Long-Term Interest Rates and Inflation: A Fisherian Approach

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In recent years, Federal Reserve (Fed) policymakers have come to rely on long-term bond yields to measure the public's long-term inflationary expectations. The long-term bond rate plays a central role in Goodfriend's (1993) narrative account of Fed behavior, 1979–1992, which links policy-related movements in the federal funds rate to changes in the yield on long-term U.S. Treasury bonds. According to Goodfriend, Fed officials interpreted rapid increases in long-term bond rates as the product of rising inflationary expectations, reflecting a deterioration in the credibility of their fight against inflation. To restore that credibility, they responded by tightening monetary policy, that is, by raising the federal funds rate. Mehra (1995) presents statistical results that support Goodfriend's view. Using an econometric model, he demonstrates that changes in long-term bond rates help explain movements in the federal funds rate during the 1980s.

While these studies provide convincing evidence of a link between Fed policy and long-term bond rates, both start with the untested hypothesis that movements in such rates primarily reflect changes in long-term inflationary expectations. And while economic theory does identify expected inflation as one determinant of nominal bond yields, it suggests that there are other determinants as well. Using theory as a guide, this article seeks to measure the contribution each determinant makes in accounting for movements in long-term bond yields. By doing so, it attempts to judge the extent to which Fed policymakers are justified in using these bond yields as indicators of inflationary expectations.

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Irving Fisher (1907) presents what is perhaps the most famous theory of nominal interest rate determination.[1] According to Fisher's theory of interest, movements in nominal bond yields originate in two sources: changes in real interest rates and changes in expected inflation. Thus, Fisher's theory provides a guide for investigating the extent to which long-term bond yields serve as reliable indicators of long-term inflationary expectations. Specifically, it implies that movements in long-term bond yields provide useful signals of changes in inflationary expectations if and only if their other determinant, the long-term real interest rate, is stable.

Although Fisher acknowledges the potential importance of risk in outlining his theory, he stops short of explicitly considering the effects of uncertainty in his graphical and mathematical treatment of interest rate determination. Recognizing that risk can play a key role in determining interest rates, and exploiting advances in mathematical economics made since Fisher's time, Lucas (1978) develops a model that extends the relationships obtained by Fisher to a setting where future economic magnitudes are uncertain.

In addition to real interest rates and expected inflation, Lucas's model identifies a third determinant of nominal bond yields: a risk premium that compensates investors for holding dollar-denominated bonds in a world of uncertainty. Thus, Lucas's model provides a more exhaustive set of conditions under which movements in long-term bond rates provide useful signals of changes in long-term inflationary expectations: it indicates that the long-term real interest rate must be stable and that the risk premium must be small.

This article draws on Fisherian theory to assess the practical usefulness of long-term bond yields as indicators of long-term inflationary expectations. It begins, in Section 1, by outlining Fisher's original theory of interest. It then shows, in Section 2, how Lucas's model generalizes the relationships derived by Fisher to account for the effects of uncertainty. Section 3 uses Lucas's model to decompose the nominal bond yield into its three components: the real interest rate, the risk premium, and the expected inflation rate. Section 4 applies this procedure to estimate the relative importance of the expected inflation component in explaining movements in long-term U.S. Treasury bond rates. Finally, Section 5 concludes the article.

1. FISHER'S THEORY OF INTEREST

To derive a relationship between the yield on a nominal bond and its determinants, Fisher (1907) considers the behavior of an investor in a simple model economy. The economy has two periods, labelled $t = 0$ and $t = 1$, and a single

[1] Although Fisher is usually identified as the inventor of this theory, Humphrey (1983) argues that its origins extend back to the eighteenth century writings of William Douglass.

consumption good. The consumption good sells for P_0 dollars in period $t = 0$ and is expected to sell for P_1^e dollars in period $t = 1$.

Fisher's investor chooses between two types of assets. The first asset, a nominal bond, costs the investor one dollar in period $t = 0$ and pays him a gross return of R dollars in period $t = 1$. The yield R on this nominal bond measures the economy's nominal interest rate. The second asset, a real bond, costs the investor one unit of the consumption good in period $t = 0$ and returns r units of the good in period $t = 1$. The gross yield r on this bond represents the economy's real interest rate.

In order to purchase a nominal bond in period $t = 0$, the investor must first acquire one dollar; he can do so by selling $1/P_0$ units of the consumption good. When it matures in period $t = 1$, the nominal bond returns R dollars, which the investor expects will buy R/P_1^e units of the good. Measured in terms of goods, therefore, the expected return on the nominal bond equals the investor's receipts, R/P_1^e, divided by his costs $1/P_0$. Letting $\pi^e = P_1^e/P_0$ denote the economy's expected gross rate of inflation, one can write this goods-denominated return as R/π^e.

In equilibrium, the goods-denominated returns on nominal and real bonds must be the same. For suppose the return R/π^e on the nominal bond were to exceed the return r on the real bond. Then every investor could profit by selling the real bond and using the proceeds to purchase the nominal bond. The resulting decrease in the demand for real bonds would raise the return r, while the increase in the demand for nominal bonds would depress the return R/π^e, until the two were brought back into equality. Similarly, any excess in the return r over R/π^e would be eliminated as investors attempted to sell nominal bonds and purchase real bonds. Thus, Fisher concludes that $R/\pi^e = r$ or, equivalently,

$$R = r\pi^e. \tag{1}$$

Fisher's equation (1) expresses the nominal interest rate R as the product of two terms: the real interest rate r and the expected inflation rate π^e. It therefore describes the circumstances under which the nominal bond yield serves as a reliable indicator of inflationary expectations. In particular, it implies that one can be sure that a movement in the nominal interest rate reflects an underlying change in inflationary expectations if and only if the real interest rate is stable.

The nominal bond in Fisher's model resembles a U.S. Treasury bond since, upon maturity, it returns a fixed number of dollars. Thus, the yield on Treasury bonds measures the economy's nominal interest rate R. Unfortunately, assets resembling Fisher's real bond do not currently trade in U.S. financial markets. As a result, it is not possible to directly observe the real interest rate r and then use equation (1) to determine the extent to which movements in Treasury bonds reflect movements in real interest rates rather than inflationary expectations.[2] However, Fisher's theory also links an economy's real interest rate to its growth rate of consumption. Hence, the theory suggests that the real interest rate may be observed indirectly using data on aggregate consumption.

To derive a relationship between the real rate of interest and the growth rate of consumption, Fisher returns to his model economy and uses a graph like that shown in Figure 1. The graph's horizontal axis measures consumption in period $t = 0$, and its vertical axis measures consumption in period $t = 1$.

Fisher's investor receives an income stream consisting of y_0 units of the consumption good in period $t = 0$ and y_1 units of the consumption good in period $t = 1$. He continues to trade in real bonds, which allow him to borrow or lend goods at the real interest rate r. In particular, if y_1 is large relative to y_0, the investor borrows by selling a real bond; this transaction gives him one more unit of the good in period $t = 0$ but requires him to repay r units of the good in period $t = 1$. Conversely, if y_1 is small relative to y_0, the investor lends by purchasing a real bond; this gives him one less unit of the good in period $t = 0$ but pays him a return of r units of the good in period $t = 1$. Thus, the real interest rate r serves as an intertemporal price; it measures the rate at which financial markets allow the investor to exchange goods in period $t = 1$ for goods in period $t = 0$. In Figure 1, the investor's budget line A, which passes through the income point (y_0, y_1), has slope r.

Fisher's investor has preferences over consumption in the two periods that may be described by the utility function

$$U(c_0, c_1) = \ln(c_0) + \beta \ln(c_1), \tag{2}$$

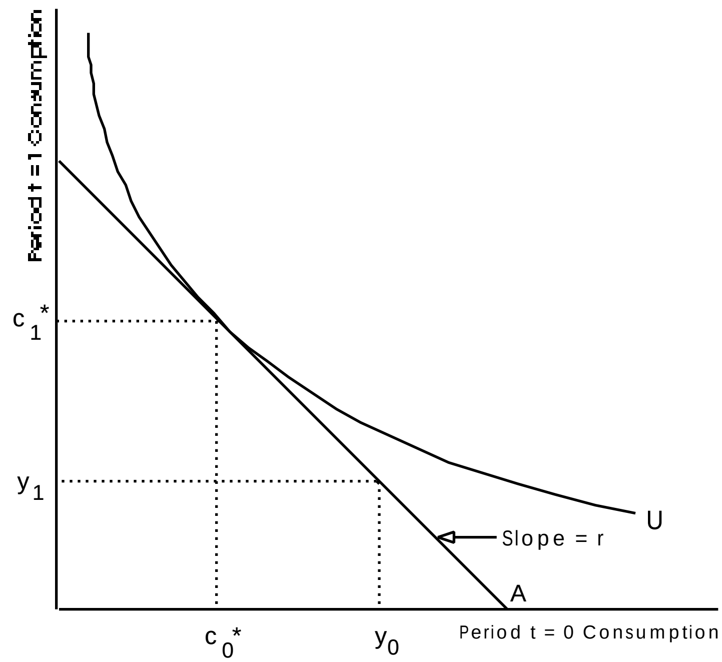
where c_0 denotes his consumption in period $t = 0$, c_1 denotes his consumption in period $t = 1$,ln is the natural logarithm, and the discount factor $\beta < 1$ implies that the investor receives greater utility from a given amount of consumption in period $t = 0$ than from the same amount of consumption in period $t = 1$.[3] In Figure 1, these preferences are represented by the indifference curve U, which traces out the set of all pairs (c_0, c_1) that yield the investor a constant level of utility as measured by equation (2).

The slope of the investor's indifference curve is determined by his marginal rate of intertemporal substitution, the rate at which he is willing to substitute consumption in period $t = 1$ for consumption in period $t = 0$, leaving his utility unchanged. Mathematically, the investor's marginal rate of intertemporal

[2] For exactly this reason, Hetzel (1992) proposes that the U.S. Treasury issue bonds paying a fixed return in terms of goods. Until Hetzel's proposal is implemented, however, only indirect measures of the real interest rate will exist.

[3] Although Fisher does not use a specific utility function to describe his investor's preferences, equation (2) helps to sharpen the implications of his theory by allowing the relationships shown in Figure 1 to be summarized mathematically by equations (3) and (4) below.

Figure 1 Fisher's Diagram



substitution equals the ratio of his marginal utility in period $t = 0$ to his marginal utility in period $t = 1$:

$$\frac{\partial U(c_0, c_1)/\partial c_0}{\partial U(c_0, c_1)/\partial c_1} = \frac{c_1}{\beta c_0}. \tag{3}$$

To maximize his utility, the investor chooses the consumption pair (c_0^*, c_1^*), where the budget line A is tangent to the indifference curve U. At (c_0^*, c_1^*), the slope of the budget line equals the slope of the indifference curve. The former is given by r; the latter is given by equation (3). Hence,

$$r = x/\beta, \tag{4}$$

where $x = c_1^*/c_0^*$ denotes the optimal growth rate of consumption.

Equation (4) shows how Fisher's theory implies that even when the real interest rate r cannot be directly observed, it can still be estimated by computing the growth rate x of aggregate consumption and dividing by the discount factor β. With this estimate in hand, one can use equation (1) to assess the usefulness of Treasury bond yields as indicators of expected inflation. Specifically, if the estimated real rate turns out to be fairly stable, then equation (1) implies that movements in Treasury bond yields primarily reflect changes in inflationary expectations.

2. LUCAS'S GENERALIZATION OF FISHERIAN THEORY

While Fisher recognized that the presence of risk may affect interest rates in important ways, he lacked the tools to incorporate uncertainty formally into his analysis and therefore assumed that his investor receives a perfectly known income stream and faces perfectly known prices and interest rates. More than seventy years later, advances in mathematical economics allowed Lucas (1978) successfully to generalize Fisher's theory to account for the effects of risk.

Lucas's model features an infinite number of periods, labelled $t = 0,1, 2, \ldots,$ and a single consumption good that sells for P_t dollars in period t. Lucas's investor receives an income stream consisting of y_t units of the consumption good in each period t and consumes c_t units of the good in each period t.

Lucas's investor, like Fisher's, trades in two types of assets. A nominal bond costs Lucas's investor one dollar in period t and returns R_t dollars in period $t+1$. Hence, R_t denotes the gross nominal interest rate between periods t and $t+1$. A real bond costs him one unit of the consumption good in period t and returns r_t units of the good in period $t+1$. Hence, r_t denotes the gross real interest rate between periods t and $t+1$. During each period t, the investor purchases B_t nominal bonds and b_t real bonds.

Unlike Fisher's investor, however, Lucas's investor may be uncertain about future prices, income, consumption, interest rates, and bond holdings. That is, he may not learn the exact values of P_t, y_t, c_t, R_t, r_t, B_t, and b_t until the beginning of period t; before then, he regards these variables as random.

As sources of funds during each period t, the investor has y_t units of the consumption good that he receives as income and $r_{t-1}b_{t-1}$ units of the consumption good that he receives as payoff from his maturing real bonds. He also has $R_{t-1}B_{t-1}$ dollars that he receives as payoff from his maturing nominal bonds; he can exchange these dollars for $R_{t-1}B_{t-1}/P_t$ units of the consumption good. As uses of funds, the investor has his consumption purchases, equal to c_t units of the good, and his bond purchases. His real bond purchases cost b_t units of the good, while his nominal bond purchases cost B_t/P_t units of the good. During period t, the investor's sources of funds must be sufficient to cover his uses of funds. Hence, he faces the budget constraint

$$y_t + r_{t-1}b_{t-1} + R_{t-1}B_{t-1}/P_t \geq c_t + b_t + B_t/P_t, \tag{5}$$

which is Lucas's analog to the budget line A in Fisher's Figure 1.

Lucas's investor chooses c_t, B_t, and b_t in each period t to maximize the utility function

$$E_t\left[\sum_{j=0}^{\infty}\beta^j \ln(c_{t+j})\right], \tag{6}$$

subject to the budget constraint (5), where E_t denotes the investor's expectation at the beginning of period t. Equation (6) simply generalizes Fisher's utility function (2) to Lucas's setting with an infinite number of periods and uncertainty. The solution to the investor's problem dictates that

$$1/r_t = \beta E_t\left[1/x_{t+1}\right] \tag{7}$$

and

$$1/R_t = \beta E_t\left[(1/x_{t+1})(1/\pi_{t+1})\right], \tag{8}$$

in each period $t = 0,1,2, \ldots$, where $x_{t+1} = c_{t+1}/c_t$ denotes the gross rate of consumption growth and $\pi_{t+1} = P_{t+1}/P_t$ denotes the gross rate of inflation between periods t and $t+1$.

Lucas's equation (7) generalizes Fisher's equation (4); it is analogous to the tangency between the investor's budget line and indifference curve shown in Figure 1. As in equation (3), the investor's marginal rate of intertemporal substitution is x_{t+1}/β. Hence, equation (7) shows that, under uncertainty, the investor chooses his consumption path so that the expected inverse of his marginal rate of intertemporal substitution equals the inverse of the real interest rate. Also, like Fisher's equation (4), Lucas's equation (7) suggests that while the real interest rate cannot be directly observed, it can still be estimated using data on aggregate consumption.

For any two random variables a and b,

$$E\left[ab\right] = Cov\left[a,b\right] + E\left[a\right]E\left[b\right], \tag{9}$$

where $Cov[a,b]$ denotes the covariance between a and b. Using this fact, one can rewrite equation (8) as

$$1/R_t = \beta Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right] + \beta E_t\left[1/x_{t+1}\right]E_t\left[1/\pi_{t+1}\right], \tag{10}$$

where Cov_t denotes the covariance based on the investor's period t information. In light of equation (7), equation (10) simplifies to

$$1/R_t = \beta Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right] + (1/r_t)E_t\left[1/\pi_{t+1}\right]. \tag{11}$$

Lucas's equation (11) generalizes Fisher's equation (1); it shows how, under uncertainty, the nominal interest rate R_t depends on the real interest rate r_t and the expected inflation term $E_t[1/\pi_{t+1}]$.

The covariance term in equation (11) captures the effect of risk on the nominal interest rate. It appears because random movements in inflation make the goods-denominated return on a nominal bond uncertain. To see this, recall from Section 1 that the return on the nominal bond, measured in terms of the consumption good, equals R_t/π_{t+1}. Since the inflation rate π_{t+1} remains unknown until period $t+1$, so too does R_t/π_{t+1}. Hence, random inflation makes the nominal bond a risky asset.

Equation (11) shows that inflation uncertainty may either increase or decrease the nominal interest rate, depending on whether the covariance term is negative or positive. In particular, inflation uncertainty increases the nominal interest rate if the covariance between $1/x_{t+1}$ and $1/\pi_{t+1}$ is negative, that is, if periods of low consumption growth coincide with periods of high inflation. In this case, high inflation erodes the nominal bond's return R_t/π_{t+1} precisely when the investor, suffering from low consumption growth, finds this loss most burdensome. Hence, the higher nominal yield R_t compensates the investor for this extra risk. Conversely, uncertainty decreases the nominal interest rate if the covariance term in equation (11) is positive, so that periods of high consumption growth coincide with periods of high inflation.

Thus, Lucas's model, like Fisher's, identifies real interest rates and expected inflation as two main determinants of nominal bond yields. Lucas's model goes beyond Fisher's, however, by identifying a third determinant: a risk premium, represented by the covariance term in equation (11), that compensates investors for holding dollar-denominated bonds in the presence of inflation uncertainty. According to Lucas's model, therefore, movements in long-term bond yields accurately reflect changes in expected inflation if and only if the real interest rate is stable and the risk premium is small.

3. DERIVING BOUNDS ON EXPECTED INFLATION

Lucas's equation (7), like Fisher's equation (4), suggests that the unobservable real interest rate can be estimated using data on aggregate consumption. Like the real interest rate, however, the risk premium component of nominal bond yields cannot be directly observed. Hence, without further manipulation, Lucas's equation (11) cannot be used to assess the extent to which movements in nominal bond yields reflect changes in expected inflation rather than changes in their other two components.

Fortunately, as shown by Smith (1993), Lucas's model also places bounds on the plausible size of the risk premium. These bounds, together with estimates of the real interest rate constructed from the consumption data, can be used to determine the extent to which movements in nominal bond yields reflect changes in inflationary expectations.[4]

[4] Smith (1993) takes the opposite approach: he uses the bounds on risk premia, along with estimates of expected inflation, to characterize the behavior of real interest rates.

Recall that the effects of risk enter into Lucas's equation (11) through the covariance term. Smith rewrites this term as

$$\beta Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right] = \beta\rho_t Std_t\left[1/x_{t+1}\right] Std_t\left[1/\pi_{t+1}\right], \tag{12}$$

where

$$\rho_t = Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right]/\left\{Std_t\left[1/x_{t+1}\right] Std_t\left[1/\pi_{t+1}\right]\right\} \tag{13}$$

denotes the correlation between $1/x_{t+1}$ and $1/\pi_{t+1}$ based on the investor's period t information and Std_t denotes the standard deviation based on period t information.

Equation (12) conveniently decomposes the covariance term into three components. The first component, the correlation coefficient ρ_t, can be negative or positive but must lie between -1 and 1. Hence, this component captures the fact that the covariance term may be of either sign. The second and third components, the standard deviations of $1/x_{t+1}$ and $1/\pi_{t+1}$, must be positive. Hence, these terms govern the absolute magnitude of the covariance term, regardless of its sign. Thus, equation (12) places bounds on the size of the covariance term:

$$\begin{aligned}\beta Std_t\left[1/x_{t+1}\right] Std_t\left[1/\pi_{t+1}\right] &\geq \beta Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right] \\ &\geq -\beta Std_t\left[1/x_{t+1}\right] Std_t\left[1/\pi_{t+1}\right],\end{aligned} \tag{14}$$

where the upper bound is attained in the extreme case where $\rho_t = 1$, the lower bound is attained at the opposite extreme where $\rho_t = -1$, and the tightness of the bounds depends on the size of the standard deviations.

Evidence presented in the appendix justifies the additional assumption that inflation volatility in the United States is limited in the sense that the coefficient of variation of $1/\pi_{t+1}$ conditional on period t information is less than one:

$$Std_t\left[1/\pi_{t+1}\right]/E_t\left[1/\pi_{t+1}\right] \leq 1. \tag{15}$$

This assumption allows equation (14) to be rewritten

$$\begin{aligned}\beta Std_t\left[1/x_{t+1}\right] E_t\left[1/\pi_{t+1}\right] &\geq \beta Cov_t\left[(1/x_{t+1}),(1/\pi_{t+1})\right] \\ &\geq -\beta Std_t\left[1/x_{t+1}\right] E_t\left[1/\pi_{t+1}\right],\end{aligned} \tag{16}$$

which, along with equations (7) and (11), implies

$$\begin{aligned}\beta Std_t\left[1/x_{t+1}\right] E_t\left[1/\pi_{t+1}\right] &\geq 1/R_t - \beta E_t\left[1/x_{t+1}\right] E_t\left[1/\pi_{t+1}\right] \\ &\geq -\beta Std_t\left[1/x_{t+1}\right] E_t\left[1/\pi_{t+1}\right],\end{aligned} \tag{17}$$

or, equivalently,

$$\begin{aligned}\beta R_t\left\{E_t\left[1/x_{t+1}\right] + Std_t\left[1/x_{t+1}\right]\right\} &\geq 1/E_t\left[1/\pi_{t+1}\right] \\ &\geq \beta R_t\left\{E_t\left[1/x_{t+1}\right] - Std_t\left[1/x_{t+1}\right]\right\}.\end{aligned} \tag{18}$$

Since

$$1/E_t\left[1/\pi_{t+1}\right] \approx E_t\left[\pi_{t+1}\right], \tag{19}$$

equation (18) places bounds on the expected inflation component that is embedded in the nominal interest rate R_t. Again, these bounds arise because the covariance term in Lucas's equation (11) may be negative or positive and because the absolute magnitude of the covariance term depends on the standard deviation of $1/x_{t+1}$. In particular, the bounds will be tight if this standard deviation—and hence the magnitude of the risk premium—is small. Equation (18) also indicates that these bounds may be estimated using data on aggregate consumption.

Together, therefore, equations (7) and (18) show how one may use data on aggregate consumption to assess the usefulness of nominal bond yields as indicators of inflationary expectations. If the estimates provided by equation (7) show that the real interest rate is stable, and if the bounds provided by equation (18) indicate that the risk premium is small, then Lucas's model implies that most of the variation in the nominal bond yield reflects underlying changes in expected inflation.

4. ESTIMATING THE REAL INTEREST RATE AND BOUNDS ON EXPECTED INFLATION

In order to estimate the real interest rate and the bounds on expected inflation using equations (7) and (18), one must first obtain estimates of the quantities $E_t[1/x_{t+1}]$ and $Std_t[1/x_{t+1}]$. Suppose, in particular, that the evolution of $g_{t+1} = 1/x_{t+1}$, the inverse growth rate of aggregate consumption, is described by the linear time series model

$$g_{t+1} = \gamma + \Gamma(L)g_t + \epsilon_{t+1}, \tag{20}$$

where γ is a constant, $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + ... + \Gamma_k L^k$ is a polynomial in the lag operator L, and ϵ_{t+1} is a random error that satisfies

$$E\left[\epsilon_{t+1}\right] = 0, Std\left[\epsilon_{t+1}\right] = \sigma, E_t\left[\epsilon_{t+1}\epsilon_{t-j}\right] = 0, E\left[\epsilon_{t+1}g_{t-j}\right] = 0 \tag{21}$$

for all $t = 0,1,2,\ \ldots$ and $j = 0,1,2,\ \ldots$. One may then use estimates of γ, $\Gamma(L)$, and σ to compute $E_t[1/x_{t+1}] = E_t[g_{t+1}]$ and $Std_t[1/x_{t+1}] = Std_t[g_{t+1}]$ as

$$E_t\left[g_{t+1}\right] = \gamma + \Gamma(L)g_t \tag{22}$$

and

$$Std_t\left[g_{t+1}\right] = \sigma. \tag{23}$$

Here, as in Mehra (1995), the long-term nominal interest rate is measured by the yield on the ten-year U.S. Treasury bond. This choice for R_t identifies each period in Lucas's model as lasting ten years. In this case, $g_{t+1} = 1/x_{t+1}$

corresponds to the inverse ten-year growth rate of real aggregate consumption of nondurables and services in the United States, converted to per-capita terms by dividing by the size of the noninstitutional civilian population, ages 16 and over. The data are quarterly and run from 1959:1 through 1994:4.

Hansen and Hodrick (1980) note that using quarterly observations of ten-year consumption growth to estimate equation (20) by ordinary least squares yields consistent estimates of γ and the coefficients of $\Gamma(L)$. But since the sampling interval of one quarter is shorter than the model period of ten years, the least squares estimate of σ is biased. Thus, the results reported below are generated using the ordinary least squares estimates of γ and $\Gamma(L)$ and Hansen and Hodrick's consistent estimator of σ, modified as suggested by Newey and West (1987). The limited sample size and the extended length of the model period imply that only one lag of g_{t+1} can be included on the right-hand side of equation (20).

Finally, equation (18) indicates that the discount factor β determines the location of the bounds on expected inflation. Thus, β may be chosen so that the midpoint between the lower and upper bounds, averaged over the sample period, equals the actual inflation rate, averaged over the sample period. This procedure yields the estimate $\beta = 0.856$, which corresponds to an annual discount rate of about 1.5 percent.

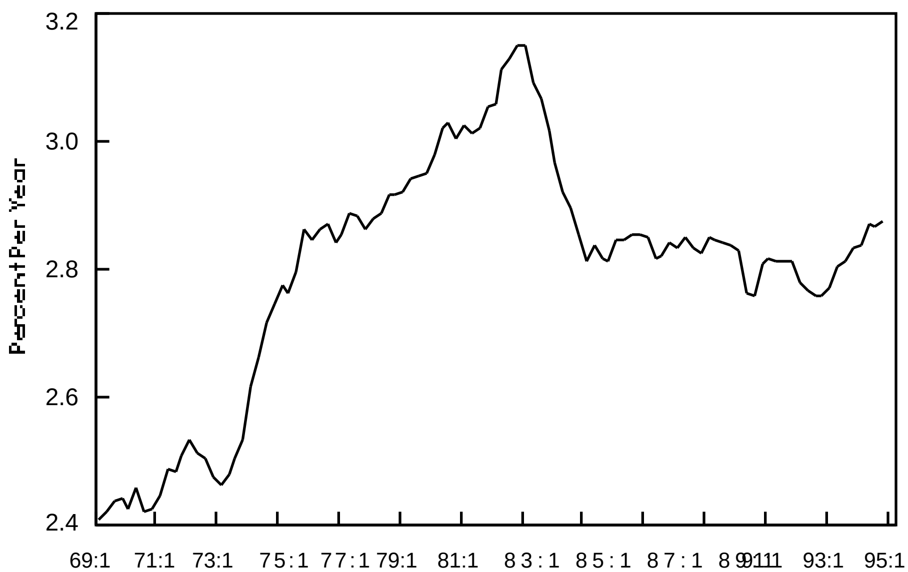
Figure 2 illustrates the behavior of the ten-year real interest rate, estimated using equations (7) and (22).[5] The real interest rate climbs steadily from 1969 until 1983 before falling sharply between 1983 and 1985. But despite these variations, the long-term real interest rate remains within a narrow, 75 basis point range throughout the entire 26-year period for which estimates are available. The average absolute single-quarter movement in the real interest rate is just two basis points; the largest absolute single-quarter move occurs in 1973:4, when the real interest rate increased by only eight basis points. Thus, Figure 2 suggests that the long-term interest rate in the United States is remarkably stable.

Figure 3 plots the bounds on ten-year expected inflation estimated using equations (18), (22), and (23). The bounds are very tight. Even at their widest, in 1981:4, they limit the expected rate of inflation to a 28 basis point band, implying that changes in the risk premium cannot account for movements in the ten-year bond rate larger than 28 basis points. Thus, Figure 3 suggests that the risk premium in the ten-year Treasury bond is very small.

Some intuition for the results shown in Figures 2 and 3 follows from equations (7), (18), (20), (22), and (23). Equation (22), along with equation (7), links variability in the long-term real interest rate to variability in the

[5] Although the sample used to estimate equation (20) extends back to 1959:1, the ten-year model period and the presence of one lag of g_{t+1} on the right-hand side imply that estimates of the real rate can only be constructed for the period beginning in 1969:1.

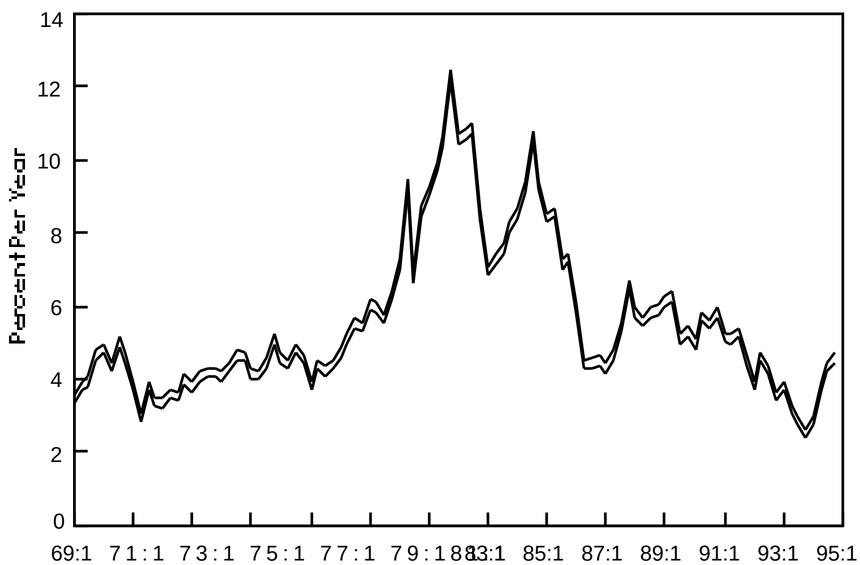
Figure 2 Ten-Year Real Interest Rate



predictable part of consumption growth, measured by $\gamma + \Gamma(L)g_t$ in equation (20). Equation (23), along with equation (18), links the size of the risk premium to variability in the unpredictable part of consumption growth, measured by ϵ_{t+1} in equation (20). In the U.S. data, aggregate consumption growth varies little over ten-year horizons. And since total consumption growth is quite stable, both of its components—the predictable and unpredictable parts—are quite stable as well. Thus, given the stability in aggregate consumption growth, Lucas's model implies that the long-term real interest rate must be quite stable and that the risk premium must be quite small.

According to Lucas's model, the stability of the real interest rate and the small size of the risk premium shown in Figures 2 and 3 imply that most of the variation in the ten-year Treasury bond rate reflects underlying changes in the third component, expected inflation. Indeed, as the largest quarterly real interest rate movement shown in Figure 2 is eight basis points, and as the bounds in Figure 3 are at most 28 basis points wide, the results suggest that any quarterly change in the ten-year bond rate in excess of 36 basis points almost certainly signals a change in inflationary expectations.

Figure 3 Bounds on Ten-Year Expected Inflation



5. CONCLUSION

Although Federal Reserve officials use the yield on long-term Treasury bonds to gauge the public's inflationary expectations, contemporary versions of Fisher's (1907) theory of interest suggest that variations in bond yields can originate in other sources as well. In particular, Lucas's (1978) model indicates that movements on long-term bond yields will accurately signal changes in long-term inflationary expectations if and only if long-term real interest rates are stable and risk premia are small.

Unfortunately, neither real interest rates nor risk premia can be directly observed. However, Lucas's model also shows how these unobservable components of nominal bond yields can be estimated using data on aggregate consumption.

This article lets Lucas's model guide an empirical investigation of the determinants of the ten-year U.S. Treasury bond yield. The results indicate that, indeed, the ten-year real interest rate is quite stable and the ten-year risk premium is quite small. Hence, according to Lucas's model, movements in the long-term bond rate primarily reflect changes in long-term inflationary expectations. Evidently, the Federal Reserve has strong justification for using long-term bond yields as indicators of expected inflation.

APPENDIX

The bounds on expected inflation given by equation (18) were derived in Section 3 under the extra assumption that equation (15) holds. Thus, this appendix provides some justification for (15).

Consider the following linear time series model for the inverse inflation rate $q_{t+1} = 1/\pi_{t+1}$:

$$q_{t+1} = \theta + \Theta(L)q_t + \eta_{t+1}, \tag{24}$$

where the random error η_{t+1} satisfies

$$E\left[\eta_{t+1}\right] = 0, Std\left[\eta_{t+1}\right] = \omega, E\left[\eta_{t+1}\eta_{t-j}\right] = 0, E\left[\eta_{t+1}q_{t-j}\right] = 0 \tag{25}$$

for all $t = 0,1,2, \ldots$ and $j = 0,1,2, \ldots$. One can use this model to estimate $E_t[1/\pi_{t+1}] = E_t[q_{t+1}]$ and $Std_t[1/\pi_{t+1}] = Std_t[q_{t+1}]$, just as equation (20) was used to estimate $E_t[1/x_{t+1}] = E_t[g_{t+1}]$ and $Std_t[1/x_{t+1}] = Std_t[g_{t+1}]$. In the U.S. data, q_{t+1} corresponds to the inverse ten-year growth rate of the price deflator for the aggregate consumption of nondurables and services.

Estimates of (24) using quarterly data from 1959:1 through 1994:4 reveal that $Std_t[q_{t+1}] = \omega = 0.0363$. The smallest estimate of $E_t[q_{t+1}]$ is 0.441, for 1969:1. Thus, for the entire sample period, estimates of $Std_t[1/\pi_{t+1}]/E_t[1/\pi_{t+1}]$ never exceed 0.0823, which suggests that the upper bound of unity imposed by equation (15) is an extremely conservative one.

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