The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy
Supplementary Material

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Contents

These supplementary notes summarize the technical aspect of the paper with some additional details and derivations.

1   Economic Environment

The economy has N regions and J sectors. We denote a particular region by $n \in \{1, ..., N\}$ (or i) and a particular sector by $j \in \{1, ..., J\}$ (or k) . There are two factors of production, aggregate labor L and regional "structures" and land, H_n. Labor moves freely across regions and sectors, while structures are a region-specific factor. Sectors are of two types, either tradeables (T), or non-tradeables (NT).

1.1   Households

Agents in each location $n \in \{1, ..., N\}$ order consumption baskets according to Cobb-Douglas preferences, with shares, α^j, over their consumption of final domestic goods, c_n^j, bought at prices, P_n^j, in all sectors $j \in \{1, ..., J\}$. Preferences are homothetic of degree one, so $\sum_{j=1}^{J} \alpha^j = 1$.

Agents supply one unit of labor inelastically. The income of an agent residing in region n is

$$I_n = (1 - \iota_n)r_n\frac{H_n}{L_n} + w_n + \chi,$$

where w_n is the wage, r_n is the rental rate of structures and land, and $r_n H_n / L_n$ is the per capita income from renting land and structures to firms in region n. The term χ represents the return per household on a national portfolio of land and structures from all regions,

$$\chi = \frac{\sum_{n=1}^{N} \iota_n r_n H_n}{\sum_{n=1}^{N} L_n},$$

where ι_n denotes the fraction of income from land and structures in region n contributed to the national portfolio. Income from land and structures not contributed to the national portfolio, $(1 - \iota_n) r_n \frac{H_n}{L_n}$, can be thought of as being earned and distributed by local governments to state residents. Thus, total income in region n is

$$L_n I_n = r_n H_n + w_n L_n - \Upsilon_n, \tag{1}$$

where $\Upsilon_n = \iota_n r_n H_n - \chi L_n$ denotes a regional trade imbalance stemming from interregional transfers implied by the national portfolio.

The problem of an agent in region n is given by

$$v_n \equiv \max_{\{c_n^j\}_{j=1}^{J}} \prod_{j=1}^{J} \left(c_n^j \right)^{\alpha^j}, \text{ subject to } \sum_{j=1}^{J} P_n^j c_n^j = I_n.$$

Total demand of final good j in region n is then

$$L_n c_n^j = \alpha^j \frac{L_n I_n}{P_n^j}. \tag{2}$$

Agents move freely across regions. The value of locating in a particular region n is

$$v_n = \frac{(1 - \iota_n) r_n H_n / L_n + w_n + \chi}{P_n},$$

where $P_n = \prod_{j=1}^{J} \left(P_n^j / \alpha^j \right)^{\alpha^j}$ is the ideal price index in region n. In equilibrium, households are indifferent between living in any region so that

$$v_n = \frac{I_n}{P_n} = U \tag{3}$$

for all $n \in \{1, ..., N\}$, for some U determined in equilibrium.

1.2 Firms

1.2.1 Intermediate Goods

Representative firms, in each region n and sector j, produce a continuum of varieties of intermediate goods that differ in their *idiosyncratic* productivity level, z_n^j, drawn randomly from a Fréchet distribution with shape parameter θ^j. Draws are independent across goods, sectors, and regions. The productivity of all firms producing varieties in a region-sector pair (n, j) is also determined by a deterministic productivity level, T_n^j, specific to that region and sector. The production function for a variety associated with idiosyncratic productivity z_n^j in (n, j) is given by

$$q_n^j(z_n^j) = z_n^j \left[T_n^j h_n^j(z_n^j)^{\beta_n} l_n^j(z_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod\nolimits_{k=1}^{J} M_n^{jk}(z_n^j)^{\gamma_n^{jk}}, \tag{4}$$

where $h_n^j(\cdot)$ and $l_n^j(\cdot)$ denote the demand for structures and labor respectively, $M_n^{jk}(\cdot)$ is the demand for final material inputs by firms in sector j from sector k (variables representing final goods are denoted with capital letters), $\gamma_n^{jk} \geqslant 0$ is the share of sector j goods spent on materials from sector k, and $\gamma_n^j \geqslant 0$ is the share of value added in gross output. The production function has constant returns to scale, $\sum_{k=1}^{J} \gamma_n^{jk} = 1 - \gamma_n^j$.

The unit cost of producing varieties with draw z_n^j in (n, j) is given by

$$\min_{h_n^j(z_n^j), l_n^j(z_n^j), \left\{ M_n^{jk}(z_n^j) \right\}_{k=1}^{J}} w_n l_n^j(z_n^j) + r_n h_n^j(z_n^j) + \sum\nolimits_{k=1}^{J} P_n^k M_n^{jk}(z_n^j),$$

subject to

$$z_n^j \left[T_n^j h_n^j(z_n^j)^{\beta_n} l_n^j(z_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod\nolimits_{k=1}^{J} M_n^{jk}(z_n^j)^{\gamma_n^{jk}} = 1,$$

where P_n^k is the price of final goods in industry k in region n. Let x_n^j denote the cost of the input bundle needed to produce intermediate good varieties in (n, j). Then

$$x_n^j = B_n^j \left[r_n^{\beta_n} w_n^{1-\beta_n} \right]^{\gamma_n^j} \prod\nolimits_{k=1}^{J} \left(P_n^k \right)^{\gamma_n^{jk}}, \tag{5}$$

where

$$B_n^j = \left[(1 - \beta_n)^{(\beta_n - 1)} (\beta_n)^{-\beta_n} \right]^{\gamma_n^j} \left[\prod\nolimits_{k=1}^{J} \left(\gamma_n^{jk} \right)^{-\gamma_n^{jk}} \right] \left(\gamma_n^j \right)^{-\gamma_n^j}.$$

The unit cost of an intermediate good with idiosyncratic draw z_n^j in region-sector pair (n, j) is then given by

$$\frac{x_n^j}{z_n^j \left(T_n^j \right)^{\gamma_n^j}}. \tag{6}$$

Firms located in region n and operating in sector j will be motivated to produce the variety whose productivity draw is z_n^j as long as its price matches or exceeds $x_n^j / z_n^j \left(T_n^j \right)^{\gamma_n^j}$.

Let $p_n^j(z^j)$ represent the equilibrium price of a variety for which the vector of idiosyncratic productivity draws in all N regions is given by $z^j = (z_1^j, z_2^j, ...z_N^j)$. The determination of this price in equilibrium is discussed in detail below. Since the production function is Cobb-Douglas, profit maximization implies that input demands, $h_n^j(z_n^j)$, $l_n^j(z_n^j)$, and $M_n^{jk}(z_n^j)$ for all k, satisfy

$$\frac{h_n^j(z_n^j)r_n}{p_n^j(z^j)q_n^j(z_n^j)} = \gamma_n^j \beta_n, \tag{7}$$

$$\frac{l_n^j(z_n^j)w_n}{p_n^j(z^j)q_n^j(z_n^j)} = \gamma_n^j (1 - \beta_n), \tag{8}$$

$$\frac{P_n^k M_n^{jk}(z_n^j)}{p_n^j(z^j)q_n^j(z_n^j)} = \gamma_n^{jk}. \tag{9}$$

These conditions imply that

$$r_n h_n^j(z_n^j) = w_n \frac{\beta_n}{1 - \beta_n} l_n^j(z_n^j), \tag{10}$$

1.2.2   Final Goods

Denote the quantity of final goods in (n, j) by Q_n^j, and denote by $\tilde{q}_n^j(z^j)$ the quantity demanded of an intermediate good of a given variety such that, for that variety, the particular vector of productivity draws received by the different n regions is $z^j = (z_1^j, z_2^j, ...z_N^j)$. The production of final goods is given by

$$Q_n^j = \left[\int \tilde{q}_n^j(z^j)^{1-1/\eta_n^j} \phi^j\left(z^j\right) dz^j \right]^{\eta_n^j/\left(\eta_n^j - 1\right)}, \tag{11}$$

where $\phi^j(z^j)$ denotes the joint density function associated with the CDF $\exp\left\{ -\sum_{n=1}^N (z_n^j)^{-\theta^j} \right\}$ for the vector z^j, with marginal CDFs given by $\exp\left\{ -(z_n^j)^{-\theta^j} \right\}$, and the integral is over $\mathbb{R}_+^N$. For non-tradeable sectors, the only relevant density is $\phi_n^j(z_n^j)$ since final good producers use only locally produced goods.

Producers of composite sectoral goods then solve

$$\max_{\left\{\tilde{q}_n^j(z^j)\right\}_{\mathbb{R}_+^N}} P_n^j Q_n^j - \int p_n^j(z^j)\tilde{q}_n^j(z^j)\phi^j\left(z^j\right) dz^j.$$

where $p_n^j(z^j)$ denotes the price of intermediate goods. Then, the demand function is given by

$$\tilde{q}_n^j(z^j) = \left(\frac{p_n^j(z^j)}{P_n^j} \right)^{-\eta_n^j} Q_n^j,$$

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where P_n^j is a price index for sector j in region n,

$$P_n^j = \left[\int p_n^j(z^j)^{1-\eta_n^j} \phi^j \left(z^j \right) dz^j \right]^{1/\left(1-\eta_n^j\right)}.$$

There is free entry in the production of final goods with competition implying zero profits.

1.3   Derivation of Prices

One unit of any intermediate good in sector j shipped from region i to region n requires producing $\kappa_{ni}^j \geq 1$ units in i, with $\kappa_{nn}^j = 1$ and, for intermediate goods in non-tradable sectors, $\kappa_{ni}^j = \infty$. The price paid for a particular variety whose vector of productivity draws is z^j, $p_n^j(z^j)$, is given by the minimum of the unit costs across locations, adjusted by the transport costs κ_{ni}^j,

$$p_n^j \left(z^j \right) = \min_i \left\{ \frac{\kappa_{ni}^j x_i^j}{z_i^j \left(T_i^j \right)^{\gamma_i^j}} \right\}. \tag{12}$$

We follow Eaton and Kortum (2002) in solving for the distribution of prices. Given the distribution of prices, when sector j is tradeable, the price of final good j in region n solves

$$\left(P_n^j \right)^{1-\eta_n^j} = \int p_n^j(z^j)^{1-\eta_n^j} \phi^j \left(z^j \right) dz^j, \tag{13}$$

which is the expected value of the random variable $p_n^j(z^j)^{1-\eta_n^j}$.

Given the assumptions on the distribution of z_i^j, and the unit cost of producing and shipping goods, we have that $\Pr\left[p_{ni}^j \leq p \right] = \Pr\left[\frac{\kappa_{ni}^j x_i^j}{z_i^j \left(T_i^j \right)^{\gamma_i^j}} \leq p \right] = \Pr\left[\frac{1}{z_i^j} \leq \frac{p\left(T_i^j \right)^{\gamma_i^j}}{\kappa_{ni}^j x_i^j} \right] =$ $\Pr\left[z_i^j \geq \frac{\kappa_{ni}^j x_i^j}{p\left(T_i^j \right)^{\gamma_i^j}} \right]$ or

$$\Pr\left[p_{ni}^j \leq p \right] = 1 - e^{-\lambda_{ni}^j p^{\theta^j}},$$

where $\lambda_{ni}^j = \left[\kappa_{ni}^j x_i^j \left(T_i^j \right)^{-\gamma_i^j} \right]^{-\theta^j}$. It follows that $\Pr\left[p_n^j \leq p \right] = \Pr\left[\min_i \left\{ p_{ni}^j(z^j) \right\} \leq p \right] = 1 - \Pr[\{p_{n1}^j(z^j), p_{n2}^j(z^j), \dots , p_{nN}^j(z^j)\} > p] = 1 - \Pr\left[p_{n1}^j(z^j) > p \right] \Pr\left[p_{n2}^j(z^j) > p \right] \dots \Pr[p_{nN}^j(z^j)$ $> p] = 1 - \Pr\left[\frac{\kappa_{n1}^j x_1^j}{z_1^j\left(T_1^j \right)^{\gamma_1^j}} > p \right] \Pr\left[\frac{\kappa_{n2}^j x_2^j}{z_2^j\left(T_2^j \right)^{\gamma_2^j}} > p \right] \dots \Pr\left[\frac{\kappa_{nN}^j x_N^j}{z_N^j\left(T_N^j \right)^{\gamma_N^j}} > p \right] = 1 - \Pr\left[z_1^j \leq \frac{\kappa_{n1}^j x_1^j}{p\left(T_1^j \right)^{\gamma_1^j}} \right] \Pr$ $\left[z_2^j \leq \frac{\kappa_{n2}^j x_2^j}{p\left(T_2^j \right)^{\gamma_2^j}} \right] \dots \Pr\left[z_N^j \leq \frac{\kappa_{nN}^j x_N^j}{p\left(T_N^j \right)^{\gamma_N^j}} \right] = 1 - e^{-\lambda_{n1}^j p^{\theta^j}} e^{-\lambda_{n2}^j p^{\theta^j}} \dots e^{-\lambda_{nN}^j p^{\theta^j}}$ or

$$\Pr\left[p_n^j \leq p \right] = 1 - e^{-\Phi_n^j p^{\theta^j}},$$

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where $\Phi_n^j = \sum_{i=1}^N \lambda_{ni}^j = \sum_{i=1}^N \left[\kappa_{ni}^j x_i^j\right]^{-\theta^j} \left(T_i^j\right)^{\gamma_i^j \theta^j}$ (Φ_n^j does not depend on i because we are integrating out the regional dimension).

Let $F_{p_n^j}(p)$ denote the CDF, $\Pr\left[p_n^j \leq p\right] = 1 - e^{-\Phi_n^j p^{\theta^j}}$. Then, the associated pdf, denoted $f(p)$, is $\Phi_n^j \theta^j p^{\theta^j - 1} e^{-\Phi_n^j p^{\theta^j}}$, and the expectation in (13) may be written as

$$\left(P_n^j\right)^{1-\eta_n^j} = \int p^{1-\eta_n^j} f(p) dp = \int p^{1-\eta_n^j} \Phi_n^j \theta^j p^{\theta^j - 1} e^{-\Phi_n^j p^{\theta^j}} dp. \tag{14}$$

For our purposes, it will be convenient to work with the random variable p^{θ^j} rather than p. To determine the distribution of p^{θ^j}, let $y = g(p) = p^{\theta^j}$ with density $f_Y(y)$ where

$$f_Y(y) = f(g^{-1}(y)) \left|\frac{dg^{-1}(y)}{dy}\right|.$$

Then, given that $g^{-1}(y) = y^{\frac{1}{\theta^j}}$ with $\frac{dg^{-1}(y)}{dy} = \frac{1}{\theta^j} y^{\frac{1-\theta^j}{\theta^j}}$, we have that

$$\begin{aligned}
f_Y(y) &= \Phi_n^j \theta^j \left(y^{\frac{1}{\theta^j}}\right)^{\theta^j - 1} e^{-\Phi_n^j y} \frac{1}{\theta^j} y^{\frac{1-\theta^j}{\theta^j}} \\
&= \Phi_n^j e^{-\Phi_n^j y}.
\end{aligned}$$

We may then re-write the expectation (14) as

$$\left(P_n^j\right)^{1-\eta_n^j} = \int \left(p^{\theta^j}\right)^{\frac{1-\eta_n^j}{\theta^j}} \Phi_n^j \theta^j p^{\theta^j-1} e^{-\Phi_n^j p^{\theta^j}} dp = \int y^{\frac{1-\eta_n^j}{\theta^j}} \Phi_n^j e^{-\Phi_n^j y} dy.$$

Now, consider the change of variables, $u = \Phi_n^j y$. Then, $du = \Phi_n^j dy$ and

$$\left(P_n^j\right)^{1-\eta_n^j} = \left(\Phi_n^j\right)^{\frac{-(1-\eta_n^j)}{\theta^j}} \int u^{\frac{1-\eta_n^j}{\theta^j}} e^{-u} du$$

or

$$P_n^j = \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}} \left(\Phi_n^j\right)^{\frac{-1}{\theta^j}},$$

where $\Gamma(\xi_n^j)$ is the Gamma function evaluated at $\xi_n^j = 1 + (1-\eta_n^j)/\theta^j$. The price of composite sectoral goods in tradeable sector j may then also be expressed as

$$P_n^j = \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}} \left[\sum_{i=1}^N \left[x_i^j \kappa_{ni}^j\right]^{-\theta^j} \left(T_i^j\right)^{\theta^j \gamma_i^j}\right]^{-\frac{1}{\theta^j}}. \tag{15}$$

In a given non-tradeable sector j, $\kappa_{ni}^j = \infty \ \forall i \neq n$ so that equation (15) reduces to

$$P_n^j = \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}} x_n^j \left(T_n^j\right)^{-\gamma_n^j}.$$

1.4 Trade Shares

Let X_n^j denote total expenditures on final goods j in region n (or total revenue), $X_n^j = P_n^j Q_n^j$. Recall that because of zero profits in the final goods sectors, total expenditures on intermediate goods in a given sector exhaust total revenue from final goods in that sector, $\int p_n^j(z^j) \tilde{q}_n^j(z^j) \phi^j(z^j) \, dz^j = P_n^j Q_n^j$. Let π_{ni}^j denote the share of region n's expenditures on sector j composite goods purchased from region i,

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j},$$

and observe that

$$X_{ni}^j = \Pr \left[p_{ni}^j(z^j) \leq \min_{m \neq i} \left\{ p_{nm}^j(z^j) \right\} \right] X_n^j.$$

We derived above that $\Pr \left[p_{ni}^j(z^j) \leq p \right] = 1 - e^{-\lambda_{ni}^j p^{\theta^j}}$, in which case $p_{ni}^j(z^j)^{\theta^j} \sim \exp(\lambda_{ni}^j)$. Furthermore, it also follows that $\Pr \left[\min_{m \neq i} \left\{ p_{nm}^j(z^j) \right\} \leq p \right] = 1 - e^{-\overline{\Phi}_n^j p^{\theta^j}}$ so that $\min_{m \neq i} \left\{ p_{nm}^j(z^j) \right\} \sim \exp(\overline{\Phi}_n^j)$, where $\overline{\Phi}_n^j = \sum_{m \neq i} [\kappa_{nm}^j x_m^j]^{-\theta^j} (T_m^j)^{\gamma_m^j \theta^j}$.

- Suppose $x \sim \exp(\lambda)$, $y \sim \exp(\mu)$, and x and y independent, then $\Pr(x < y) = \frac{\lambda}{\lambda + \mu}$.

Thus, we have that

$$
\begin{aligned}
\pi_{ni}^j &= \Pr \left[p_{ni}^j(z^j) \leq \min_{m \neq i} \left\{ p_{nm}^j(z^j) \right\} \right] \\
&= \Pr \left[p_{ni}^j(z^j)^{\theta^j} \leq \min_{m \neq i} \left\{ p_{nm}^j(z^j) \right\}^{\theta^j} \right] \\
&= \frac{\lambda_{ni}^j}{\Phi_n^j} \\
&= \frac{\left[\kappa_{ni}^j x_i^j \left(T_i^j \right)^{-\gamma_i^j} \right]^{-\theta^j}}{\sum_{i=1}^N \left[\kappa_{ni}^j x_i^j \right]^{-\theta^j} \left(T_i^j \right)^{\gamma_i^j \theta^j}}
\end{aligned}
$$

From equation (15), we have that $\sum_{i=1}^N \left[x_i^j \kappa_{ni}^j \right]^{-\theta^j} \left(T_i^j \right)^{\theta^j \gamma_i^j} = (P_n^j)^{-\theta^j} \Gamma(\xi_n^j)^{\frac{\theta^j}{1-\eta_n^j}}$. Therefore, we may also write the trade share π_{ni}^j as

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \left[\frac{\kappa_{ni}^j x_i^j \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}}}{\left(T_i^j \right)^{\gamma_i^j} P_n^j} \right]^{-\theta^j}.$$

In non-tradeable sectors, $\kappa_{ni}^j = \infty \; \forall i \neq n$ so that $\pi_{nn}^j = 1$.

1.5　Market Clearing Conditions

Regional labor market clearing requires that

$$\sum_{j=1}^{J} L_n^j = \sum_{j=1}^{J} \int l_n^j(z)\phi_n^j(z)dz = L_n, \ n = 1,...N$$

and national market clearing then gives

$$\sum_{n=1}^{N} L_n = L.$$

Market clearing for land and structures in each region imply that

$$\sum_{j=1}^{J} H_n^j = \sum_{j=1}^{J} \int h_n^j(z)\phi_n^j(z)dz = H_n, \ n = 1,...,N$$

Final goods market clearing means that

$$L_n c_n^j + \sum_{k=1}^{J} M_n^{kj} = L_n c_n^j + \sum_{k=1}^{J} \int M_n^{kj}(z)\phi_n^k(z)dz = Q_n^j.$$

Given our definition of total expenditures X_j^n above, as well as firm and household optimality conditions, this last expression can also be written as

$$\alpha^j L_n I_n + \sum_{k=1}^{J} \gamma_n^{kj} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^k X_i^k\right)}_{X_{in}^k} = X_n^j.$$

Observe that $\sum_{i=1}^{N} X_{in}^k$ represents the value of sector k's gross production in region n, $\int p_n^k(z)q_n^k(z)\phi_n^k(z)dz$. Given that the technology in the production of intermediate goods is CRS, these revenues exactly offset payments to labor, land and structures, and materials. In particular, we have that $w_n L_n^k = \gamma_n^k (1 - \beta_n) \sum_{i=1}^{N} X_{in}^k$, $r_n H_n^k = \gamma_n^k \beta_n \sum_{i=1}^{N} X_{in}^k$, and, as indicated above, $P_n^j M_n^{kj} = \gamma_n^{kj} \sum_{i=1}^{N} X_{in}^k$.

In equilibrium, total expenditures by a given region n on intermediates purchased from all other regions must equal region n's total revenue from selling intermediates to all other regions,

$$\sum_{j=1}^{J}\sum_{i=1}^{N} \pi_{ni}^j X_n^j = \sum_{j=1}^{J}\sum_{i=1}^{N} \pi_{in}^j X_i^j.$$

1.6 Free mobility Condition

Utility of a household in region n is given by $U = \frac{I_n}{P_n}$. Using equation (10), market clearing conditions for labor and land and structures imply that

$$r_n H_n = \frac{\beta_n}{1 - \beta_n} w_n L_n. \tag{16}$$

Equation (16) implies that

$$\frac{w_n}{1 - \beta_n} = \frac{r_n}{\beta_n} \frac{H_n}{L_n}$$

or

$$\left(\frac{w_n}{1 - \beta_n} \right)^{\beta_n} = \left(\frac{r_n}{\beta_n} \right)^{\beta_n} \left(\frac{H_n}{L_n} \right)^{\beta_n},$$

$$\frac{w_n}{1 - \beta_n} = \underbrace{\left(\frac{w_n}{1 - \beta_n} \right)^{1 - \beta_n} \left(\frac{r_n}{\beta_n} \right)^{\beta_n}}_{\omega_n} \left(\frac{H_n}{L_n} \right)^{\beta_n},$$

so that

$$\frac{w_n}{1 - \beta_n} = \omega_n \left(\frac{H_n}{L_n} \right)^{\beta_n},$$

where $\omega_n = \left(\frac{w_n}{1 - \beta_n} \right)^{1 - \beta_n} \left(\frac{r_n}{\beta_n} \right)^{\beta_n}$.

Income of a household in region n is given by

$$
\begin{aligned}
I_n &= (1 - \iota_n) \frac{r_n H_n}{L_n} + w_n + \chi \\
&= \frac{r_n H_n}{L_n} + w_n - \frac{\Upsilon_n}{L_n},
\end{aligned}
$$

and its utility can thus be expressed as

$$U = \frac{\omega_n}{P_n} \left(\frac{H_n}{L_n} \right)^{\beta_n} - \frac{u_n}{P_n},$$

where $u_n = \frac{\Upsilon_n}{L_n} = \frac{(\iota_n r_n H_n - \chi L_n)}{L_n}$.

The free mobility condition may then be written as

$$L_n = \left(\frac{\omega_n}{U P_n + u_n} \right)^{\frac{1}{\beta_n}} H_n,$$

or, alternatively,

$$L_n = \frac{\left(\frac{\omega_n}{U P_n + u_n} \right)^{\frac{1}{\beta_n}} H_n}{L} L.$$

Using aggregate labor market clearing, this last expression also becomes

$$L_n = \frac{\left(\frac{\omega_n}{U P_n + u_n} \right)^{\frac{1}{\beta_n}} H_n}{\sum_{i=1}^{N} \left(\frac{\omega_i}{U P_i + u_n} \right)^{\frac{1}{\beta_i}} H_i} L.$$

2 Solving the Model and Counterfactuals

To carry out meaningful quantitative assessments of the effects of productivity changes in the U.S. economy, we first need to write a variant of the economic environment that can be matched against observed regional trade imbalances. We can then calibrate this model variant using observations on the actual economy with trade imbalances, and use the calibrated model to calculate what counterfactual allocations would have been in the U.S. absent trade imbalances. The counterfactual economy without imbalances may then be used as a benchmark from which to assess the effects of productivity and other fundamental changes.

2.1 Equilibrium Conditions with Regional Trade Deficits

Contributions from each region, ι_n, to the national portfolio are chosen to minimize the squared differences between observed trade imbalances in practice and the trade imbalances that emerge through interregional transfers implied by the national portfolio. Specifically, to account for actual trade imbalances, denote the observed per capita trade surplus of region n by $\left(\frac{\Upsilon_n + S_n}{L_n}\right)$, where $\frac{S_n}{L_n}$ can be thought of as a measure of regional surplus unexplained by the model. We then choose ι_n to minimize the square of these deviations from the data.

With regional trade deficits and surpluses, household income in region n is given by

$$I_n = r_n \frac{H_n}{L_n} + w_n - u_n - s_n.$$

The free mobility condition then becomes

$$U = \frac{r_n H_n / L_n + w_n - u_n - s_n}{P_n}.$$

Given that $r_n \frac{H_n}{L_n} = \frac{\beta_n}{1-\beta_n} w_n$ in equilibrium, and following the same steps as above, we have that

$$
\begin{aligned}
U &= \left(\frac{w_n}{1-\beta_n}\right)\frac{1}{P_n} - \frac{u_n}{P_n} - \frac{s_n}{P_n} \\
&= \frac{\omega_n}{P_n}\left(\frac{H_n}{L_n}\right)^{\beta_n} - \frac{u_n}{P_n} - \frac{s_n}{P_n},
\end{aligned}
$$

where $\omega_n = \left(\frac{r_n}{\beta_n}\right)^{\beta_n}\left(\frac{w_n}{1-\beta_n}\right)^{1-\beta_n}$. Solving for L_n gives

$$
\begin{aligned}
L_n &= \left(\frac{\omega_n}{P_n U + u_n + s_n}\right)^{\frac{1}{\beta_n}} H_n \\
&= \frac{\left(\frac{\omega_n}{P_n U + u_n + s_n}\right)^{\frac{1}{\beta_n}} H_n}{L} L
\end{aligned}
$$

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or, using the aggregate labor market clearing condition,

$$L_n = \frac{\left(\frac{\omega_n}{P_n U + u_n + s_n}\right)^{\frac{1}{\beta_n}} H_n}{\sum_{i=1}^{N} \left(\frac{\omega_i}{P_i U + u_n + s_i}\right)^{\frac{1}{\beta_i}} H_i} L.$$

Expressions for the cost of the input bundle and prices are unchanged,

$$x_n^j = B_n^j \left[r_n^{\beta_n} w_n^{1-\beta_n}\right]^{\gamma_n^j} \prod_{k=1}^{J} \left(P_n^k\right)^{\gamma_n^{jk}},$$

$$P_n^j = \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}} \left[\sum_{i=1}^{N} \left[x_i^j \kappa_{ni}^j\right]^{-\theta^j} \left(T_i^j\right)^{\theta^j \gamma_i^j}\right]^{-\frac{1}{\theta^j}},$$

as are expressions for the trade shares,

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \left[\frac{\kappa_{ni}^j x_i^j \Gamma(\xi_n^j)^{\frac{1}{1-\eta_n^j}}}{\left(T_i^j\right)^{\gamma_i^j} P_n^j}\right]^{-\theta^j}.$$

Regional market clearing in final goods now becomes

$$\alpha^j L_n \left(r_n \frac{H_n}{L_n} + w_n - u_n - s_n\right) + \sum_{k=1}^{J} \gamma_n^{kj} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^k X_i^k\right)}_{X_{in}^k} = X_n^j$$

or, following the usual steps,

$$\alpha^j L_n \left(\frac{w_n}{1-\beta_n} - s_n\right) + \sum_{k=1}^{J} \gamma_n^{kj} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^k X_i^k\right)}_{X_{in}^k} = X_n^j,$$

$$\alpha^j L_n \left(\omega_n \left(\frac{H_n}{L_n}\right)^{\beta_n} - u_n - s_n\right) + \sum_{k=1}^{J} \gamma_n^{kj} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^k X_i^k\right)}_{X_{in}^k} = X_n^j,$$

$$\alpha^j \left(\omega_n H_n^{\beta_n} L_n^{1-\beta_n} - \Upsilon_n - S_n\right) + \sum_{k=1}^{J} \gamma_n^{kj} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^k X_i^k\right)}_{X_{in}^k} = X_n^j. \tag{17}$$

Regional trade imbalances are given by

$$\sum_{j=1}^{J} X_n^j + \Upsilon_n + S_n = \sum_{j=1}^{J} \sum_{i=1}^{N} \pi_{in}^j X_i^j. \tag{18}$$

Observe that combining (17) and (18) also give the following equilibrium condition,

$$\omega_n H_n^{\beta_n} L_n^{1-\beta_n} = \sum_{j=1}^{J} \gamma_j^n \sum_{i=1}^{N} \pi_{in}^j X_i^j. \tag{19}$$

This last expression may be derived by first summing equation (17) across all sectors,

$$\left(\omega_n H_n^{\beta_n} L_n^{1-\beta_n} - \Upsilon_n - S_n\right) \underbrace{\sum_{j=1}^{J} \alpha^j}_{1} + \underbrace{\sum_{j=1}^{J}\sum_{k=1}^{J} \gamma_n^{jk}}_{1-\gamma_n^j} \sum_{i=1}^{N} \underbrace{\left(\pi_{in}^j X_i^j\right)}_{X_{in}^j} = \sum_{j=1}^{J} X_n^j.$$

Using equation (18), this gives

$$\omega_n H_n^{\beta_n} L_n^{1-\beta_n} - \Upsilon_n - S_n + \sum_{j=1}^{J}\sum_{i=1}^{N} \pi_{in}^j X_i^j - \sum_{j=1}^{J}\gamma_n^j \sum_{i=1}^{N} \pi_{in}^j X_i^j = \sum_{j=1}^{J}\sum_{i=1}^{N} \pi_{in}^j X_i^j - \Upsilon_n - S_n,$$

which then gives equation (19).

2.2 Equilibrium Conditions in Relative Terms

For any variable x, we denote the change in x following a change in the economic environment as $\widehat{x} = \frac{x'}{x}$.

We saw earlier that an expression for regional labor was

$$L_n = H_n \left(\frac{\omega_n}{P_n U + u_n + s_n}\right)^{1/\beta_n},$$

so that

$$\hat{L}_n = \frac{L_n'}{L_n} = \left(\frac{\hat{\omega}_n}{\frac{P_n' U' + u_n' + s_n'}{P_n U + u_n + s_n}}\right)^{1/\beta_n}.$$

Let $b_n = u_n + s_n$. Observe that

$$\begin{aligned}
\frac{P_n' U' + u_n' + s_n'}{P_n U + u_n + s_n} &= \frac{P_n' U'}{P_n U + b_n}\frac{P_n U}{P_n U} + \frac{b_n'}{P_n U + b_n}\frac{b_n}{b_n}\\
&= \hat{P}_n \hat{U}\frac{P_n U}{P_n U + b_n} + \widehat{b}_n \frac{b_n}{P_n U + b_n}
\end{aligned}$$

Hence, it follows that

$$\begin{aligned}
\frac{P_n' U' + u_n' + s_n'}{P_n U + u_n + s_n} &= \hat{P}_n \hat{U}\frac{1}{1 + \frac{b_n}{P_n U}} + \widehat{b}_n \frac{\frac{b_n}{P_n U}}{1 + \frac{b_n}{P_n U}}\\
&= \varphi_n \hat{P}_n \hat{U} + (1 - \varphi_n)\widehat{b}_n
\end{aligned}$$

where $\varphi_n = 1/\left(1 + \frac{b_n}{P_n U}\right)$, and we have

$$\hat{L}_n = \left(\frac{\hat{\omega}_n}{\varphi_n \hat{P}_n \hat{U} + (1 - \varphi_n)\widehat{b}_n}\right)^{1/\beta_n}.$$

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Recall also from the previous section that

$$U = \frac{\omega_n}{P_n}\left(\frac{H_n}{L_n}\right)^{\beta_n} - \frac{b_n}{P_n}.$$

Therefore, an expression for $\hat{U}$ is

$$\hat{U} = \frac{1}{\varphi_n}\frac{\hat{\omega}_n}{\hat{P}_n}\left(\hat{L}_n\right)^{-\beta_n} - \frac{1-\varphi_n}{\varphi_n}\frac{\widehat{b}_n}{\hat{P}_n}. \tag{20}$$

The aggregate labor market clearing condition is

$$L = \sum_{n=1}^{N} L_n,$$

so that

$$\hat{L} = \frac{L'}{L} = 1 = \sum_{n=1}^{N} \hat{L}_n\frac{L_n}{L}$$

or

$$L = \sum_{n=1}^{N} L_n\hat{L}_n.$$

Then, when expressed in changes, labor in region n becomes

$$\hat{L}_n = \frac{\hat{L}_n}{L} = \frac{\left[\frac{\hat{\omega}_n}{\varphi_n\hat{P}_n\hat{U}+(1-\varphi_n)\widehat{b}_n}\right]^{1/\beta_n}}{\sum_{n=1}^{N} L_n\left[\frac{\hat{\omega}_n}{\varphi_n\hat{P}_n\hat{U}+(1-\varphi_n)\widehat{b}_n}\right]^{1/\beta_n}}, \qquad N \text{ equations.}$$

Since $L = \sum_{n=1}^{N} L_n\hat{L}_n$, we have that

$$\hat{U}L = \sum_{n=1}^{N} \hat{U}L_n\hat{L}_n$$

or, from (20),

$$\begin{aligned}
\hat{U} &= \frac{1}{L}\sum_{n} L_n\hat{L}_n\left(\frac{1}{\varphi_n}\frac{\hat{\omega}_n}{\hat{P}_n}\left(\hat{L}_n\right)^{-\beta_n} - \frac{1-\varphi_n}{\varphi_n}\frac{\widehat{b}_n}{\hat{P}_n}\right) \\
&= \frac{1}{L}\sum_{n} L_n\frac{1}{\varphi_n}\frac{\hat{\omega}_n}{\hat{P}_n}\left(\hat{L}_n\right)^{1-\beta_n} - \frac{1}{L}\sum_{n} L_n\frac{1-\varphi_n}{\varphi_n}\frac{\hat{L}_n\widehat{b}_n}{\hat{P}_n}
\end{aligned}$$

In changes, the cost of the input bundle becomes

$$\widehat{x}_n^j = (\widehat{\omega}_n)^{\gamma_n^j}\prod_{k=1}^{J}\left(\widehat{P}_n^k\right)^{\gamma_n^{jk}}, \qquad JN \text{ equations,}$$

prices take the form,

$$\widehat{P}_n^j = \left[\sum_{i=1}^{N} \pi_{ni}^j\left[\widehat{x}_i^j\widehat{\kappa}_{ni}^j\right]^{-\theta^j}\left(\widehat{T}_i^j\right)^{\theta^j\gamma_i^j}\right]^{-\frac{1}{\theta^j}}, \qquad JN \text{ equations,}$$

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while trade share after an exogenous change are

$$\left(\pi_{ni}^j\right)' = \pi_{ni}^j \left[\frac{\widehat{\kappa}_{ni}^j \widehat{x}_i^j}{\widehat{P}_n^j}\right]^{-\theta^j} \left(\widehat{T}_i^j\right)^{\gamma_i^j \theta^j}, \qquad JN^2 \text{ equations.}$$

Market clearing for final goods implies that following an exogenous change in fundamentals,

$$
\begin{aligned}
X_n^{j'} &= \alpha^j \left(\omega_n' H_n^{\beta_n} (L_n')^{1-\beta_n} - S_n'\right) + \sum_{k=1}^J \gamma_n^{kj} \sum_{i=1}^N \pi_{in}^{k'} X_i^{k'} \\
&= \alpha^j \left(\widehat{\omega}_n \left(\widehat{L}_n\right)^{1-\beta_n} \omega_n H_n^{\beta_n} (L_n)^{1-\beta_n} - \Upsilon' - S_n'\right) + \sum_{k=1}^J \gamma_n^{kj} \sum_{i=1}^N \pi_{in}^{k'} X_i^{k'}
\end{aligned}
$$

Now, recall that

$$
\begin{aligned}
\omega_n H_n^{\beta_n} L_n^{1-\beta_n} &= \frac{w_n L_n}{1 - \beta_n} \\
&= r_n H_n + w_n L_n \\
&= I_n L_n + \Upsilon_n + S_n.
\end{aligned}
$$

Therefore, we have that

$$X_n^{j'} = \alpha^j \left(\widehat{\omega}_n \left(\widehat{L}_n\right)^{1-\beta_n} [I_n L_n + \Upsilon_n + S_n] - \Upsilon'_n - S_n'\right) + \sum_{k=1}^J \gamma_n^{kj} \sum_{i=1}^N \pi_{in}^{k'} X_i^{k'}, \qquad JN \text{ equations.}$$

Following similar steps, the trade balance condition becomes

$$\widehat{\omega}_n \left(\widehat{L}_n\right)^{1-\beta_n} \omega_n H_n^{\beta_n} (L_n)^{1-\beta_n} = \sum_{j=1}^J \gamma_j^n \sum_{i=1}^N \pi_{in}^{j'} X_i^{j'} \qquad N \text{ equations.}$$

The system to be solved consists of $2N + 3JN + JN^2$ equations. The unknowns, and their quantity in parenthesis, are $\widehat{\omega}_n$ (N), $\widehat{L}_n$ (N), $(X_n^j)'$ (JN), $\widehat{P}_n^j$ (JN), $\left(\pi_{in}^j\right)'$ (JN^2), and $\widehat{x}_n^j$ (JN).

2.3 Algorithm for Computing Counterfactuals

Exogenous changes in fundamentals are given by any combination of S_n', $\widehat{\kappa}_{ni}^j$ and/or $\widehat{T}_{ni}^j$. To solve for the equilibrium resulting from any of these changes using the system above, we proceed as follows:

Guess a relative change in regional factor prices $\widehat{\omega}$.

Step 1. Obtain $\widehat{P}_n^j$ and $\widehat{x}_n^j$ consistent with $\widehat{\omega}$ using

$$\widehat{x}_n^j = (\widehat{\omega}_n)^{\gamma_n^j} \prod_{k=1}^J (\widehat{P}_n^k)^{\gamma_n^{kj}}, \ N \times J,$$

and

$$\hat{P}_n^j = \left(\sum_{i=1}^{N} \pi_{ni}^j \left[\hat{\kappa}_{ni}^j \widetilde{x}_i^j \right]^{-\theta^j} \left(\hat{T}_i^j \right)^{\theta^j \gamma_i^j} \right)^{-1/\theta^j} , \quad N \times J.$$

Step 2. Solve for the trade shares, $\pi_{ni}^{j'}(\hat{\boldsymbol{\omega}})$, consistent with the change in factor prices using $\hat{P}_n^j(\hat{\boldsymbol{\omega}})$ and $\widehat{x}_n^j(\hat{\boldsymbol{\omega}})$ as well as the definition of trade shares,

$$\pi_{ni}^{j'}(\hat{\boldsymbol{\omega}}) = \pi_{ni}^j \left(\frac{\widetilde{x}_i^j(\hat{\boldsymbol{\omega}})}{\hat{P}_n^j(\hat{\boldsymbol{\omega}})} \widehat{\kappa}_{ni}^j \right)^{-\theta^j} \hat{T}_i^{j\ \theta^j \gamma_i^j}.$$

Step 3. Solve for the change in labor across regions, $\hat{L}_n(\hat{\boldsymbol{\omega}})$, consistent with $\hat{\boldsymbol{\omega}}$ given $\hat{P}_n^j(\hat{\boldsymbol{\omega}})$ and $\hat{x}_n^j(\hat{\boldsymbol{\omega}})$. This step is carried out by iterating between two substeps:

Step 3a. Define $\hat{L}_n(\hat{\boldsymbol{\omega}}, \widehat{\mathbf{b}})$ as

$$\hat{L}_n = \frac{\left(\frac{\hat{\omega}_n}{\varphi_n \hat{P}_n \hat{U} + (1-\varphi_n)\widehat{b}_n} \right)^{1/\beta_n}}{\sum_n L_n \left(\frac{\hat{\omega}_n}{\varphi_n \hat{P}_n \hat{U} + (1-\varphi_n)\widehat{b}_n} \right)^{1/\beta_n}} L,$$

where $\hat{P}_n(\hat{\boldsymbol{\omega}}) = \prod_{j=1}^{J} \hat{P}_n^j(\hat{\boldsymbol{\omega}})^{\alpha^j}$, and $\hat{U} = \frac{1}{L} \sum_n L_n \frac{1}{\varphi_n} \frac{\hat{\omega}_n}{\hat{P}_n} \left(\hat{L}_n \right)^{1-\beta_n} - \frac{1}{L} \sum_n L_n \frac{1-\varphi_n}{\varphi_n} \frac{\hat{L}_n \widehat{b}_n}{\hat{P}_n}$.

Step 3b. Define $\widehat{b}_n(\hat{\boldsymbol{\omega}}, \widehat{\mathbf{L}})$ as follows: $\widehat{b}_n = \frac{u_n'}{u_n + s_n}$ where $u_n' = \frac{\Upsilon_n'}{L_n'} = \frac{\widehat{\Upsilon}_n}{\hat{L}_n} \frac{\Upsilon_n}{L_n}$. We have that

$$\widehat{\Upsilon}_n = \frac{\iota_n r_n' H_n' + \chi_n'}{\iota_n r_n H_n + \chi_n},$$

where $\iota_n r_n H_n + \chi_n$ is known. Then, $r_n' H_n' = \frac{\beta_n}{1-\beta_n} w_n' L_n' = \left(\frac{\beta_n}{1-\beta_n} w_n L_n \right) \widehat{w}_n \widehat{L}_n$ where, from the definition of ω, $\widehat{w}_n \widehat{L}_n = \widehat{\omega}_n \widehat{L}_n^{1-\beta_n}$. Therefore,

$$r_n' H_n' = \left(\frac{\beta_n}{1-\beta_n} w_n L_n \right) \widehat{\omega}_n \widehat{L}_n^{1-\beta_n},$$

where $\frac{\beta_n}{1-\beta_n} w_n L_n$ is known, and $\chi_n' = \sum_{i-1}^{N} \iota_n r_n' H_n' = \sum_{i-1}^{N} \left(\frac{\beta_n}{1-\beta_n} w_n L_n \right) \widehat{\omega}_n \widehat{L}_n^{1-\beta_n}$.

Thus, to carry out step 3, given $\hat{\boldsymbol{\omega}}$, set $\widehat{\mathbf{L}} = 1$ and compute $\widehat{b}_n(\hat{\boldsymbol{\omega}}, 1)$ using step 3b. Then compute $\hat{L}_n(\hat{\boldsymbol{\omega}}, \widehat{\mathbf{b}})$ using step 3a. Use these updated values of $\hat{L}_n$ to compute new values of $\widehat{b}_n(\hat{\boldsymbol{\omega}}, \widehat{\mathbf{L}})$ and so on iterating between steps 3a and 3b.

Step 4. Solve for expenditures in the counterfactual equilibrium consistent with the change in factor prices $X_n^{j'}(\hat{\boldsymbol{\omega}})$.

$$\begin{aligned}
X_n^{j'}(\hat{\boldsymbol{\omega}}) &= \sum_{k=1}^{J} \gamma_n^{k,j} \left(\sum_{i=1}^{N} \pi_{in}^{k'}(\hat{\boldsymbol{\omega}}) X_i^{k'}(\hat{\boldsymbol{\omega}}) \right) \\
&+ \alpha^j \left(\widehat{\omega}_n \left(\hat{L}_n(\hat{\boldsymbol{\omega}}) \right)^{1-\beta_n} (I_n L_n + \Upsilon_n + S_n) - \Upsilon_n' - S_n' \right),
\end{aligned}$$

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which constitutes $N \times J$ linear equations in $N \times J$ unknowns, $\{X_n^{j'}(\hat{\boldsymbol{\omega}})\}_{N \times J}$. This can be solved through simple matrix inversion. Observe that carrying out this step first requires having solved for $\hat{L}_n(\hat{\boldsymbol{\omega}})$.

Step 5. Obtain a new guess for the change in factor prices, $\hat{\omega}_n^*$, using

$$\hat{\omega}_n^* = \frac{\sum_j \gamma_n^j \sum_i \pi_{in}^{'j}(\hat{\boldsymbol{\omega}}) X_i^{'j}(\hat{\boldsymbol{\omega}})}{\hat{L}_n(\hat{\boldsymbol{\omega}})^{1-\beta_n}(L_n I_n + \Upsilon_n + S_n)}.$$

Repeat Steps 1 through 5 until $||\hat{\boldsymbol{\omega}}^* - \hat{\boldsymbol{\omega}}|| < \varepsilon$.

To assess the quantitative effects of changes in fundamental productivity, $\widehat{T}_n^j$, and/or regional trade barriers, $\widehat{\kappa}_{ni}^j$, we must first take account of observed regional trade imbalances through S_n, and establish benchmark counterfactual allocations without such imbalances. This is done by solving the algorithm above while matching S_n to observed trade imbalances $\Upsilon_n + S_n$, setting $S_n' = 0$, $\widehat{T}_n^j = 1$ and $\widehat{\kappa}_{ni}^j = 1$. The resulting changes can be used to calculate new equilibrium allocations, $(X_n^j)'$, $(\pi_{in}^j)'$, $L_n' = \widehat{L}_n L_n$, etc., that are then used to define the baseline economy from which to assess any productivity or other changes to the environment.

3 Measuring TFP, GDP, and Welfare Changes

3.1 TFP in a given region-sector pair (n, j)

Measured total factor productivity (TFP) in a region-sector pair (n, j) is commonly calculated as

$$\ln A_n^j = \ln \frac{Y_n^j}{P_n^j} - (1 - \beta_n)\gamma_n^j \ln L_n^j - \beta_n \gamma_n^j \ln H_n^j - \sum_{k=1}^{J} \gamma_n^{jk} \ln M_n^{jk},$$

where Y_n^j denotes intermediate goods production in sector j. As noted above, zero profits imply that total production of intermediates is exactly offset by factor payments, $w_n L_n^j + r_n H_n^j + \sum_{k=1}^{J} P_n^k M_n^{jk} = \int p_n^j(z_n^j) q_n^j(z_n^j) \phi_n^j(z_n^j) dz_n^j = Y_n^j$. From firms' optimality conditions in section 1.2.1, we have that

$$\frac{w_n l_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)} = \gamma_n^j(1 - \beta_n)$$

or

$$w_n L_n^j = \gamma_n^j(1 - \beta_n) \int p_n^j(z_n^j) q_n^j(z_n^j) \phi_n^j(z_n^j) dz_n^j,$$

so that

$$\frac{w_n L_n^j}{Y_n^j} = \gamma_n^j(1 - \beta_n) = \frac{w_n l_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)}.$$

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Similarly, it follows that

$$\frac{r_n H_n^j}{Y_n^j} = \gamma_n^j \beta_n = \frac{r_n h_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)}$$

and

$$\frac{P_n^k M_n^{jk}}{Y_n^j} = \gamma_n^{jk} = \frac{P_n^k M_n^{jk}(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)}.$$

Therefore,

$$l_n^j(z_n^j) = \frac{p_n^j(z_n^j) q_n^j(z_n^j) L_n^j}{Y_n^j}, \ h_n^j(z_n^j) = \frac{p_n^j(z_n^j) q_n^j(z_n^j) H_n^j}{Y_n^j} \ \text{and} \ M_n^{jk}(z_n^j) = \frac{p_n^j(z_n^j) q_n^j(z_n^j) M_n^{jk}}{Y_n^j}.$$

Substituting these expressions in the production of intermediate goods gives

$$q_n^j(z_n^j) = \frac{p_n^j(z_n^j) q_n^j(z_n^j)}{Y_n^j} z_n^j \left[T_n^j (H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{J} (M_n^{jk})^{\gamma_n^{jk}}$$

which, given that $p_n^j(z_n^j) = x_n^j/z_n^j (T_n^j)^{\gamma_n^j}$, implies

$$q_n^j(z_n^j) = \frac{x_n^j q_n^j(z_n^j)}{Y_n^j} \left[(H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{J} (M_n^{jk})^{\gamma_n^{jk}}.$$

Therefore,

$$\frac{Y_n^j}{P_n^j} = A_n^j \left[(H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{J} (M_n^{jk})^{\gamma_n^{jk}}, \tag{21}$$

where TFP in region-sector pair (n, j), A_n^j, is given by

$$A_n^j = \frac{x_n^j}{P_n^j},$$

and

$$\ln \widehat{A}_n^j = \ln \frac{\widehat{x}_n^j}{\widehat{P}_n^j}. \tag{22}$$

Since, from section 2.2,

$$\left(\widehat{\pi}_{nn}^j \right) = \left[\frac{\widehat{x}_n^j}{\widehat{P}_n^j} \right]^{-\theta^j} \left(\widehat{T}_n^j \right)^{\gamma_n^j \theta^j},$$

it follows that TFP changes in a given region-sector pair may also be expressed as

$$\ln \widehat{A}_n^j = \ln \frac{\left(\widehat{T}_n^j \right)^{\gamma_n^j}}{\left(\widehat{\pi}_{nn}^j \right)^{\frac{1}{\theta^j}}}$$

3.2 Aggregate, Regional, and Sectoral TFP

Given equation (21), we have that changes in regional TFP are given by

$$\widehat{A}_n = \sum_{j=1}^{J} \left(\frac{Y_n^j}{\sum_{j=1}^{J} Y_n^j} \right) \widehat{A}_n^j = \sum_{j=1}^{J} \left(\frac{\frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}}{\sum_{j=1}^{J} \frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}} \right) \widehat{A}_n^j,$$

where $w_n L_n^j$ and $\gamma_n^j (1-\beta_n)$ are both known in the base year. Similarly, changes in sectoral TFP may be obtained according to

$$\widehat{A}^j = \sum_{n=1}^{N} \left(\frac{Y_n^j}{\sum_{j=1}^{J} Y_n^j} \right) \widehat{A}_n^j = \sum_{n=1}^{N} \left(\frac{\frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}}{\sum_{j=1}^{J} \frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}} \right) \widehat{A}_n^j,$$

while aggregate changes in TFP are given by

$$\widehat{A} = \sum_{n=1}^{N} \sum_{j=1}^{J} \left(\frac{Y_n^j}{\sum_{j=1}^{J} Y_n^j} \right) \widehat{A}_n^j = \sum_{n=1}^{N} \sum_{j=1}^{J} \left(\frac{\frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}}{\sum_{j=1}^{J} \frac{w_n L_n^j}{\gamma_n^j (1-\beta_n)}} \right) \widehat{A}_n^j.$$

3.3 GDP in a given region-sector pair (n, j)

GDP in a given region-sector pair is the difference between gross production and expenditures on materials, $\frac{w_n L_n^j + r_n H_n^j}{P_n^j}$. Since, in equilibrium, $r_n H_n^j = \frac{\beta_n}{1-\beta_n} w_n L_n^j$, GDP in (n, j) is also $\left(\frac{1}{1-\beta_n} \right) \frac{w_n L_n^j}{P_n^j}$, from which it follows that

$$\ln \widehat{GDP}_n^j = \ln \widehat{w}_n + \ln \widehat{L}_n^j - \ln \widehat{P}_n^j.$$

Recall that

$$\widehat{P}_n^j = \widehat{x}_n^j \left(\widehat{T}_n^j \right)^{-\gamma_n^j} \left(\widehat{\pi}_{nn}^j \right)^{1/\theta^j}, \tag{23}$$

therefore

$$\begin{aligned} \ln \widehat{GDP}_n^j &= \ln \frac{\left(\widehat{T}_n^j \right)^{\gamma_n^j}}{\left(\widehat{\pi}_{nn}^j \right)^{1/\theta^j}} + \ln \widehat{L}_n^j + \ln \frac{\widehat{w}_n}{\widehat{x}_n^j} \\ &= \ln \widehat{A}_n^j + \ln \widehat{L}_n^j + \ln \frac{\widehat{w}_n}{\widehat{x}_n^j}. \end{aligned}$$

3.4 Aggregate, Regional, and Sectoral GDP

Changes in regional real GDP arising from a change in fundamentals are given by

$$\widehat{GDP}_n = \sum_{j=1}^{J} \left(\frac{w_n L_n^j + r_n H_n^j}{\sum_{j=1}^{J} \left(w_n L_n^j + r_n H_n^j \right)} \right) \widehat{GDP}_n^j.$$

Similarly, changes in sectoral real GDP may be expressed as

$$\widehat{GDP}^j = \sum_{n=1}^{N} \left(\frac{w_n L_n^j + r_n H_n^j}{\sum_{n=1}^{N} \left(w_n H_n^j + r_n H_n^j \right)} \right) \widehat{GDP}_n^j.$$

Finally, aggregate change in GDP is given by

$$\widehat{GDP} = \sum_{j=1}^{J} \sum_{n=1}^{N} \left(\frac{w_n L_n^j + r_n H_n^j}{\sum_{j=1}^{J} \sum_{n=1}^{N} \left(w_n L_n^j + r_n H_n^j \right)} \right) \widehat{GDP}_n^j.$$

3.5 Welfare

From the free mobility condition, using the fact that $I_n = (1 - \iota_n)r_n H_n + w_n + \chi = \frac{\beta_n}{1-\beta_n}(1 - \iota_n)w_n + w_n + \chi$, we have that

$$U = \left[\frac{(1 - \beta_n \iota_n)w_n + (1 - \beta_n)\chi}{1 - \beta_n} \right] \frac{1}{P_n}$$

so that

$$\widehat{U} = \frac{\varpi \widehat{w}_n + (1 - \varpi)\widehat{\chi}}{\widehat{P}_n},$$

where $\varpi = \frac{(1-\beta_n \iota_n)w_n}{(1-\beta_n \iota_n)w_n+(1-\beta_n)\chi}$.

Recall from the household problem that $P_n = \prod_{j=1}^{J} (P_n^j/\alpha^j)^{\alpha^j}$ and, using equation (22), we have that

$$
\begin{aligned}
\ln \widehat{U} &= \ln \left(\varpi \widehat{w}_n + (1 - \varpi)\widehat{\chi} \right) - \ln \widehat{P}_n \\
&= \ln \left(\varpi \widehat{w}_n + (1 - \varpi)\widehat{\chi} \right) - \sum_{j=1}^{J} \alpha_j \ln \widehat{P}_n^j \\
&= \ln \left(\varpi \widehat{w}_n + (1 - \varpi)\widehat{\chi} \right) - \sum_{j=1}^{J} \alpha_j \left(\ln \widehat{x}_n^j - \ln \widehat{A}_n^j \right) \\
&= \sum_{j=1}^{J} \alpha_j \ln \left(\varpi \widehat{w}_n + (1 - \varpi)\widehat{\chi} \right) + \sum_{j=1}^{J} \alpha_j \left(\ln \widehat{A}_n^j - \ln \widehat{x}_n^j \right) \\
&= \sum_{j=1}^{J} \alpha_j \left[\ln \widehat{A}_n^j + \ln \left(\varpi \frac{\widehat{w}_n}{\widehat{x}_n^j} + (1 - \varpi)\frac{\widehat{\chi}}{\widehat{x}_n^j} \right) \right].
\end{aligned}
$$

Observe that when $\iota_n = 0\ \forall n$, then $\chi_n = 0$ and $\varpi_n = 1\ \forall n$. In that case,

$$
\begin{aligned}
\ln \widehat{U} &= \sum_{j=1}^{J} \alpha_j \left[\ln \widehat{A}_n^j + \ln \frac{\widehat{w}_n}{\widehat{x}_n^j} \right] \\
&= \sum_{j=1}^{J} \alpha_j \left(\ln \widehat{GDP}_n^j - \ln \widehat{L}_n^j \right).
\end{aligned}
$$

4   Data and Calibration

The model is calibrated to match key features of the 50 U.S. states (with Virginia and the District of Columbia combined into a single region) using a total of 26 sectors—15 tradable goods sectors, 10 service sectors, and construction—according to the North American Industry Classification System (NAICS). We assume that all service sectors and construction are non-tradable. Later in this section, we present a list of the sectors that we use and describe how we combine a subset of these sectors to ease computations. As stated earlier in the supplementary material, carrying out structural quantitative exercises on the effects of disaggregated fundamental changes requires data on $\left\{L_n, I_n, \pi_{ni}^j, S_n\right\}_{n=1,i=1,j=1}^{N,N,J}$ and values for the parameters $\left\{\gamma_n^j, \gamma_n^{jk}, \alpha^j, \beta_n, \theta^j\right\}_{n=1,j=1,k=1}^{N,J,J}$. To describe how we obtain these data and parameter values, we start with a discussion of the trade data used for this paper.

4.1   Trade Tables

The paper uses newly released data from the Commodity Flow Survey (CFS), jointly produced by the U.S. Census and the Bureau of Transportation. This dataset tracks pairwise trade flows across all 50 states and the District of Columbia (we combine Virginia and D.C.) for 21 manufacturing sectors of the U.S. economy (we aggregate several of these for a total of 15 tradable goods sectors, as detailed in the "List of Sectors" subsection). The CFS records a total of 5.2 trillion (2007) dollars of trade across all states in these manufacturing sectors. The most recent CFS data, initially released in December 2010 and last revised in 2012, covers the year 2007, thus explaining our choice of 2007 as the baseline year for our analysis. Using this 2007 CFS data, we construct 15 trade tables. Each trade table corresponds to a particular sector and is a 50×50 matrix whose entries represent pairwise trade flows in that sector between U.S. states. In a trade table for a given sector j, summing a row n gives total exports of sector j goods from state n to all other states, while summing column n gives total imports of sector j goods to state n from all other states.

While the CFS aims to quantify only domestic trade, some foreign imports that are subsequently traded in a domestic transaction are potentially included. To exclude this imported part of gross output, we calculate U.S. domestic consumption of domestic goods by subtracting exports from gross production for each NAICS sector using sectoral measures of gross output from the Bureau of Economic Analysis (BEA) and of exports from the U.S. Census. As expected, for each sector, the domestic shipment of goods implied by the CFS is larger than our measure of domestic consumption by a factor ranging from 1 to 1.4. We thus adjust the CFS tables proportionally so that they represent the total amount of domestic

consumption of domestic goods.

4.2 Regional Employment and Income

We set $L = 1$ so that, for each $n \in \{1, 2, ..., N\}$, L_n is the share of state n's employment in total U.S. employment. Regional employment data is obtained from the BEA, with aggregate employment across all states summing to 137.3 million in 2007. We calculate I_n using total value added in each state provided by the BEA and then dividing the result by total population for that state in 2007.

4.3 Interregional Trade Flows and Surpluses

To measure the share of state n's total sector j intermediate goods expenditures that go to state i, π_{ni}^j, we use the CFS trade table corresponding to sector j. State n's total expenditures on sector j intermediate goods is given by summing column n in this trade table, while state n's expenditures on sector j intermediate goods purchased from state i is given by the (i, n) entry of the trade table. So, we obtain π_{ni}^j simply by dividing the (i, n) entry by the sum of column n.

To compute state n's trade balance S_n, we sum across row n for all tradable sectors to obtain total exports, sum across column n for all tradable sectors to obtain total imports, and subtract total imports from total exports to obtain S_n.

4.4 Value Added Shares and Shares of Material Use

In order to obtain value added shares, observe that, for a particular sector j, summing row n of the corresponding adjusted CFS trade table yields gross output for sector j in region n, $\left\{\sum_{i=1}^{N} \pi_{in}^j X_i^j\right\}_{n=1}^{N}$. Hence, for each region-sector pair (n, j) where j is tradable, we divide value added from the BEA by gross output from the trade table to obtain the share of value added in gross output by region and sector, $\left\{\overline{\gamma}_n^j\right\}_{n=1, j=1}^{N, 15}$. For the 11 non-tradable sectors, gross output is not available at the sectoral level by state, so for a given non-tradable sector j, we assume that the value added share is constant across states and equal to that sector's aggregate value added share of aggregate gross output (both aggregate measures are obtained from the BEA), $\overline{\gamma}_n^j = \overline{\gamma}^j \ \forall n \in \{1, 2, ..., N\}$ and $j > 15$. However, regardless of the sector, these $\overline{\gamma}_n^j$'s generally overestimate the γ_n^j's as defined in our model—i.e. the value added share related to labor and fixed structures. Specifically, in practice, equipment is another factor of production that contributes significantly to value added. Greenwood, Hercowitz, and Krusell (1997) measure the shares of labor, structures, and equipment in value added

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for the U.S. economy at 70, 13, and 17 percent, respectively. Since our model takes explicitly account of materials, we assign the share of equipment to that of materials and adjust the share of value added accordingly, $\gamma_n^j = 0.83\overline{\gamma}_n^j$

While material input shares are available from the BEA by sector, they are not disaggregated by state. Given the structure of our model, it is nevertheless possible to infer region-specific material input shares from a national input-output (IO) table and other available data. The BEA "use table" gives the value of inputs from each industry used by every other industry at the aggregate level. This use table is available at 5 year intervals, the most recent of which uses 2002 data. A column sum of the use table gives total dollar payments from a given sector to all other sectors. Therefore, at the national level, we can compute γ^{jk}, the share of sector j's total payments to materials that goes to sector k material inputs. Since $\sum_{k=1}^{N} \gamma^{jk} = 1$, one may then construct the state-specific share of payments from sector j to material inputs from sector k in each state n as $\gamma_n^{jk} = (1 - \gamma_n^j)\gamma^{jk}$.

4.5   Share of Final Good Expenditures

The share of income spent on goods from sector j is calculated as follows:

$$\alpha^j = \frac{Y^j + M^j - E^j - \sum_k \gamma^{kj}(1 - \gamma^k)Y^k}{\sum_j \left[Y^j + M^j - E^j - \sum_k \gamma^{kj}(1 - \gamma^k)Y^k \right]},$$

where E^j denotes total sector j exports from the U.S. to the rest of the world, M^j denotes total sector j imports to the U.S., Y^j is gross production in sector j, and all intermediate input and value added shares are national averages.

4.6   Payments to Labor and Structure Shares

As noted previously, we assume that state n's share of payments to labor in value added, $1 - \beta_n$, is constant across sectors; data on compensation of employees from the BEA is not available by individual sector in every state. To calculate $1 - \beta_n$, we first sum data on compensation of employees across all available sectors in state n and divide this sum by value added in state n. The resulting measure, which we denote $1 - \overline{\beta}_n$, generally overestimates β_n, the value added share of land and structures since, as discussed earlier, part of the remaining (i.e. non-labor) factor used in production involves equipment. Thus, to adjust the value added shares of land and structures, we take the share of non-labor value added by region, $\overline{\beta}_n$, subtract the share of equipment, 0.17, and re-normalize so that the new shares add to one. That is, we calculate the value added share of land and structures as $\beta_n = (\overline{\beta}_n - 0.17)/0.83$ and that of labor as $1 - \beta_n = (1 - \overline{\beta}_n)/0.83$.

4.7 List of Sectors

The NAICS manufacturing sectors included in the CFS trade data are: Food (NAICS 311); Beverage and Tobacco Products (312); Textile Mills (313); Textile Product Mills (314); Apparel (315); Leather and Allied Products (316); Wood Products (321); Paper (322); Printing and Related Support Activities (323); Petroleum and Coal Products (324); Chemical (325); Plastics and Rubber Products (326); Nonmetallic Mineral Product (327); Primary Metal (331); Fabricated Metal Product (332); Machinery (333); Computer and Electronic Products (334); Electrical Equipment, Appliance, and Components (335); Transportation Equipment (336); Furniture and Related Products (337); and Miscellaneous (339). We aggregate Food and Beverage and Tobacco Products (311-312); Textile Mills, Textile Product Mills, Apparel, and Leather and Allied Products (313-316); Wood Products and Paper (321-322); and Primary Metal and Fabricated Metal Products (331-332). This leaves us with 15 tradable sectors.

The 11 non-tradable sectors we include in our analysis are: Construction (23); Wholesale and Retail Trade (42, 44-45); Transportation (481-488); Select Information (511, 515, 517-518); Finance and Insurance (52); Real Estate and Rental and Leasing (53); Educational Services (61); Health Care and Social Assistance (62); Arts, Entertainment, and Recreation (71); Accommodation and Food Services (72); and Other Services (493, 541, 55, 561-562, 811-814).

4.8 Sectoral Distribution of Productivities

The parameters $\{\theta^j\}_{j=1}^J$ determine the dispersion of productivities in each sector. A larger value of θ^j indicates a higher dispersion of productivity within sector j, which in turn implies that sector j goods are less substitutable and, therefore, respond less to changes in, for example, trade costs κ_{ni}^j. We obtain these $\{\theta^j\}_{j=1}^J$ parameters from Caliendo and Parro (2012), where they are calculated for 20 tradable sectors using data at the two-digit level of the third revision of the International Standard Industrial Classification (ISIC Rev. 3). We match the sectors in Caliendo and Parro (2012) to our NAICS 2007 sectors using information available in concordance tables. In five of our sectors, estimates in Caliendo and Parro (2012) are calculated at either a higher or lower level of aggregation. When separate estimates are reported for subsectors aggregated into a single larger sector j in our framework, we use their data to compute our aggregate θ^j. When one of our sectors represents instead a subset of a larger sector in Caliendo and Parro (2012), we simply use their estimate of the larger sector's dispersion parameter.

Here, we detail the five analogous cases just mentioned. For our sectors, "Wood Products

and Paper" (NAICS 321-322); "Primary Metal and Fabricated Metal Products" (NAICS 331-332); and "Transportation Equipment"(NAICS 336), Caliendo and Parro separately estimated dispersion parameters for, respectively, "wood products" and "paper products;" "primary metals" and "fabricated metals;" and "motor vehicles," "trailers and semi-trailers," and "other transportation equipment." In each of these three sectors, we use their data to compute an aggregate dispersion parameter. On the other hand, the dispersion parameter for our sector "Printing and Related Support Activities" (NAICS 323) is estimated together with "pulp and paper products" (ISIC3 21-22) in Caliendo and Parro, and that for "Furniture and Related Products" (NAICS 337) is estimated together with "other manufacturing" (ISIC3 36-37). Thus, in these last two cases, we use their estimate of the larger sector's parameter for our less aggregated sectors.

4.9   Average Miles per Shipment by Sector

Data on average mileage of all shipments from one state to another by NAICS manufacturing industries is obtained from a special release of the Commodity Flows Survey.