Price Stability
Under Long-Run
Monetary Targeting

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Price-level stability is widely recognized as the principal goal of monetary policy (see, for example, Black 1990, Carlstrom and Gavin 1991, and Hoskins 1991). A program for price stability actually has two distinct objectives: achieving each objective has its own distinct benefits. The first objective is to reduce the expected rate of price inflation to zero. The second objective is to eliminate uncertainty about long-run changes in the price level. When these two objectives are met, monetary policy ceases to have disruptive effects in the real economy and in financial markets.

A simple model of the demand for money, such as the inventory model presented by Barro (1984), indicates that a rate of inflation that is expected to be positive provides consumers and firms with an incentive to engage in costly cash management activities in order to economize on their money holdings. In addition, because the federal income tax system is not completely indexed for changes in the price level, a positive rate of inflation causes some tax rates to increase automatically over time (Altig and Carlstrom 1991). Under zero inflation, costly cash management activities are unnecessary and unlegislated tax increases do not occur. These are the benefits of achieving the first objective of price stability.

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Economic theory also indicates that uncertainty about long-run changes in the price level disrupts the functioning of capital markets. When there is long-run price-level uncertainty, lenders must worry that the real value of their savings will be depreciated by unexpected inflation. Similarly, borrowers must worry that the real value of their debt burdens will be increased by unexpected deflation. Fischer (1984) describes the problems that long-run price-level uncertainty causes for individuals who are saving for their retirement. Klein (1975) argues that long-run price-level uncertainty increases firms’ dependence on short-term borrowing by making long-term debt financing riskier. Irving Fisher (1925, p. 65) points to these disruptive effects of long-run uncertainty as the greatest evil resulting from unstable prices:

The chief indictment, then, of our present dollar is that it is uncertain. As long as it is used as a measuring stick, every contract is necessarily a lottery; and every contracting party is compelled to be a gambler in gold without his own consent.

Credit markets operate more efficiently when uncertainty about long-run changes in the price level is eliminated. This is the benefit of achieving the second objective of price stability.

This article demonstrates how a simple rule for monetary policy proposed by Milton Friedman (1960) provides for price stability. Friedman’s k-percent rule requires that the nominal money supply, defined by the broad aggregate M2, grow at an annual rate of k percent, where k is a constant between three and five. Broaddus and Goodfriend (1984) interpret Friedman’s k-percent rule as calling for a long-run monetary targeting procedure that eliminates the permanent changes away from trend in the money supply known as base drift. The most recent argument in favor of the k-percent rule is made by Hetzel (1989).

The ability of the k-percent rule to achieve the first objective of price stability, zero inflation, is seldom questioned. The ability of the k-percent rule to achieve the second objective of price stability, the elimination of long-run price-level uncertainty, is a more controversial issue. Thus, while this article shows how the k-percent rule can achieve both objectives of price stability, its argument focuses on the problem of long-run uncertainty that is also emphasized by Fisher (1925). Accordingly, the first section draws on time-series methods to obtain a working definition of long-run uncertainty. Section 2 derives a simple model of the price level that explains why economic theory gives no clear answer as to whether the k-percent rule can eliminate long-run price-level uncertainty: the model indicates that the answer hinges critically on the time-series properties of the demand for money. Section 3 goes on to examine the properties of M2 demand and concludes that the k-percent rule will, in fact, greatly reduce the degree of uncertainty concerning the long-run behavior of the price level. Section 4 concludes with a discussion, following Broaddus and Goodfriend (1984), of how the k-percent rule can actually be implemented.
1. LONG-RUN PRICE-LEVEL UNCERTAINTY

The idea of long-run price-level uncertainty can be formalized by reference to the time-series concepts of trend stationarity and difference stationarity. These concepts are introduced to economists by Nelson and Plosser (1982).

Denote the natural logarithm of an economic time series at time $t$ by $w_t$. Suppose this series fluctuates over time about a long-run trend according to

$$ w_t = \alpha + \beta t + c_t, \tag{1} $$

where $c_t$ is a stationary, mean zero deviation from trend. Since $c_t$ is stationary, it can be represented as the linear combination of past and present identically and independently distributed (iid) innovations $\epsilon_t$,

$$ c_t = \theta(L)\epsilon_t, \tag{2} $$

where $\theta(L) = \theta_0 + \theta_1 L + \theta_2 L^2 + \cdots$ is a polynomial in the lag operator $L$ (for details, see Sargent 1987, Ch. 11). Equations (1) and (2) define $w_t$ as a trend stationary process, since it always tends to revert to the linear trend $\alpha + \beta t$.

Consider a second time series, with natural logarithm $x_t$ at time $t$. Suppose that the first differences of $x_t$ are stationary. Then

$$ (1 - L)x_t = \gamma + d_t, \tag{3} $$

where $d_t$ is a stationary, mean zero process with the representation

$$ d_t = \phi(L)\delta_t, \tag{4} $$

$\delta_t$ is an iid innovation, and $\phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + \cdots$. Equations (3) and (4) define $x_t$ as a difference stationary process, since its first differences tend to revert to a constant mean $\gamma$.

Equations (1) and (2) imply that, with $\lambda(L) = (1 - L)\theta(L)$,

$$ (1 - L)w_t = \beta + \lambda(L)\epsilon_t. \tag{5} $$

Similarly, equations (3) and (4) imply that

$$ (1 - L)x_t = \gamma + \phi(L)\delta_t. \tag{6} $$

Equations (5) and (6) reveal that the first difference of both trend stationary and difference stationary processes can be expressed as the sum of a constant and a mean zero deviation.

Beveridge and Nelson (1981) demonstrate that any process that can be written in the form of equations (5) and (6) can be decomposed into a long-run, trend, or permanent component and a short-run, cyclical, or transitory component. For $w_t$ and $x_t$, let $w^t_t$ and $x^t_t$ denote the trend components and let $w^c_t$ and $x^c_t$ denote the cyclical components.

The Beveridge-Nelson decomposition defines the trend components $w^t_t$ and $x^t_t$ to be random walks with drift (see part 1 of Appendix A for details), so that

$$ E_{t-1}(w^t_t) = \beta + w^t_{t-1} \tag{7} $$
and

\[ E_{t-1}(w_t^i) = \gamma + w_{t-1}^i \]  

(8)

where \( E_{t-1}(w_t^i) \) is the expectation, or forecast, of \( w_t^i \) conditional on \( \{\epsilon_{t-1}, \epsilon_{t-2}, \epsilon_{t-3}, \ldots\} \) and \( E_{t-1}(x_t^i) \) is the expectation, or forecast, of \( x_t^i \) conditional on \( \{\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \ldots\} \).

The variances of the trend components \( w_t^i \) and \( x_t^i \) conditional on \( \{\epsilon_{t-1}, \epsilon_{t-2}, \epsilon_{t-3}, \ldots\} \) and \( \{\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \ldots\} \) are given by

\[
V_{t-1}(w_t^i) = \sigma_\epsilon^2 (\lambda_0 + \lambda_1 + \lambda_2 + \cdots)^2 = \sigma_\epsilon^2 [\lambda(1)]^2
\]  

(9)

and

\[
V_{t-1}(x_t^i) = \sigma_\delta^2 (\phi_0 + \phi_1 + \phi_2 + \cdots)^2 = \sigma_\delta^2 [\phi(1)]^2,
\]  

(10)

where \( \sigma_\epsilon^2 \) is the unconditional variance of \( \epsilon_t \) and \( \sigma_\delta^2 \) is the unconditional variance of \( \delta_t \). The cyclical components \( w_t^i \) and \( x_t^i \), meanwhile, are stationary, mean zero processes.

The conditional, or forecast, variance of the Beveridge-Nelson trend component serves as a working definition of long-run uncertainty, since it indicates exactly how much variation is expected in a series’ long-run trend. From the definition \( \lambda(L) = (1 - L)\theta(L) \), \( \lambda(1) = 0 \). Thus, equation (9) implies that the trend stationary process \( w_t \) has zero forecast variance in its long-run component. Since \( w_t \) always tends to revert to the linear trend \( \alpha + \beta t \), there is never any uncertainty about its long-run behavior. In general, \( \phi(1) \neq 0 \), so equation (10) indicates that the difference stationary process \( x_t \) has a long-run component with positive forecast variance. Since \( x_t \) shows no tendency to revert to a linear trend, there is always some uncertainty about its long-run behavior.

Bordo, Choudhri, and Schwartz (1990) suggest that the Beveridge-Nelson decomposition can be used to measure the degree of long-run price-level uncertainty that is present under any given monetary policy. A monetary policy that makes the price level trend stationary will, in light of equation (9), completely eliminate long-run uncertainty. A monetary policy that makes the price level difference stationary, in contrast, fails to completely eliminate long-run uncertainty. In this latter case, the forecast variance \( \sigma_\delta^2 [\phi(1)]^2 \) of the price level’s trend component can be used to measure the degree of long-run uncertainty that remains.

2. A SIMPLE MODEL OF THE PRICE LEVEL

A simple model of the price level begins with the national income version of the equation of exchange,

\[
M_t V_t = P_t Y_t,
\]  

(11)

where \( M_t \) is nominal money, \( V_t \) is the income velocity of money, \( P_t \) is the price level, and \( Y_t \) is real income. Taking logs in (11) and rearranging yields

\[
p_t = m_t + v_t - y_t,
\]  

(12)
where lowercase letters denote the natural logarithms of the variables represented by the corresponding uppercase letters.

By definition, the income velocity of money is equal to nominal income divided by nominal money. Hence,

\[ v_t = \ln(V_t) = \ln(P_t Y_t/M_t) = y_t - \ln(M_t/P_t). \]  

Substituting equation (13) into equation (12) yields

\[ p_t = m_t - \ln(M_t/P_t). \]  

Monetary theory indicates that in the long run, nominal money is determined by supply, while real money is determined by demand (Friedman 1969, p. 8). Monetary theory, therefore, says that equation (14) can be written as

\[ p_t = m_s^t - (m/p)_d^t. \]  

where \( m_s^t \) is the log of the nominal money supply and \( (m/p)_d^t \) is the log of real money demand. Equation (15) shows how the price level is determined by the interaction of money supply and money demand.

Monetary theory also indicates that the demand for real money \( (m/p)_d^t \) depends on real income \( y_t \) and the opportunity cost \( R^*_t \) of holding real balances (McCallum and Goodfriend 1988):

\[ (m/p)_d^t = f(y_t, R^*_t, \xi_t). \]  

In equation (16), the opportunity cost \( R^*_t \) is defined as the difference between the rate of interest paid on nonmonetary assets and the rate of interest paid on monetary assets. Even when explicit interest payments on monetary assets are regulated or prohibited, as they were in the United States from 1933 through the early 1980s, interest payments may still be made implicitly in the form of services to depositors, and \( R^*_t \) must be measured so as to account for these implicit payments (Klein 1974; Dotsey 1983). The third term in the money demand function (16), \( \xi_t \), captures variation in money demand that cannot be attributed to either changes in \( y_t \) or changes in \( R^*_t \).

The neutrality of money implies that variations in the nominal money supply have no long-run effect on real income \( y_t \) or shifts in real money demand \( \xi_t \). In addition, as noted by Moore, Porter, and Small (1990), competition in the banking system forces deposit rates to adjust one-for-one with changes in market rates of interest in the long run, so that if money is defined (as it is in Friedman 1960) by the broad aggregate M2, then changes in the nominal money supply will have no long-run effect on the opportunity cost \( R^*_t \). In this case, changes in the growth rate of the money supply that translate into changes in the rate of inflation have no long-run effect on \( R^*_t \); that is, money is not only neutral, but superneutral as well. Equation (16), along with these extra assumptions about the neutrality of money and competition in the banking system, therefore implies that long-run changes in the nominal money supply have no effect on long-run changes in real money demand. In equation (15), it
is possible to consider the long-run behavior of $m_s$ separately from the long-run behavior of $(m/p)_d^t$.

Milton Friedman’s (1960) k-percent rule calls for steady long-run growth in the broad monetary aggregate. In terms of the time-series concepts introduced in section 1, the k-percent rule can be interpreted as a proposal to make the supply of M2 a trend stationary process, with an average annual growth rate of k percent. Suppose first that the demand for real M2 is a trend stationary process. Then equation (15) indicates that the k-percent rule will achieve the first objective of price stability, zero inflation, when k is set equal to the trend rate of growth in the demand for M2. Moreover, because the sum of two trend stationary processes is itself trend stationary, the k-percent rule will also achieve the second objective of price stability by completely eliminating long-run price-level uncertainty.

Now suppose that the demand for real M2 is difference stationary. In this case, the k-percent rule can still provide for zero expected inflation when k is set equal to the drift in real M2 demand. Since the sum of a trend stationary process and a difference stationary process is difference stationary, however, equation (15) indicates that the price level will be difference stationary. That is, considerable long-run uncertainty about the behavior of the price level may remain. Only the first objective of price stability will be achieved; the second objective will not be met.

As emphasized by Walsh (1986) and as shown by equation (15), the question of whether the k-percent rule will provide for price stability cannot be answered on theoretical grounds; instead, it is an empirical question. The k-percent rule can always achieve zero expected inflation, but it will eliminate long-run price-level uncertainty only if the demand for real M2 is trend stationary. Thus, the next section examines the time-series behavior of real M2 in order to determine whether real M2 is trend stationary or difference stationary and thereby to determine whether or not the k-percent rule can achieve both objectives of price stability.

3. THE TIME-SERIES PROPERTIES OF REAL M2

A number of empirical studies, including those by Moore, Porter, and Small (1990), Hafer and Jansen (1991), and Mehra (1991), document the existence of a stable econometric relationship between the demand for real M2 and its determinants, income and interest rates, based on a linearized version of equation (16):

$$(m/p)_d^t = a_1 y_t + a_2 R^*_t + \xi_t. \quad (17)$$

These studies also find evidence that income, and perhaps interest rates as well, are best described as difference stationary processes. Together, the stability of the demand function and the difference stationarity of the determinants imply
that real M2 is itself difference stationary. Corroborating evidence is supplied by Nelson and Plosser (1982), who find that M2 velocity and national income are difference stationary. Equation (13), which can be rearranged to read

$$\ln(M_t/P_t) = y_t - v_t,$$

implies that if income and velocity are both difference stationary, then real M2 will be, in general, difference stationary as well.

These earlier studies draw their conclusions from the results of Dickey-Fuller (1979) tests of the null hypotheses that the relevant economic time series are difference stationary. One version of the Dickey-Fuller test uses the t-statistic on the OLS estimate of $\rho_2$ in the regression equation

$$z_t = \rho_0 + \rho_1 t + \rho_2 z_{t-1} + e_t. \quad (19)$$

If the null hypothesis that $\rho_2 = 1$ can be rejected in favor of the alternative that $\rho_2 < 1$, then there is evidence that the series $z_t$ is trend stationary. If the null that $\rho_2 = 1$ cannot be rejected, then there is evidence that $z_t$ is difference stationary.

Under the null that $\rho_2 = 1$, the t-statistic has a nonstandard distribution; its critical values are given by Fuller (1976, p. 373). Also, if the regression errors $e_t$ are autocorrelated, then the t-statistic must be adjusted as shown by Phillips and Perron (1988).

Table 1 presents the results of Dickey-Fuller tests for real M2 over two sample periods, a long period that runs from 1915 through 1990 and a postwar period that runs from 1947 through 1990. The data are annual; their sources are given in Appendix B. The Phillips-Perron adjustments to the t-statistics use Newey and West’s (1987) method to estimate the variance of the regression error along with Andrews’ (1991) method (described in part 3 of Appendix A) to determine the number of nonzero autocorrelations that need to be allowed for.

**Table 1 Dickey-Fuller Tests for Real M2**

<table>
<thead>
<tr>
<th>Period</th>
<th>Equation</th>
<th>$\ln(M_2/P_t)$</th>
<th>$\ln(M_{2t-1}/P_{t-1})$</th>
<th>DW</th>
<th>DF</th>
<th>lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>1915–1990</td>
<td>$\ln(M_2/P_t) = 0.178 + 0.00510 t + 0.839 \ln(M_{2t-1}/P_{t-1})$</td>
<td>(0.0608)</td>
<td>(0.00209)</td>
<td>1.27</td>
<td>-2.97</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$\ln(M_{2t-1}/P_{t-1})$</td>
<td>(0.0650)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1947–1990</td>
<td>$\ln(M_2/P_t) = 0.439 + 0.00663 t + 0.788 \ln(M_{2t-1}/P_{t-1})$</td>
<td>(0.175)</td>
<td>(0.00277)</td>
<td>1.11</td>
<td>-2.66</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$\ln(M_{2t-1}/P_{t-1})$</td>
<td>(0.0895)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors of the OLS estimates are given in parentheses. DW is the Durbin-Watson statistic. DF is the Dickey-Fuller statistic, corrected for autocorrelated errors as suggested by Phillips and Perron (1988). The variance of the regression error is estimated as suggested by Newey and West (1987). The number of nonzero autocorrelations, lags, is chosen as suggested by Andrews (1991).
For both sample periods, the Dickey-Fuller test fails to reject the null hypothesis that real M2 is difference stationary at the 10 percent significance level. These results, like those from earlier studies, can be interpreted as evidence that the k-percent rule will fail to eliminate long-run price-level uncertainty.

The results of the Dickey-Fuller tests have an alternative interpretation, however, that is more favorable for the k-percent rule. A statistical test can fail to reject its null hypothesis for two reasons. One reason is that the null is, in fact, true. The other is that although the null is false, the data do not contain enough information to allow the test to statistically reject it. The low power of Dickey-Fuller tests to distinguish between difference stationarity as the null and trend stationarity as the alternative is the subject of a number of recent papers, including those by Diebold and Rudebusch (1989), Christiano and Eichenbaum (1990), Stock (1991), DeJong et al. (1992), and Rudebusch (1992). These papers argue that Nelson and Plosser (1982) fail to reject the hypotheses that velocity and income are difference stationary not because the hypotheses are true, but because there is simply not enough information in the data to reject them.

Kwiatkowski, Phillips, Schmidt, and Shin (1992) suggest a way to determine why a Dickey-Fuller test fails to reject its null that a series is difference stationary. They recommend a direct test of the null hypothesis of trend stationarity that complements the Dickey-Fuller test. If their direct test rejects its null of trend stationarity, then there is good reason to believe that the Dickey-Fuller test fails to reject because the series is truly difference stationary. But if the direct test does not reject trend stationarity, then there is reason to conclude that the Dickey-Fuller test fails to reject because there is not enough information in the data. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, which is based on the properties of the residuals from the regression equation

\[ z_t = \rho_0 + \rho_1 t + e_t, \]

is described in detail in part 2 of Appendix A.

Table 2 presents the results of the KPSS test for real M2. For neither sample period can the null of trend stationarity be rejected at the 10 percent significance level. These results suggest that the Dickey-Fuller tests do not reject the null of difference stationarity because there is not enough information in the M2 data for them to do so, not because M2 is necessarily difference stationary.

When applied to real M2, the Dickey-Fuller test is a test of the null hypothesis that the forecast variance of the Beveridge-Nelson trend component of real M2 is greater than zero (recall the discussion at the end of section 1). The KPSS test is a test of the null hypothesis that the forecast variance of the Beveridge-Nelson trend component of real M2 is equal to zero. Since
Table 2 Kwiatkowski-Phillips-Schmidt-Shin Tests for Real M2

1915–1990:

\[
\ln\left(\frac{M_2}{P}\right) = 0.917 + 0.0318 t
\]

\[
\text{DW} = 0.326 \quad \text{KPSS} = 0.113 \quad \text{lags} = 14
\]

1947–1990:

\[
\ln\left(\frac{M_2}{P}\right) = 1.96 + 0.0304 t
\]

\[
\text{DW} = 0.357 \quad \text{KPSS} = 0.0977 \quad \text{lags} = 9
\]

Notes: Standard errors of the OLS estimates are given in parentheses. DW is the Durbin-Watson statistic. KPSS is the Kwiatkowski-Phillips-Schmidt-Shin (1992) statistic. The variance of the regression error is estimated as suggested by Newey and West (1987). The number of nonzero autocorrelations, lags, is chosen as suggested by Andrews (1991).

these two tests indicate only that there is not enough information in the data to distinguish between their two hypotheses, a different approach is needed to assess the ability of the k-percent rule to achieve both objectives of price stability. Such an approach is developed by Bordo, Choudhri, and Schwartz (1990). Bordo, Choudhri, and Schwartz simply estimate the size of the forecast variance of the Beveridge-Nelson trend component of real M2. They use this point estimate, rather than a hypothesis test, to measure the amount by which the k-percent rule will reduce long-run price-level uncertainty.

Bordo, Choudhri, and Schwartz perform the following experiment. The degree of long-run price-level uncertainty that has actually prevailed in the United States during the two sample periods can be measured by the forecast variance of the Beveridge-Nelson trend component of the GNP deflator. Recall that equation (16), along with the assumptions about monetary neutrality and competition in the banking system discussed in section 2, implies that the Beveridge-Nelson component of real M2 is invariant to changes in monetary policy. Equation (15) expresses the log of the price level as the difference between the log of nominal money supply and the log of real money demand. Because the k-percent rule makes the money supply trend stationary, equation (15) implies that the degree of long-run uncertainty that would have prevailed if the k-percent rule had guided monetary policy during the sample periods can be measured by the forecast variance of the trend component of real M2. Thus, Bordo, Choudhri, and Schwartz estimate the amount by which the k-percent rule can reduce long-run uncertainty by comparing the forecast variance of the trend component of the actual GNP deflator to the forecast variance of the trend component of real M2.

Cochrane (1988) shows that the forecast variance of the Beveridge-Nelson trend component of a time series \( z_t \) can be estimated by \( \hat{V}_n(z) = 1/n \times \)
variance of the series’ $n$-differences:

$$V_n(z) = (1/n) \text{var}(z_t - z_{t-n}).$$  \hspace{1cm} (21)

In equation (21), $n$ is a large constant. Cochrane and Sbordone (1988) recommend using a value of 20 or 30 for $n$, but caution that $n$ must not exceed half of the sample size $T$. Cochrane (1988) shows that $V_n(z)$ has an asymptotic variance that can be estimated by $(4n/3T)V_n(z)$.

Table 3 compares $V_n$ for real M2 and the GNP deflator over both sample periods; $n = 30$ is used for the longer sample and $n = 20$ is used for the postwar data. The estimates indicate that the forecast variance of the trend component of real M2 is quite small, so that the long-run behavior of real M2 closely resembles that of a trend stationary process. In fact, the forecast variance of the trend component of real M2 is of an order of magnitude smaller than the forecast variance of the trend component of the GNP deflator.

Implementing the Bordo-Choudhri-Schwartz (BCS) experiment using the results from Table 3 indicates that long-run price-level uncertainty would have been reduced by 82 percent had the k-percent rule been used throughout the period from 1915 to 1990. The k-percent rule would have reduced uncertainty by more than 92 percent during the postwar years.

Taken together, the results presented in Tables 1–3 reveal that earlier studies overstate the case against the k-percent rule by suggesting that there is strong evidence that real M2 is difference stationary. In fact, the results indicate that Dickey-Fuller tests fail to reject their null hypothesis that real M2 is difference stationary because the data do not contain enough information to discriminate among various hypotheses, not because their null is necessarily true.

Table 3  Forecast Variances of Beveridge-Nelson Trend Components, Real M2 and GNP Deflator

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>$V_n(M2/P) = 0.000644$</td>
<td>$V_n(M2/P) = 0.000273$</td>
</tr>
<tr>
<td></td>
<td>$(0.000467)$</td>
<td>$(0.000213)$</td>
</tr>
<tr>
<td>$V_n(M2/P)/V_n(P)$</td>
<td>$= 0.180$</td>
<td>$= 0.0767$</td>
</tr>
<tr>
<td>$n = 30$</td>
<td></td>
<td>$n = 20$</td>
</tr>
<tr>
<td>$V_n(P) = 0.00358$</td>
<td>$(0.00260)$</td>
<td>$V_n(P) = 0.00356$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.00277)$</td>
</tr>
</tbody>
</table>

Notes: $V_n(M2/P)$ is the forecast variance of the Beveridge-Nelson trend component of the log of real M2, estimated as $1/n$ times the variance of $n$-differences as suggested by Cochrane (1988). $V_n(P)$ is the forecast variance of the trend component of the log of the GNP deflator. Asymptotic standard errors are given in parentheses.
Dickey-Fuller and KPSS hypothesis tests indicate that it is not possible to draw firm conclusions about the trend stationarity or difference stationarity of real M2. Furthermore, the point estimates used in the BCS experiments suggest that adopting the k-percent rule will reduce long-run price-level uncertainty by at least 80 percent. These results provide good reason to believe that Friedman’s k-percent rule can achieve both objectives of price stability by providing for zero expected inflation and by greatly reducing long-run price-level uncertainty.

4. IMPLEMENTING THE k-PERCENT RULE

Broaddus and Goodfriend (1984) recommend that the Federal Reserve implement the k-percent rule by adopting a long-run monetary targeting procedure. The differences between this alternative targeting procedure and the annual M2 targeting procedure that is presently used by the Federal Open Market Committee are illustrated in Figures 1 and 2.

Figure 1B shows that the long-run procedure starts from a single base (here, the base is chosen to be the actual level of M2 in the fourth quarter of 1981) and specifies a constant growth rate target from that base over many years, just as called for by the k-percent rule. Figure 1A shows that the annual procedure, in contrast, starts from a new base in each year and specifies a growth rate target from the new base over a single year only. According to

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Figure 1A  FOMC M2 Targets, 1981:4–1984:4

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the annual procedure, the new base for each year is given by the actual level of M2, rather than the previous year’s target level. As a result, the deviations of actual M2 from target at the end of each year are turned into permanent changes away from trend—known as base drift—in the nominal money supply.

One important feature of the long-run targeting procedure is that it eliminates base drift. While Figure 1A shows that M2 fell within the FOMC’s target cone during 1984, so that the Federal Reserve was not required to make up for rapid M2 growth in the previous year, Figure 1B shows that under the Broaddus-Goodfriend targeting procedure, the Federal Reserve would have been required in 1984 to correct for its past mistake by bringing M2 back into its long-run target band.

Broaddus and Goodfriend’s suggestion to eliminate base drift is criticized by Walsh (1986). Walsh uses a somewhat more elaborate model of the price level than the one used here in section 2 to demonstrate that if the demand for real money is difference stationary, then the k-percent rule will fail to completely eliminate uncertainty about the long-run behavior of the price level. Of course, this is exactly the same message that is contained in equation (15). Walsh goes on to point out that Nelson and Plosser’s (1982) results imply that real M2 is, in fact, difference stationary. Thus, according to Walsh, the k-percent rule will not achieve an important objective of price stability.

The empirical results obtained here in section 3, however, indicate that Nelson and Plosser’s conclusions about the time-series properties of real M2,
which are based in Dickey-Fuller tests, only reflect the fact that the data do not
contain enough information to discriminate between alternative hypotheses. The KPSS tests show that there is as much evidence that real M2 is trend
stationary as there is that it is difference stationary. In other words, the results
show that Walsh is too quick to conclude that the elimination of base drift is
inconsistent with the objectives of price stability.

Walsh also notes that if real M2 is difference stationary, then the forecast
variance of its Beveridge-Nelson trend component is positive. In this case, there
are permanent shocks to real M2 that prevent the series from reverting to a long-
run trend. In principle, the permanent changes in the nominal money supply
caused by base drift can act to offset these permanent shocks to real M2 and
thereby prevent them from translating into long-run shocks to the price level.

The calculations associated with the Bordo-Choudhri-Schwartz experiment
performed in section 3 show that the long-run component of real M2 has a
small, but positive, forecast variance. This means that the k-percent rule can
largely, but not completely, eliminate long-run price-level uncertainty. Walsh
is correct, therefore, in noting that base drift could in theory act to offset the
small permanent shocks to real M2, thereby improving on the k-percent rule
by completely eliminating long-run uncertainty.

In practice, however, base drift has exacerbated, rather than offset, the
effects of long-run variation in real M2 on the price level. If base drift offset
the long-run effects of shocks to real M2, then Table 3 would show that the
long-run forecast variance of the GNP deflator is smaller than the long-run
forecast variance of real M2. Instead, Table 3 reveals that the long-run forecast
variance of prices is of an order of magnitude larger than the long-run forecast
variance of real M2. Thus, the long-run effects of base drift on the price
level have been exactly the opposite of those predicted by Walsh. In fact, the
Bordo-Choudhri-Schwartz experiment indicates that the k-percent rule, which
eliminates base drift, will reduce the amount of long-run price-level uncertainty
by at least 80 percent.

In addition to eliminating base drift, there is a second way in which the
Broaddus-Goodfriend targeting procedure improves on the current annual tar-
inging procedure. As can be seen in Figures 1 and 2, the long-run targeting
procedure sets bounds around the monetary growth rate target in the form of a
band of constant width. The annual procedure, in contrast, sets bounds around
the target in the form of a cone. In this way, the long-run targeting procedure
gives the Federal Reserve more room than does the annual procedure to pur-
sue its short-term policy objectives throughout the entire year. Figure 2, for
example, shows that while actual M2 dipped below the FOMC’s target cone in
the first and second quarters of 1989, potentially constraining short-run policy,
actual M2 continued to stay well within the long-run target band throughout
this period. By using the k-percent rule to guide monetary policy, therefore,
the Federal Reserve would not only achieve its long-run goal of price stability,
but would have considerably more room to pursue its short-run goals as well.
Figure 2A  FOMC M2 Targets, 1988:4–1991:4

Figure 2B  Alternative M2 Targets, 1988:4–1991:4
APPENDIX A: TECHNICAL ANALYSIS

1. The Beveridge-Nelson Decomposition

Let the trend stationary process \( w_t \) and the difference stationary process \( x_t \) be as defined by equations (1)–(4) in the text, so that

\[
(1 - L)w_t = \beta + \lambda(L)\epsilon_t \tag{22}
\]

and

\[
(1 - L)x_t = \gamma + \phi(L)\delta_t. \tag{23}
\]

When applied to \( w_t \) and \( x_t \), the Beveridge-Nelson decomposition yields

\[
w_t = w^f_t + w^c_t, \tag{24}
\]

where

\[
w^f_t = \beta + w^f_{t-1} + \sum_{j=0}^{\infty} \lambda_j \epsilon_t \tag{25}
\]

and

\[
w^c_t = -\sum_{k=0}^{\infty} \sum_{j=k+1}^{\infty} \lambda_j \epsilon_{t-k} \tag{26}
\]

for \( w_t \) and

\[
x_t = x^f_t + x^c_t, \tag{27}
\]

where

\[
x^f_t = \gamma + x^f_{t-1} + \sum_{j=0}^{\infty} \phi_j \delta_t \tag{28}
\]

and

\[
x^c_t = -\sum_{k=0}^{\infty} \sum_{j=k+1}^{\infty} \phi_j \delta_{t-k} \tag{29}
\]

for \( x_t \).

Equations (25) and (28) reveal that the Beveridge-Nelson decomposition defines the trend components \( w^f_t \) and \( x^f_t \) to be random walks with drift, so that equations (7) and (8) hold. Equations (9) and (10) also follow from (25) and (28). Equations (26) and (29), meanwhile, show that the Beveridge-Nelson cyclical components \( w^c_t \) and \( x^c_t \) are stationary, mean zero processes.
In addition to the Beveridge-Nelson decomposition, there may be other ways to write either \( w_t \) or \( x_t \) as the combination of a random walk and a stationary component. However, the random walk component defined by any such decomposition will have the same forecast variance as the Beveridge-Nelson trend component (see Cochrane 1988 for a proof of this fact). In this sense, the forecast variance of the Beveridge-Nelson trend component is a fairly general measure of long-run uncertainty.

Quah (1992) examines a broader class of decompositions, including those that define the trend component as an arbitrary ARIMA process instead of a pure random walk with drift. He shows that those decompositions (like the Beveridge-Nelson decomposition) that define the trend component as a random walk maximize the importance of the trend component in the series as a whole. Quah’s result implies that if a given monetary policy completely eliminates long-run price-level uncertainty as defined by the Beveridge-Nelson decomposition, then it completely eliminates long-run price-level uncertainty as defined by any decomposition of the price level into long-run and short-run ARIMA components.

2. The Kwiatkowski-Phillips-Schmidt-Shin Test

Kwiatkowski et al. (1992) test the null hypothesis that the series \( z_t \) is trend stationary by estimating the regression equation

\[
    z_t = \rho_0 + \rho_1 t + e_t. \tag{30}
\]

They apply Newey and West’s (1987) method to estimate the variance of the residuals from this regression as

\[
    s^2(\tau) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{\tau} b(s, \tau) \sum_{t=j+1}^{T} e_t e_{t-s}, \tag{31}
\]

where the sample size is \( T \), the weighting function \( b(s, \tau) \) is given by

\[
    b(s, \tau) = 1 - \frac{s}{1 + \tau}, \tag{32}
\]

and the lag truncation parameter \( \tau \) is chosen according to the method developed by Andrews (1991), which is described in part 3 of this appendix.

With \( S_t \) defined as

\[
    S_t = \sum_{i=1}^{t} e_t, \tag{33}
\]
the KPSS test statistic is

$$KPSS = \frac{1}{s(\tau)T^2} \sum_{i=1}^{T} S_i^2,$$  \hspace{1cm} (34)

Kwiatkowski et al. show that this test statistic has a nonstandard distribution; its critical values are given in their Table 1 (p. 166).

### 3. Choosing the Lag Truncation Parameter

The Phillips-Perron (1988) correction to the Dickey-Fuller (1979) test statistic requires an estimate of the variance of the residuals from the regression equation

$$z_t = \rho_0 + \rho_1 t + \rho_2 z_{t-1} + e_t.$$  \hspace{1cm} (35)

Similarly, the KPSS test requires an estimate of the variance of the residuals from the regression equation

$$z_t = \rho_0 + \rho_1 t + e_t.$$  \hspace{1cm} (36)

In both cases, it is appropriate to use Newey and West’s (1987) method to estimate the variance of \(e_t\) as

$$s^2(\tau) = T^{-1} \sum_{i=1}^{T} e_i^2 + 2T^{-1} \sum_{s=1}^{\tau} b(s, \tau) \sum_{i=s+1}^{T} e_i e_{i-s},$$  \hspace{1cm} (37)

where \(T\) is the sample size and the weighting function \(b(s, \tau)\) is given by

$$b(s, \tau) = 1 - \frac{s}{1 + \tau}.$$  \hspace{1cm} (38)

In equation (37), the lag truncation parameter \(\tau\) determines how many nonzero autocorrelations in \(e_t\) are allowed for. Use the residuals from (35) or (36) to estimate the regression equation

$$e_t = \pi e_{t-1} + u_t.$$  \hspace{1cm} (39)

Andrews (1991) shows that the best choice for \(\tau\) is given by

$$\tau = 1.1447(\alpha T)^{\frac{1}{3}} - 1,$$  \hspace{1cm} (40)

where

$$\alpha = \frac{4\pi^2}{(1 - \pi)^2(1 + \pi)^2}.$$  \hspace{1cm} (41)

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**APPENDIX B: DATA SOURCES**
REFERENCES


Christiano, Lawrence J., and Martin Eichenbaum. “Unit Roots in Real GNP: Do We Know and Do We Care?” Carnegie-Rochester Conference Series on Public Policy, vol. 32 (Spring 1990), pp. 7–61.


