Tail Dependence in the US Financial Sector
and Measures of Systemic Risk\textsuperscript{1}

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Abstract:

Tail dependence among international stock markets is widely studied using measures of extremal dependence. In this study, we employ this approach to investigate the extreme loss tail dependence among stock prices of US depository institutions. We find that they exhibit strong loss dependence even in their limiting joint extremes. Motivated by this result, we derive extremal dependence-based systemic risk measures. The proposed systemic risk measures capture downturns in the US financial industry very well. We also develop a set of firm-level extremal dependence-based systemic interconnectedness measures and demonstrated their usefulness as potential early warning signals. Finally, we identify drivers of extremal dependence for the US depository institutions in a panel regression setting. Strength of extremal dependence increases with asset size and similarity of financial fundamentals. On the other hand, strength of extremal dependence decreases with capitalization, liquidity, funding stability and asset quality. We believe the proposed measures have the potential to inform the prudential supervision of systemic risk.

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1. INTRODUCTION

Financial crises over the last 20 years spread beyond the borders, industries or firms they originated in, causing widespread disruption to the broader economy. In the 1990s, the Asian financial crisis of 1997 and the Russian default of 1999 spurred turmoil around the world, and the collapse of the hedge fund LTCM created similar concerns for the US economy. More recently, the global financial meltdown of 2007-09 has reverberated across countries and sectors, resulting in countless regulatory interventions. It has become clear that some institutions play a critical role in the financial system, due to their size, leverage, and interconnectedness with the rest of the financial industry. Therefore, measuring systemic risk in the financial system and identifying systemically interconnected financial institutions are critical tasks for policy makers and regulators.

There have been many attempts to measure aggregate systemic risk. A popular approach to quantify systemic risk is the use of balance sheet or market information to generate early-warning indicators of systemic risk. Good examples of this approach can be found in Hart and Zingales (2011), who use credit default swap (CDS) spreads to construct an indicator of systemic instability, or in Huang, Zhou and Zhu (2009), who use CDS spreads and equity returns to derive the price of joint insurance for banking sector liabilities. In this paper, we contribute to this literature by developing new indicators of systemic risk based on the extremal dependence of large depository institutions’ equity returns. We focus on extreme co-movement of equity returns.
because the information contained in equity prices is generally viewed as forward looking. Forward looking indicators facilitate monitoring of systemic risk.

During all episodes of financial crises, extreme co-movement in asset prices and financial sector stock values was pervasive. As a result, correlations among different asset classes in general and among different banks in particular had huge spikes, reaching unprecedented levels. Patro, Qi and Sun (2013), explored the potential of stock return correlations of financial institutions as systemic risk indicators. They found that correlations capture the downturns in the U.S. financial system well. Nevertheless, Pearson correlation only provides partial information regarding the co-movement of asset prices during financial crises, because it only measures linear dependence between assets’ returns, therefore disregarding nonlinearities. More importantly, Pearson correlation measures only “average” dependence and is dominated by the observations around the mean. This leads to the dependence of extreme values being underrepresented, if not ignored. We believe that the information content of extremal dependence measures is crucial in understanding systemic risk. It is essential to measure co-movement of financial institutions under conditions of stress, as this co-movement can be very different from the co-movement under normal times.

In this study, we construct indicators of systemic risk based on extremal dependence measures developed by Ledford and Tawn (1996, 1997 and 1998) and Coles, Heffernan, and Tawn (1999). These studies separated extremal dependence structures into two broad categories, asymptotically dependent and asymptotically independent, based on the asymptotic behavior of joint loss distributions. The extremal dependence measure $\chi$ is defined as

$$\chi = \lim_{q \to 1} \Pr(L_1 > L_{1,q} \mid L_2 > L_{2,q})$$
where $L_1, L_2$ stand for two loss variables and $L_{1,q}, L_{2,q}$ stand for their respective marginal $q^{th}$ quantiles. If $\chi > 0$, $L_1$ and $L_2$ are said to be asymptotically dependent. If $\chi = 0$, $L_1$ and $L_2$ are said to be asymptotically independent.

We propose two systemic risk indicators driven from the extremal dependence measures. The first indicator measures the proportion of asymptotically dependent financial institution pairs to the total number of financial institution pairs in our sample. Chan-Lau, Mathieson and Yao (2004) investigated the strength of financial contagion in international stock markets by retrospectively monitoring extremal dependence during the Mexican peso crisis and the Asian crisis. Thus, we borrow this measure from the international financial contagion literature to measure the systemic risk in the US banking system. Billio et al. (2012) proposed as a systemic risk measure the share of bank pairs whose stock returns exhibit granger causality among all bank pairs in their dataset. In our study, we use extremal dependence measures to quantify the interconnectedness of financial institutions and, therefore, our measure is an extension of Billio et al. (2012) to tail co-movement. The other measure we consider is the average $\chi$ across all financial institutions. Average $\chi$ can provide insights regarding the strength of interconnectedness in the financial system, beyond the proportion of asymptotically dependent institutions. Our results show that these indicators behave as expected both when the financial market has been in turmoil and when it has been stable.

Besides overall systemic risk measurement, another challenge for regulators attempting to minimize systemic risk is identifying systemically important financial institutions (SIFIs). Appropriately measuring the contributions of individual institutions to systemic risk would allow regulators to better target policies to contain systemic risk. In the aftermath of the 2007-09 financial crisis, US and international policy makers are enacting new regulations for financial
institutions perceived as “too big to fail”, due to their size, interconnectedness, complexity, lack of substitutability, or global scope.²

Recent debates suggest that systemic risk cannot be reduced to a simple formula. It is, therefore, unsurprising that many indicators of systemic importance have been proposed in the literature, such as the CoVaR (Adrian and Brunnermeier 2011), the Marginal Expected Shortfall (Acharya et al. 2010) and the granger causality based measures of Billio et al. (2012). We believe that the co-existence of multiple approaches to measuring systemic importance and interconnectedness could enhance the supervision and regulation of systemic risk. We add to the literature by developing two firm-level measures of systemic interconnectedness, based on the stock price co-movements under conditions of joint stress. These measures of tail co-movement can help understand a financial institution’s interconnectedness in a potential crisis. Our first measure is the proportion of other institutions that are asymptotically dependent with an institution. Our second measure is the average $\chi$ of an institution, across all the institution-pairs involving itself.

We explore how our measures of tail co-movement can shed light on a financial institution’s interconnectedness in a potential crisis. We calculated these systemic interconnectedness measures as of end of June 2007 and investigated their predictive power for the crisis period (July 2007 - December 2008) returns. We found that these measures are significant predictors of crisis period stock returns. Since our measures are driven by the

² Both the Financial Stability Oversight Council (FSOC) (a creation of the Dodd-Frank Act in the United States) and the Basel Committee weighed in on the criteria for the designation of systemically important institutions in Q4 2011. For the United States, the Dodd-Frank Act (Section 113) lists statutory considerations for the designation of systemically important institutions. For additional details on global designation, see the Basel Committee’s “Globally Systemically Important Banks: Assessment Methodology and the Additional Loss Absorbency Requirements.” November 2011. We include many of the FSOC and Basel Committee proposed designation criteria in Section 4 when we look at the firm-specific financial indicators that can predict systemic interconnectedness for financial institutions.
historical co-movement of stock prices during stress, they do a good job in predicting the stock returns during the recent financial crisis. Similarly to Acharya et al. (2010), we compare the forecasting performance of our measures with a more conventional risk measure, which we choose to be CAPM Beta. We find that our measures provide additional information on the performance of bank stocks beyond what Beta alone would have provided on the eve of the financial crisis. Note that our measures do not take into account the size and leverage of the firms so they cannot be directly translated to an estimate of capital shortfall under stressful conditions.

The extremal dependence-based measures of systemic risk and interconnectedness that we develop in this paper perform very well in capturing the stock market downturns and the institutions most affected by them. Moore and Zhou (2012) suggest a methodology to identify “systemically important” institutions with similar measures to the systemic interconnectedness we propose in this paper. We believe our approach to measure systemic interconnectedness is preferable to theirs since our methodology explicitly tests for the existence of asymptotic dependence. In Moore and Zhou (2012) asymptotic dependence is assumed to exist if $\chi > .15$, independently of the precision of the estimate.

In this paper, we take a step beyond measuring systemic risk and interconnectedness. We try to predict periods of stress, by identifying the fundamental drivers of the extreme loss tail dependence. To determine potential drivers of extremal dependence, we consider an array of institutional characteristics traditionally used in the bank performance literature. We find that strength of extremal dependence increases with asset size and similarity of financial fundamentals. On the other hand, strength of extremal dependence decreases with capitalization, liquidity, funding stability and asset quality.
The remainder of the paper is organized as follows. Section 2 introduces our measures of systemic risk. In section 3, we explore the potential of our measures as early warning indicators. In Section 4, we analyze how balance sheet variables can be used to explain and predict the measures of systemic contribution. Section 5 concludes.

2. EXTREMAL DEPENDENCE AND SYSTEMIC RISK MEASURES

2.1 AN INTRODUCTION TO THE MEASUREMENT OF ASYMPTOTIC DEPENDENCE.

Let \( \chi_q \) be the probability of loss variable \( L_1 \) being above the \( q \)th quantile of its marginal distribution, \( L_{1,q} \), conditional on the loss variable \( L_2 \) being above its \( q \)th quantile, \( L_{2,q} \).

\[
\chi_q = \Pr(L_1 > L_{1,q} \mid L_2 > L_{2,q})
\]

The tail dependence measure \( \chi \) is defined as

\[
\chi = \lim_{q \to 1} \chi_q \quad (1)
\]

If \( \chi > 0 \), the two loss variables, \( L_1 \) and \( L_2 \), are said to be asymptotically dependent, while if \( \chi = 0 \), they are said to be asymptotically independent. Asymptotic dependence between two variables means that no matter how far into the joint tail we go, there is still dependence between the variables.

As an example of an asymptotically dependent pair of institutions, in Figure 1, we plot \( \chi_q \) as a function of the quantile level \( q \), for JP Morgan Chase and Goldman Sachs. The unconditional probability of a loss greater than the \( q \)th quantile is also plotted as the line \( p = 1 - q \).
There is a positive association between the extremes of these two institutions as the conditional probability of an institution experiencing a loss larger than $L_{i,q}$ is always greater than the unconditional probability. Also, as the quantile level increases, this positive association of extremes never dies away. Instead, as $q$ approaches 1, the conditional probability is converging to a positive value around .6.

As an example of an asymptotically independent pair of institutions, we plot in Figure 2 $\chi_q$ for Fifth Third Bancorp (FITB) and Hudson City Bancorp (HCBK). Again, there is positive association between these two institutions as $\chi_q$ is always higher than the unconditional probability. However, for this pair, as $q$ gets closer to 1, the conditional probability approaches the unconditional probability and, therefore, 0.
Figure 2: An example of an asymptotically independent institution pair.

Figures 1 and 2 are presented to introduce the concept of asymptotic dependence in a graphical way. However, determining the extremal dependence structure for two variables requires rigorous statistical analysis. In this paper, we follow the methodology to estimate asymptotic dependence developed by Poon, Rockinger and Tawn (2004). First, two loss variables, $L_1$ and $L_2$, are converted to unit Frechet margins by the probability integral transform

$$X_1 = -\frac{1}{\log F_{L_1}(l_1)} \quad \text{and} \quad X_2 = -\frac{1}{\log F_{L_2}(l_2)}$$

Where $F_{L_1}$ and $F_{L_2}$ are the distribution functions for loss variables $L_1$ and $L_2$.\(^3\) These conversions have no impact on the quantile-based extremal dependence measures considered in this article.

\(^3\) As the distribution function for losses, the empirical cdf is used.
because these are monotonically increasing transformations, and thus do not impact the order of the data.

When the marginal variables $X_1$ and $X_2$ are unit Frechet distributed, it is straightforward to confirm that the definition of $\chi$ reduces to

$$\chi = \lim_{r \to \infty} \Pr(X_1 > r \mid X_2 > r) \quad (2)$$

by applying the change of variable $q = 1 - 1/r$, letting $r \to \infty$ and using the property that, if $X$ is unit Frechet distributed,

$$\lim_{r \to \infty} \Pr(X \leq r) = 1 - 1/r \quad (3)$$

Ledford and Tawn (1996) showed that the joint tail region of two unit Frechet distributed variables satisfies

$$\Pr(X_1 > r, X_2 > r) = l(r)r^{-1/\eta} \quad (4)$$

where $l(r)$ is a slowly varying function\(^4\) and $\eta \in (0,1]$ is called the coefficient of tail dependence. Using (3) and (4), $\chi$ can be written as

$$\chi = \lim_{r \to \infty} l(r) \times (r)^{-1/\eta} \quad (5)$$

Therefore, $\eta = 1$ corresponds to asymptotic dependence and $\eta < 1$ corresponds to asymptotic independence. Hence, the problem reduces to estimating the coefficient of tail dependence $\eta$.

In order to estimate $\eta$, we construct the variable $T = \min(X_1, X_2)$. Being the minimum of two Unit-Frechet distributions, $T$ is also a fat tailed distribution. In the EVT literature, it is

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\(^4\) A slowly varying function is defined by $\lim_{r \to \infty} \frac{l(a r)}{l(r)} = 1 \ \forall \ a > 0$.\]
established that fat tailed distributions follow a power law beyond a high threshold, which can be written as

$$\bar{F}(t) = \Pr(T > t) \approx l(t) t^{-\xi} \text{ for } t > u$$

(6)

where \(l(r)\) is a slowly varying function, \(u\) is a high threshold and \(\xi\) is the tail parameter. Taking (4) and (6) together, it can be seen that the coefficient of tail dependence for \(X_1\) and \(X_2, \eta\), is the tail parameter for the univariate variable \(T\), which can be estimated by the Hill’s estimator.

In order to test for asymptotic dependence, we test the hypothesis \(\eta = 1\). Based on MLE properties of the Hill’s estimator, it can be shown that the variance of \(\eta\) can be estimated by

$$\hat{\sigma}_\eta^2 = \frac{\hat{\eta}^2}{N_u}$$

where \(N_u\) is the number of observations above the threshold \(u\) (i.e. the number of observations used in the Hill’s estimation). In this study, the threshold is chosen as the 95\(^{th}\) quantile of the univariate variable \(T\). If the null hypothesis of asymptotic dependence (\(\eta = 1\)) cannot be rejected, then \(\chi\) is calculated as

$$\chi = \frac{N_u}{N} u$$

where \(u\) is the threshold, \(N_u\) is the number of observations above the threshold in Hill’s estimation and \(N\) is the total number of observations in the sample.  

2.2 SYSTEMIC RISK AND INTERCONNECTEDNESS INDICATORS

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We follow Acharya et al (2010) and Brownlees and Engle (2011) in creating a panel containing all U.S. depository firms with a market capitalization greater than $5 billion as of the end of June 2007. We obtain daily returns and market capitalization from CRSP. The panel of firms is unbalanced, as not all companies have continuously been trading during the sample period of January 2nd 1990 to March 30th 2012. The list of firms included is shown in Appendix A.

In this section we describe our two systemic risk indicators, as well as two firm-level interconnectedness measures that rely on the extremal dependence measures.

Let \( \text{AsympDep}_{ij,t} = \begin{cases} 1 & \text{if } \chi_{ij,t} > 0 \\ 0 & \text{if } \chi_{ij,t} = 0 \end{cases} \).

The first systemic risk indicator, AsympDepRate, is defined as the ratio of asymptotically dependent institution pairs to the total number of institution pairs in our dataset.

\[
\text{AsympDepRate}_t = \frac{\sum_i \sum_{j \neq i} \text{AsympDep}_{ij,t}}{N \times (N-1)}
\]

In Figure 3, we present a time series plot of AsympDepRate. To obtain the asymptotic dependence for time t, data from the six year period ending at time t is used. When we repeat the calculation for time t+1, we simply roll the sample one quarter forward. For example, asymptotic dependence is estimated for each pair of institutions in the sample using data from January 1990 to December 1995, and the AsympDepRate calculated from this period is plotted at December 1995. Then, the procedure is repeated using data from April 1990 to March 1996 and the AsympDepRate is plotted at March 1996, and so on. We have chosen a six year sample window for each iteration because extreme value theory applications generally require large sample sizes.
in order to accurately measure tail dependence.\textsuperscript{6} The methods only use loss events that lie in the tails of the data and, therefore, enough extreme loss events are necessary to obtain precise estimates of asymptotic dependence.

![Figure 3: AsympDepRate over time using six year rolling window samples.](image)

The AsympDepRate systemic risk indicator captures financial crises well. At the end of 1995 the indicator was below 10% but started climbing, reaching nearly 25% after the Asian financial crisis. Afterwards, there is a more significant increase around the collapse of the hedge fund Long Term Capital Management (LTCM) on September 1998 and a slow decline, up until

\textsuperscript{6} With a sample of 6 years, we have approximately 1,500 daily returns, which in turn result in approximately 75 extreme tail observations to be used in Hill’s estimation for testing the asymptotic dependence hypothesis.
2004. Finally, the indicator gradually increased during the 2005-2008 period, peaking at an unprecedented 95% during the heights of the financial crisis.

The second systemic risk measure, $AvgChi$, is defined as the average of $\chi$ over all possible institution pairs in our dataset.

$$AvgChi_i = \frac{\sum_{j \neq i} \chi_{ij}}{N \times (N-1)}$$

In Figure 4, we plot the $AvgChi$ measure, calculated using a six year rolling window.

![AvgChi over time using six year rolling window samples.](image)

Figure 4: $AvgChi$ over time using six year rolling window samples.
In terms of direction, $AvgChi$ is very similar to $AsympDepRate$. $AvgChi$ captures all notable financial downturns as well as $AsympDepRate$. The main difference between the two is their scale. $AsympDepRate$ is calculated from a binary variable that indicates existence of asymptotic dependence. $AvgChi$ measures the strength of that dependence, if it exists. Analysis of the $AvgChi$ measure illustrates that the latest financial crisis was substantially more severe than the previous crises in the end of the 1990’s.

In order to measure the systemic interconnectedness of individual institutions, we develop similar indicators to the systemic risk measures. A limitation of using extremal dependence to measure the systemic interconnectedness of an institution is that they do not allow us to ascertain the direction of asymptotic dependence. Despite this limitation, we believe that it is useful to determine which institutions are more interconnected under conditions of market stress as these institutions are more likely to be vulnerable to market swings.

The first measure of interconnectedness considered reflects the percentage of institutions that are asymptotically dependent with a given institution.

$$AsympDepRate_{i,j} = \frac{\sum_{j \neq i} AsympDep_{ij}}{(N - 1)}$$

The second interconnectedness measure we propose results from averaging $\chi$ for all institution pairs an institution is involved in.

$$AvgChi_{i,j} = \frac{\sum_{j \neq i} \chi_{ij}}{(N - 1)}$$

These two measures are studied in detail in the next section.
3. PREDICTIVE POWER AND POTENTIAL AS EARLY WARNING INDICATORS

In this section we explore the potential of our proposed interconnectedness measures as early warning signals. If our measures have explanatory power over the performance of financial institutions during financial crises, they can be used by regulators as early warning signals on the health of financial institutions. In testing the out-of-sample predictive power of our measures, we present a case study, where we follow the empirical approach taken by Acharya et al. (2010) of calculating financial institutions’ “systemicness” for a pre-crisis period (in our case, July 2001 to June 2007), and then evaluating the measures’ out-of-sample performance by examining how they predicted banks’ stock returns during the crisis (assumed to span from July 2007 to December 2008, as in most of the literature).\(^7\)

When a financial institution is asymptotically dependent with many institutions, and the average strength of this asymptotic dependence (the average \(\chi\) of an institution) is large, we expect the institution to be very vulnerable to financial crises due to its strong interconnection with the financial system. Below, we test the ability of our systemic interconnectedness measures to predict the performance of US banks during the last financial crisis.

In all regressions of Table 1, the dependent variable is the cumulative return of banks during the crisis period. In regression (1), the explanatory variable is the percentage of other banks that are asymptotically dependent with the bank for which returns are being predicted. The coefficient of this variable is negative as expected, and it is statistically significant at the 5% significance level. The economic impact implied by this coefficient is quite large. Being

\(^7\) This approach was also followed by Billio et al. (2011) and seems to have become a standard way of testing the performance of systemic risk measures.
asymptotically dependent with one more bank implies a 2.07%\(^8\) decline in stock price during the crisis period. In regression (2), the explanatory variable is the average \(\chi\) of a bank taken across all other banks. The coefficient associated with this measure is also negative and statistically significant at the 5% significance level. The adjusted \(R^2\) of the first two regressions are, respectively, 12.7% and 15.3%. In a similar analysis, Acharya et al. (2010) obtained an adjusted \(R^2\) of 6.7%, in a regression where the same 18 month crisis returns are explained by their proposed Marginal Expected Shortfall (MES) measure and industry dummies.\(^9\)

**Table 1 Performance during the Financial Crisis**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.098</td>
<td>-0.100</td>
<td>-0.155</td>
<td>-0.040</td>
<td>-0.064</td>
</tr>
<tr>
<td>AsympDepRate</td>
<td>-0.539** (0.247)</td>
<td>-0.511* (0.275)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AvgChi</td>
<td>-1.169** (0.491)</td>
<td>-1.133** (0.550)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SystBeta</td>
<td>-0.312 (0.298)</td>
<td>-0.080 (0.311)</td>
<td>-0.049 (0.308)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.160</td>
<td>0.185</td>
<td>0.042</td>
<td>0.162</td>
<td>0.186</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.127</td>
<td>0.153</td>
<td>0.004</td>
<td>0.093</td>
<td>0.118</td>
</tr>
</tbody>
</table>

Estimated standard errors in parenthesis. ** = p-value < 0.05; * = p-value < 0.1.

Similarly to Acharya et al. (2010), we compare the forecasting performance of our measures with a more conventional risk measure, the CAPM Beta.\(^10\) For this purpose, we regress the crisis period returns on the Beta calculated from the six year pre-crisis period (regression

\(^8\) We have N=27 banks in this regression. Therefore being asymptotically dependent with one other bank increases the AsympDepRate by 1/26 which results in -0.539/26= -0.0207 change in crisis return.

\(^9\) The Acharya et al. (2010) study includes financial firms from depositories, insurance firms, broker dealers and others. We only include bank holding companies in this study.

\(^10\) The market portfolio we consider in computing Beta is the CRSP value weighted market returns.
(3)). Regressions (4) and (5) in Table 1 include both Beta and one of our measures. The estimated coefficients for our measures decrease in the regressions including Beta. Nevertheless, _AsympDepRate_ is still significant at 10% significance level and _Avgchi_ is still significant at 5% confidence level. Moreover, the results seem to indicate that our measures provide additional information on the performance of bank stocks beyond what Beta alone would have provided on the eve of the financial crisis.

**4. PREDICTING ASYMPTOTIC DEPENDENCE**

In this section we analyze how balance sheet variables can be used to explain and predict the strength of asymptotic dependence. We assume that the expected value of the strength of asymptotic dependence between two banks, $\chi_{ij,t+1}$, follows the following linear structure:

$$E[\chi_{ij,t+1} | \chi_{ij,t+1} > 0] = c_1 + c_2 \cdot 1(\chi_{ij,t} = 0) + \theta_{ij} + \tau_{t+1} + \rho \chi_{ij,t} + \beta X_{ij,t}$$

$\theta_{ij} = $ Bank pair fixed effect  
$\tau_{t} = $ Quarter fixed effect  
$X_{ij,t} = $ Balance sheet indicators for institutions $i$ and $j$ in quarter $t$

We estimated this linear structure through a fixed effects regression. The aim of this regression is to understand the drivers of the strength of asymptotic dependence, when asymptotic dependence exists, and thus we exclude observations for which $\chi_{ij,t+1}$ equals zero. We have adopted linear estimation, despite $\chi_{ij,t+1}$ being bounded between zero and one, because it allows us to include bank pair fixed effects – which prove crucial to properly identify the drivers of extremal dependence – while avoiding the incidental variables problem. This comes at the cost of having a slightly misspecified model.
The balance sheet indicators we consider in our forecasting regressions are meant to differentiate financial institutions according to criteria typically considered in the literature. We distinguish institutions according to size, capital, liquidity, funding stability, earnings and asset quality using data from COMPUSTAT.\textsuperscript{11} We measure size by the log of the bank’s total assets; capital by equity capital divided by total assets; liquidity by cash divided by total assets; funding stability by the ratio between long-term deposits and liabilities; earnings by net income divided by total assets; and, finally, asset quality by the ratio between non-performing assets and total assets. A priori we expect extremal dependence to increase with size and to decrease with capitalization, liquidity, funding stability (where we take larger reliance on deposits to mean higher funding stability), profitability and asset quality.

Assessing the effect of balance sheet indicators on the measures of asymptotic dependence is challenging in our empirical setup because for each observation we have two sets of balance sheet indicators. We have opted to include two measures for each indicator, the minimum value for the indicator between the two institutions, and an index of similarity between the institutions. The similarity indexes are calculated by the expression\textsuperscript{12}:

$$\text{Similarity}(Y_{i,t}, Y_{j,t}) = 1 - \frac{|Y_{i,t} - Y_{j,t}|}{Y_{i,t} + Y_{j,t}}$$

The two measures complement each other. The minimum measure allows us to assess how both institutions being of at least of a certain size, having at least certain leverage, and so on, affects the strength of asymptotic dependence. On the other hand, the similarity indexes

\textsuperscript{11} Data is used between 1995Q3 and 2011Q4. With a 6-year rolling window approach and stock price data starting from January 1990, the first Chi estimated corresponds to 1995Q4.

\textsuperscript{12} Except in the case of Net Income over Assets, where similarity is equal to $1 - |(\text{Net Income/Assets})_{i,t} - (\text{Net Income/Assets})_{j,t}|$. 

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allow us to analyze how much the differences between the institutions contribute to asymptotic dependence. In general, we expect increases in similarity indices to be associated with an increase in asymptotic dependence, as we expect more similar institutions to experience more tail co-movement.

We include time effects in the regression. Thus, the coefficients we estimate capture how variation of fundamentals across institutions within time periods correlates with asymptotic dependence. While this choice results in our regressions not being pure forecasting regressions, as part of the “time t + 1” variation is absorbed through the time fixed effect, we believe this approach leads to more relevant results. The usefulness of this modeling approach for a prudential supervisor rests in its ability to determine which institutions are more likely to suffer an increase in asymptotic dependence under conditions of stress. This ability is enhanced by not confounding the effect of a given characteristic in asymptotic dependence with the effect of aggregate shocks.

When calculating standard errors for our estimates, we have opted to use time period clustering. We have experimented with different standard error specifications, including robust standard errors and clustering by pair of institutions, and concluded that time clustering produces the most conservative (largest) standard errors. The increase in standard errors associated with using time clustering likely results from different error volatilities persisting for different time periods, even when time effects are accounted for. The next section presents the empirical results of our analysis.
### 4.1 RESULTS

**Table 1: \(X_{ij,t+1}\) Forecasts for Large US Financial Institutions**

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Forecast window</strong></td>
<td>1 quarter</td>
</tr>
<tr>
<td><strong>Min</strong> {(\ln(\text{Assets})<em>{i,t}, \ln(\text{Assets})</em>{j,t})}</td>
<td>.011***</td>
</tr>
<tr>
<td></td>
<td>( .003)</td>
</tr>
<tr>
<td>Similarity((\ln(\text{Assets})_{ij,t}))</td>
<td>.111***</td>
</tr>
<tr>
<td></td>
<td>( .035)</td>
</tr>
<tr>
<td><strong>Min</strong> {(\text{Capital}/\text{Assets}<em>{i,t}, \text{Capital}/\text{Assets}</em>{j,t})}</td>
<td>-.093*</td>
</tr>
<tr>
<td></td>
<td>( .048)</td>
</tr>
<tr>
<td>Similarity((\text{Capital}/\text{Assets}_{ij,t}))</td>
<td>-.001</td>
</tr>
<tr>
<td></td>
<td>( .007)</td>
</tr>
<tr>
<td><strong>Min</strong> {(\text{Cash}/\text{Assets}<em>{i,t}, \text{Cash}/\text{Assets}</em>{j,t})}</td>
<td>-.075***</td>
</tr>
<tr>
<td></td>
<td>( .026)</td>
</tr>
<tr>
<td>Similarity((\text{Cash}/\text{Assets}_{ij,t}))</td>
<td>.011***</td>
</tr>
<tr>
<td></td>
<td>( .003)</td>
</tr>
<tr>
<td><strong>Min</strong> {(\text{Deposits}/\text{Liabilities}<em>{i,t}, \text{Deposits}/\text{Liabilities}</em>{j,t})}</td>
<td>-.033**</td>
</tr>
<tr>
<td></td>
<td>( .015)</td>
</tr>
<tr>
<td>Similarity((\text{Deposits}/\text{Liabilities}_{ij,t}))</td>
<td>.016</td>
</tr>
<tr>
<td></td>
<td>( .014)</td>
</tr>
<tr>
<td><strong>Min</strong> {(\text{Net Income}/\text{Assets}<em>{i,t}, \text{Net Income}/\text{Assets}</em>{j,t})}</td>
<td>.319</td>
</tr>
<tr>
<td></td>
<td>( .507)</td>
</tr>
<tr>
<td>Similarity((\text{Net Income}/\text{Assets}_{ij,t}))</td>
<td>-.137</td>
</tr>
<tr>
<td></td>
<td>( .405)</td>
</tr>
<tr>
<td><strong>Min</strong> {(\text{Non-Performing Assets}/\text{Assets}<em>{i,t}, \text{Non-Performing Assets}/\text{Assets}</em>{j,t})}</td>
<td>1.698***</td>
</tr>
<tr>
<td></td>
<td>( .203)</td>
</tr>
<tr>
<td>Similarity((\text{Non-Performing Assets}/\text{Assets}_{ij,t}))</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>( .002)</td>
</tr>
</tbody>
</table>
$\chi_{ij,t}$ \hspace{1cm} .481***

$(.032)$

$AD_{ij,t}$ \hspace{1cm} -.186***

$(.015)$

Constant \hspace{1cm} .230

$(.408)$

Note: $N = 9,457$. The regression includes quarter dummies. Estimated standard errors, clustered by quarter, in parenthesis. *** = p-value < 0.01; ** = p-value < 0.05; * = p-value < 0.1.

These are the main findings of our analysis:

a. Size, as measured by total assets, is a key driver of asymptotic dependence. Pairs of institutions, where both institutions are large, experience stronger asymptotic dependence, than pairs where at least one institution is small. This is an intuitive finding, and in line with larger financial institutions being more systemically important, as most systemic risk literature shows. Also, we find that similarity between the size of both institutions leads to an increase in extremal dependence.

b. Capitalization is also a driver of extremal dependence. As we could expected, pairs of institutions where both institutions have high capital ratios are less likely to show strong asymptotic dependence. On the other hand, similarity of capital levels does not seem to have a statistically significant impact on the level of asymptotic dependence between two institutions.

c. Liquidity also has the expected impact on the strength of extremal dependence. Banks pairs were both institutions have a large proportion of very liquid assets, like cash, are less likely to suffer from strong asymptotic dependence. Also, similarity in the ratio of cash to assets leads to larger extremal dependence.
d. The stability of funding sources, as proxied by the share of deposits on total liabilities, has the expected impact on asymptotic dependence. When both institutions enjoy a large share of deposit funding, they are less likely to suffer strong asymptotic dependence. On the other hand, similarity of deposit ratios does not influence the strength of asymptotic dependence.

e. Profitability, as measured by the ratio between net income and total assets does not significantly impact the strength of asymptotic dependence.

f. Low asset quality, as expected, leads to increased extremal dependence. When both banks have high levels of non-performing assets, they experience stronger asymptotic dependence. Similarity in regard to asset performance does not significantly impact the strength of asymptotic dependence.

g. Overall, similarity between institutions has a positive impact on the strength of this asymptotic dependence.

5. CONCLUSION

By applying extreme value theory-based estimations to US bank holding company stocks, we contribute new measures to the literature on systemic risk and financial firm interconnectedness. We find that large US financial institutions’ equity prices exhibit strong dependence even in their limiting joint extremes. Because of this finding, we propose two systemic risk indicators driven from extremal dependence measures. The first indicator measures the proportion of asymptotically dependent financial institution pairs to the total number of financial institution pairs in our sample. Our second indicator is the average of $\chi$ across all financial institutions. Our results show that these indicators perform as expected during both periods of financial turmoil and stability.
We also develop two firm-level measures of banks’ interconnectedness, based on a bank’s stock price co-movement with other banks, under conditions of joint stress. Our first measure of systemic interconnectedness of an institution is the proportion of institutions that are asymptotically dependent with that institution. Our second measure is the average $\chi$ of an institution across all the bank-pair combinations involving that institution.

Lastly, we explore what institutional information can be used to explain and predict asymptotic dependence. We consider criteria identified in the systemic risk literature and designated by the Dodd Frank Act (section 113) in the United States and the Basel Committee internationally. Size, capitalization, liquidity, funding sources and asset quality of financial institutions are good predictors of asymptotic dependence. Similarity between institutions has a positive impact on asymptotic dependence. We believe the proposed measures have the potential to inform the prudential supervision of systemically important firms, an area currently of great relevance in supervisory policy.

REFERENCES


APPENDIX A- List of firms included in our dataset

Bank of America
Bank of New York Mellon
BB&T
Citigroup
Comerica
Commerce Bancorp
Hudson City Bancorp
Huntington Bancshares
JPMorgan Chase
Key Bank
M&T Bank
Marshall & Ilsley
National City Corp
New York Community Bancorp
Northern Trust
Peoples United Financial
PNC
Regions Financial
Sovereign Bancorp
State Street
Suntrust Banks
Synovus Financial
Unionbancal Corp
US Bancorp
Wachovia
Washington Mutual
Wells Fargo
Western Union
Zion