13]
F(x-H)	1.74	-0.11	0.43	1.57	-0.08	4.20	2.60	0.97	0.10	-0.16
	(1.45, 2.05)	(-0.60, 0.47)	(0.17, 1.02)	(0.70, 2.58)	(-0.87, 0.30)	(1.00, 28.40)	(1.00, 28.40)	(0.83, 1.00)	(0.01, 0.42)	(-0.71, 0.48)
	[1.22, 2.32]	[-0.94, 1.00]	[0.10, 1.66]	[0.21, 3.73]	[-2.40, 1.06]	[0.00, 73.88]	[0.00, 119.16]	[0.56, 1.00]	[0.00, 0.69]	[-0.90, 0.88]
PBS	1.82	0.64	1.68	3.01	-4.71	10.92	4.20	0.63	0.09	-0.89
	(1.52, 2.15)	(0.27, 1.04)	(0.68, 3.38)	(1.76, 4.66)	(-13.32, -1.20)	(2.60, 28.40)	(1.00, 10.92)	(0.35, 0.83)	(0.01, 0.35)	(-0.98, -0.65)
	[1.29, 2.42]	[-0.05, 1.40]	[0.38, 4.85]	[0.74, 6.53]	[-26.17, -0.44]	[1.00, 45.80]	[0.00, 45.80]	[0.20, 0.92]	[0.00, 0.61]	[-1.00, -0.37]
EdHe	0.75	0.94	1.70	3.28	-3.28	4.20	4.20	0.35	0.11	-0.67
	(0.43, 1.03)	(0.35, 1.51)	(0.71, 2.63)	(0.98, 4.99)	(-9.61, -0.70)	(1.00, 28.40)	(1.00, 45.80)	(0.08, 0.66)	(0.01, 0.35)	(-0.88, -0.30)
	[0.09, 1.27]	[-0.09, 2.02]	[0.25, 3.50]	[0.29, 6.82]	[-18.18, -0.18]	[0.00, 119.16]	[0.00, 192.20]	[0.01, 0.82]	[0.00, 0.61]	[-0.94, -0.12]
AEFS	0.63	-0.97	0.71	1.87	-0.91	45.80	1.61	0.13	0.46	-0.37
	(0.14, 1.17)	(-1.37, -0.28)	(0.26, 1.84)	(1.23, 2.80)	(-2.87, -0.20)	(10.92, 119.16)	(0.00, 6.77)	(0.01, 0.41)	(0.07, 0.78)	(-0.69, -0.13)
	[-0.17, 1.58]	[-1.68, 0.67]	[0.14, 3.11]	[0.55, 4.15]	[-6.25, -0.01]	[2.60, 192.20]	[0.00, 45.80]	[0.00, 0.65]	[0.01, 0.90]	[-0.87, -0.01]
OthS	0.16	1.14	2.88	0.84	-0.43	6.77	73.88	0.04	0.08	-0.24
	(-0.49, 0.88)	(0.41, 1.89)	(0.40, 5.18)	(0.23, 3.39)	(-4.26, 0.01)	(1.00, 119.16)	(10.92, 192.20)	(0.00, 0.20)	(0.01, 0.27)	(-0.61, 0.01)
	[-0.95, 1.49]	[-0.18, 2.44]	[0.16, 7.67]	[0.12, 7.14]	[-15.19, 0.89]	[0.00, 310.00]	[2.60, 500.00]	[0.00, 0.40]	[0.00, 0.50]	[-0.79, 0.29]
Hous	0.36	1.50	10.92	1.88	-14.68	10.92	2.60	0.01	0.63	-0.74
	(-0.59, 1.33)	(1.18, 1.89)	(3.12, 18.23)	(1.13, 2.87)	(-41.27, -2.72)	(1.00, 73.88)	(1.00, 10.92)	(0.00, 0.04)	(0.30, 0.86)	(-0.97, -0.29)
	[-1.25, 2.04]	[0.80, 2.28]	[1.01, 25.59]	[0.39, 4.04]	[-87.76, -0.40]	[0.00, 192.20]	[0.00, 45.80]	[0.00, 0.10]	[0.09, 0.94]	[-0.99, -0.05]
Fac			0.49	1.29	-0.31	28.40	4.20	0.77	0.49	-0.47
			(0.19, 1.19)	(0.50, 2.13)	(-1.30, -0.05)	(10.92, 73.88)	(1.00, 28.40)	(0.58, 0.89)	(0.27, 0.72)	(-0.37, -0.79)
			[0.11, 1.92]	[0.19, 3.09]	[-3.13, 0.01]	[6.77, 192.20]	[0.00, 119.16]	[0.43, 0.94]	[0.14, 0.85]	[-0.33, -0.34]

Notes: See Notes for Table A1.

2 A Growth Model with Sectoral Linkages in Materials and Investment

In this section, we describe an economy where different sectors produce materials and investment goods for other sectors in a way that mimics the U.S. make-use and capital flow tables. These production linkages give rise to *sectoral multipliers* that summarize the influence that different sectors have on the aggregate economy. In general, a sector that has a significant role in producing capital goods and intermediate inputs for other sectors will be associated with a large sectoral multiplier.

2.1 Economic Environment

The representative household has preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t C_t,$$

$$C_t = \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \ \sum_{j=1}^{n} \theta_j = 1, \ \theta_j \geq 0,$$

where C_t represents an aggregate consumption bundle taken to be the numéraire good. The production side of the economy is described as follows:

Gross output in sector j is produced according to the technology,

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{(1-\gamma_j)}, \ \gamma_j \in [0,1].$$

Materials and value added in sector j are produced respectively with the technologies,

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \ \sum_{i=1}^{n} \phi_{ij} = 1, \ \phi_{ij} \geq 0,$$

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j}, \ \alpha_j \in [0,1],$$

where the definitions of variables are those given in the main text.

Capital accumulation in each sector follows

$$k_{j,t+1} = x_{j,t} + (1-\delta_j) k_{j,t},$$

$$x_{j,t} = \prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}}, \ \sum_{i=1}^{n} \omega_{ij} = 1, \ \omega_{ij} \geq 0.$$

Goods market clearing requires that

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + \sum_{i=1}^{n} x_{ji,t} = y_{j,t}.$$

Finally, for now observed labor input is taken to be exogenous so that we define

$$A_{j,t} = z_{j,t} \left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j},$$

and express value added in sector j as

$$v_{j,t} = A_{j,t} \left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j}.$$

We then express the driving process for $A_{j,t}$ as

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1-\alpha_j)\Delta \ln \ell_{j,t},$$

where $\Delta \ln z_{j,t}$ and $\Delta \ln \ell_{j,t}$ grow at exogenous rates.

Throughout this appendix, we use the following notation: $\Theta = (\theta_1, ..., \theta_n)_{1\times n}$, $\Gamma_d = diag\{\gamma_j\}_{n\times n}$, $\Phi = \{\phi_{ij}\}_{n\times n}$, $\Omega = \{\omega_{ij}\}_{n\times n}$, $\alpha_d = diag\{\alpha_j\}_{n\times n}$, $\delta_d = diag\{\delta_j\}_{n\times n}$.

2.2 The Planner's Problem

Because the economy we have just described has no explicit frictions, the equilibrium and first-best allocations coincide. Lagrange multipliers in the solution to the planner's problem will correspond to prices in the decentralized equilibrium. Hence, we denote the price of gross output in sector j by $p^y_{j,t}$, the price of the composite materials bundle in sector j by $p^m_{j,t}$, etc.

The planner then solves

$$\begin{aligned} \max \ \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j}\right)^{\theta_j} \\ & + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^y_{j,t} \left[\left(\frac{v_{j,t}}{\gamma_j}\right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j}\right)^{(1-\gamma_j)} - c_{j,t} - \sum_{i=1}^{n} m_{ji,t} - \sum_{i=1}^{n} x_{ji,t} \right] \end{aligned}$$

$$
\begin{aligned}
&+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{m}\left[\prod_{i=1}^{n}\left(\frac{m_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}}-m_{j,t}\right]\\
&+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{v}\left[A_{j,t}\left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j}-v_{j,t}\right]\\
&+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{x}\left[\prod_{i=1}^{n}\left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}}+(1-\delta_j)k_{j,t}-k_{j,t+1}\right].
\end{aligned}
$$

The first-order conditions from the planner's problem give a solution described by:

$$\frac{\theta_j C_t}{c_{j,t}}=p_{j,t}^{y},$$

which also defines the ideal price index,

$$1=\prod_{j=1}^{n}\left(p_{j,t}^{y}\right)^{\theta_j}.$$

Moreover, we have that

$$\gamma_j\frac{p_{j,t}^{y}y_{j,t}}{v_{j,t}}=p_{j,t}^{v},$$

and

$$(1-\gamma_j)\frac{p_{j,t}^{y}y_{j,t}}{m_{j,t}}=p_{j,t}^{m},$$

which define a price index for gross output,

$$p_{j,t}^{y}=\left(p_{j,t}^{v}\right)^{\gamma_j}\left(p_{j,t}^{m}\right)^{1-\gamma_j}.$$

In addition,

$$\phi_{ij}\frac{p_{j,t}^{m}m_{j,t}}{m_{ij,t}}=p_{i,t}^{y},$$

which gives material prices in terms of gross output prices,

$$p_{j,t}^{m}=\prod_{i=1}^{n}\left(p_{i,t}^{y}\right)^{\phi_{ij}},$$

and

$$\omega_{ij}\frac{p_{j,t}^{x}x_{j,t}}{x_{ij,t}}=p_{i,t}^{y},$$

which gives prices for capital in each sector in terms of gross output prices,

$$p_{j,t}^{x}=\prod_{i=1}^{n}\left(p_{i,t}^{y}\right)^{\omega_{ij}}.$$

Finally, we have an Euler equation associated with optimal investment in each sector j,

$$p_{j,t}^{x}=\beta\mathbb{E}_{t}\left[\alpha_{j}\frac{p_{j,t+1}^{v}v_{j,t+1}}{k_{j,t+1}}+p_{j,t+1}^{x}(1-\delta_{j})\right].$$

Value added in sector j in this economy is $p_{j,t}^{y}y_{j,t}-\sum_{i}p_{i,t}^{y}m_{ij,t} = p_{j,t}^{y}y_{j,t}-\sum_{i}(1-\gamma_{j})\phi_{ij}p_{j,t}^{y}y_{j,t} = \gamma_{j}p_{j,t}^{y}y_{j,t} = p_{j,t}^{v}v_{j,t}$. GDP is then given by $\sum_{j}p_{j,t}^{v}v_{j,t}$. It is also the case that $p_{j,t}^{y}y_{j,t}-\sum_{i}p_{j,t}^{y}m_{ji,t}=p_{j,t}^{y}c_{j,t}+\sum_{i}p_{j,t}^{y}x_{ji,t}$.

2.3 The Full Set of Equilibrium Conditions

For clarity, we collect in this subsection the full set of equilibrium conditions,

$$c_{j,t}+\sum_{i=1}^{n}m_{ji,t}+\sum_{i=1}^{n}x_{ji,t}=y_{j,t},\ \forall j,$$

$$x_{j,t}=\prod_{i=1}^{n}\left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}},\ \forall j,$$

$$k_{j,t+1}=x_{j,t}+(1-\delta)k_{j,t},\ \forall j,\ \text{and } k_{j,0}\text{ given},$$

$$v_{j,t}=A_{j,t}\left(\frac{k_{j,t}}{\alpha_{j}}\right)^{\alpha_{j}},\ \forall j,$$

$$m_{j,t}=\prod_{i=1}^{n}\left(\frac{m_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}},\ \forall j,$$

$$y_{j,t}=\left(\frac{v_{j,t}}{\gamma_{j}}\right)^{\gamma_{j}}\left(\frac{m_{j,t}}{1-\gamma_{j}}\right)^{1-\gamma_{j}},\ \forall j.$$

The first-order conditions from the planner's problem are,

$$\frac{\theta_{j}C_{t}}{c_{j,t}}=p_{j,t}^{y},\ \forall j,$$

$$C_{t}=\prod_{j=1}^{n}\left(\frac{c_{j,t}}{\theta_{j}}\right)^{\theta_{j}},$$

$$\gamma_j \frac{p^y_{j,t} y_{j,t}}{v_{j,t}} = p^v_{j,t},\ \forall j,$$

$$(1-\gamma_j)\frac{p^y_{j,t} y_{j,t}}{m_{j,t}} = p^m_{j,t},\ \forall j,$$

$$\phi_{ij} \frac{p^m_{j,t} m_{j,t}}{m_{ij,t}} = p^y_{i,t},\ \forall i,j,$$

$$\omega_{ij} \frac{p^x_{j,t} x_{j,t}}{x_{ij,t}} = p^y_{i,t},\ \forall i,j,$$

$$p^x_{j,t} = \beta \mathbb{E}_t \left[\alpha_j \frac{p^v_{j,t+1} v_{j,t+1}}{k_{j,t+1}} + p^x_{j,t+1}(1-\delta_j) \right]\ \forall j$$

The exogenous sectoral processes driving the scale of value added in sector j, $A_{j,t}$, are embedded in

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1-\alpha_j)\Delta \ln \ell_{j,t}. \tag{4}$$

Thus, we have $2n^2 + 11n + 1$ equations with unknowns given by: $y_{j,t}, c_{j,t}, m_{j,t}, x_{j,t}, v_{j,t}, k_{j,t+1}$, $A_{j,t}$, $p^y_{j,t}, p^v_{j,t}, p^m_{j,t}, p^x_{j,t}, j = 1, ..., n$; $m_{ij,t}, x_{ij,t}, i, j = 1, ..., n$; and C_t.

2.4 Balanced Growth and Sectoral Multipliers

This section describes how, in the long run, changes in the growth rates of TFP or labor in different sectors affect GDP (aggregate value added) growth. We describe how this effect may be summarized in the form of *sectoral multipliers* for different sectors.

Consider a balanced growth path where the growth rates of TFP and labor in sector j are given by g^z_j and g^ℓ_j respectively. From equation (4), it follows that along that path,

$$\Delta \ln A_{j,t} = g^a_j = g^z_j + (1-\alpha_j)\, g^\ell_j.$$

Furthermore, as highlighted in the empirical section of the main text, we let

$$g^z_j = \lambda^z_j g^z_f + g^z_{u,j} \text{ and } g^\ell_j = \lambda^\ell_j g^\ell_f + g^\ell_{u,j}. \tag{5}$$

In other words, composite sources of sectoral growth in the steady state, g^a_j, reflect steady state sectoral TFP growth, g^z_j, and sectoral labor growth, g^ℓ_j. The growth rates of these inputs in turn reflect both common (aggregate) factors, $(\lambda^z_j g^z_f, \lambda^\ell_{j,} g^\ell_f)$, and unique idiosyncratic components, $(g^z_{u,j}, g^\ell_{u,j})$.

Then,

$$\Delta \ln A_{j,t} \equiv g_j^a = \lambda_j^z g_f^z + g_{u,j}^z + (1-\alpha_j)\left(\lambda_j^\ell g_f^\ell + g_{u,j}^\ell\right), \tag{6}$$

and we denote by $\widetilde{A}_{j,t}$ the gross growth rate of $A_{j,t}$,

$$\widetilde{A}_{j,t} = \frac{A_{j,t}}{A_{j,t-1}} = e^{g_j^a} \approx 1 + g_j^a.$$

The balanced growth path of the economy is one in which, given the constant growth rates of TFP, $\lambda_j^z g_f^z + g_{u,j}^z$, and labor input, $\lambda_j^\ell g_f^\ell + g_{u,j}^\ell$, all other variables grow at constant rates and all shares are constant. Thus, to derive the aggregate balanced growth path, we need to normalize the model's variables in such a way that these 'detrended' variables (generally denoted by a '$\sim$' over the variable) are constant along that path. Because different sectors will generally grow at different rates along the balanced growth path, the factors used to normalize variables will be sector-specific. Hence, we generally denote these normalizing factors by $\mu_{j,t}$ (or functions thereof). Solving for those factors below will yield a system of equations that is stationary in the normalized variables along the economy's steady state growth path and, importantly, the growth rates of all variables along that path.

2.4.1 Making the Model Stationary

If all growth rates are constant, the resource constraint in any individual sector implies that all the variables in that constraint must grow at the same rate. Thus, define $\widetilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$, $\widetilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$, $\widetilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\widetilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$. The goal in this subsection is to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters only (and in particular the constant growth rates of TFP and labor input).

The economy's resource constraint becomes

$$\widetilde{c}_{j,t} + \sum_{i=1}^{n} \widetilde{m}_{ji,t} + \sum_{i=1}^{n} \widetilde{x}_{ji,t} = \widetilde{y}_{j,t}.$$

Given the above definitions, the production of investment goods may be re-written as

$$\widetilde{x}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{x}_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}},$$

where $\widetilde{x}_{j,t} = x_{j,t}/\prod_{i=1}^{n}\mu_{i,t}^{\omega_{ij}}$. Under this normalization, the capital accumulation equation is

$$k_{j,t+1} = \widetilde{x}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\omega_{ij}} + (1-\delta_j)k_{j,t},$$

and so becomes

$$\widetilde{k}_{j,t+1} = \widetilde{x}_{j,t} + (1-\delta_j)\widetilde{k}_{j,t}\prod_{i=1}^{n}\left(\frac{\mu_{i,t-1}}{\mu_{i,t}}\right)^{\omega_{ij}},$$

where $\widetilde{k}_{j,t+1} = k_{j,t+1}/\prod_{i=1}^{n}\mu_{i,t}^{\omega_{ij}}$.

The expression for value added may be written as

$$v_{j,t} = A_{j,t}\left(\frac{\widetilde{k}_{j,t}\prod_{i=1}^{n}\mu_{i,t-1}^{\omega_{ij}}}{\alpha_j}\right)^{\alpha_j},$$

so that defining

$$\widetilde{v}_{j,t} = \frac{v_{j,t}}{A_{j,t}\left(\prod_{i=1}^{n}\mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}}, \tag{7}$$

where $A_{j,t}\left(\prod_{i=1}^{n}\mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}$ is the scaling factor that makes normalized value added, $\widetilde{v}_{j,t}$, constant along the balanced growth path, we have

$$\widetilde{v}_{j,t} = \left(\frac{\widetilde{k}_{j,t}}{\alpha_j}\right)^{\alpha_j}.$$

The composite bundle of materials used in sector j may be expressed as

$$\widetilde{m}_{j,t} = \prod_{i=1}^{n}\left(\frac{\widetilde{m}_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}},$$

with $\widetilde{m}_{j,t} = m_{j,t}/\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}$.

Under our normalization, gross output may be written as

$$\widetilde{y}_{j,t}\mu_{j,t} = \left(\frac{\widetilde{v}_{j,t}A_{j,t}\prod_{i=1}^{n}\mu_{i,t-1}^{\alpha_j\omega_{ij}}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}}{1-\gamma_j}\right)^{1-\gamma_j},$$

which, collecting terms, gives

$$\widetilde{y}_{j,t} = \left(\frac{\widetilde{v}_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}\left[\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\right].$$

We can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\left(\prod_{i=1}^{n}\frac{\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}}{\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}}\right)\left(\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}\right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Thus, without loss of generality with respect to growth rates, we choose $\mu_{j,t}$ in every sector such that, on the steady state growth path,[1]

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}+(1-\gamma_j)\phi_{ij}} = 1.$$

2.4.2 Sectoral Value Added Growth

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln A_{j,t} - \ln\mu_{j,t} + \sum_{i=1}^{n}\left(\gamma_j\alpha_j\omega_{ij} + (1-\gamma_j)\phi_{ij}\right)\ln\mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d \ln A_t - \ln\mu_t + \Gamma_d\alpha_d\Omega'\ln\mu_t + (I-\Gamma_d)\Phi'\ln\mu_t = 0,$$

which gives us

$$\ln\mu_t = \Xi'\ln A_t, \tag{8}$$

[1]This is without loss of generality since in the derivations of growth rates below, any constant κ may be used instead of 1.

where

$$\Xi' = \left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d,$$

with $\Xi = \{\xi_{ij}\}$ is the generalized Leontief inverse.

Going back to equation (6), and writing the vector of productivity growth rates as $\Delta \ln A_t = g^a$ where $g^a = (g_1^a, ..., g_n^a)$, it follows that[2]

$$\Delta \ln \mu_t = \Xi' g^a = \Xi' \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d)\big(\lambda^\ell g_f^\ell + g_u^\ell\big)\right),$$

where, given equation (5), g_f^z and g_f^ℓ are common sources of TFP and labor growth, λ^z and λ^ℓ are loading vectors and g_u^z and g_u^ℓ are vectors of (unique) idiosyncratic TFP and labor growth rates.

Recall from equation (7) above that the normalizing factor for value added in sector j is $A_{j,t}\left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}$. Thus, define this factor by $\mu_{j,t}^v$,

$$\mu_{j,t}^v = A_{j,t}\left(\prod_{j=1}^{n} \mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}.$$

In particular, since $\mu_{j,t}^v$ is the normalizing factor that makes value added in sector j constant along the balanced growth path, it follows that $\mu_{j,t}^v$ grows at the same rate as j's value added along that path, denoted g_j^v. Then, using equation (8), we have that

$$\ln \mu_t^v = \ln A_t + \alpha_d \Omega' \Xi' \ln A_{t-1},$$

or

$$g^v = \left[I + \alpha_d \Omega' \Xi'\right] \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d)\left(\lambda^\ell g_f^\ell + g_u^\ell\right)\right), \tag{9}$$

where $g^v = (g_1^v, ..., g_n^v)$ is a vector that summarizes value added growth in every sector.

2.4.3 GDP Growth and Sectoral Multipliers

In equation (9), the generalized Leontief inverse, $\Xi' = \left(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d$, captures the importance of production linkages across sectors. In particular, changes in the growth rates of productivity or labor in sectors that produce capital and materials for a wide range of other sectors will have a greater influence on the path of GDP growth compared to sectors that have few linkages to the rest of the economy.

[2]Note: $\Gamma_d^{-1}\left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi'\right)\mathbf{1} = (I - \alpha_d)\mathbf{1}$.

The Divisia index describing aggregate GDP growth is

$$\Delta \ln V_t = \sum_{j=1}^{n} s_j^v \Delta \ln v_{j,t},$$

where $\Delta \ln v_{j,t}$ denotes the growth rate of real value added in sector j and s_j^v is the share of j's value added in GDP,

$$s_j^v = \frac{p_j^v v_j}{\sum_{j=1}^{n} p_j^v v_j}.$$

In the next section we show that the sectoral value added shares in GDP are constant on the BGP. Hence, from equation (9), the balanced growth rate of real aggregate GDP, denoted g^V, is

$$g^V = s^{v\prime} \left[I + \alpha_d \Omega' \Xi'\right] \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d) \left(\lambda^\ell g_f^\ell + g_u^\ell\right)\right),$$

where s^v is the vector of value added sectoral shares, $(s_1^v, ..., s_1^v)$. Alternatively, we have that

$$g^V = \sum_{j=1}^{n} s_j^v \left[g_j^a + \sum_{i=1}^{n} \alpha_j \omega_{ij} \sum_{k=1}^{n} \xi_{ki} g_k^a\right],$$

where $g_j^a = \lambda_j^z g_f^z + g_{u,j}^z + (1 - \alpha_j) \left(\lambda_j^\ell g_f^\ell + g_{u,j}^\ell\right),\ j = 1, ..., n.$ This implies that

$$\frac{\partial g^V}{\partial g_j^a} = s_j^v + s_j^v \alpha_j \sum_{k=1}^{n} \omega_{kj} \xi_{jk} + \sum_{i \neq j} s_i^v \alpha_i \sum_{k=1}^{n} \omega_{ki} \xi_{jk}. \tag{10}$$

In equation (10), the first term captures the direct effects of changes in sources of input growth in sector j by way of productivity or labor on GDP growth. This direct effect is simply j's value added share in GDP reminiscent of Hulten (1978) but now in growth rates. The second and third terms reflect the indirect effects of changes in j's productivity or labor growth on aggregate value added through j's production linkages to other sectors. In other words, disturbances in sector j percolate to sectors that rely on it for inputs and thus amplify j's effects on the aggregate economy.

We define sector j's direct and indirect effects on GDP growth in (10) as j's *sectoral multiplier* given by the j^{th} element of

$$s^{v\prime} \left[I + \alpha_d \Omega' \Xi'\right].$$

2.5 Values, Shares, and Prices Along the Balanced Growth Path

We now characterize the evolution of values, shares, and prices along the BGP. Along that path, all values grow at the same rate. Furthermore, just as productivity growth in any one sector potentially affects value added growth in multiple other sectors (recall equation (9) above), productivity growth in a sector also potentially helps determine price changes in multiple other sectors. As in equation (9), the relationship between price changes in different sectors and sectoral productivity growth will involve the generalized Leontief inverse, Ξ', which summarizes the effects of production linkages in the economy. We assume that, analogous to quantities, all prices grow at possibly different but constant rates along the BGP.

From the conditions determining optimal consumption, it follows that

$$p_i^y c_i = \theta_i p^C C, \text{ with } \ln p^C = \Theta \ln p^y \tag{11}$$

where $p^y = (p_1^y, ..., p_n^y)'$, p^C is the unit price of the consumption bundle C, and $p^C C$ is then total spending on consumption.[3] We choose the consumption bundle to be our numéraire good, $p^C = 1$, and all nominal values below are measured in units of the consumption bundle index.

From the optimal capital use condition, the Euler equation for capital, and the capital accumulation equation, c.f. section 2.3, we have that on the BGP,

$$\alpha_i p_i^v v_i = u_i k_i = \left[\frac{1}{\beta\left(1+g_i^{p^x}\right)} - (1-\delta_i)\right] p_i^x k_i = \Delta_i^{-1} p_i^x x_i, \text{ with } \Delta_i = \frac{\beta\left(g_i^k + \delta_i\right)}{1/\left(1+g_i^{p^x}\right) - (1-\delta_i)\beta},$$

where u is the rental rate of capital. With constant growth rates along the BGP, this means that in any given sector, the nominal values of payments to capital, investment, and value added all grow at the same rate. Similarly, from the conditions determining the optimal use of value-added and intermediate inputs, we have that

$$\begin{aligned} p_i^v v_i &= \gamma_i p_i^y y_i, \\ p_j^y m_{ji} &= \phi_{j,i}\left(1-\gamma_i\right) p_i^y y_i. \end{aligned}$$

Thus, the nominal values of gross output and intermediate inputs in a sector grow at the same rate as that of value-added.

Finally, expressing the resource equations in common nominal terms (i.e., in units of C),

[3] Note that $p^C C = \sum_{i=1}^n p_i^y c_i$.

we obtain

$$\frac{p_j^v v_j}{\gamma_j} = \theta_j C + \sum_{i=1}^{n} \phi_{ji}(1-\gamma_j)\frac{p_i^v v_i}{\gamma_i} + \sum_{i=1}^{n} \omega_{ji}\Delta_i \alpha_i p_i^v v_i.$$

On a BGP, therefore, the value of total consumption must grow at the same rate as that of sectoral value-added. Alternatively, the ratios of nominal value-added, $p_j^v v_j$, to that of total consumption, C, must be constant.

In vector form, the resource constraints become

$$\Gamma_d^{-1}\left(p^v. \times v\right) = \Theta' C + \Phi(I-\Gamma_d)\Gamma_d^{-1}\left(p^v. \times v\right) + \Omega\Delta_d\alpha_d\left(p^v. \times v\right),$$

where $(p^v. \times v)$ represents the vector of sectoral nominal value added. Thus, we can solve for the ratios of nominal value-added to that of the consumption index,

$$\frac{(p^v. \times v)}{C} = \left(\left[I - \Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\Delta_d\alpha_d\right)^{-1}\Theta' = \psi. \tag{12}$$

It follows that the vector of sectoral nominal value-added shares in nominal GDP is given by

$$s^v = \psi/(\mathbf{1}'\psi).$$

We saw above that real GDP growth may be defined as a Divisa index aggregating sectoral value-added growth rates, g^v, using the constant sectoral value-added shares, s^v, as weights,

$$g^V = s^{v\prime} g^v.$$

It remains, therefore, to determine the growth rates of nominal values and prices.

2.5.1 Real Sectoral Growth and Sectoral Price Growth

The full set of equilibrium conditions outlined in Section 2.3 above immediately implies the following set of relationships for prices (unit costs) of sectoral gross output, investment, consumption, and value added,

$$\ln p^y = \left[I - (I-\Gamma_d)\Phi'\right]^{-1}\Gamma_d \ln p^v = \Upsilon \ln p^v, \tag{13}$$

$$\ln p^x = \Omega' \ln p^y, \tag{14}$$

where $p^v = (p^v_1, ..., p^v_n)'$, and $p^x = (p^x_1, ..., p^x_n)'$. In turn, these relations immediately imply that the rates of change for sectoral prices on the BGP are

$$g^{p^y} = \Upsilon g^{p^v},\ g^{p^x} = \Omega' g^{p^y}, \text{ and } 0 = \Theta g^{p^y}. \tag{15}$$

Observe that the same relationships hold for the quantity growth rates of sectoral gross output, investment, consumption, and value added along the BGP,

$$g^y = \Upsilon g^v,\ g^x = \Omega' g^y, \text{ and } g^C = \Theta g^y. \tag{16}$$

We are now in a position to derive how the growth rates of sectoral prices relate to sectoral input growth, here in the form of sectoral TFP growth and labor growth.

From equation (12), it follows that nominal sectoral value-added, $p^v. \times v$, grows at the same rate as that of the consumption bundle, C. That is,

$$g^{p^v} = \mathbf{1} g^C - g^v.$$

Combining this expression with equation (15), it follows that sectoral gross output prices grow at rate

$$g^{p^y} = \Upsilon\left(\mathbf{1} g^C - g^v\right) = \Upsilon\left(\mathbf{1}\Theta g^y - \Upsilon^{-1} g^y\right) = \left(\Upsilon \mathbf{1}\Theta - I\right) g^y.$$

Observe also that $\Upsilon \mathbf{1} = \left[I - (I - \Gamma_d)\Phi'\right]^{-1} \Gamma_d \mathbf{1} = \mathbf{1}$.[4] Therefore on the BGP, sectoral gross output prices grow at rates that reflect drivers of real sectoral growth as follows,

$$g^{p^y} = \left(\mathbf{1}\Theta - I\right) g^y = \left(\mathbf{1}\Theta - I\right) \Xi' g^a, \tag{17}$$

where $\Xi' = \left(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d$ is the general Leontief inverse that, as in equation (9) for sectoral growth, describes the network effects of production linkages and determine sectoral multipliers. Note also that since expenditure shares, Θ, are less than one, increases in TFP growth through g^a lower producer prices, g^{p^y}.

Finally, note that

$$\Theta g^{p^y} = \Theta \mathbf{1} \Theta g^y - \Theta g^y = 0,$$

which confirms that, given the construction of gross output prices, the consumption price index indeed remains constant along the BGP.

[4] Let $J^{-1} = \left[I - (I - \Gamma_d)\Phi'\right]^{-1}$ and note that $J\mathbf{1} = \Gamma_d \mathbf{1}$. It follows that $\mathbf{1} = J^{-1}\Gamma_d \mathbf{1} = \left[I - (I - \Gamma_d)\Phi'\right]^{-1} \Gamma_d \mathbf{1}$.

3 Examples and Relationship to Previous Work

This section provides examples of sectoral multipliers by relating our analysis to previous work, in particular Greenwood, Hercowitz, and Krusell (1997) (henceforth GHK) and variations thereof. We also discuss briefly Ngai and Pissarides (2007). These examples help underscore the role of capital-producing sectors for the strength of sectoral multipliers. In these examples, goods and factor markets are perfectly competitive and factors of production are freely mobile across sectors. However, as we also make clear, the way in which sectoral sources of growth are amplified at the aggregate level is invariant to the assumption of factor mobility.

3.1 Greenwood, Hercowitz, and Krusell (1997)

To relate the economic environment in GHK (1997) to that of Section 2 above, note first that the one-sector environment with an aggregate production function in GHK (1997) also has an interpretation as a two-sector economy. As highlighted in Section V. A. of GHK (1997), under that interpretation, one sector produces consumption goods (sector 1) and the other investment goods (sector 2), and each sector's production function has the same capital elasticity, α. For simplicity, we focus on the discussion in Section III of GHK (1997) which abstracts from the distinction between equipment and structures.

3.1.1 Interpretation of GHK (1997) as a Two-Sector Economy

Consider an economy whose production structure is described by

$$c_t = y_{1,t} = z_{1,t} k_{1,t}^{\alpha} \ell_{1,t}^{1-\alpha},$$

$$x_t = y_{2,t} = z_{2,t} k_{2,t}^{\alpha} \ell_{2,t}^{1-\alpha},$$

$$k_t = k_{1,t} + k_{2,t},$$

$$\ell_t = \ell_{1,t} + \ell_{2,t},$$

$$k_{t+1} = x_t + (1-\delta) k_t,$$

where the constant scale factors in the production functions (which simplify the algebra) in the main text have been dropped. We now show that under the maintained assumptions, this two-sector environment indeed allows for an aggregate production function and the one-sector framework of GHK (1997).

To see this, observe that the FOCs for optimal production in the two-sector economy are,

$$p_{2,t} = (1+r_t)^{-1}\left[u_{t+1} + (1-\delta)p_{2,t+1}\right],$$

$$w_t = (1-\alpha)p_{1,t}z_{1,t}\left(\frac{k_{1,t}}{\ell_{1,t}}\right)^{\alpha} = (1-\alpha)p_{2,t}z_{2,t}\left(\frac{k_{2,t}}{\ell_{2,t}}\right)^{\alpha},$$

$$u_t = \alpha p_{1,t}z_{1,t}\left(\frac{k_{1,t}}{\ell_{1,t}}\right)^{\alpha-1} = \alpha p_{2,t}, z_{2,t}\left(\frac{k_{2,t}}{\ell_{2,t}}\right)^{\alpha-1},$$

Equality of factor rentals then implies equal capital-labor ratios across sectors,

$$\frac{w_t}{u_t} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{k_{i,t}}{\ell_{i,t}}\right) \rightarrow \frac{k_{i,t}}{\ell_{i,t}} = \frac{k_t}{\ell_t}.$$

Production of the two goods can then be rewritten as

$$c_t = z_{1,t}\left(\frac{k_t}{\ell_t}\right)^{\alpha}\ell_{1,t} \text{ and } x_t = z_{2,t}\left(\frac{k_t}{\ell_t}\right)^{\alpha}\ell_{2,t} = \frac{z_{2,t}}{z_{1,t}}z_{1,t}\left(\frac{k_t}{\ell_t}\right)^{\alpha}\ell_{2,t} = \frac{z_{1,t}}{q_t}\left(\frac{k_t}{\ell_t}\right)^{\alpha}\ell_{2,t},$$

where $q_t = \frac{z_{1,t}}{z_{2,t}}$ is the relative price of investment goods. In particular, adding the two resource constraints, we have that

$$c_t + q_t x_t = z_{1,t}\left(\frac{k_t}{\ell_t}\right)^{\alpha}(\ell_{1,t}+\ell_{2,t}) = z_{1,t}k_t^{\alpha}\ell_t^{1-\alpha}.$$

This gives us an expression for aggregate output (in units of consumption goods) as a function of total factor endowment only, which is also equation (24) in GHK (1997). Thus, to the extent that technical progress in the investment sector is generally more pronounced than in the consumption sector, the relative price of investment goods will decline over time as emphasized by GHK (1997).

3.1.2 Amplification of Sectoral Growth Along the Balanced Growth Path

We now derive the balanced growth path (BGP) in GHK (1997) and discuss its implications for sectoral multipliers. That is, we highlight how capital accumulation amplifies sectoral drivers of growth. Therefore, capital producing sectors will tend to have an outsize effect on the aggregate economy relative to sectors that produce mainly consumption goods.

Along the BGP, all variables grow at constant but potentially different rates. From the market clearing conditions and the form of production technologies, it follows that sectoral output growth rates, g_i^v, are given by (in terms of the notation introduced above),

$$g_i^v = g_i^z + (1-\alpha)g^\ell + \alpha g^k = g_i^a + \alpha g^k,\ i = 1, 2. \tag{18}$$

Equation (18) makes clear that any amplification of sectoral sources of growth, g_i^a, can only take place through capital accumulation. In this case, it follows from the capital accumulation equation that along the BGP, capital grows at the same rate as investment which, in the capital goods producing sector, is also that of output. Thus, we have that

$$g_2^v = g^k = \frac{1}{1-\alpha} g_2^a, \tag{19}$$

and

$$g_1^v = g_1^a + \frac{\alpha}{1-\alpha} g_2^a. \tag{20}$$

Note that the assumption of factor mobility across sectors has only minor implications for the characterization of the BGP. First, even with sector-specific investment, the resource constraint for investment implies that investment and capital grow at the same rate in each sector. Second, with sector-specific labor, the expression for output growth remains as in equation (18) with the only difference being that sector-specific labor growth rates, g_i^ℓ, now replace the aggregate labor growth rate, g^ℓ,

$$g_i^a = g_i^z + (1-\alpha) g_i^\ell.$$

Aggregate GDP growth is defined as the Divisia index of sectoral value-added growth rates weighted by their respective value added shares. Because GHK (1997) do not consider intermediate goods, there is no distinction between gross output and value added in equation (18). Thus, from equations (19) and (20), aggregate GDP growth is

$$g^V = s_1^v \left(g_1^a + \frac{\alpha}{1-\alpha} g_2^a \right) + s_2^v \frac{1}{1-\alpha} g_2^a, \tag{21}$$

or alternatively,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{\alpha}{1-\alpha} g_2^a, \tag{22}$$

where s_i^v is sector i's value-added share in GDP.

In this economy, sector 2 is the sole producer of capital for both sectors 1 and 2 and has both a direct and indirect effect on the aggregate economy. As explained above, the indirect effect stems from the fact that capital accumulation amplifies the role of sectoral sources of growth. In equation (21), sector 2 contributes $\frac{\alpha}{1-\alpha} g_2^a > 0$ to value added growth in sector 1 and scales its contributions from TFP and labor to its own value added growth by $\frac{1}{1-\alpha} > 1$. Thus, the direct effect of an expansion in sector 2, by way of TFP or labor growth, on GDP growth is simply its share, s_2^v, while its aggregate indirect effect is $\frac{\alpha}{1-\alpha} > 0$. Sector 2's sectoral multiplier, therefore, is $s_2^v + \frac{\alpha}{1-\alpha}$. In contrast, because sector 1 produces goods that are only fit for final consumption, it only has a direct effect on the aggregate economy. Its sectoral multiplier is then simply its share in GDP, s_1^v. The sum of sectoral multipliers is larger than 1 so that in principle, sectoral changes in TFP growth that leave aggregate TFP growth unchanged will nevertheless have an effect on GDP.

3.1.3 Relationship to FHSW (2022) with Two Sectors

We now show that a straightforward application of the framework laid out in Section 2 produces the same balanced growth path and sectoral multipliers for sectors 1 and 2 we have just discussed. In particular, the GHK (1997) economy is a special case of Section 2 where $n = 2$ and, since sector 2 is the only sector producing investment goods, $\omega_{2j} = 1, j = 1, 2$ (and $\omega_{1j} = 0, j = 1, 2$). In addition, each good is produced without intermediate inputs, $\gamma_j = 1, j = 1, 2$, and the sectors use the same production functions, $\alpha_j = \alpha, j = 1, 2$, except for the scale factors, $z_j, j = 1, 2$.

With these restrictions, the parameters of the model are summarized by $\Gamma_d = I$, $\alpha_d = \alpha I$ and

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

where the last matrix reflects the production structure whereby all capital in the economy is produced by sector 2.

Then, the generalized Leontief inverse is

$$\Xi' = (I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi')^{-1} \Gamma_d = (I - \alpha\Omega')^{-1} = \begin{pmatrix} 1 & -\alpha \\ 0 & 1-\alpha \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{\alpha}{1-\alpha} \\ 0 & \frac{1}{1-\alpha} \end{pmatrix},$$

and the BGP equations reduce to

$$g^v = (I + \alpha\Omega'\Xi')g^a = \begin{pmatrix} 1 & \frac{\alpha}{1-\alpha} \\ 0 & \frac{1}{1-\alpha} \end{pmatrix} \begin{pmatrix} g_1^a \\ g_2^a \end{pmatrix},$$

where

$$g_i^a = g_i^z + (1-\alpha)g_i^\ell, \; i = 1, 2.$$

It follows that value added growth in sectors 1 and 2 are, respectively,

$$g_1^v = g_1^a + \frac{\alpha}{1-\alpha} g_2^a,$$

and

$$g_2^v = \frac{1}{1-\alpha} g_2^a.$$

The Divisia aggregate index of GDP growth is then $g^V = s^{v\prime} g^v = s^{v\prime} g^a + s^{v\prime} \alpha \Omega' \Xi' g^a$ or

$$\begin{aligned} g^V &= s_1^v(g_1^a + \alpha\omega_{21}(\xi_{12} g_1^a + \xi_{22} g_2^a)) + s_2^v(g_2^a + \alpha\omega_{22}(\xi_{12} g_1^a + \xi_{22} g_2^a)), \\ &= s_1^v g_1^a + s_2^v g_2^a + (s_1^v + s_2^v)\frac{\alpha}{1-\alpha} g_2^a, \\ &= s_1^v g_1^a + s_2^v g_2^a + \frac{\alpha}{1-\alpha} g_2^a. \end{aligned}$$

Observe that this last expression is exactly equation (22) above.

Moreover (holding shares constant), the sectoral multipliers, which summarize the effects of sectoral growth by way of TFP or labor, on GDP growth are given by

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v,$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{\alpha}{1-\alpha}.$$

Alternatively, sectoral multipliers are given by the elements of $s^{v\prime}\left[I + \alpha_d \Omega' \Xi'\right]$. As discussed above, as the sole producer of capital goods, sector 2 has both a direct effect, s_2^v, and an indirect effect, $\frac{\alpha}{1-\alpha}$, on GDP growth. In contrast, sector 1 only has a direct effect on GDP growth, s_1^v.

3.2 Greenwood, Hercowitz, and Krusell (1997) with Different Factor Shares

Actual production linkages are generally more involved than those we have just discussed. Thus, consider the case where factor income shares differ, $\alpha_1 \neq \alpha_2$, while the rest of the production side of the economy remain as in Section 3.1. Importantly, even in the context of two sectors and no materials, the simple fact that factor income shares differ across sectors is enough to prohibit an aggregate production function and thus a one-sector interpretation of the economic environment.[5] The implications of unequal factor income shares, however, are

[5] See also GHK (1997), Section V. A.

relatively straightforward to work out in our framework. As before, it is still the case that sources of growth in the capital goods sector are amplified relative to sectors that mainly produce consumption goods. However, this amplification now depends on a value-added-share-weighted average of capital elasticities.

3.2.1 Amplification of Sectoral Growth along the Balanced Growth Path

With perfect factor mobility, sectoral output growth rates along the BGP are given by

$$g_i^v = g_i^z + (1-\alpha_i)g^\ell + \alpha_i g^k = g_i^a + \alpha_i g^k, \tag{23}$$

and the indirect effect of sectoral growth now depends on a sector's specific capital elasticity, cf. equation (18). As before, the investment goods producing sector determines capital accumulation

$$g_2^v = g^k = \frac{1}{1-\alpha_2} g_2^a, \tag{24}$$

and value added growth in sector 1 is

$$g_1^v = g_1^a + \frac{\alpha_1}{1-\alpha_2} g_2^a,$$

cf. equations (19) and (20). In the absence of factor mobility across sectors, the sector-specific labor growth rates, g_i^ℓ, simply replace the aggregate labor growth rate, g^ℓ,

$$g_i^a = g_i^z + (1-\alpha_i)g_i^\ell.$$

The long-run growth rate of GDP is now given by

$$g^V = s_1^v \left(g_1^a + \frac{\alpha_1}{1-\alpha_2} g_2^a \right) + s_2^v \frac{1}{1-\alpha_2} g_2^a,$$

or alternatively,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2} g_2^a, \tag{25}$$

where s_i^v is sector i's value-added share in GDP.

Thus, the sectoral multipliers for sectors 1 and 2 are respectively s_1^v and $s_2^v + \frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2}$. In this economy, the indirect effect of changes in sector 2 on GDP growth, $\frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2}$, depends for the most part on α_2. As $\alpha_2 \to 0$, this effect tends to $s_1^v\alpha_1 < 1$. Thus, even when sector 2 uses mostly labor in production, it nevertheless has an indirect effect on aggregate growth (over and above its direct effect through its own value added share, s_2^v) since it remains a supplier of (new) capital goods to sector 1. In this case, however, this indirect effect is entirely determined by the parameters of sector 1, specifically its importance in the economy as measured by its value added share in GDP, s_1^v, scaled by the intensity with which it uses capital to produce consumption goods, α_1.[6]

3.2.2 Relationship to FHSW (2022) with Two Sectors

With different factor shares, GHK (1997) may no longer be interpreted as a one-sector economy. Then, a direct application of our framework to the two-sector version of GHK (1997) with different factor shares,

$$\alpha_d = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix},$$

and all other parameters defined as in the previous section, immediately gives the generalized Leontief inverse as

$$\Xi' = (I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi')^{-1} \Gamma_d = (I - \alpha_d \Omega')^{-1} = \begin{pmatrix} 1 & -\alpha_1 \\ 0 & 1 - \alpha_2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{\alpha_1}{1-\alpha_2} \\ 0 & \frac{1}{1-\alpha_2} \end{pmatrix},$$

so that the BGP equations reduce to

$$g^v = (I + \alpha \Omega' \Xi') g^a = \begin{pmatrix} 1 & \frac{\alpha_1}{1-\alpha_2} \\ 0 & \frac{1}{1-\alpha_2} \end{pmatrix} \begin{pmatrix} g_1^a \\ g_2^a \end{pmatrix},$$

where

$$g_i^a = g_i^z + (1 - \alpha_i) g_i^\ell, \ i = 1, 2.$$

It follows that value added growth in sectors 1 and 2 are, respectively,

$$g_1^v = g_1^a + \frac{\alpha_1}{1 - \alpha_2} g_2^a,$$

and

$$g_2^v = \frac{1}{1 - \alpha_2} g_2^a.$$

[6] As $\alpha_2 \to 1$, the indirect effect becomes ill-defined since our derivation of the BGP assumes exogenous forces, g_i^z and g_i^ℓ, whereas in the limit where the capital elasticity is one, the model becomes an AK-type endogenous growth model.

The Divisia aggregate index of GDP growth is then $g^V = s^{v\prime}g^v = s^{v\prime}g^a + s^{v\prime}\alpha\Omega'\Xi' g^a$ or

$$
\begin{aligned}
g^V &= s_1^v(g_1^a + \alpha_1\omega_{21}(\xi_{12}g_1^a + \xi_{22}g_2^a)) + s_2^v(g_2^a + \alpha_2\omega_{22}(\xi_{12}g_1^a + \xi_{22}g_2^a)),\\
&= s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2} g_2^a,
\end{aligned}
$$

which is indeed equation (25). The corresponding sectoral multipliers in $s^{v\prime}\left[I + \alpha_d\Omega'\Xi'\right]$ are,

$$
\frac{\partial g^V}{\partial g_1^a} = s_1^v,
$$

and

$$
\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2},
$$

discussed above.

3.3 Greenwood, Hercowitz, Krusell (1997) with Intermediate Inputs

Actual production linkages are more involved still in that they also reflect a network of materials between sectors. Thus, we now introduce intermediate goods into the GHK (1997) environment. With intermediate inputs, additional sectoral contributions to value-added growth continue to arise through the capital growth rate. However, when the consumption sector (sector 1) also produces materials for the investment goods sector (sector 2), the growth rate of capital depends on conditions in both sectors 1 and 2. This means that in contrast to the previous two examples, both sectors 1 and 2 will have indirect effects on long-run GDP growth over and above their share in the economy.

We illustrate these points via a simple network of intermediate goods. Here, sector 1 produces not only consumption goods but also materials, $m_{1,t}$, used by sector 2. Similarly, sector 2 still produces capital goods for both sectors but also materials, $m_{2,t}$ used by sector 1. Since sector 1 now produces consumption goods and intermediate goods, we refer to sector 1 as the non-durables sector. Thus, in terms of the notation used in Section 2, we have that $\gamma_i \neq 1$ and $\omega_{2,i} = 1$ for $i = 1, 2$. Moreover, the relevant resource constraints in sectors 1 and 2 are now

$$
c_t + m_{1,t} = y_{1.t} = \left[z_{1,t}k_{1,t}^{\alpha_1}\ell_{1,t}^{1-\alpha_1}\right]^{\gamma_1} m_{2,t}^{1-\gamma_1}, \tag{26}
$$

and

$$
x_t + m_{2,t} = y_{2,t} = \left[z_{2,t}k_{2,t}^{\alpha_2}\ell_{2,t}^{1-\alpha_2}\right]^{\gamma_2} m_{1,t}^{1-\gamma_2}, \tag{27}
$$

while the rest of the production side of the economy is as in Section 3.1.

3.3.1 Amplification of Sectoral Growth along the Balanced Growth Path

As before, with perfect factor mobility it follows that

$$g^x = g^k = g_1^k = g_2^k \text{ and } g^\ell = g_1^\ell = g_2^\ell,$$

while from the goods market clearing conditions, we have that

$$g_1^y = g^c = g_1^m \text{ and } g_2^y = g^x = g_2^m.$$

From the production of goods, which now uses intermediate inputs, it follows that gross output growth rates are

$$\begin{aligned} g_1^y &= \gamma_1 \left[g_1^z + \alpha_1 g_2^y + (1-\alpha_1) g^\ell \right] + (1-\gamma_1) g_2^y, \\ g_2^y &= \gamma_2 \left[g_2^z + \alpha_2 g_2^y + (1-\alpha_2) g^\ell \right] + (1-\gamma_2) g_1^y. \end{aligned}$$

Therefore, solving for the growth rate of new capital goods, we obtain

$$g_2^y = \frac{(1-\gamma_2)\gamma_1 g_1^a + \gamma_2 g_2^a}{\Delta} = g^k. \tag{28}$$

where $\Delta = 1 - \gamma_2\alpha_2 - (1-\gamma_2)\left[\gamma_1\alpha_1 + (1-\gamma_1)\right]$. With intermediate inputs, sectoral value added growth now differs from gross output growth. Using the definition of the value added index, sectoral value added growth is still determined as in the two previous examples without intermediate inputs, equations (18) and (23),

$$g_i^v = g_i^z + \alpha_i g^k + (1-\alpha_i) g^\ell = g_i^a + \alpha_i g^k,$$

Aggregate GDP growth is then given by

$$g^V = s_1^v g_1^a + s_2^v g_2^a + (s_1^v \alpha_1 + s_2^v \alpha_2) g^k, \tag{29}$$

where g^k follows from equation (28). Two important observations emerge relative to the previous examples. First, since the non-durable goods sector now produces intermediate inputs for the investment goods sector, the growth rate of (new) capital goods in equation (28) reflects sources of growth in both sectors, g_1^a and g_2^a. Unlike in the previous examples, therefore, both sectors 1 and 2 will have an indirect effect on long-run GDP growth in equation (29), $(s_1^v\alpha_1 + s_2^v\alpha_2)\frac{\partial g^k}{\partial g_1^a}$ and $(s_1^v\alpha_1 + s_2^v\alpha_2)\frac{\partial g^k}{\partial g_2^a}$ respectively, over and above their shares in the economy, s_1^v and s_2^v. Second, from equation (29), the indirect effect from sector 2 on GDP growth will dominate that from sector 1 if and only if its contributions to overall capital growth, $\frac{\partial g^k}{\partial g_2^a}$, are larger than the corresponding contributions from sector 1, $\frac{\partial g^k}{\partial g_1^a}$. Going back to equation (28), this condition holds if and only if

$$\gamma_2 > (1-\gamma_2)\gamma_1.$$

Put differently, this condition implies that the effect from a one percent change in TFP growth in sector 2 on overall capital growth, g^k, is larger than the corresponding effect from sector 1 transmitted through intermediate inputs. It will fail to hold, for example, in economies where the value added share in gross output of the capital sector, γ_2, is small. In that case, the main input into the production of capital goods in equation (27) are intermediate inputs from the non-durables sector. Therefore, it is that sector's conditions that matter most.

More generally, in a multi-sector environment, the amplification of a non-durable goods sector's sources of growth depends on how much that sector contributes intermediate inputs, however indirectly, to capital goods sectors.

3.3.2 Relationship to FHSW (2022) with Two Sectors

In the general framework we lay out, the relevant parameterization is now

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

since sector 2 is still the sole producer of capital goods, while

$$\alpha_d = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix},\ \Gamma_d = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix},\ \text{and } \Phi' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

In this case, the general Leontief inverse, $\Xi' = (I - \alpha_d\Gamma_d\Omega' - (I-\Gamma_d)\Phi')^{-1}\,\Gamma_d$, reduces to

$$\Xi' = \frac{1}{\Delta}\begin{pmatrix} \gamma_1 - \alpha_2\gamma_1\gamma_2 & \gamma_2 - (1-\alpha_1)\gamma_1\gamma_2 \\ \gamma_1(1-\gamma_2) & \gamma_2 \end{pmatrix},$$

where $\Delta = 1-\gamma_2\alpha_2-(1-\gamma_2)(\gamma_1\alpha_1+1-\gamma_1)$ is the determinant of $(I-\alpha_d\Gamma_d\Omega'-(I-\Gamma_d)\Phi')$. Then the vector of value added growth is given by

$$g^v = I + \frac{1}{\Delta}\begin{pmatrix}\alpha_1\gamma_1(1-\gamma_2) & \alpha_1\gamma_2\\ \alpha_2\gamma_1(1-\gamma_2) & \alpha_2\gamma_2\end{pmatrix}\begin{pmatrix}g_1^a\\ g_2^a\end{pmatrix}.$$

so that

$$g_1^v = \frac{1}{\Delta}\left(\alpha_1\gamma_1(1-\gamma_2)g_1^a + \alpha_1\gamma_2 g_2^a\right) + g_1^a,$$

and

$$g_2^v = \frac{1}{\Delta}(\alpha_2\gamma_1(1-\gamma_2)g_1^a + \alpha_2\gamma_2 g_2^a) + g_2^a.$$

With intermediate inputs added to the GHK (1997) economy, GDP growth becomes,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v\alpha_1\gamma_1(1-\gamma_2)+s_2^v\alpha_2\gamma_1(1-\gamma_2))}{\Delta}g_1^a + \frac{s_1^v\alpha_1\gamma_2+s_2^v\alpha_2\gamma_2}{\Delta}g_2^a.$$

It follows that the sectoral multipliers, $s^{v\prime}(I+\alpha_d\Omega'\Xi')$, in this case are given by

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v + \frac{(s_1^v\alpha_1\gamma_1(1-\gamma_2)+s_2^v\alpha_2\gamma_1(1-\gamma_2))}{\Delta},$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{s_1^v\alpha_1\gamma_2+s_2^v\alpha_2\gamma_2}{\Delta},$$

which reproduces the intuition given above.

3.4 Ngai and Pissarides (2007), or Greenwood, Hercowitz, Krusell (1997) with Multiple Consumption Goods

The economy described in Ngai and Pissarides (2007) extends the two-sector interpretation of GHK (1997) to multiple consumption goods, $i = 1, ..., m$. As before, all of the capital used by these m sectors is produced in a single sector, in this case sector m. In terms of our notation, this implies the following resource constraints,

$$c_{i,t} = y_{i,t} = z_{i,t}k_{i,t}^{\alpha_i}\ell_{i,t}^{1-\alpha_i},\ i = 1, ..., m-1,$$

and

$$x_t + c_{m,t} = y_{m,t} = z_{m,t}k_{m,t}^{\alpha_m}\ell_{m,t}^{1-\alpha_m}.$$

Thus, value added growth is determined as in the previous sections without intermediate inputs, with a similar sectoral multiplier for the capital goods sector. However, consumption goods may now grow at different rates. Therefore, a balanced growth path with constant shares now requires unitary elasticity of substitution between goods in preferences.[7] Put another way, constant differences in consumption growth need to be consistent with a market equilibrium.

To see this, observe that for a general utility function, $U(C_t)$, optimal consumer demand implies that the marginal rate of substitution between 2 goods i and j must be equal to their relative prices,

$$\frac{\partial U(C_t)/\partial c_{i,t}}{\partial U(C_t)\ \partial c_{j,t}} = \frac{p^y_{i,t}}{p^y_{j,t}}.$$

Therefore, on the balance growth path, we have that

$$\sigma_i g_{c_i} - \sigma_j g_{c_j} = g^y_{p_i} - g^y_{p_j},$$

where σ_i and σ_j denote the elasticity of utility with respect to the i^{th} and j^{th} consumption goods respectively. From the resource constraints, consumption and output must grow at the same rate in each sector along a BGP. Since nominal values of all sectors grow at the same rate, so that value shares remain constant, the price growth differential must be the negative of the output growth differential. It follows that

$$\sigma_i g_{y_i} - \sigma_j g_{y_j} = g_{y_j} - g_{y_i},$$

or

$$(1+\sigma_i) g_{y_i} = (1+\sigma_j) g_{y_j}.$$

This condition will hold for arbitrary growth rates if and only if the utility function is of the form,

$$U(c) = u\left(\sum_{i=1,2} \phi_i \ln c_i\right),$$

where $u(.)$ is an increasing concave function. This also means a unitary elasticity of substitution between different consumption goods which applies, for example, to the preferences

[7] Ngai and Pissarides (2007) work out conditions on preferences that allow for a more flexible balanced growth path but in a more restrictive model of production. They maintain the assumption of equal factor income shares across all goods which yields an aggregate production function as in Section 3.1. However, they then show the existence of a balanced growth path for aggregate output, consumption, and capital that can coincide with non-constant individual consumption shares. These changing consumption shares then lead to changing implicit labor shares in the production of the consumption goods.

in Section 2 of this appendix.

3.5 Greenwood, Hercowitz, Krusell (1997) with Traded Capital Goods

While our analysis focuses on capital accumulation in a closed economy, a large portion of international trade in goods consists of the import and export of capital goods. This section develops the implications for the BGP when we introduce traded capital goods into the GHK (1997) framework using the approach of Basu, Fernald, Fisher, and Kimball (2013).

We start from a GHK (1997) two-sector economy with different capital shares and where both capital and labor can move freely between the consumption and investment goods sectors. We now differentiate between domestically produced and imported investment goods. Domestically produced investment goods, x_t^d, can be used at home, x_t^{dd}, or exported, x_t^{df}. Both, domestically produced investment goods, x_t^d, and imported investment goods, x_t^f, are used in the production of new domestic capital goods, x_t. Thus, the resource constraints are

$$\begin{aligned}
c_t &= y_{1,t} = z_{1,t} k_{1,t}^{\alpha_1} \ell_{1,t}^{1-\alpha_1}, \\
x_t^d &= y_{2,t} = z_{2,t} k_{2,t}^{\alpha_2} \ell_{2,t}^{1-\alpha_2} = x_t^{dd} + x_t^{df}, \\
x_t &= (x_t^{dd})^{\omega} (x_t^f)^{1-\omega}, \\
k_{t+1} &= x_t + (1-\delta) k_t, \\
k_t &= k_{1,t} + k_{2,t}, \\
\ell_t &= \ell_{1,t} + \ell_{2,t}.
\end{aligned}$$

Let the price of domestic and foreign investment goods be respectively p_t^{xd} and p_t^{xf}, and the price of new domestic capital goods, p_t^x. Here, investment goods are the only traded goods and the terms of trade, therefore, are $\tau_t = p_t^{xd}/p_t^{xf}$. Net-exports, denoted NX_t, is the value of exported domestic investment goods less the value of imported foreign investment goods. Aggregate GDP is then the value of final demand: consumption goods, investment goods, and net-exports,

$$\begin{aligned}
p_t^x x_t &= p_t^{xd} x_t^{dd} + p_t^f x_t^f, \\
NX_t &= p_t^{xd} x_t^{df} - p_t^{xf} x_t^f, \\
GDP_t &= p_t^c c_t + p^x x_t + NX_t.
\end{aligned}$$

In this exercise, as in Basu et al. (2013), we take the terms of trade and the initial level of net-exports are exogenous.

3.5.1 Balanced Growth Rates

Along a BGP, all variables grow at constant but potentially different rates while value shares are constant. Under these conditions, the GDP equation above implies that

$$g^{p^c} + g^c = g^{p^x} + g^x = g^{NX},$$

while the definition of net-exports means that

$$g^{NX} = g^{p^{xd}} + g^{x^{df}} = g^{p^{xf}} + g^{x^f}.$$

We can use the expression for the growth rate of net-exports to write the (exogenous) growth rate of the terms of trade in terms of domestic and foreign investment growth rates,

$$g^{\tau} = g^{p^{xd}} - g^{p^{xf}} = g^{x^f} - g^{x^{df}} = g^{x^f} - g^{x^d}.$$

Thus, we can now obtain the growth rate of capital from the BGP expressions for the production of capital goods,

$$\begin{aligned} g^k = g^x &= \omega g^{x^{dd}} + (1-\omega) g^{x^f}, \\ &= \omega g^{x^d} + (1-\omega)\left(g^{x^d} + g^{\tau}\right), \\ &= g^{x^d} + (1-\omega) g^{\tau}, \\ &= \omega\left[g_2^z + \alpha_2 g^k + (1-\alpha_2)\, g^{\ell}\right] + (1-\omega) g^{\tau}. \end{aligned}$$

which, when solved in terms of exogenous forces only, gives

$$g^k = \frac{g_2^z + (1-\omega) g^{\tau} + (1-\alpha_2)\, g^{\ell}}{1-\alpha_2}. \tag{30}$$

Observe that this last expression corresponds to expression (24) in the closed-economy version, except for the presence of the terms of trade whose growth rate is weighted by the share of foreign capital goods, $1-\omega$.

After we substitute for net exports, the equation for aggregate GDP becomes

$$V_t = p_t^c c_t + p_t^{xd} x_t^d,$$

so that the Divisia index for real GDP growth along the BGP is

$$g^V = s_c g^c + s_{x^d} g^{x^d}.$$

In other words, the value of final expenditures is also the value of domestic production as in the closed economy version without trade in capital goods. Put another way, the presence of traded investment goods leaves the basic formulation of GDP growth unchanged and, from equation (30), it follows that its effect on capital accumulation along the BGP will be small when the share of foreign capital goods, $1-\omega$, is small or as long as terms of trade do not have a pronounced trend.

To acquire some perspective on the role of imported capital goods, we plot the terms of trade for capital goods and the share of imported capital goods in domestic equipment investment in Figure A2. We consider capital goods broadly defined excluding automotive and obtain values of capital exports and imports, as well as their respective price indices, from the Foreign Transactions Tables 4.2 of the National Income Accounts. The terms of trade are defined as the price index of exports relative to the price index of imports. The share of capital goods imports is defined as the ratio of imports to equipment investment, excluding automotive.

Figure A2 shows that the import share of capital goods has increased from being unimportant in 1970, at less than 10 percent, to about 60 percent in 2010 (right-hand side scale of Figure A2). At the same time, the terms of trade increased at about a 1 percent annual growth rate (left-hand side scale of Figure A2). Relative to the pace of TFP growth in the durable goods producing sector, which grew at an annual rate of almost 3 percent over the same time period, the smaller terms of trade change scaled down by an import share generally less than 1/2 imply a relatively muted effect (though increasing) on the trend growth rate of GDP growth.

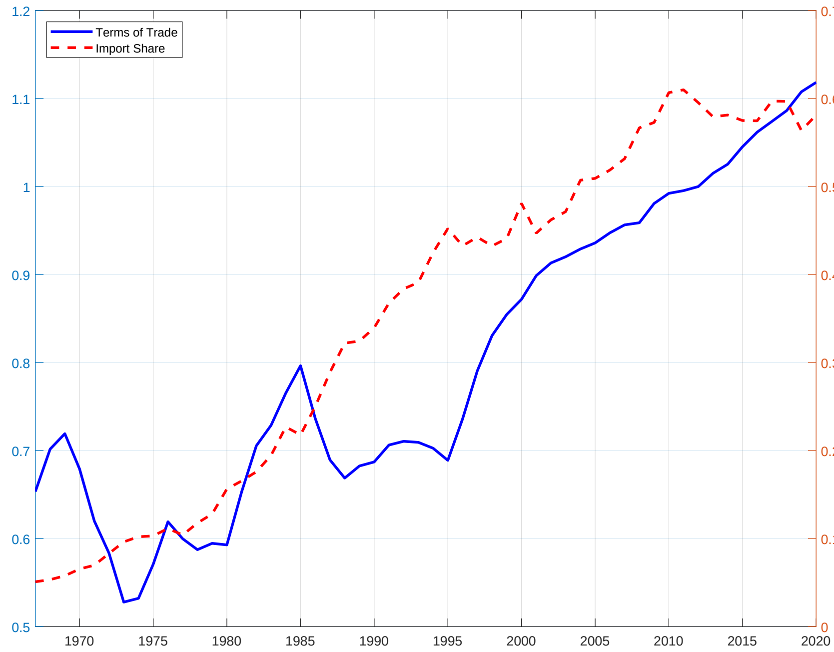
3.5.2 Balanced Growth Equilibrium with Terms of Trade and Net Exports

The environment we have just described embodies some important implicit assumptions. One such assumption treats the terms of trade as exogenous. Another has in the background a foreign net-asset accumulation equation which determines assets, a_t, as a residual,

$$a_{t+1} = (1 + r_t^f) a_t + NX_t,$$

given a rate of return, r_t^f, at which the economy can borrow or lend abroad, and the evolution of net-exports, NX_t. For completeness, therefore, we end this section by laying out the full

Figure A2: Capital Goods, excluding Automotive



equilibrium with balanced growth conditional on the terms of trade and net exports as in Basu et al. (2013) for an economy that starts out on the BGP. We show that in this equilibrium, the initial net-export ratio determines the consumption-output ratio.

For simplicity, we assume equal capital shares in the consumption and investment goods sectors as in the original GHK (1997) analysis. Thus, as in Section 3.1 above, the resource constraint for consumption and investment goods collapses to

$$c_t + q_t x_t^d = y_{1,t} = z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha} \text{ where } q_t = \frac{p_t^{x^d}}{p_t^c} = \frac{z_{1,t}}{z_{2,t}}.$$

We take consumption goods to be the numéraire so that $p_t^c = 1$ and q_t denotes the relative price of domestic investment goods. From the unit cost of the investment goods aggregate, it follows that capital goods prices change at the rate

$$\begin{aligned} g^{p^k} = g^{p^x} &= \omega g^q + (1-\omega) g^{p^{xf}}, \\ &= \omega g^q + (1-\omega)\left(g^q - g^{\tau}\right), \\ &= g^q - (1-\omega) g^{\tau}. \end{aligned}$$

Along a BGP, consumption grows at the rate

$$g^c = g_1^z + \alpha g^k + (1-\alpha)\, g^\ell .$$

From the Euler equation for investment in domestic assets along the BGP, we have that

$$1 = (1+r)^{-1}\, \beta \left(1+g_c\right)^{-\sigma}.$$

Note that the return on domestic assets, r_t need not equal the interest rate on borrowing and lending from abroad, r_t^f.

The Euler equation describing optimal investment is

$$p_t^x = (1+r_t)^{-1} \left[\alpha \frac{y_{1,t+1}}{k_{t+1}} + (1-\delta)\, p_{t+1}^x \right],$$

which, along the BGP, becomes

$$\frac{1+r}{1+g^{p^x}} - 1 + \delta = \alpha \frac{z_c}{p^x} \left(\frac{k}{\ell} \right)^{\alpha-1},$$

and thus determines the initial capital stock, k_0, on that BGP, conditional on employment, ℓ_0.

From the capital accumulation equation, we then obtain the investment aggregate conditional on the capital stock,

$$\frac{x_0}{k_0} = g_k + \delta,$$

while the FOCs for the investment aggregate determine, for given prices q_0 and p_0^{xf}, the input ratio of domestic to foreign investment goods,

$$\frac{\omega p_0^x x_0 / x_0^{dd}}{(1-\omega) p_0^x x_0 / x_0^f} = \frac{\omega}{1-\omega} \cdot \frac{x_0^f}{x_0^{dd}} = \frac{q_0}{p_0^{xf}}.$$

Together with the level of aggregate investment and the investment aggregator, this equation then determines domestic and foreign investment inputs,

$$\begin{aligned} x_0 &= \left(\frac{p_0^{xf}}{q_0} \frac{\omega}{1-\omega} \right)^{\omega} x_0^f \\ &= \left(\frac{x_0^{dd}}{x_0^f} \right)^{\omega} x_0^f . \end{aligned}$$

Finally, we substitute net-exports in the resource constraint to obtain

$$
\begin{aligned}
y_0 &= z_{c,0} k_0^{\alpha} \ell_0^{1-\alpha} \\
&= c_0 + q_0 \left(x_0^{dd} + x_0^{df} \right) \\
&= c_0 + q_0 x_0^{dd} + NX_0 + p_0^{xf} x_0^{f} \\
&= c_0 + NX_0 + q_0 \left(x_0^{dd} + \frac{p_0^{xf}}{q_0} x_0^{f} \right) \\
&= c_0 + NX_0 + q_0 \left(x_0^{dd} + \frac{1}{\tau} x_0^{f} \right).
\end{aligned}
$$

Thus, the initial level of consumption is determined by the initial level of net-exports.

4 Endogenous Labor Supply

In this section, we extend the model described in Section 2 above to include more general preferences including an endogenous labor supply decision in each sector. A conventional treatment of labor supply produces a growth formula that is isomorphic to that presented in the main text. In particular, the way in which the network features of production and capital accumulation determine the influence of different sectors on aggregate growth is unchanged, as are the effects of long-run changes in TFP growth on GDP growth. The key difference is that with endogenous labor supply, the common and idiosyncratic components of labor input now carry a structural interpretation. Specifically, in the example below, the common component is associated with broad demographics such as population growth and how they affect labor input in each sector. The idiosyncratic component reflects sector-specific factors such as those which determine the disutility cost of working in different sectors, including a sector-specific Frisch elasticity, or sector-specific labor quality adjustments.

We underscore two observations in this context. First, the structural interpretation of variations in labor will necessarily be model dependent. In contrast, our focus in the main text is on growth accounting given the behavior of labor input whatever its underlying forces. Second, in this vein, we provide a historical decomposition of our findings but refrain from speculating on counterfactuals. As this section now makes clear, the upper and lower bound calculations provided in an earlier version of the paper, Foerster et al. (2019), reduce in part to making assumptions about sector-specific elasticities and other drivers of labor supply (which is not the focus of this paper).

4.1 A Model with Labor Choice

The observation in the 1980's that aggregate per capita hours worked in the post-WWII United States appeared more or less stationary motivated the traditional assumption that per capita employment should be constant along a BGP. This in turn motivated restrictions on preferences consistent with constant per capita employment along a BGP (see King, Plosser, and Rebelo (1988)). More recently, however, Boppart and Krusell (2020) have argued that over longer time periods, per capita hours worked are not actually stationary. In particular, they have declined over time in several advanced economies. Consequently, they propose preferences that generalize those in King et al. (1988) and allow for constant growth of per capita employment along the BGP.

Our data suggest variations in the trend growth rate of aggregate labor in the U.S., in part driven by quality improvements, as well as disparate trend variations in labor growth across sectors. Therefore, we introduce endogenous labor supply along the lines of Boppart and Krusell (2020) which allows for sectoral per capita labor to grow (or decline) at different rates in steady state.[8]

At each date t, the economy is populated by a continuum of identical households uniformly distributed on $[0,1]$ consisting of N_t family members. Each household supplies $h_{j,t}$ hours to sector j. Labor hours are quality adjusted according to a sector-specific factor, $q_{j,t}$. Therefore, total effective labor in sector j, $\ell_{j,t}$, is given by $\ell_{j,t} = q_{j,t}h_{j,t}$.

In each period, a family member derives utility from their share of an aggregate consumption bundle, C_t/N_t, and experiences a disutility cost from supplying labor to different sectors according to

$$\ln\left(\frac{C_t}{N_t}\right) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j},\ \nu_j \geq 0,$$

where $e_{j,t}$ scales the disutility of labor supplied to sector j and ν_j is a sector-specific Frisch elasticity of labor supply.

The planner then maximizes the utility of households,

$$\sum_{t=0}^{\infty} \beta^t N_t \left[\ln\left(\frac{C_t}{N_t}\right) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j}\right],$$

[8] Ngai and Pissarides (2007) explore an alternative framework where the reallocation of labor among consumption goods sectors is an outcome of unbalanced growth among those goods while preserving balanced growth at the aggregate level. Absent from their work, however, are the network considerations and the role of capital in determining network multipliers that are central to this paper.

where

$$\ln(C_t) = \sum_{j=1}^{n} \theta_j \ln\left(\frac{c_{j,t}}{\theta_j}\right), \ \sum_{j=1}^{n} \theta_j = 1, \ \theta_j \geq 0.$$

We let N_t grow at exogenous rate g_t^N to account for common or aggregate demographic forces in the economy (that raise the overall working age population). Moreover, to the degree that $e_{j,t}$ grows or declines at a constant rate over time along a balanced growth path, per capita labor, $h_{j,t}/N_t$, will decline or grow at a rate inversely proportional to $e_{j,t}^{\frac{1}{1+\nu_j}}$ so as to leave $e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}$ constant along that path. Finally, the rate of quality adjustment of labor is given by $g_{j,t}^q$.

As before, we let

$$\Delta \ln z_{j,t} = \lambda_j^z g_{f,t}^z + g_{u,j,t}^z,$$

where $g_{f,t}^z$ and $g_{u,j,t}^z$ denote respectively the common and sector-specific components of TFP growth in sector j, and λ_j^z is a loading that captures the effect of the common TFP component on sector j's productivity. We allow the exogenous drivers of labor supply in each sector, $\Delta \ln e_{j,t}$, to have their own (unique) idiosyncratic component, denoted $g_{u,j,t}^e$ and to differentially reflect the effects of demographics, $\lambda_j^N g_t^N$, so that

$$\Delta \ln e_{j,t} = \lambda_j^N g_t^N + g_{u,j,t}^e. \tag{31}$$

This specification allows common demographics in the working age population, such as the baby boom or the rising female labor force participation rate, to affect different sectors in different ways. In addition, it is also conceivable that labor quality adjustments in different sectors, $q_{j,t}$, are also driven by a common component reflecting, say, the overall state of education (which would introduce a second common factor that we abstract from for transparency). All other aspects of the economic environment are as described as in Section 2.

4.2 The Planner's Problem

The planner's problem now is

$$\begin{aligned}\max \ \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t N_t \left[\sum_{j=1}^{n} \theta_j \ln\left(\frac{c_{j,t}}{\theta_j}\right) - \ln(N_t) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j}\right] \\ & + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p_{j,t}^y \left[\left(\frac{v_{j,t}}{\gamma_j}\right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j}\right)^{(1-\gamma_j)} - c_{j,t} - \sum_{i=1}^{n} m_{ji,t} - \sum_{i=1}^{n} x_{ji,t}\right]\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^m_{j,t} \left[\prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}} - m_{j,t} \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^v_{j,t} \left[z_{j,t} \left(\frac{\ell_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} - v_{j,t} \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^x_{j,t} \left[\prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}} + (1-\delta_j) k_{j,t} - k_{j,t+1} \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} w_{j,t} \left[q_{j,t} h_{j,t} - \ell_{j,t} \right].
\end{aligned}
$$

The key differences with the first-order conditions presented in Section 2 are,

$$
\frac{N_t \theta_j}{c_{j,t}} = p^y_{j,t},
$$

which also defines the ideal price index,

$$
1 = \frac{C_t}{N_t} \underbrace{\prod_{j=1}^{n} \left(p^y_{j,t} \right)^{\theta_j}}_{P_t}.
$$

Labor supply and labor demand satisfy respectively,

$$
w_{j,t} = \frac{e_{j,t}}{q_{j,t}} \left(\frac{h_{j,t}}{N_t} \right)^{\nu_j},
$$

and

$$
w_{j,t} = (1-\alpha_j) \frac{p^v_{j,t} v_{j,t}}{\ell_{j,t}},
$$

while labor market clearing implies

$$
\ell_{j,t} = q_{j,t} h_{j,t}.
$$

Together these equations give:

$$
\ell_{j,t} = q_{j,t} (1-\alpha_j) \frac{p^v_{j,t} v_{j,t}}{e_{j,t}} \left(\frac{h_{j,t}}{N_t} \right)^{-\nu_j},
$$

It follows that

$$h_{j,t} = \left((1-\alpha_j)\frac{p^v_{j,t}v_{j,t}}{e_{j,t}}N_t^{\nu_j}\right)^{\frac{1}{1+\nu_j}} = \left((1-\alpha_j)\gamma_j\frac{p^y_{j,t}y_{j,t}}{e_{j,t}}N_t^{\nu_j}\right)^{\frac{1}{1+\nu_j}} = N_t\left((1-\alpha_j)\gamma_j\frac{\theta_j y_{j,t}}{c_{j,t}e_{j,t}}\right)^{\frac{1}{1+\nu_j}}$$

so that along a balanced growth path where $y_{j,t}$ and $c_{j,t}$ grow at the same rate, $\frac{h_{j,t}}{N_t}$ will grow at a rate inversely proportional to that of $e_{j,t}^{\frac{1}{1+\nu_j}}$ with $\frac{h_{j,t}}{N_t}e_{j,t}^{\frac{1}{1+\nu_j}}$ constant. The remaining equations are as in Section 2.

4.3 The Full Set of Equilibrium Conditions

The full set of equilibrium conditions includes the description of the economic environment,

$$c_{j,t} + \sum_{i=1}^n m_{ji,t} + \sum_{i=1}^n x_{ji,t} = y_{j,t},\ \forall j,$$

$$x_{j,t} = \prod_{i=1}^n \left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}},\ \forall j,$$

$$k_{j,t+1} = x_{j,t} + (1-\delta_j)k_{j,t},\ \forall j,\ \text{and } k_{j,0} \text{ given},$$

$$v_{j,t} = z_{j,t}\left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}\left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j},\ \forall j,$$

$$m_{j,t} = \prod_{i=1}^n \left(\frac{m_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}},\ \forall j,$$

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{m_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j},\ \forall j.$$

$$\ell_{j,t} = q_{j,t}h_{j,t},\ \forall j.$$

From the planner's problem, we have

$$\frac{N_t\theta_j}{c_{j,t}} = p^y_{j,t},\ \forall j,$$

$$\ln(C_t) = \sum_{j=1}^n \theta_j \ln\left(\frac{c_{j,t}}{\theta_j}\right)$$

$$\ell_{j,t} = q_{j,t}h_{j,t},\ \forall j,$$

$$h_{j,t} = N_t\left((1-\alpha_j)\gamma_j\frac{\theta_j y_{j,t}}{c_{j,t}e_{j,t}}\right)^{\frac{1}{1+\nu_j}},\ \forall j,$$

$$w_{j,t} = (1-\alpha_j)\,\frac{p^v_{j,t} v_{j,t}}{q_{j,t} h_{j,t}},\ \forall j,$$

$$\gamma_j \frac{p^y_{j,t} y_{j,t}}{v_{j,t}} = p^v_{j,t},\ \forall j,$$

$$(1-\gamma_j)\frac{p^y_{j,t} y_{j,t}}{m_{j,t}} = p^m_{j,t},\ \forall j,$$

$$\phi_{ij} \frac{p^m_{j,t} m_{j,t}}{m_{ij,t}} = p^y_{i,t},\ \forall i, j,$$

$$\omega_{ij} \frac{p^x_{j,t} x_{j,t}}{x_{ij,t}} = p^y_{i,t},\ \forall i, j,$$

$$p^x_{j,t} = \beta \mathbb{E}_t \left[\alpha_j \frac{p^v_{j,t+1} v_{j,t+1}}{k_{j,t+1}} + p^x_{j,t+1}(1-\delta_j) \right]\ \forall j.$$

4.4 Balanced Growth

The derivation of the balanced growth path follows as in Section 2. As before, TFP growth is constant in the long run and allows for both common and sector-specific components,

$$\Delta \ln z_{j,t} \equiv \lambda^z_j g^z_f + g^z_{u,j},$$

so that

$$\widetilde{z}_{j,t} = \frac{z_{j,t}}{z_{j,t-1}} = e^{g^z_j} \approx 1 + g^z_j.$$

In addition, population growth and the rate of labor quality adjustment are given by

$$\Delta \ln N_t \equiv g^N \rightarrow \widetilde{N}_t = \frac{N_t}{N_{t-1}} = e^{g^N} \approx 1 + g^N,$$

and

$$\Delta \ln q_{j,t} \equiv g^q_j \rightarrow \widetilde{q}_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = e^{g^q_j} \approx 1 + g^q_j,$$

while the growth rates of the exogenous drivers of labor supply follow equation (31),

$$\Delta \ln e_{j,t} \equiv g^e_j = \lambda^N_j g^N + g^e_{u,j}$$

so that

$$\widetilde{e}_{j,t} = \frac{e_{j,t}}{e_{j,t-1}} = e^{g^e_j} \approx 1 + g^e_j.$$

4.4.1 Making the Model Stationary

Following the steps in Section 2, we normalize the model's variables using sector-specific factors, $\mu_{j,t}$, that we need to solve for to characterize the BGP. As before, the resource constraint in any individual sector implies that all variables in that constraint must grow at the same rate. Thus, define $\widetilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$, $\widetilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$, $\widetilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\widetilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$. Then, the economy's resource constraint becomes

$$\widetilde{c}_{j,t} + \sum_{i=1}^{n} \widetilde{m}_{ji,t} + \sum_{i=1}^{n} \widetilde{x}_{ji,t} = \widetilde{y}_{j,t}.$$

The production of investment goods may be re-written as

$$\widetilde{x}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{x}_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}},$$

where $\widetilde{x}_{j,t} = x_{j,t}/\prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$. Under this normalization, the capital accumulation equation is

$$k_{j,t+1} = \widetilde{x}_{j,t} \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}} + (1-\delta_j) k_{j,t},$$

and so becomes

$$\widetilde{k}_{j,t+1} = \widetilde{x}_{j,t} + (1-\delta_j) \widetilde{k}_{j,t} \prod_{i=1}^{n} \left(\frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where $\widetilde{k}_{j,t+1} = k_{j,t+1}/\prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$.

For the normalization of the labor market equations when labor supply is endogenous, observe that using the normalized first-order demand condition,

$$\frac{\theta_j}{\widetilde{c}_{j,t}} = \widetilde{p}^{y}_{j,t} \equiv \frac{p^{y}_{j,t} \mu_{j,t}}{N_t},$$

with

$$\ell_{j,t} = q_{j,t} h_{j,t} = q_{j,t} N_t \left((1-\alpha_j) \gamma_j \frac{\theta_j y_{j,t}}{c_{j,t} e_{j,t}} \right)^{\frac{1}{1+\nu_j}} = q_{j,t} N_t \left((1-\alpha_j) \gamma_j \frac{\theta_j \widetilde{y}_{j,t}}{\widetilde{c}_{j,t} e_{j,t}} \right)^{\frac{1}{1+\nu_j}},$$

we obtain

$$\widetilde{\ell}_{j,t} = \frac{\ell_{j,t} e_{j,t}^{\frac{1}{1+\nu_j}}}{q_{j,t} N_t} = \left((1-\alpha_j)\gamma_j \widetilde{p}_{j,t}^{y} \widetilde{y}_{j,t}\right)^{\frac{1}{1+\nu_j}} .$$

$$\widetilde{h}_{j,t} = \frac{h_{j,t} e_{j,t}^{\frac{1}{1+\nu_j}}}{N_t} = \left((1-\alpha_j)\gamma_j \widetilde{p}_{j,t}^{y} \widetilde{y}_{j,t}\right)^{\frac{1}{1+\nu_j}} .$$

Labor market clearing thus gives us a solution for wages,

$$\begin{aligned} w_{j,t} &= (1-\alpha_j)\,\frac{p_{j,t}^{v} v_{j,t}}{q_{j,t} h_{j,t}} = (1-\alpha_j)\,\gamma_j \frac{p_{j,t}^{y} y_{j,t}}{\ell_{j,t}} \\ &= e_{j,t}^{\frac{1}{1+\nu_j}} ((1-\alpha_j)\gamma_j \frac{\widetilde{p}_{j,t}^{y} \widetilde{y}_{j,t}}{q_{j,t} \widetilde{\ell}_{j,t}} \end{aligned}$$

where normalized wages are then given by

$$\widetilde{w}_{j,t} = ((1-\alpha_j)\gamma_j \widetilde{p}_{j,t}^{y} \widetilde{y}_{j,t})^{\frac{\nu_j}{1+\nu_j}} = \frac{q_{j,t} w_{j,t}}{e_{j,t}^{\frac{1}{1+\nu_j}}}.$$

Following the steps in Section 2, the expression for value added may be written as

$$v_{j,t} = z_{j,t} \left(\frac{\widetilde{\ell}_{j,t} q_{j,t} N_t}{(1-\alpha_j) e_{j,t}^{\frac{1}{1+\nu_j}}} \right)^{1-\alpha_j} \left(\frac{\widetilde{k}_{j,t} \prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}}}{\alpha_j} \right)^{\alpha_j} ,$$

so that, defining $\widetilde{v}_{j,t} = v_{j,t} e_{j,t}^{\frac{1-\alpha_j}{1+\nu_j}} / z_{j,t} (q_{j,t} N_t)^{1-\alpha_j} \left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$, it becomes

$$\widetilde{v}_{j,t} = \left(\frac{\widetilde{\ell}_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j} \left(\frac{\widetilde{k}_{j,t}}{\alpha_j} \right)^{\alpha_j} .$$

The composite bundle of materials used in sector j may be expressed as

$$\widetilde{m}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{m}_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}} ,$$

with $\widetilde{m}_{j,t} = m_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\phi_{ij}}$.

Under the normalization, gross output may be written as

$$\widetilde{y}_{j,t}\mu_{j,t} = \left(\frac{\widetilde{v}_{j,t}z_{j,t}(q_{j,t}N_t)^{1-\alpha_j}\prod_{i=1}^{n}\mu_{i,t-1}^{\alpha_j\omega_{ij}}}{e_{j,t}^{\frac{1-\alpha_j}{1+\nu_j}}\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}}{1-\gamma_j}\right)^{1-\gamma_j},$$

which, collecting terms, gives

$$\widetilde{y}_{j,t} = \left(\frac{\widetilde{v}_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}\left[\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\right].$$

As before, we can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\left(\prod_{i=1}^{n}\frac{\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}}{\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}}\right)\left(\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}\right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Then, without loss of generality with respect to growth rates, we choose $\mu_{j,t}$ such that

$$\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}+(1-\gamma_j)\phi_{ij}} = 1.$$

4.4.2 Sectoral Value Added Growth

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln z_{j,t}q_{j,t}^{1-\alpha_j}N_t^{1-\alpha_j}e_{j,t}^{-\frac{1-\alpha_j}{1+\nu_j}} - \ln\mu_{j,t} + \sum_{i=1}^{n}(\gamma_j\alpha_j\omega_{ij} + (1-\gamma_j)\phi_{ij})\ln\mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d\ln z_t + \Gamma_d(1-\alpha_d)(\ln N_t + \ln q_t) - \Gamma_d(I-\alpha_d)(I+\nu_d)^{-1}\ln e_t - \ln\mu_t + \Gamma_d\alpha_d\Omega'\ln\mu_t + (I-\Gamma_d)\Phi'\ln\mu_t = 0,$$

which gives us

$$\ln \mu_t = \Xi'(\ln z_t + (I - \alpha_d)(\ln N_t + \ln q_t) - (I - \alpha_d)(I + \nu_d)^{-1} \ln e_t), \tag{32}$$

where

$$\Xi' = \left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d,$$

with $\Xi = \{\xi_{ij}\}$ is the same generalized Leontief inverse as in Section 2.

Going back to equation (32), and writing the vector of productivity growth rates as $\Delta \ln z_t = g^z$, population growth as $\Delta \ln N_t = g^N$, the rates of labor quality adjustment as $\Delta \ln q_t = g^q$, and the growth rates of drivers of labor supply as $\Delta \ln e_t = g^e$, it follows that

$$\begin{aligned}
\Delta \ln \mu_t &= \Xi'(g^z + (I - \alpha_d)g^N + (I - \alpha_d)g^q - (I - \alpha_d)(I + \nu_d)^{-1} g^e) \\
&= \Xi' \left(\underbrace{\lambda^z g^z_f + g^z_u}_{g^z} + (I - \alpha_d)g^N + (I - \alpha_d)g^q - (I - \alpha_d)(I + \nu_d)^{-1} (\underbrace{\lambda^N g^N + g^e_u}_{g^e}) \right).
\end{aligned}$$

Let $\mu^v_{j,t}$ denote the normalizing factor for value added in sector j, and recall from the definition of normalized or detrended value added above that

$$\mu^v_{j,t} = z_{j,t}(q_{j,t}N_t)^{1-\alpha_j} e_{j,t}^{-\frac{1-\alpha_j}{1+\nu_j}} \left(\prod_{j=1}^{n} \mu^{\omega_{ij}}_{i,t-1} \right)^{\alpha_j}.$$

In addition, since $\mu^v_{j,t}$ makes normalized value added in sector j constant along the steady state growth path, it must grow at the same rate as j's value added along that path, denoted g^v_j. Then, using equation (32), we have that

$$\begin{aligned}
\ln \mu^v_t = &(\ln z_t + (I - \alpha_d)\ln N_t + (I - \alpha_d)\ln q_t - (I - \alpha_d)(I + \nu_d)^{-1} \ln e_t) \\
&+ \alpha_d \Omega' \Xi' (\ln z_{t-1} + (I - \alpha_d)\ln N_{t-1} + (I - \alpha_d)\ln q_{t-1} - (I - \alpha_d)(I + \nu_d)^{-1} \ln e_{t-1}),
\end{aligned}$$

or

$$g^v = [I + \alpha_d \Omega' \Xi'] \left(\lambda^z g^z_f + g^z_u + (I - \alpha_d) \left(g^N + g^q - (I + \nu_d)^{-1} (\lambda^N g^N + g^e_u) \right) \right). \tag{33}$$

Comparing equation (33) to (9), the basic expression is unchanged. In particular, the production features that determine the influence that different sectors have on GDP both directly, via I, and indirectly, via $\alpha_d \Omega' \Xi'$, are the same as before as are the effects of long-run TFP growth on GDP growth. The key difference is that with endogenous labor supply, the common and idiosyncratic components of sectoral labor input now have a structural interpretation. Thus, the common component of labor input growth in equation (9), g^{ℓ}_{f}, is associated with overall demographics, g^N, and individual sector loadings in equation (9), λ^{ℓ}_{j}, are given by $(1-\alpha_j)(1-(1+\nu_j)^{-1}\lambda^N_j)$ which capture the way these demographics affect labor in individual sectors. Similarly, the idiosyncratic component of labor input growth in equation (9), g^{ℓ}_{j}, corresponds to $(1-\alpha_j)g^q_j-(1-\alpha_j)(1+\nu_j)^{-1}g^e_j$. In other words, in each sector, our estimates of sector-specific labor input growth reflect sector-specific capital shares and Frisch elasticities as well as idiosyncratic labor quality adjustments and the disutility costs of providing labor to sector j (a lower disutility cost increases labor input in that sector).

5 Capital and the Effects of Sectoral Disturbances on the Level of GDP

In this section, we relate our work to that of Acemoglu et al. (2012) and, more recently, Baqaee and Farhi (2019). In a static multisector environment which abstracts from the production of capital, Acemoglu et al. (2012) show that the effects of a TFP change scaling value added in a given sector on GDP is that sector's value added share in GDP (it would be its Domar weight if TFP scaled gross output rather than value added). It may be natural, therefore, to interpret this outcome in terms of Hulten (1978)'s aggregation result and Baqaee and Farhi (2019) refer to it as Hulten's theorem.

Baqaee and Farhi (2019) then explore the role of non-linearities in generating aggregate effects from sectoral disturbances over and above Hulten's theorem. Our work focuses instead on other key assumptions, unexplored in previous work, that nevertheless are central for understanding the aggregate implications of sectoral trends. One is the role that capital plays as part of a production network in amplifying the effects of sectoral changes on long-run GDP growth. Here, the long-run dynamics of capital accumulation are central to that role. Another is that while Acemoglu et al. (2012) and much subsequent work focuses mostly on level effects, $\partial \ln V/\partial \ln A$, our empirical motivation lies in the implications of sectoral (trend) growth rates for long-run GDP growth, $\partial\Delta \ln V/\partial\Delta \ln A$.

Naturally, the finding in Acemoglu et al. (2012) that Baqaee and Farhi (2019) refer to as Hulten's theorem remains nested by a static version of our economic environment without capital and where the focus is on levels rather than growth rates. Below, we show how this 'levels' insight changes in the steady state of a dynamic economy with capital.[9]

[9] Hulten's theorem then emerges as a special case where shares of capital in value added in every sector, α_d, are set to zero.

5.1 The Model with Capital Goods but No Steady State Growth

We saw in Section 2 that the full set of equilibrium conditions immediately implied a set of relationships (in levels) between the prices of materials, investment, and value added,

$$\Theta \ln p^y = 0,$$

$$\ln p^v = \Gamma_d^{-1} \left[I - (I - \Gamma_d)\Phi' \right] \ln p^y,$$
$$\ln p^x = \Omega' \ln p^y,$$

where $p^v = (p_1^v, ..., p_n^v)'$, $p^x = (p_1^x, ..., p_n^x)'$, and $p^y = (p_1^y, ..., p_n^y)'$, and where recall that Θ is a $1 \times n$ vector of consumption shares, Γ_d is an $n \times n$ matrix of value added shares in gross output, Φ is an $n \times n$ matrix of materials input shares and Ω is an $n \times n$ capital flow matrix.

Furthermore, from the definition of value added, we have that

$$\ln v_j = \ln A_j + \alpha_j \ln \left(\frac{k_j}{\alpha_j} \right),$$

and from the Euler equation governing the optimal choice of capital in each sector,

$$\frac{k_j}{\alpha_j} = \left(\frac{p_j^v v_j}{p_j^x} \right) \left(\frac{\beta}{1 - \beta(1 - \delta_j)} \right).$$

Combining these expressions gives

$$\ln v_j = \ln A_j + \alpha_j \ln \left(p_j^v v_j \right) - \alpha_j \ln p_j^x + \alpha_j \ln \left(\frac{\beta}{1 - \beta(1 - \delta_j)} \right),$$

or in matrix form,

$$(I - \alpha_d) \ln (p^v . \times v) = \ln A + \ln p^v - \alpha_d \ln p^x + \alpha_d \ln \Delta_d,$$

where $(p^v . \times v)$ represents the vector of nominal value added, $\left\{ p_j^v v_j \right\}$, $\alpha_d = diag(\alpha_j)$, and $\Delta_d = diag \left(\frac{\beta}{1-\beta(1-\delta_j)} \right)$. Substituting for investment and value added prices on the right-hand-side of this last expression, we obtain

$$\ln p^y = \left(\Gamma_d^{-1} \left[I - (I - \Gamma_d)\Phi' \right] - \alpha_d \Omega' \right)^{-1} \left[(I - \alpha_d) \ln (p^v . \times v) - \ln A - \alpha_d \ln \Delta_d \right]. \qquad (34)$$

From the resource constraints in each sector j, and the optimal allocation of materials and investment in the economy, it follows that

$$\frac{p_j^v v_j}{\gamma_j} = \theta_j C + \sum_{i=1}^{n} \phi_{ji}(1-\gamma_j)\frac{p_i^v v_i}{\gamma_i} + \sum_{i=1}^{n} \omega_{ji}\frac{\beta\delta_j}{1-\beta(1-\delta_j)}\alpha_i p_i^v v_i,$$

or in matrix form,

$$\Gamma_d^{-1}\left(p^v.\times v\right) = \Theta' C + \Phi(I-\Gamma_d)\Gamma_d^{-1}\left(p^v.\times v\right) + \Omega\Delta_d\delta_d\alpha_d\left(p^v.\times v\right), \tag{35}$$

so that

$$\frac{(p^v.\times v)}{C} = \left(\left[I-\Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d\right)^{-1}\Theta'. \tag{36}$$

Aggregate GDP (in units of the final consumption bundle) is then given by

$$V = \mathbf{1}'\left(p^v.\times v\right) = \mathbf{1}'\psi C, \tag{37}$$

where $\psi = \left(\left[I-\Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d\right)^{-1}\Theta'$.

Substituting the expression for value added in (2.5) into the equation for gross output prices, (34), and using the fact that the ideal price index for the final consumption bundle implies $\Theta \ln p^y = 0$, we obtain

$$\ln C = \frac{\Theta\left(\Gamma_d^{-1}\left[I-(I-\Gamma_d)\Phi'\right] - \alpha_d\Omega'\right)^{-1}\left[\ln A + \alpha_d \ln\Delta_d - (I-\alpha_d)\ln\psi\right]}{\Theta\left(\Gamma_d^{-1}\left[I-(I-\Gamma_d)\Phi'\right] - \alpha_d\Omega'\right)^{-1}(I-\alpha_d)\mathbf{1}},$$

Note: In the denominator, $\Theta\left(\Gamma_d^{-1}\left[I-(I-\Gamma_d)\Phi'\right] - \alpha_d\Omega'\right)^{-1}(I-\alpha_d)\mathbf{1} = 1$.

It follows that

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{\left(\left[I-\Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\alpha_d\right)^{-1}\Theta'\right\}_j. \tag{38}$$

5.1.1 Interpretation in Terms of Sectoral Shares

To give an an interpretation to this result, observe from equations (36) and (37) above that the vector of value added shares s^v, is simply given by

$$\frac{(p^v.\times v)}{V} = \frac{\left(\left[I-\Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d\right)^{-1}\Theta'}{\mathbf{1}'\left(\left[I-\Phi(I-\Gamma_d)\right]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d\right)^{-1}\Theta'}. \tag{39}$$

We now examine the limit case where $\beta \to 1$ so that $\Delta_d \delta_d \to I$. Then, we can write equation (38) as

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v,$$

where $\eta = \mathbf{1}' \left([I - \Phi(I - \Gamma_d)] \Gamma_d^{-1} - \Omega\alpha_d \right)^{-1} \Theta'$.

In particular, η is approximately the inverse of the mean labor share in value added across sectors. To see this, observe that η can also be expressed as $\eta = \frac{\mathbf{1}'\left([I-\Phi(I-\Gamma_d)]\Gamma_d^{-1}-\Omega\alpha_d\right)^{-1}\Theta'}{\mathbf{1}'(I-\alpha_d)\left([I-\Phi(I-\Gamma_d)]\Gamma_d^{-1}-\Omega\alpha_d\right)^{-1}\Theta'}$ since the denominator equals 1. Thus, when $\alpha_j = \alpha \; \forall j$, $\eta = \frac{1}{1-\alpha}$.

Equation (38) then tells us that in the steady state of a dynamic economy with capital (and no sectoral or aggregate growth), the effect of a change in productivity in a given sector on GDP is that sector's value added share in GDP scaled by the inverse of the mean labor share in the economy.

5.1.2 Recovering Hulten's Theorem as a Special Case without Capital

To recover the starting insight in Acemoglu et al. (2012), it suffices to get rid of capital in the economy and set the corresponding shares, α_d, to zero across sectors. Then equation (38) becomes

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{ \Gamma_d \left[I - \Phi(I - \Gamma_d) \right]^{-1} \Theta' \right\}_j .$$

At the same time, equation (39) which defines sectoral value added shares in GDP becomes

$$\frac{(p^v . \times v)}{V} = \frac{\Gamma_d \left[I - \Phi(I - \Gamma_d) \right]^{-1} \Theta'}{\mathbf{1}'\Gamma_d \left[I - \Phi(I - \Gamma_d) \right]^{-1} \Theta'},$$

where the denominator is simply equal to 1. In other words, in this case, the effect of a change in productivity in a given sector on GDP is simply that sector's value added share in GDP, $\partial \ln V / \partial \ln A_j = s_j^v$. Alternatively, Acemoglu et al. (2012) refer to the matrix, $\Gamma_d \left[I - \Phi(I - \Gamma_d) \right]^{-1} \Theta'$, in equation (38) as the *influence vector*. In that expression, the matrix $\left[I - \Phi(I - \Gamma_d) \right]^{-1}$ is the Leontief inverse, $\mathscr{L}$, which can also be expressed as the Neumann series $\mathscr{L} = I + \Phi(I - \Gamma_d) + \Phi(I - \Gamma_d)^2 + \dots$. This series summarizes the way in which a disturbance in sector j percolates to other sectors. The initial change affects sector j's purchases of inputs from other sectors and, since those sectors may themselves purchase inputs from sector j to produce their own output, the process kicks off another round of knock-on effects and so on.

6 Measurement Error and Bias

6.1 Capital Aggregation

While the framework we laid out allows for a relatively tractable characterization of long-run growth when intricate production linkages are present, both in materials and investment goods, it also contains a discrepancy in the way that capital aggregation is treated relative to NIPA. In our framework, multiple investment goods are aggregated into one investment good, and the capital stock reflects the undepreciated component of this aggregate investment good. In NIPA, the services of multiple capital stocks are instead aggregated into one aggregate capital service flow, and each type of capital evolves according to its corresponding investment decisions.[10] Gourio and Rognlie (2020) point out that the difference in setups can bias estimates of the contributions from capital to growth in the model with multiple investment goods. The importance of this bias hinges on different capital types having substantially different rates of depreciation and price changes, e.g., structures versus equipment.

It is easiest to illustrate these points in the context of an aggregate model that nevertheless allows for multiple capital goods as well as one consumption good, similar to Greenwood, Hercowitz, and Krusell (1997) discussed above. Thus, consider the following model with aggregate resource constraint,

$$c_t + \sum_{j=2}^{n} q_{j,t} x_{j,t} = y_{1,t} = z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha}, \tag{40}$$

$$q_{j,t} = z_{1,t}/z_{j,t}, \;\; j = 2, \ldots, n.$$

Here, aggregate capital in the production function, k_t, reflects the services of multiple capital stocks, $k_{j,t}$ according to weights ϕ_j, and the evolution of each capital type is determined by a type-specific accumulation equation. In particular,

$$k_t = \prod_{j=2}^{n} k_{j,t}^{\phi_j}, \;\; \text{with} \;\; \sum_{j=2}^{n} \phi_j = 1, \tag{41}$$

$$k_{j,t+1} = x_{j,t} + (1 - \delta_j) k_{j,t}, \;\; j = 2, \ldots, n.$$

We refer to this set up as the 'capital-aggregate' model. The capital pricing equations in

[10] In an analysis of transition dynamics or impulse responses to disturbances, our approach has the dimension of the capital state vector increasing linearly with the number of sectors, rather than quadratically as in the NIPA procedure.

this case are

$$(1+r_t)q_{j,t} = u_{j,t+1} + (1-\delta_j)q_{j,t+1}, \quad j = 2,\ldots,n, \tag{42}$$

and

$$u_{j,t} = \alpha z_{1,t}\left(\frac{k_t}{\ell_t}\right)^{\alpha-1}\phi_j\frac{k_t}{k_{j,t}}, \quad j = 2,\ldots,n.$$

Along a BGP, all variables grow at constant but potentially different rates. Specifically, the resource constraint implies that

$$g_1^y = g^c = g_j^q + g_j^x = g_1^z + \alpha g^k + (1-\alpha)g^\ell = g_1^a + \alpha g^k,$$

while from the capital aggregation and accumulation equations, it also follows that

$$g^k = \sum_{j=2}^{n}\phi_j g_j^k \text{ where } g_j^k = g_j^x.$$

Therefore, along the BGP, output growth (in terms of consumption units) is

$$g_1^y = \frac{g_1^a - \alpha\sum_{j=2}^{n}\phi_j g_j^q}{1-\alpha}. \tag{43}$$

6.1.1 Bias from Alternative Aggregation

The framework we exploit in the main text allows for a relatively tractable characterization of long-run growth in the presence of different types of production linkages. However, its aggregation properties with respect to capital differ somewhat from those we have just discussed. In particular, in the context of the multi-capital goods model just introduced, while the resource constraint is consistent with equation (40), multiple investment goods, x_t^j, are aggregated into one investment good, x_t, and the capital stock reflects the undepreciated component of this aggregate investment good. Thus, we have that

$$x_t = \prod_{j=2}^{n} x_{j,t}^{\omega_j}, \text{ with } \sum_{j=2}^{n}\omega_j = 1, \tag{44}$$

with

$$k_{t+1} = x_t - (1-\delta)k_t.$$

We refer to this set-up as the 'investment-aggregate' model.

Along the BGP, the growth rate of output (in consumption units) is now

$$g_1^y = \frac{g_1^a - \alpha \sum_{j=2}^{n} \omega_j g_j^q}{1-\alpha}. \tag{45}$$

Therefore, the extent to which the growth rate in equation (45) differs from (or is biased relative to) that in equation (43) depends on the degree to which changes in relative prices, g_j^q, are weighted differently. The weights in equation (43) refer to capital income shares in production, ϕ_j, while the weights in equation (45) refer to investment shares, ω_j.

Along the BGP, capital income share weights, ϕ_j, and investment expenditure share weights, ω_j, in the 'capital-aggregate' model are related through the capital rental and Euler equations as well as investment-capital ratios. In particular, from equation (42) we have that

$$u_j k_j = \left[\frac{1+r}{1+g_j^q} - (1-\delta_j)\right] q_j k_j \approx \left(r - g_j^q + \delta_j\right) q_j k_j \tag{46}$$

Dividing through by the aggregate capital income share and using the capital accumulation equation for each type, we obtain

$$\phi_j = \frac{r - g_j^q + \delta_j}{g_j^k + \delta_j} \cdot \frac{q_j x_j}{\alpha y_1} = \frac{r - g_j^q + \delta_j}{g_1^y - g_j^q + \delta_j} \cdot \frac{q_j x_j}{\alpha y_1}.$$

Therefore,

$$q_j x_j = \left[\alpha y_1 \frac{g_1^y - g_j^q + \delta_j}{r - g_j^q + \delta_j}\right] \phi_j = \psi_j \phi_j,$$

which establishes the relationship between sectoral investment shares, ω_j, and capital income shares, ϕ_j, conditional on sectoral depreciation rates and the steady state evolution of relative prices,

$$\omega_j = \frac{\phi_j \psi_j}{\sum_{s>1} \phi_s \psi_s}.$$

Note that when depreciation rates, δ_j, and the steady state evolution of relative prices, q_j, are close, the differences between ψ_j's are small which implies that differences between investment and capital income shares will also be small.

6.1.2 Quantitative Assessment

We now gauge how the potential for misspecification bias plays out in the non-financial corporate sector of the U.S. economy for the period 1950-2016.

The BEA's Fixed Asset Tables gives us information on the three main capital aggregates: structures, equipment and intellectual property products (IPP). For each capital type, we have information on nominal and real net-stocks, depreciation, and investment. To better highlight the effects of differences in depreciation rates and relative price changes on BGP calculations using investment expenditure shares rather than capital income shares, we aggregate the two high depreciation types, equipment and IPP, into a single category, 'E&I.' We then contrast this category with structures. Repeating the exercise with the three capital types yields essentially the same findings.

From the National Income Accounts (NIAs), we obtain nominal gross value added (GVA) and the price indices for non-durable consumption goods and services. We construct a joint price index for non-durable consumption goods and services which we use to deflate nominal GVA and investment goods prices to obtain aggregate output, y_1, and investment good prices, q_j, in units of consumption goods. From the BLS Productivity and Cost Tables, we obtain labor input as total hours worked and the labor compensation share.

Assuming zero profits, we first allocate non-labor compensation to the two capital types assuming that the rates of return are equalized. This is a standard procedure in productivity accounting, Organisation for Economic Co-operation and Development (2009). Summing the capital rental equation (46) across capital types, we have that

$$\alpha y_1 = \sum_{j \in \{S, E\&I\}} u_j k_j = \sum_{j \in \{S, E\&I\}} \left(r + \delta_j - g_j^q \right) q_j k_j$$

which allows us to solve for the implicit rate of return on capital,

$$r = \frac{\alpha y_1 - \sum_j \left(\delta_j - g_j^q \right) q_j k_j}{\sum_j q_j k_j}.$$

Given data on capital compensation, depreciation, and the value of the net-stock of capital, we can calculate the implicit rate of return on capital, r. Given r we can then calculate income shares for the different capital types.

For the full sample, the average capital income share and its allocation among the two capital types are

$$\alpha = 0.37, \;\; \phi_S = 0.44 \text{ and } \phi_{E\&I} = 0.56.$$

This compares with the average allocation of investment among the two capital types of

$$\omega_S = 0.26 \text{ and } \omega_{E\&I} = 0.74.$$

Because structures depreciate at a lower rate than does E&I, the net-stock of structures is relatively high despite the smaller investment share for structures. The higher net-stock of structures in turn implies a higher implicit share in capital income for structures.

Carrying out a growth accounting exercise similar to Greenwood, Hercowitz, and Krusell (1997), we calculate the average growth rates for employment, consumption-specific TFP, and the relative TFP for the production of investment goods,

$$g^{\ell} = 1.5\%,\ g_1^z = 0.6\%,\ g_S^q = 0.8\%,\ g_{E\&I}^q = -1.6\%$$

Using these primitives, we can calculate the implied growth rates along the BGP using either of the two average capital allocation shares,

$$\phi : g^k = 3.3\% \text{ and } g_1^y = 2.8\%$$
$$\omega : g^k = 4.0\% \text{ and } g_1^y = 3.0\%$$

Using investment shares instead of capital shares puts relatively more weight on the faster growing E&I TFP component and, therefore, overstates somewhat the BGP contributions from capital, g^k. However, the implied bias for output growth is about 0.2 ppts, 3.0 percent vs. 2.8 percent. This bias is noticeable but not overly so. Moreover, it would have immaterial effects on our findings regarding the relative importance of sector-specific factors in driving long-run aggregate growth rates.

6.2 Mismeasurement of Output

One challenge with the analysis of multisector environments is that output is more easily measured in some sectors, for example Durable Goods, than others, for example Professional and Business Services (PBS) or Financial Services. We now discuss the implications from systematic output mismeasurement on the BGP, and derive expressions for carrying out counterfactuals given varying degrees of measurement error.

A systematic bias in measured gross output growth rates on the BGP means that there is also a systematic bias in the measured growth rates of capital and intermediate inputs. Taking these biases into account, the production network and capital accumulation structure of an economy impart any bias from one sector's output growth to measured TFP growth rates in potentially all sectors. The model structure can be used to calculate the effect of measurement error in sectoral output growth on aggregate GDP growth directly, without going through the intermediate steps of adjusting the measured growth rates of capital, intermediate inputs, and implied TFP growth rates.

Consider a BGP for our model where the growth rates of gross output, capital, and employment are respectively given by $\left\{g^y, g^k, g^{\ell}\right\}$. The parameters of the model, $\{\alpha, \Gamma, \Omega, \Phi\}$, are assumed known and so are employment growth rates in all sectors. However, we observe biased measures of gross output growth rates, denoted $g^{y,m}$, where the second superscript m stands for 'measured.' This measurement error then results in biased growth measures of intermediate inputs and capital,

$$\begin{aligned} g^{y,m} &= g^y + \epsilon, \\ g^{k,m} &= \Omega' g^{y,m} = g^k + \Omega'\epsilon, \\ g^{m,m} &= \Phi' g^{y,m} = g^m + \Phi'\epsilon. \end{aligned}$$

Since the parameters of the environment are known, we can also calculate the resulting measurement error in productivity growth from the production functions,

$$\begin{aligned} g^{y,m} &= \Gamma\left[g^{z,m} + \alpha_d g^{k,m} + (I-\alpha_d)\, g^{\ell}\right] + (I-\Gamma)\, g^{m,m}, \\ g^y + \epsilon &= \Gamma\left[g^{z} + \alpha_d g^{k} + (I-\alpha_d)\, g^{\ell}\right] + (I-\Gamma)\, g^{m} \\ &\quad + \Gamma\left(g^{z,m} - g^z\right) + \Gamma\alpha_d\Omega'\epsilon + (I-\Gamma)\,\Phi'\epsilon. \end{aligned}$$

Therefore,

$$\begin{aligned} g^{z,m} &= g^z + \Gamma^{-1}\left[I - \Gamma\alpha_d\Omega' - (I-\Gamma)\,\Phi'\right]\epsilon \\ &= g^z + \Xi'^{-1}\epsilon, \end{aligned}$$

so that measurement bias in any sector's output growth rate, ϵ, is generally reflected in all sectors' measured TFP growth, $g^{z,m}$, through the generalized Leontief inverse. In particular, to the degree that output growth in PBS is under measured for example ($\epsilon_{PBS} < 0$), so is its TFP growth rate, while TFP growth in other sectors tends to be overstated (because the off-diagonal elements of Ξ'^{-1} are generally negative).[11] This last expression then allows us to carry out counterfactuals exploring the implications of measurement error in sectoral gross output growth.

Taking measured TFP growth rates as given, the implied BGP growth rates for sectoral gross output are

$$g^y\left(g^{z,m}\right) = \Xi'\left(g^z + \Xi'^{-1}\epsilon\right) = g^y + \epsilon = g^{y,m},$$

which just confirms the mismeasurement of gross output growth along the BGP. It also

[11] The net-effect may well be that aggregate TFP, defined as the value-added share weighted average of sectoral TFP growth rates is overstated. See the discussion below.

follows that removing the measurement error, ϵ, changes measured GDP growth by

$$- s^{v\prime}(I + \alpha_d \Omega' \Xi') \Xi'^{-1} \epsilon = -s^{v\prime} \Xi'^{-1} \epsilon - s^{v\prime} \alpha_d \Omega' \epsilon. \tag{47}$$

Thus, correcting any downward bias in the measurement of output in PBS, the first term on the RHS of the above expression, $-s^{v} \Xi'^{-1} \epsilon$ increases the contributions to GDP growth from PBS and lowers the contributions from other sectors (because measured TFP growth is now higher in PBS and lower in other sectors). The second term, $-s^{v\prime} \alpha_d \Omega' \epsilon$, generally increases all sectors' contributions to GDP growth to the degree that PBS sells some investment goods to these sectors. The net effect of correcting for understated output growth in a sector, therefore, is an increase in that sector's contributions to GDP growth, and either an increase or decrease in the contributions from other sectors.

6.2.1 Unmeasured capital growth

Our discussion of mismeasured output growth is related to that of unmeasured capital inputs in Basu et al. (2004) and Byrne et al. (2016). They argue that unmeasured output growth coming from unmeasured investment growth leads to an overstatement of aggregate TFP growth, that is the value-added share weighted average of sectoral TFP growth rate.

The common elements in Basu et al. (2004) and our analysis are easiest to see in the two sector GHK (1997) economy that differentiates between investment and consumption goods. Thus, consider again a BGP and suppose that only investment goods are mismeasured, that is,

$$g^{x,m} = g^{x} + \epsilon = g^{k.m}.$$

Then measured TFP growth for the investment and consumption goods sectors are respectively,

$$\begin{aligned} g^{z_x,m} &= g^{z_x} + (1-\alpha)\epsilon, \\ g^{z_c,m} &= g^{z_c} - \alpha\epsilon. \end{aligned}$$

If measured investment growth is understated, $\epsilon < 0$, then measured TFP growth in the investment goods sector is understated, and measured TFP growth in the consumption sector is overstated, since capital input growth is understated.

Then, aggregate TFP growth is the value-added-share-weighted average of sectoral TFP growth rates,

$$g^{z_V,m} = g^{z_V} + \left[s_x(1-\alpha) - s_c\alpha\right]\epsilon = g^{z_V} + (s_x - \alpha)\epsilon.$$

In this simple two-sector example without intermediate goods, the value-added shares of consumption and investment correspond to their shares in GDP. Therefore, since the investment share, s_x, tends to be smaller than the capital income share, α, understating output growth in the investment goods sector, $\epsilon < 0$, leads to an overstatement of aggregate TFP growth, $g^{z_V,m} > g^{z_V}$.

6.3 Misclassification of Goods

Aside from the mismeasurement of output, the other key challenge to measurement is the classification of goods. In some sectors, such as Professional Business Services, and in particular Computer System Design, the distinction between materials and investment goods is not always unambiguous. Over time the BEA has in several instances come to recognize expenditures on goods as investment rather than payments for intermediate inputs. This is the case, for example, in the comprehensive National Income Accounts (NIA) revisions of 1999 for software expenditures, and that of 2013 regarding expenditures on R&D and entertainment originals. Therefore, in this section, we explore in detail the implications of misclassifying goods.

Consider a commodity c that is used as both an intermediate input in industry i, $U_{c,i}$, and as an investment good in final demand, f, $e_{c,f}$.[12] Suppose that a fraction, ν, of c is misclassified as intermediate goods. Correcting this error then requires rescaling the contribution of c as an intermediate good across industries,

$$U_{c,i} \to (1-\nu)U_{c,i},$$

and reallocating it to the use of c in the production of final demand f,

$$e_{c,f} \to e_{c,f} + \nu \sum_i U_{c,i}.$$

One then needs to adjust the use of investment goods, f, across industries. A natural way to reallocate c among industries is to use the original distribution of good c as an intermediate good among industries,

$$x_{i,f} \to x_{i,f} + \frac{U_{c,i}}{\sum_j U_{c,j}} \nu \sum_j U_{c,j}.$$

Having adjusted the commodity-by-industry use tables, one can construct the adjusted industry-by-industry use tables, since the make tables are not affected by the misclassifi-

[12]The notation in this section follows the notation introduced for input-output tables in section 8.1 below.

cation of commodities.

6.4 An Application: Professional and Business Services

As mentioned above, some sectors such as PBS are more likely prone to measurement problems than others. In the case of PBS, this matters for two reasons. First, PBS is a large supplier of intermediate inputs to other sectors. Second, after the Information sector, PBS is the second largest producer of Intellectual Property Products (IPP). In the 2015 input-output use tables, PBS accounts for more than 20 percent of all intermediate inputs produced and three quarters of the value of new IPP capital produced, Table A5.

Table A5: Contributions from Professional and Business Services

	Intermediates	IPP
PBS - IPP	1,326	550
PBS - non IPP	1,652	–
Total	13,360	718

Notes: Values of intermediates and IPP produced, in billions of dollars, from the 2015 input-output use tables. PBS-IPP consists of Computer System Design (5415), and Miscellaneous Professional, Scientific, and Technical Services (5412OP). PBS-non IPP consists of the remaining PBS industries: Legal Services (5411), Management of Companies, 55, Administrative and Support Services (561), and Waste Management and Remediation Services (562). BEA industry codes in parentheses.

PBS potentially suffers from both types of mismeasurement discussed above. First, service price deflators that account for quality changes are notoriously difficult to obtain. Therefore, given the role of PBS as a major intermediate input supplier, there is the risk of incorrectly attributing sources of productivity growth across sectors. We may also misstate its contributions to IPP capital accumulation and growth. Second, our analysis relies on capital flow tables from 1997 to determine the sources of investment goods in different sectors. While these tables already include software as an investment good, they do not line up exactly with the broader IPP definition used in the construction of capital stocks in the KLEMS data. Our capital requirements matrix, Ω, therefore, likely does not capture all of the investment contributions from PBS.

To explore the role of possible output mismeasurement in the PBS sector, we consider the possibility that price growth in the two IPP related sub-sectors of PBS, namely Computer System Design and Miscellaneous Professional, Scientific, and Professional Services, in the following Miscellaneous Services for short, is overstated in KLEMS.[13] Alternatively, gross output growth in those sectors is understated. In particular, we modify observed price

[13]The BEA industry codes for Computer System Designs and Miscellaneous Professional, Scientific, and Professional Services, are 5415 and 5412OP, respectively. The BEA industry codes generally follow NAICS.

measures in the two IPP related subsectors of PBS to be more closely aligned with price measures of IPPs (that cover similar commodities as Computer System Design and Miscellaneous Services) in the National Income Accounts (NIAs). The NIA price indices indicate less rapid price growth and, therefore, imply higher productivity. By using closely related NIA price indices, we interpret this exercise as a reasonable first pass at correcting for suspected bias in the KLEMS price indices, or at least providing a sense of robustness with respect to measurement. It is comparable to that carried out by Byrne et al. (2016) who extrapolate price changes for products with known quality adjustments to similar products for which no explicit quality adjustment exists.

Table A6: Alternative Price Indices for Computer System Design and Miscellaneous Services

	Computer System Design					Miscellaneous Services		
	KLEMS Data	Total	Software Prepack.	Cust.	Modified Data	KLEMS	R&D	Modified
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1964-1985	6.3	0.4	–	–	0.4	5.2	4.7	2.7
1986-2000	2.3	-2.5	-9.0	0.1	-6.0	3.3	2.6	0.6
2001-2018	-0.8	-1.7	-3.9	-0.3	-2.7	1.9	1.7	-0.3

Notes: The columns display mean price growth rates for price indices related to the Computer System Design sector (BEA Industry Code 5415) and the Miscellaneous Professional, Scientific, and Professional Services sector (BEA Industry Code 5412OP), columns (1) through (5) and columns (6) through (8), respectively. Column (1) is the price index used in the KLEMS data sets for Computer System Design, Columns (2) through (3) are NIA software price indices for total, prepackaged, and custom software, respectively. Column (5) is a weighted average of prepackaged and custom software price changes that is used as the modified price index for Computer System Design. Column (6) is the price index used in the KLEMS data sets for Miscellaneous Services, column (7) is the price index for aggregate R&D investment from the NIAs, and column (8) is the modified price index for Miscellaneous Services. The NIA price indices are from Table 5.6.4 of the U.S. NIAs.

Since Computer System Design produces predominantly software related products, we construct a modified price index as an alternative to the KLEMS price index for that sector based on other available software price indices. We follow Byrne et al. (2016) and define the new modified price index as a weighted average of the price indices for prepackaged and custom software with weights of two-thirds and one third respectively. Separate price indices for prepackaged and custom software are available from the NIAs for the period post-1985. Observe in Table A6 that prices of prepackaged software declined at a notably faster rate than those of custom software during that period, and also faster than the price used in the KLEMS data set (see Table A6). For the period covering $1964-1985$, we use the available price index for all software in the NIAs.

Miscellaneous Services produces mainly R&D related services and we define our modified alternative price index for that sector in relation to the observed aggregate R&D price index from the NIAs.[14] For the period 1964 – 2018, the price index used in KLEMS for Miscellaneous Services tends to increase at about half a percentage point faster than the R&D price index from the NIAs. We assume as an upper bound for any measurement error in KLEMS that in the post-1964 sample, the modified Miscellaneous Services price index increases at a rate that is 2 percentage points lower than the R&D price index.

Given our price adjustments to Computer System Design and Miscellaneous Services, and no further adjustments to other sub-sectors of PBS, modified PBS prices increase at a rate that is one percentage point lower than corresponding KLEMS prices for the period post-1964, (see the two LHS columns of Table A7). Given the lack of data availability prior to 1964, we assume that modified prices in PBS prior to 1964 also increase at a rate that is one percentage point lower than KLEMS prices. The converse of these adjustments is that actual gross output growth in PBS exceeds measured output growth in KLEMS by about one percentage point.

One could calculate a modified value-added TFP growth measure by simply recalculating real gross output and value added growth without taking into account the implications for capital input growth, the two RHS columns in Table A7. Since true capital accumulation is likely to be faster because of faster output growth, this simpler calculation would likely overstate TFP growth as noted in section 6.2. Therefore, here we use the procedure outlined in section 6.2 to correct aggregate GDP growth for the implied mismeasurement of output growth in PBS.

The dashed black line in Figure 13 in the main text shows the effects of higher productivity in PBS implied by the more rapidly declining prices of its Intellectual Property Products in the NIA. As explained above, higher measured productivity growth in PBS affects all sectors, including Construction and Durable Goods highlighted in Figure 13. The contribution from PBS to trend GDP growth is noticeably higher both because of the direct effect of higher measured TFP in that sector, through the corresponding element of $-s^{v\prime}\Xi^{\prime -1}\epsilon$ in equation (47) above (recall that $\epsilon_{PBS} < 0$), and because its production of capital goods benefits from its more productive IPP sectors, the corresponding element of $-s^{v\prime}\alpha_d\Omega'\epsilon$ in (47). In contrast, the quantitative contributions from Construction and Durable Goods to the trend growth rate of GDP do not change appreciably relative to their baseline. On the one hand, measured TFP growth is now smaller in those sectors (i.e., the corresponding elements of $-s^{v\prime}\Xi^{\prime -1}\epsilon$ are negative). On the other hand, those sectors also benefit from employing more productive IPP sectors in PBS in producing their own output.

The other key potential source of mismeasurement in multisector models is the mis-

[14] Differences between the aggregate R&D price index and the R&D price indices for manufacturing and non-manufacturing in the NIAs are barely noticeable.

Table A7: Alternative Price Indices and TFP for PBS

	Prices		VA TFP	
	KLEMS	Modified	KLEMS	Modified
1948-1963	2.6	1.6	1.8	2.9
1964-1985	5.6	4.6	0.1	1.6
1986-2000	3.4	2.2	-0.9	1.0
2001-2018	1.8	0.8	0.4	1.9

Notes. The table displays mean growth rates in percent for price indices and value added TFP, respectively. For each measure, the left column displays the series from the KLEMS data set, and the right column displays the series related to the modified price index.

classification of goods. Specifically, while the 1997 capital flow tables include software as investment, they do not line up exactly with the broader definition of IPPs used in KLEMS for capital. Thus, they likely miss contributions from PBS to investment stemming from its IPP industries. To explore the implications of this misclassification problem, we again separate out the two sectors producing IPPs within PBS, Computer System Design and Miscellaneous Services, from other PBS industries producing more clearly defined intermediate inputs.[15] We then construct a modified capital requirement matrix, Ω, that accounts for the possible omission of IPP contributions from PBS to the production of new capital. In particular, as an upper bound for possible mismeasurement in Ω, we reclassify 50 percent of the value of IPPs produced in PBS as final investment demand for IPP. This reclassification implies a new capital requirement matrix, Ω, that results in a sectoral multiplier for PBS of 0.36 as compared to our baseline of 0.25.

The solid blue line in Figure 13 in the main text then shows the combined effects of the new capital requirement matrix with those correcting for possible bias in KLEMS prices of IPPs in PBS. The partial reclassification of Computer System Designs, and Miscellaneous Services, in PBS from materials to capital raises its sectoral multiplier and lowers those of Construction and Durable Goods. The net effect is that contributions from PBS to trend GDP growth are now higher overall than those of Construction. While the reapportioning of 50 percent of the production value of IPP producing industries in PBS may be an upper bound on missing contributions from IPP capital in Ω, the exercise nevertheless underscores the importance of accurately classifying goods appropriately. Moreover, this section also highlights the importance of continuing efforts to address challenges associated with the measurement of IPP indices.

[15] These are Legal Services, Management of Companies and Enterprises, Administrative and Support Services, and Waste Management and Remediation Services.

7 KLEMS Data

The KLEMS dataset contains quantity and price indices for inputs and outputs across 61 private industries. The growth rate of any one industry's aggregate is defined as a Divisia index given by the value-share weighted average of its disaggregated component growth rates. Labor input is differentiated by gender, age, education, and labor status. Labor input growth is then defined as a weighted average of growth in annual hours worked across all labor types using labor compensation shares of each type as weights. Similarly, intermediate input growth reflects a weighted average of the growth rate of all intermediate inputs averaged using payments to those inputs as weights. Finally, capital input growth reflects a weighted average of growth rates across 53 capital types using payments to each type of capital as weights. Capital payments are based on implicit rental rates consistent with a user-cost-of-capital approach. Total payments to capital are the residuals after deducting payments to labor and intermediate inputs from the value of production. Put another way, there are no economic profits.

An industry's TFP growth rate is defined in terms of its Solow residual, specifically output growth less the revenue-share weighted average of input growth rates. This calculation is consistent with the canonical theoretical framework we adopt in Section 4 of the main text where all markets operate under perfect competition and production is constant-returns-to-scale. For earlier versions of Jorgenson's KLEMS data up to 1990, Basu and Fernald (1997, 2001) compute total payments to capital as the sum of rental rates implied by the user-cost-of-capital and find small industry profits on average that amount at most to three percent of gross output. In the presence of close to zero profits, elasticities to scale and markups are equivalent. More recently, an active debate has emerged on the extent to which the competitive environment has changed in the U.S. over the last two decades. On the one hand, Barkai (2017), also applying the user-cost-of-capital framework but using post 1990 data, finds substantial profit shares over that period. On the other hand, Karabarbounis and Neiman (2018) argue that the user-cost-of-capital framework, to the extent that it implies high profit shares starting in the 1990s, also implies unreasonably high profit shares in the 1950s.[16] In this paper, we maintain the assumptions of competitive markets and constant-

[16] In addition, De Loecker and Eeckhout (2017), estimating industry production functions from corporate balance sheets, present evidence of rising markups and returns to scale since the 1980s. However, Traina (2018) argues that the evidence on rising markups from corporate balance sheets depends crucially on the measurement of variable costs and weights in aggregation. Similarly, Rossi-Hansberg et al. (2020) show that while sales concentration has unambiguously risen at the national level since the 1980s, concentration has steadily declined at the Core-Based Statistical Area, county, and ZIP code levels over the same period. While these facts can seem conflicting, the authors present evidence that large firms have become bigger through the opening of more establishments or stores in new local markets, but this process has lowered concentration in those markets.

returns-to-scale as a benchmark from which to study the aggregate implications of sectoral changes in labor inputs and TFP.

Our calculations rely on the official 2020 version of the ILPA KLEMS dataset which covers the period 1987-2018, and the experimental ILPA KLEMS dataset for the period 1947-2016.[17] The experimental ILPA data from 1947-1963 cover 42 SIC private industries while the experimental ILPA data from 1963-2016, and the official ILPA data from 1987-2018, cover 61 private NAICS industries. To simplify the presentation and analysis, we carry out the empirical work using private industries at the two-digit level. In particular, we aggregate the underlying industry detail in the two KLEMS datasets into 16 two-digit private industries following the procedure in Hulten (1978).[18] Each two-digit industry contains nominal and real series for gross output, $Y_{j,t}$ and $y_{j,t}$ respectively, intermediate inputs, $M_{j,t}$ and $m_{j,t}$, capital, $K_{j,t}$ and $k_{j,t}$, and labor, $L_{j,t}$ and $l_{j,t}$. There remain minor differences between the experimental and official ILPA data but these are reflected mostly in the levels of the variables and not their growth rates. Hence, we use the growth rates calculated using the experimental ILPA data before 1987 and using the official ILPA data after that date.

From the experimental ILPA data, we have an intermediate input aggregate and the official ILPA data give us separate series for nominal and real energy, materials, and services. Thus, we construct an intermediate input aggregate corresponding to the official ILPA series as a Divisia index from these three components. The Divisia quantity index for a series of nominal and real components respectively, $X_{j,t}$ and $x_{j,t}$, with $j \in J$, is defined as

$$100 \times \Delta \ln x_t = 100 \times \sum_{j \in J} \bar{S}^x_{j,t} \Delta \ln x_{j,t},$$

where $\bar{S}^x_{j,t} = \left(S^x_{j,t} + S^x_{j,t-1}\right)/2$ and $S^x_{j,t} = X_{j,t} / \sum_{s \in J} X_{s,t}$. Both ILPA datasets give us nominal and real series for two types of labor inputs: non-college and college labor; and five types of capital inputs: IT equipment, software, R&D, entertainment related intellectual property, and others. We then construct aggregate series for labor and capital inputs by way of Divisa quantity indices using these different input types.

For each dataset, we construct growth rates of nominal and real value added, capital, labor, and value-added TFP at the level of sectoral detail available. The official ILPA data include measures of nominal and real value added, but the experimental ILPA data do not.

[17]The official ILPA dataset for 1987-2018 is downloaded from `https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems` and the experimental ILPA dataset for 1947-2016 is downloaded from `https://www.bls.gov/mfp/special_requests/tables_detail.xlsx`. See Fleck et al. (2014) and Corby et al. (2020) for a detailed description of the official ILPA data, and Eldridge et al. (2020) for the experimental ILPA data.

[18]A detailed description is provided in Section 7.2 below.

We define the growth rates of real valued added in the experimental ILPA through the Divisia index definition of real gross output. The following definitions hold:

1. Nominal Value Added,
$$V_{j,t} = Y_{j,t} - M_{j,t},$$

2. Value Added Share in Gross Output,
$$S^{VY}_{j,t} = \frac{V_{j,t}}{Y_{j,t}},$$

3. Intermediate Input Share in Gross Output,
$$S^{MY}_{j,t} = \frac{M_{j,t}}{Y_{j,t}},$$

4. Value Added Growth Rates,
$$\Delta\tilde{v}_{j,t} = 100 \times \frac{\Delta \ln y_{j,t} - \bar{S}^{MY}_{j,t} \Delta \ln m_{j,t}}{\bar{S}^{VY}_{j,t}},$$

5. Capital Share in Value Added,
$$S^{KV}_{j,t} = \frac{K_{j,t}}{V_{j,t}},$$

6. Labor Share in Value Added,
$$S^{LV}_{j,t} = \frac{L_{j,t}}{V_{j,t}},$$

7. Capital Growth Rates,
$$\Delta\tilde{k}_{j,t} = 100 \times \Delta \ln k_{j,t},$$

8. Labor Growth Rates,
$$\Delta\tilde{\ell}_{j,t} = 100 \times \Delta \ln \ell_{j,t},$$

9. Value Added TFP Growth Rates,
$$\Delta\tilde{z}_{j,t} = 100 \times \left[\Delta \ln v_{j,t} - \bar{S}^{KV}_{j,t} \Delta \ln k_{j,t} - \bar{S}^{LV}_{j,t} \Delta \ln \ell_{j,t}\right].$$

7.1 Housing

The detailed ILPA industry data include a 'Real Estate' (RE) industry, which combines residential housing (HO), both tenant and owner occupied, and 'Other Real Estate' (ORE), and the detailed data also contain a separate 'Rental and Leasing Services' industry. We separate out residential housing, and include the other real estate related industries in Finance, Insurance, and Real Estate (FIRE) ex housing.

We separate out HO and ORE from the RE industry as follows. We obtain data on nominal and real gross output, value added, and intermediate inputs for residential housing from the NIPA Supplemental Tables 7.4. We construct real gross output, value added, and intermediate inputs for ORE as Divisia indices using real and nominal gross output, value added, and intermediate inputs for RE and HO, similar to our construction of real value added from gross output and intermediate input data described above. We construct real employment and capital services for HO and ORE by splitting total employment and capital services in RE according to the wage and capital shares of HO and ORE. This procedure assumes that the factor rentals in HO and ORE are the same. We end up with real and nominal inputs and outputs for HO and ORE. We treat HO as a separate industry, and we include ORE in the FIRE ex Housing industry aggregate.

7.2 Aggregating KLEMS into Consolidated Sectors

As mentioned, we combine the disaggreated KLEMS sectors above into broader consolidated sectors. For example, we might combine sectors $j \in \{1, ..., n\}$ into a single sector J. We use the following formulas to create consolidated sectors:

1. Nominal Value Added in Consolidated Value Added Shares,

$$S_{j,t}^{VV_J} = \frac{V_{j,t}}{\sum_{s \in J} V_{s,t}},$$

2. Nominal Labor in Consolidated Labor Shares,

$$S_{j,t}^{LL_J} = \frac{L_{j,t}}{\sum_{s \in J} L_{s,t}},$$

3. Nominal Capital in Consolidated Capital Shares,

$$S_{j,t}^{KK_J} = \frac{K_{j,t}}{\sum_{s \in J} K_{s,t}},$$

4. Value Added Growth Rates,

$$\Delta \tilde{v}_{J,t} = \sum_{j \in J} \bar{S}_{j,t}^{VV_J} \Delta \tilde{v}_{j,t},$$

5. Capital Growth Rates,
$$\Delta\tilde{k}_{J,t} = \sum_{j\in J} \bar{S}^{KK_J}_{j,t} \Delta\tilde{k}_{j,t},$$

6. Labor Growth Rates,
$$\Delta\tilde{\ell}_{J,t} = \sum_{j\in J} \bar{S}^{LL_J}_{j,t} \Delta\tilde{\ell}_{j,t},$$

7. TFP Growth Rates,
$$\Delta\tilde{z}_{J,t} = \Delta\tilde{v}_{J,t} - \bar{S}^{KV_J}_{J,t} \Delta\tilde{k}_{J,t} - \bar{S}^{LV_J}_{J,t} \Delta\tilde{\ell}_{J,t}, = \sum_{j\in J} \Delta\tilde{z}_{j,t}.$$

We obtain measures of nominal and real aggregate value added, that is, GDP, capital, labor, and TFP the same way we construct these measures for consolidated sectors. Frequently we replace the time-varying shares with constant sample averages of these shares.

8 Input-Output Tables

8.1 Definitions

We use the BEA input-output tables, in particular, the make and use tables to parameterize the use of intermediate goods. The use table describes the use of commodities as intermediate inputs in private industries and in final demand, that is, consumption, investment, imports, and exports. The make table describes which industries produce what commodities. We combine the make and use tables to obtain a mapping from industry production to industry and final use. Our description and notation follows Horowitz and Planting (2009), chapter 12, and is independent of the notation used in the remainder of this Appendix and the paper.

The basic use table is displayed in Table A8.[19] The matrix $U = [U_{c,i}]$ lists the use of commodity c in industry i, with n_C commodities and n_I industries. In our application, there is an equal number of commodities and industries, $n_C = n_I$, that is, the use table is square. The matrix $e = [e_{c,f}]$ displays the contribution of commodity c to final demand f, with n_F types of final demand. The vector $q = [q_c]$ denotes the total use of commodity c; the vector $v = [v_i]$ denotes value-added in industry i, that is, the total value of primary inputs used in industry i; and the vector $g = [g_i]$ denotes gross output of industry i. All vectors are column vectors.

[19] For simplicity, we ignore the presence of scrap goods and non-allocated imports.

Table A8: Use Table

	Industry Intermediate Goods Use	Final Demand	Total Use
Commodities	U	e	q
Value added	v'		
Total production	g'		

The use matrix U covers all commodities used in production by industries, independent of whether they are domestically produced or imported. The column vectors of final demand include consumption, investment, imports, and exports. Imports show up as negative entries, such that the total use of commodities q by industries and final demand represents the use of domestically produced commodities,

$$q = U\mathbf{1}_{n_I} + e\mathbf{1}_{n_F}.$$

On the other hand, the total use of commodities and primary inputs (value added) by industries represents gross output of domestic industries, independent of whether it is domestically used or exported,

$$g = U'\mathbf{1}_{n_C} + v.$$

The basic make table is displayed in Table A9. The matrix $V = [V_{i,c}]$ lists the production of commodity c by industry i, and the vectors q and g denote the total use of commodities and the gross output of industries as defined above. Note that

$$q = V'\mathbf{1}_{n_I} \text{ and } g = V\mathbf{1}_{n_C}.$$

Table A9: Make Table

	Commodities	Total
Industry Production	V	g
Total	q'	

In our model, we do not distinguish between commodities and industries in the production of goods, rather industries produce distinct goods for intermediate and final use. To match our model to the input-output data, we transform the commodity-by-industry use table to an industry-by-industry use table through application of the make table. For this purpose define the market share matrix D,

$$D = Vq_d^{-1},$$

where lower case d denotes diagonal. That is $D_{i,c}$ denotes industry i's share in the production of commodity c. Pre-multiplying the use and final demand matrices with the market share matrix yields the industry sources of intermediate and final use

$$\tilde{U} = DU \text{ and } \tilde{e} = De$$

Notice that

$$\mathbf{1}'_{n_I}\tilde{U} = \mathbf{1}'_{n_I}DU = \mathbf{1}'_{n_C}U = v' - g'$$

and

$$\tilde{U}\mathbf{1}_{n_I} = DU\mathbf{1}_{n_I} = D\left(q - e\mathbf{1}_{n_F}\right) = V\mathbf{1}_{n_C} - De\mathbf{1}_{n_F} = g - De\mathbf{1}_{n_F}.$$

The combined make-use table is as in Table A8 with U and e replaced by $\tilde{U}$ and $\tilde{e}$.

The IO matrix is then

$$\Phi = \tilde{U}(g - v)_d^{-1},$$

the value added shares are

$$\gamma = vg_d^{-1},$$

and the consumption shares are

$$\theta = \tilde{e}_C / \left(\mathbf{1}_{n_I}\tilde{e}_C\right),$$

where e_C is the final demand use consumption.

A final comment on the use of input-output data, which include imports and exports, for the parameterization of a closed economy model. Imports are implicit in the use table to the extent that industries use commodities that are not domestically produced, but imported. We essentially assume that imported and domestically produced commodities are perfect substitutes, and that the Input-Output production structure is independent of the source of the commodities.

8.2 Summary Tables of Sectoral Linkages

Below is the capital flow matrix for the U.S. economy.

Table A10: Ω, Capital Flow Matrix

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Min	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Util	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Const	0.12	0.07	0.47	0.03	0.15	0.14	0.14	0.52	0.14	0.20	0.15	0.12	0.38	0.57	0.41	0.91
Dur Gds	0.70	0.31	0.36	0.76	0.58	0.61	0.60	0.35	0.69	0.52	0.58	0.45	0.43	0.30	0.42	0.05
Nd Gds	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00
Wh Trd	0.10	0.05	0.04	0.08	0.07	0.08	0.07	0.04	0.06	0.06	0.06	0.07	0.09	0.05	0.05	0.00
Ret Trd	0.04	0.01	0.01	0.04	0.01	0.02	0.03	0.02	0.01	0.02	0.07	0.04	0.02	0.03	0.04	0.01
T&W	0.02	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00
Inf	0.00	0.02	0.03	0.04	0.03	0.02	0.05	0.02	0.02	0.10	0.04	0.06	0.03	0.01	0.02	0.00
FIRE (x-Hous)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
PBS	0.02	0.04	0.04	0.04	0.16	0.12	0.09	0.03	0.06	0.10	0.08	0.25	0.06	0.03	0.04	0.00
Ed&H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A,E&FS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oth Serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Element ω_{ij} of matrix Ω denotes the share of investment goods used by sector j that originated in sector i. Entries are computed from the 1997 BEA capital flow tables.

Below is the IO matrix for the U.S. economy.

Table A11: Φ, Input-Output Matrix

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.39	0.00	0.00	0.00	0.01	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Min	0.01	0.27	0.32	0.02	0.02	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Util	0.01	0.02	0.02	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.04	0.01	0.02	0.02	0.01	0.00
Const	0.01	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.01	0.40
Dur Gds	0.04	0.15	0.02	0.38	0.57	0.06	0.03	0.03	0.07	0.12	0.01	0.06	0.07	0.05	0.19	0.06
Nd Gds	0.26	0.10	0.11	0.13	0.09	0.37	0.04	0.05	0.21	0.04	0.01	0.05	0.12	0.22	0.07	0.01
Wh Trd	0.10	0.05	0.03	0.09	0.09	0.07	0.08	0.05	0.07	0.04	0.00	0.03	0.04	0.04	0.04	0.01
Ret Trd	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.03	0.03
T&W	0.05	0.08	0.17	0.04	0.04	0.05	0.13	0.14	0.27	0.03	0.02	0.04	0.03	0.03	0.02	0.00
Inf	0.00	0.02	0.02	0.02	0.02	0.01	0.06	0.06	0.02	0.37	0.05	0.08	0.04	0.04	0.04	0.00
FIRE (x-Hous)	0.07	0.05	0.08	0.03	0.02	0.01	0.18	0.27	0.12	0.07	0.52	0.19	0.31	0.18	0.36	0.42
PBS	0.04	0.22	0.16	0.13	0.13	0.11	0.42	0.33	0.19	0.25	0.25	0.47	0.28	0.30	0.18	0.07
Ed&H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.03	0.00	0.01	0.00
A,E&FS	0.00	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.06	0.03	0.05	0.03	0.06	0.02	0.00
Oth Serv	0.00	0.00	0.01	0.01	0.00	0.00	0.04	0.02	0.01	0.02	0.02	0.03	0.04	0.02	0.03	0.00
Housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Element ϕ_{ij} of matrix Φ denotes the share of materials used by sector j that originated in sector i. Entries are computed from the 2015 BEA make and use tables.

The implied generalized Leontief inverse from the capital flow and IO matrices above is as follows.

Table A12: Ξ', Generalized (Weighted) Leontief Inverse

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.53	0.08	0.02	0.09	0.43	0.14	0.19	0.03	0.07	0.05	0.10	0.28	0.00	0.01	0.01	0.00
Min	0.01	1.06	0.01	0.09	0.38	0.07	0.12	0.02	0.06	0.05	0.07	0.29	0.00	0.01	0.01	0.00
Util	0.01	0.16	0.67	0.18	0.35	0.07	0.11	0.03	0.07	0.05	0.07	0.27	0.00	0.02	0.01	0.00
Const	0.01	0.05	0.01	0.61	0.37	0.07	0.12	0.05	0.05	0.04	0.06	0.24	0.00	0.01	0.01	0.00
Dur Gds	0.02	0.06	0.01	0.07	0.87	0.08	0.15	0.02	0.05	0.05	0.07	0.31	0.00	0.02	0.01	0.00
Nd Gds	0.07	0.18	0.02	0.09	0.40	0.50	0.16	0.02	0.07	0.05	0.07	0.34	0.00	0.02	0.01	0.00
Wh Trd	0.01	0.03	0.01	0.07	0.29	0.04	0.76	0.02	0.06	0.06	0.09	0.30	0.00	0.01	0.02	0.00
Ret Trd	0.01	0.03	0.01	0.10	0.23	0.04	0.08	0.60	0.06	0.05	0.12	0.27	0.01	0.01	0.02	0.00
T&W	0.01	0.05	0.01	0.07	0.31	0.09	0.11	0.02	0.60	0.05	0.10	0.28	0.00	0.01	0.01	0.00
Inf	0.01	0.04	0.01	0.10	0.38	0.06	0.12	0.02	0.05	0.69	0.09	0.35	0.00	0.03	0.02	0.00
FIRE (x-Hous)	0.01	0.03	0.03	0.09	0.31	0.05	0.09	0.03	0.04	0.07	0.73	0.32	0.00	0.02	0.02	0.00
PBS	0.01	0.02	0.01	0.05	0.21	0.04	0.07	0.02	0.03	0.05	0.09	0.93	0.00	0.02	0.02	0.00
Ed&H	0.01	0.03	0.01	0.05	0.18	0.05	0.06	0.01	0.03	0.04	0.12	0.23	0.59	0.02	0.02	0.00
A,E&FS	0.02	0.04	0.01	0.10	0.23	0.08	0.09	0.02	0.04	0.05	0.11	0.28	0.00	0.55	0.02	0.00
Oth Serv	0.01	0.03	0.01	0.07	0.23	0.04	0.07	0.02	0.03	0.04	0.12	0.20	0.00	0.01	0.65	0.00
Housing	0.01	0.05	0.01	0.54	0.39	0.06	0.12	0.06	0.05	0.04	0.10	0.25	0.00	0.01	0.01	0.90

Notes: See text for definition of Ξ.

9 Additional Model Implications and Robustness

This section covers additional implications of the analysis referred to in the main text along with associated robustness exercises. In particular, we explore the robustness of our sectoral multipliers in Table 4 of the main text to different definitions of value added shares and IO matrices. We then present the endogenous trend behavior of capital growth rates implied by our model across sectors against their data counterpart. We carry out a similar exercise for the endogenous trend behavior of growth rates in producer prices. Finally, we present the results of an exercise that explores the implications of trend variations in sectoral TFP growth rates alone.

9.1 Robustness of the Sectoral Multipliers

Tables A10, A11, and A12 above help determine the sectoral multipliers shown in Table 4 of the main text. The key take away from that table is that the influence of sectors on aggregate growth generally exceed their value added share in GDP, up to more than three times their respective share in the economy. As the table makes clear, this observation of course depends on what shares are being used and how the sectors interact through input-output and capital linkages. Table A13, therefore, explores how sectoral multipliers change with the definition of shares or IO matrix. (Data limitations require us to use the 1997 capital flow table throughout.)

The first column of Table A13 reproduces our benchmark sectoral multipliers shown in the last column of Table 4; recall that these results are based on constant value added shares computed as averages over the full sample (1950-2018) and the 2015 make and use tables. The second column of Table A13 shows the sectoral multipliers obtained using constant mean shares calculated only over the first 15 years of the sample (1950-1964). The third column shows these multipliers computed instead using constant mean shares from the last 15 years of the sample (2002-2016). The fourth and fifth columns of Table A13 shows the sectoral multipliers implied by the make and use tables from 1997 and 1960 respectively.

While there are differences across the columns of Table A13, the general lesson remains the same. The sum of the multipliers always exceeds 1 and varies from 1.7 to 1.9 across columns. Durable Goods, Professional and Business Services, and Construction consistently have an outsize influence on aggregate growth regardless of the calculation in Table A13 given their central as input suppliers. Moreover, the ranking of sectoral multipliers by sector is also generally consistent across columns. The make and use table from 1960 does not allow us to separate FIRE and Housing so that the last row of Table A13 gives a multiplier for the combined sectors, about 0.24 on average across columns.

Table A13: Sectoral Network Multipliers Under Alternative Calibrations

Sector	Benchmark	Mean Shares, First 15 Years	Mean Shares, Last 15 Years	1997 IO Matrix	1960 IO Matrix
Agriculture	0.03	0.06	0.02	0.03	0.04
Mining	0.05	0.06	0.05	0.04	0.04
Utilities	0.03	0.03	0.02	0.03	0.03
Construction	0.17	0.17	0.17	0.15	0.17
Durable Goods	0.42	0.48	0.35	0.39	0.42
Nondurable Goods	0.13	0.16	0.10	0.13	0.14
Wholesale Trade	0.15	0.15	0.14	0.14	0.13
Retail Trade	0.11	0.12	0.09	0.11	0.11
Trans. & Ware.	0.07	0.08	0.06	0.06	0.07
Information	0.08	0.07	0.09	0.08	0.07
FIRE (x-Housing)	0.14	0.12	0.17	0.14	–
PBS	0.24	0.20	0.28	0.20	0.16
Educ. & Health	0.06	0.03	0.09	0.06	0.06
Arts, Ent., & Food Svc.	0.04	0.04	0.05	0.04	0.04
Other Services (x-Gov)	0.04	0.04	0.03	0.04	0.04
Housing	0.09	0.08	0.11	0.09	–
Addendum: FIRE + Housing	0.24	0.20	0.28	0.24	0.23

Notes: This table shows the sectoral multiplier (see Table 4) for the baseline calibration (column 1), for alternative value-added share weights (columns 2 and 3) and for alternative IO matrices (columns 4 and 5).

9.2 Trend Capital Growth Rates

An important mechanism underlying our findings, and that gives rise to long-run sectoral multipliers, lies in endogenous capital accumulation. Thus, Figures A3 and A4 show a comparison of model-implied capital growth trends, g_k, against their counterparts in the data for every sector. Here, the model-implied capital growth trends are calculated using the main balanced-growth expressions to approximate long-run variations in g_v and g_k conditional on the extracted trends in TFP and labor, g_a. That is, $g_v = [I + \alpha_d \Omega' \Xi']\, g_a$ and $g_k = \Omega' \Xi' g_a$ in Section 2 above.

As shown in Figures, A3 and A4, the balanced growth expressions generally match well long-run variations in capital growth rates in both our baseline case, $q = 8$, corresponding to long-run variations with periods longer than 17 years, and $q = 6$, corresponding to periods longer than 23 years. Because the balanced growth equations represent approximations with constant growth rates, while the data is time varying, we adjust the overall level of the series to account for what would be initial capital stocks that are part of the full dynamic solution. Our baseline model captures the trend decline in the capital growth rate in almost all sectors, including Durable Goods which has by far the largest sectoral multiplier. It misses notably on the low frequency capital growth rates of the Mining sector which has a relatively small multiplier. However, in other sectors with small multipliers - Education and Health, Arts and Entertainment, or Housing - the model-implied capital trend growth rates approximate closely their counterpart in the data. In Construction, which has the second largest sectoral multiplier, the model-implied capital growth trend displays less variation than in the data in the latter part of the sample, even at these low frequencies. Here, the fact that the balanced growth approximations do not represent the full dynamic solution of the model, and thus abstract from the higher frequency components of endogenous capital accumulation may be partly responsible. In addition, the dynamics of capital accumulation likely differ across sectors, for example through time-to-build production processes with different horizons (not taken into account here), that would lead to further differences across sectors.

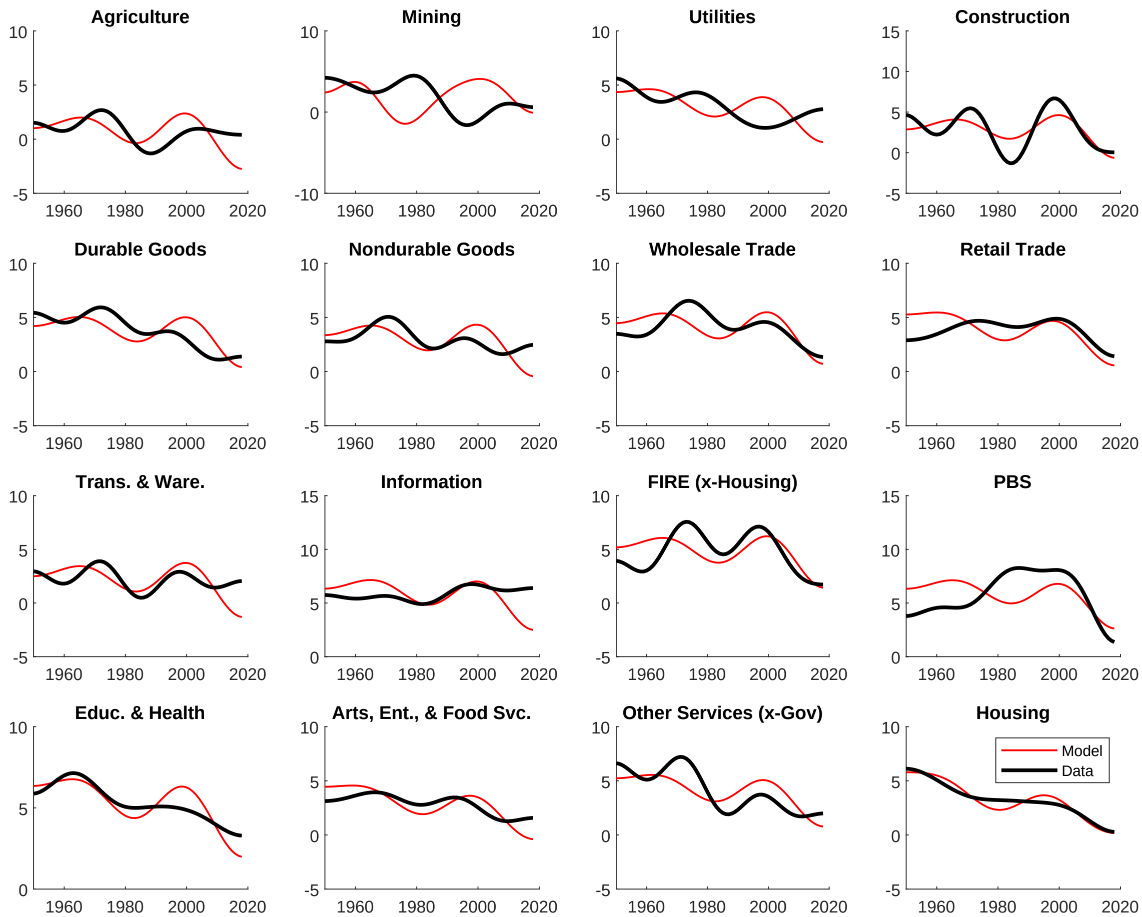
9.3 Trend Producer Price Growth Rates

Conditional on trend input growth, g^a, the model also has implications for producer prices derived in Section 2, $g^{p^y} = (\mathbf{1}\Theta - I)\, g^y = (\mathbf{1}\Theta - I)\, \Xi' g^a$. In this expression, the general Leontief inverse, $\Xi' = (I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi')^{-1}\, \Gamma_d$, amplifies the effects of sectors that play a more central role in the production network either as suppliers of capital, captured by Ω, or materials, captured by Φ. Moreover, since expenditure shares, Θ, are less than one, increases in TFP growth through g^a lower producer prices, g^{p^y}.

As shown in Figure A5, the evolution of trend growth rates in producer prices obtained from the balanced growth expressions, conditional on the trend growth rates of labor and TFP, generally match well their counterparts in the data, especially in Durable Goods, Information, Housing, and PBS for example. However, the balanced growth approximation in this case means that model-implied trend growth rates of producer prices generally display somewhat less variation than in the data across most sectors, especially Agriculture and Nondurable Goods. [20]

[20] These calculations correspond to our baseline case, $q = 8$, associated with periods longer than 17 years. As with the capital growth rates, we adjust the overall level of the series to account for initial conditions that would be part of the full dynamic solution.

Figure A3: Trend Capital Growth, $q = 8$

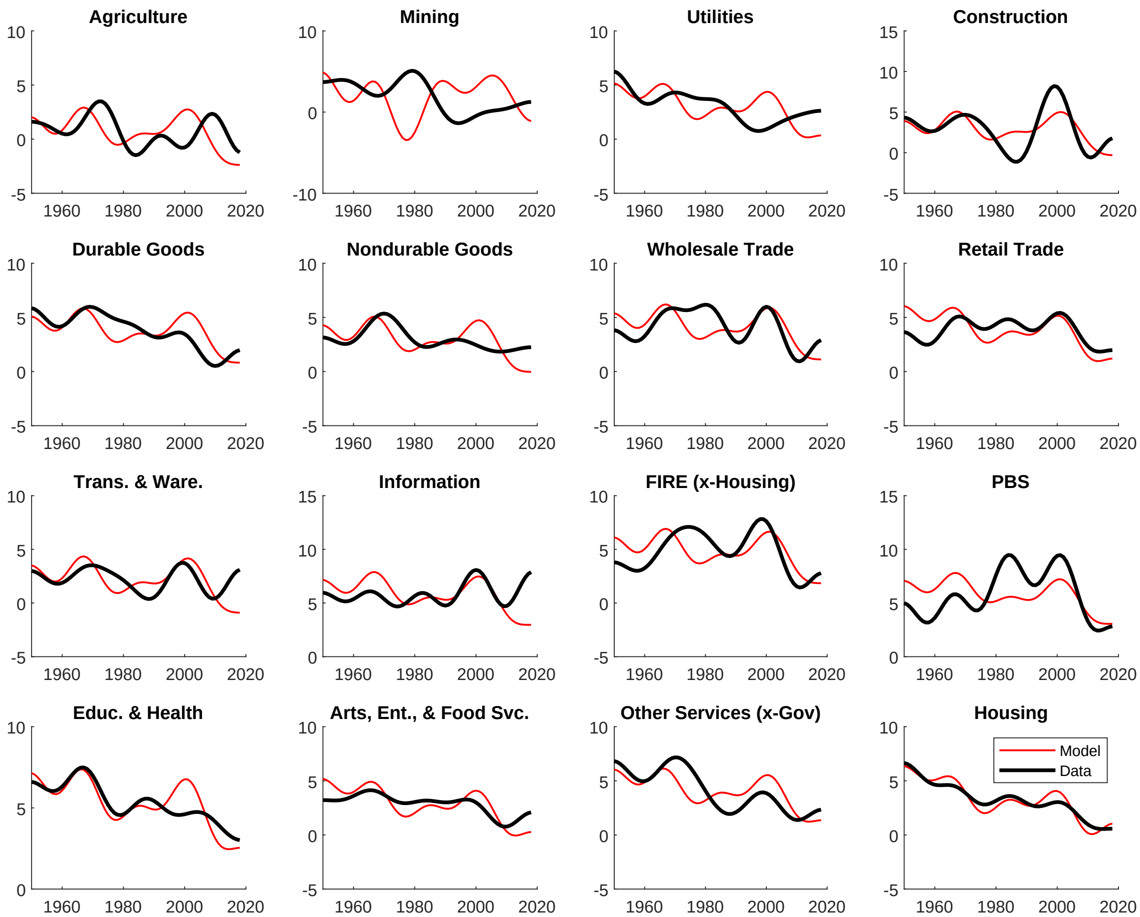


9.4 Implications of TFP Trends Alone

We now consider the case where only TFP trends are the drivers of growth, with labor responding endogenously. Ignoring exogenous factors in hours, and with King, Plosser, and Rebelo (1988) preferences and no population growth, our balanced growth equation for aggregate GDP reflects only the TFP terms. In this case, the balanced growth path equation (9) is unchanged, but the drivers are given by $g^{\ell} = 0$ and $g^{a} = g^{z}$. In other words, we repeat the analysis only considering (sector-specific and common) TFP. As a consequence, the model can be interpreted as providing implications for trend per-capita GDP growth.

Figures A6 and A7 illustrate our benchmark findings when considering only the impli-

Figure A4: Trend Capital Growth, $q = 6$



cations of variations in TFP trends for per capita GDP growth. As in the full analysis, the effects of idiosyncratic sectoral trends to dominate. The median posterior R_f^2 in Figure A6 now falls to less than 0.20 so that sector-specific TFP trends explain more than 80 percent of the long-run variation in per capita GDP growth. Idiosyncratic sources of variations continue to dominate, even more so, for two reasons. First, the distribution of sectoral multipliers is unchanged. Second, sectoral TFP growth trends are dominated by their idiosyncratic components (Figure 7 in the main text), especially in sectors with large multipliers (e.g. Durable Goods). Figure A7 then gives the sector-specific contributions to the trend growth rate of per capita GDP growth. Comparing this figure to Figure 12 in the main text, the shapes are generally similar (though magnitudes can differ somewhat).

Figure A5: Trend Producer Price Growth, $q = 8$

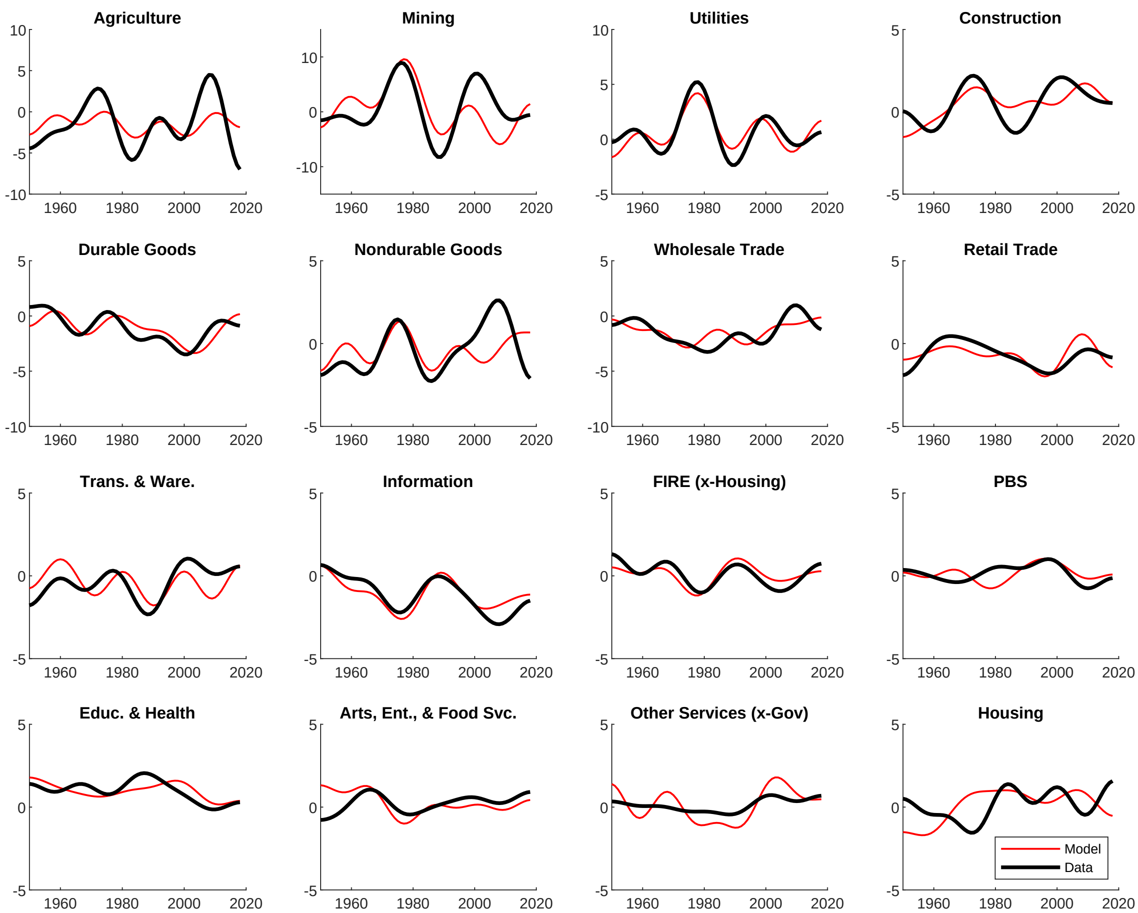


Figure A6: Decomposition of the Trend Growth Rate in Per-Capita GDP

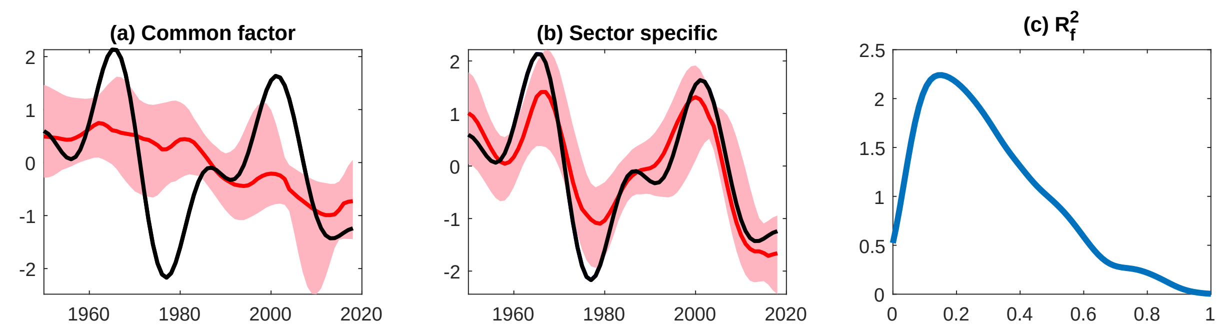
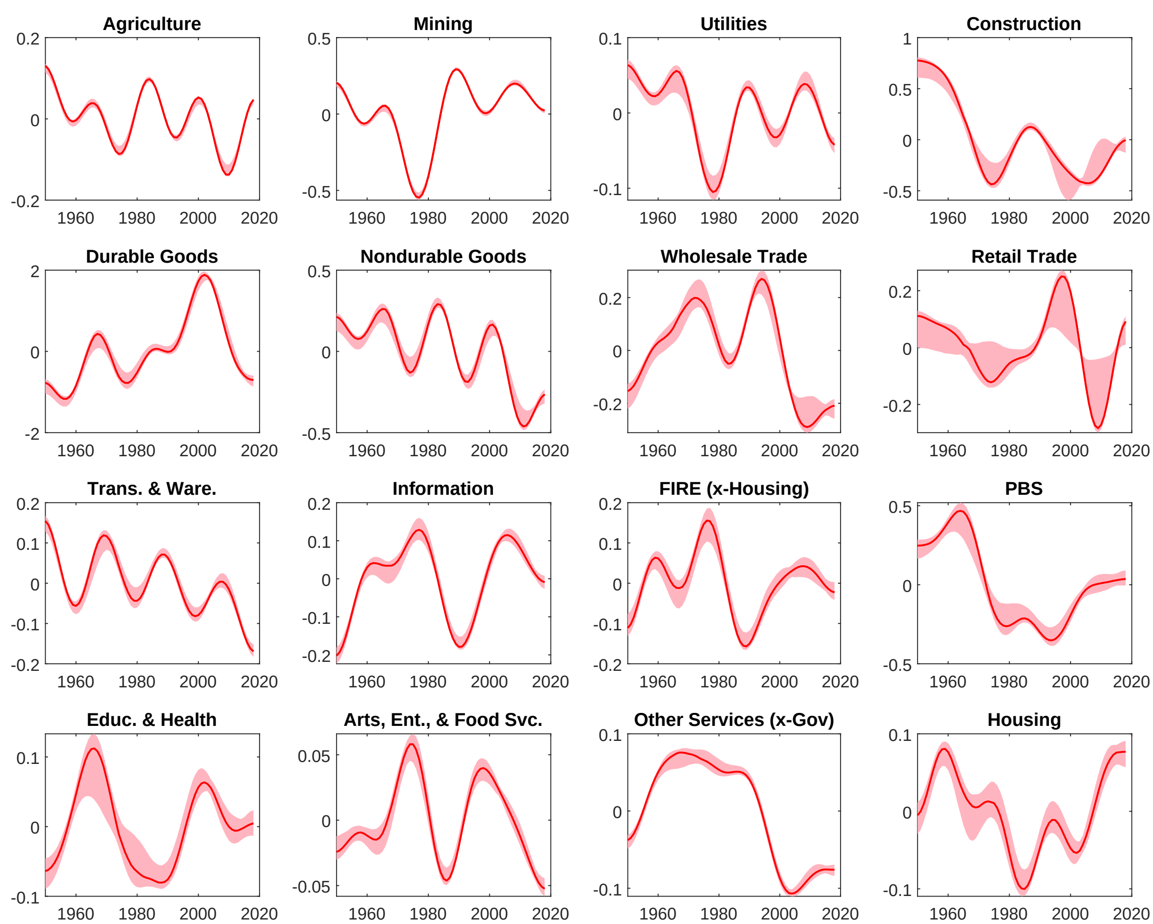


Figure A7: Sector-Specific Contributions to the Trend Growth Rate of Per-Capita GDP (percentage points at annual rate)



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