Inventory Investment and the Business Cycle

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When reporting on the current state of the economy, the business press gives considerable attention to changes in inventory investment. The reason for the media attention appears to be related to three issues. First, changes in inventory investment apparently account for a substantial fraction of changes in gross domestic product (GDP). Second, current changes in inventory investment are assumed to convey useful information about the near-term future of the economy. Third, there is a view that the inherent dynamics of inventory investment are destabilizing the economy. In this article I review some of the empirical regularities of inventory investment over the business cycle taking the first issue as a starting point.[1] The empirical regularities I choose to study are to some extent determined by particular theories of inventory investment, but any theory of inventory investment should be consistent with these regularities.

The argument that inventory investment is important for the business cycle is often based on the close relationship between changes in inventory investment and GDP during recessions. For example, Blinder (1981) and Blinder and Maccini (1991) argue that, in a typical U.S. recession, declining inventory investment accounts for most of the decline in GDP. In support of this claim, Table 1 documents the peak-to-trough decline of GDP and inventory investment during postwar U.S. recessions. This same peak-to-trough decline is apparent

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[1] When appropriate, I will make some comments on the second issue, namely, whether inventory investment is useful for forecasting GDP. In the conclusion, I will remark briefly on the third issue, namely, whether inventory investment is destabilizing the economy.

Table 1 GDP and Inventory Investment in Postwar Recessions

GDP Peak to Trough	Change in GDP	Change in Inventory Investment
1948:4 to 1949:4	−24.4	−33.3
1953:2 to 1954:2	−48.8	−20.0
1957:3 to 1958:1	−81.4	−18.4
1960:3 to 1960:4	−40.7	−47.9
1969:3 to 1970:4	−20.3	−38.4
1973:4 to 1975:1	−146.2	−77.0
1980:1 to 1980:3	−116.7	−52.7
1981:3 to 1982:3	−140.9	−43.4
1990:2 to 1991:1	−124.1	−60.7

Notes: "Dates correspond to the largest peak-to-trough decline in GDP associated with each postwar recession. Each date is within one quarter of the quarter containing the peak or trough month as defined by the National Bureau of Economic Research."

Source: Fitzgerald 1997, Table 1, p. 12.

during the 1980–97 period, as shown in Figure 1.[2] Figure 1 also shows that inventory investment is a very noisy time series. During that period, inventory investment not only declines dramatically during recessions, it also declines substantially during expansions. For example, in the expansion years 1986 and 1988, inventory investment declined by almost as much as it did during the 1990 recession. This experience suggests that, while the observation that during recessions declining inventory investment accounts for much of declining GDP is interesting, it might not be very useful when we want to evaluate the role of inventory investment over the complete business cycle.[3]

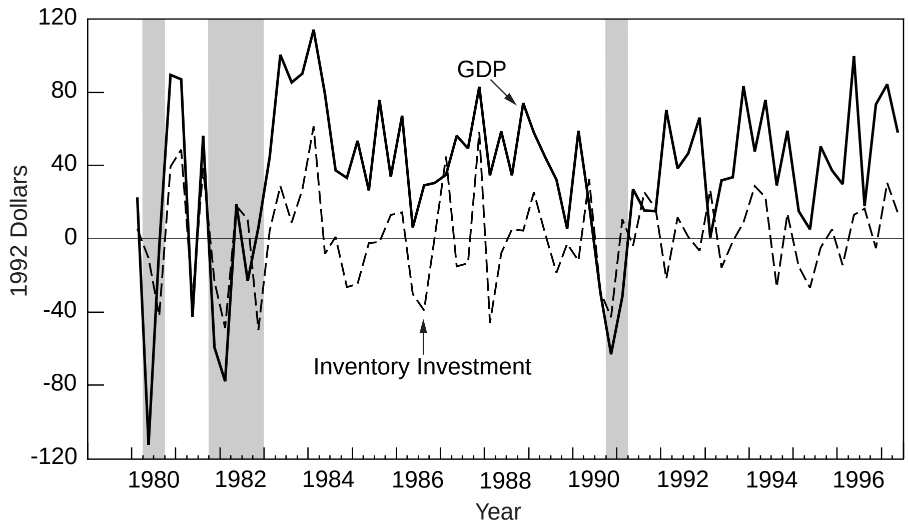
Rather than study the behavior of inventory investment for a particular phase of the business cycle, I choose to document the stylized facts of such investment over the entire business cycle using standard methods.[4] A stylized

[2] Shaded areas in Figure 1 represent National Bureau of Economic Research (NBER) recessions. We display only the last two NBER business cycles so that the graph is not overcrowded. The behavior of inventory investment and GDP from 1980 to the present is not qualitatively different from the earlier part of the postwar period.

[3] Because inventory investment is such a noisy time series, it is also unlikely that it contains useful information to forecast GDP growth. There are two additional pieces of evidence which suggest that inventory investment is not a particularly good predictor of future GDP growth. First, the Conference Board (1997) does not include inventory investment in its widely distributed list of leading economic indicators. Only the inventory/sales ratio is included and then as a lagging indicator. Second, if we forecast GDP growth using lagged GDP growth alone, we do better than if we include lagged GDP growth and changes in lagged inventory investment. That is, we do better in the sense that the first procedure has a lower mean squared forecast error.

[4] For the most part I review earlier work on inventory investment by Blinder (1981) and Blinder and Maccini (1991), using a different method to extract the business cycle components of time series.

Figure 1 Changes in GDP and Inventory Investment



fact is an observed empirical regularity between particular variables, which is of interest because economic theory predicts a certain pattern for it. One cannot look for stylized facts without the guidance of economic theory, but economic theory is also developed from the stylized facts uncovered. Since inventory investment ΔN is the difference between production Y and sales X, that is $\Delta N = Y - X$, the stylized facts discussed involve the behavior of these three interrelated variables.

1. MODELS OF INVENTORY INVESTMENT

The two leading economic theories of inventory investment are the production-smoothing model and the (S, s) inventory model. Both theories start with a single firm that solves a dynamic constrained-profit-maximization problem using inventory investment as one of the firm's decision variables.[5] The theories differ in how the implications for inventory investment, derived for an individual firm, are applied to the study of aggregate inventory investment.[6]

[5] These theories differ from the early behavioral models of inventory investment that are not explicitly based on fully specified dynamic optimization problems (Metzler 1941).

[6] For an extensive survey of theories of inventory investment, see Blinder and Maccini (1991).

A simple production-smoothing model starts with the assumption that a firm's production is subject to increasing marginal cost and that sales are exogenous. If the firm's sales are changing over time but its marginal cost schedule is constant, then the firm minimizes cost by smoothing production, and it reduces (increases) inventories whenever sales exceed (fall short of) production. Thus production is less volatile than sales, and inventory investment and sales tend to be negatively correlated. A firm with increasing marginal cost wants to use inventories to smooth production regardless of whether or not the changes in demand are foreseen. If demand changes randomly and the firm has to decide on current production before it knows what current demand is, the firm also uses inventories as a buffer stock and accordingly reduces (increases) inventory stocks whenever demand is unexpectedly high (low). This buffer-stock motive then reinforces the negative correlation between inventory investment and sales.

The previous argument assumes that the firm faces only demand variations. If, on the other hand, the firm predominantly faces supply shocks in the form of a changing marginal cost schedule, then the implications for inventory investment, production, and sales are very different. In order to minimize costs, the firm now increases (decreases) production and accumulates (reduces) inventories during times when marginal cost is low (high). Thus production is more volatile than sales, and inventory investment and production tend to be positively correlated.

So far the production-smoothing model described above applies to the behavior of an individual firm, rather than the behavior of aggregate variables. To understand the aggregate variables, one often uses the concept of a representative agent and interprets the behavior of aggregate variables in terms of the behavior of a large number of identical individual decision units. The simple production-smoothing model then predicts that production will be more (less) volatile than sales if supply shocks are more (less) important than demand shocks.[7]

A simple (S, s) inventory model assumes that the seller of a good does not himself produce the good. Instead, the seller orders the good from some producer and incurs a fixed cost when he places the order. Suppose that the marginal cost of ordering one more unit of the good is constant and that sales are exogenous. A seller who chooses the order size that minimizes total cost faces the following tradeoff. On the one hand, increasing the order size reduces the average or per-unit order cost because it spreads the fixed cost over more units of the good. On the other hand, an increased order size means that the seller forgoes additional interest income on the funds that have been used to finance the larger order. Given the optimal order size, the seller places an order

[7] Further work has studied the effects of serial correlation in demand shocks, stock-out avoidance, etc. Again, for a survey on this work, see Blinder and Maccini (1991) or Fitzgerald (1997).

whenever the inventory falls below a critical lower level s and the order brings inventories up to the higher level S. After that, sales reduce the inventory until the critical lower level s is reached again. If orders equal production, then production will be more volatile than sales. The relationship between sales and inventory investment is unclear.

Like the production-smoothing model, the (S, s) inventory model applies to an individual decision unit. Unlike the production-smoothing model, however, the notion of a representative agent cannot be used in order to understand the behavior of aggregate variables. The problem is that in the (S, s) inventory model, a firm's behavior is characterized by long periods of inactivity interrupted by short bursts of activity. While one may observe such discontinuous behavior for individual decision units, one does not observe it for aggregate variables. For this theory, aggregation has to be studied explicitly, and aggregate variables will not necessarily behave the same way as do the corresponding variables of individual decision units. In particular, in a model where individual firms follow (S, s) inventory policies, one cannot a priori say whether aggregate production or aggregate sales is more volatile or how aggregate inventory investment is correlated with aggregate sales. Fisher and Hornstein (1997) study the effects of technology and preference shocks on aggregate production, sales, and inventory investment in a general equilibrium model with a trade sector where individual firms use (S, s) inventory policies.[8] They find that for both types of shocks (1) production is more volatile than sales, and (2) inventory investment tends to be positively correlated with sales. They also find that preference and technology shocks differ in their effect on retail-price markups. In particular, retail-price markups are procyclical for technology shocks and countercyclical for preference shocks.

2. CYCLICAL COMPONENTS OF INVENTORY INVESTMENT

Up to this point I have used economic theory to identify potential stylized facts pertaining to inventory investment. To further evaluate the role of inventory investment over the business cycle, I will need an operational definition to identify business cycle movements in the data. Usually business cycles are identified with recurring expansions and contractions in economic activity that occur simultaneously over a wide range of sectors. Burns and Mitchell (1946, p. 3) state that

[8] In a general equilibrium model, shocks cannot be unambiguously classified as demand or supply disturbances. Usually shocks that affect the production technology are interpreted as supply disturbances, and shocks that affect the preferences of agents are interpreted as demand disturbances.

> . . . a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

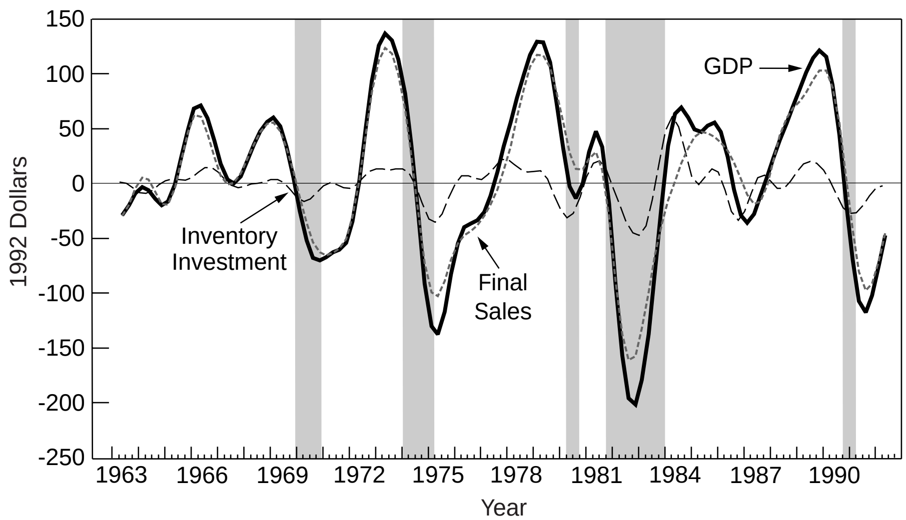
The business cycle is thus different from long-term trend and short-term irregular movements in the economy. Yet many of the economic variables are growing over time (GDP, sales) or are very erratic (inventory investment). Furthermore, since inventories can serve as a buffer stock to compensate for short-term movements in demand or supply, one may want to study the business cycle component and the short-term irregular component separately.

Bandpass filters, which essentially are moving averages, separate the time series of a variable into components with different periodicities (see Baxter and King [1995]).[9] Using this method, I construct the business cycle components of inventory investment, GDP, and final sales as displayed in Figure 2. First, one can see that, for business cycle movements, inventory investment contributes only a small part to GDP volatility. Second, GDP is more volatile than final sales, and final sales and inventory investment tend to increase and decrease together. Figure 3 plots the irregular components of inventory investment. Consistent with Figure 1's depiction of changes in GDP and inventory investment, Figure 3 shows that inventory investment accounts for a substantial fraction of the short-term volatility of GDP.

The two inventory models discussed above capture different features of the inventory holding problem. In any one sector of the economy, one of the features will play a bigger role. For example, when firms in the manufacturing sector choose the size of their finished goods inventories, the production-smoothing model seems to be more appropriate. But when firms in the trade sector make their order decisions, or firms in the manufacturing sector decide on the size of their material inventories, the (S, s) inventory model seems to be more appropriate. Our study of disaggregated data shows two things. On the one hand, it is difficult to attribute aggregate inventory investment volatility to particular sectors because inventory investment moves much the same in each sector. On the other hand, we find that although important features of the inventory holding problem differ systematically across sectors, the properties of inventory investment, production, and sales are remarkably similar across sectors; for business cycle movements, production is more volatile than sales, and inventory investment and sales are positively correlated. The only

[9] In Appendix A I describe the basic idea underlying the decomposition of a time series using bandpass filters.

Figure 2 Business Cycle Components of GDP, Final Sales, and Inventory Investment



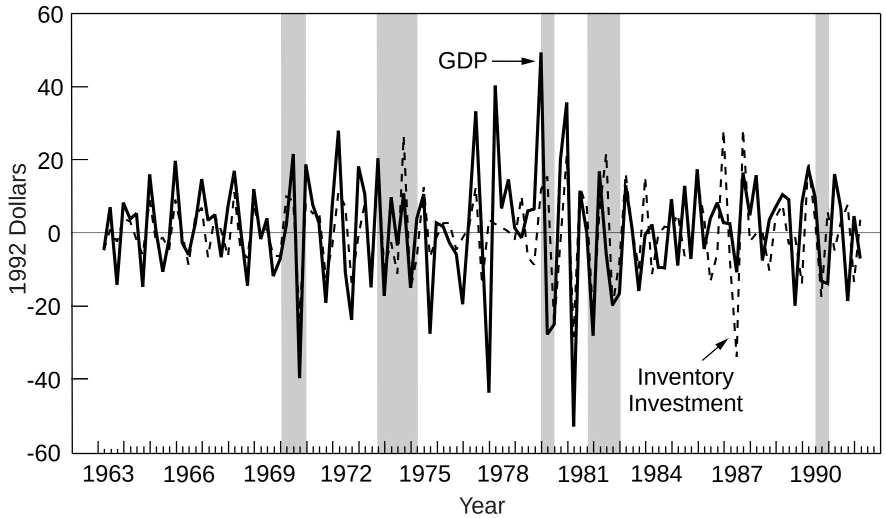
exception concerns the behavior of retail-price markups, which are not consistently procyclical or countercyclical across sectors.

3. STYLIZED FACTS OF INVENTORY INVESTMENT

The organization of the stylized facts is suggested by the predictions of the two basic models of inventory investment, the production-smoothing and the (S, s) inventory models. First, I document the behavior of aggregate variables, GDP, final sales, and inventory investment. Then I decompose aggregate inventory investment into sectors according to whether it is more likely that inventory decisions are influenced by the production-smoothing motive or the fixed order cost motive. Next, I examine the relative volatilities of production and sales and the correlation between inventory investment and sales. Finally, I study the behavior of the retail price index relative to the producer price index, that is, the retail-price markup.[10]

[10] Appendix B describes the time series used and how the business cycle and irregular component of each time series is constructed.

Figure 3 Irregular Components of GDP and Inventory Investment



Inventory Investment at the Aggregate Level

For quarterly changes in aggregate values of GDP, final sales, and inventory investment, Table 2 quantifies some of the observations made earlier in the introduction using Figure 1. The table shows that GDP is more volatile than final sales and that final sales and inventory investment are essentially uncorrelated (the correlation coefficient is 0.01). This observation is consistent with properties of the simple production-smoothing model and the (S, s) model, and for the former it implies that supply shocks must be relatively more important than demand shocks. Note that, consistent with conventional wisdom, changes in inventory investment account for a substantial part of the variance of changes in GDP, about 30 percent.[11]

When distinguishing between the business cycle and irregular components of a time series, one sees that GDP is more volatile than final sales for both components, whereas inventory investment is positively correlated with final sales for the business cycle component but negatively correlated for the irregular component. The correlation coefficients are, respectively, 0.54 and -0.2.

[11] Given that the variance of output is the sum of the variance of sales, the variance of inventory investment, and the covariance of sales and inventory investment, we can attribute output volatility to sales and inventory investment volatility because of the low sales/inventory investment correlation.

Table 2 GDP, Final Sales, and Inventory Investment

	First Difference of Levels		Business Cycle Component		Irregular Component	
NIA Component	**Variance**	**Percent**	**Variance**	**Percent**	**Variance**	**Percent**
GDP	1534.99		5479.39		255.41	
Final sales	1085.94	70.7	3961.99	72.3	190.19	74.5
Inventory investment	439.11	28.6	315.63	5.8	122.88	48.1
Covariance of final sales and inventory investment	9.94	0.6	1197.46	21.9	−57.92	−22.7

Again, the fact that production is more volatile than sales is consistent with the simple production-smoothing model when supply shocks dominate demand shocks, but now the model cannot easily account for the comovement of sales and inventory investment. In particular, the model does not predict the strong positive correlation between inventory investment and sales for the business cycle component. Moreover, the weak negative correlation for the irregular component seems to indicate that, over the short term, demand shocks dominate supply shocks, and inventories are used as a buffer stock. The (S, s) model is consistent with the properties of the business cycle component but does not predict the negative correlation between inventory investment and sales for the irregular component.

For the business cycle component, it is difficult to attribute GDP volatility to either sales or investment volatility since there is a strong positive correlation between these two components of GDP. Furthermore, relative to GDP, inventory investment is much less volatile for the business cycle component than it is for the irregular component. Inventory investment variance represents only 6 percent of GDP variance for the business cycle component but 50 percent of GDP variance for the irregular component. Thus it appears that inventory investment is less important for GDP volatility over the business cycle than it is for short-term fluctuations.[12]

[12] Since the calculation of changes in a variable, that is, its first differences, and the calculation of business cycle and irregular components of the same variable represent different data transformations, it is hardly surprising that they lead to different results. These observations are not inconsistent; they only reflect different properties of the data and the transformation used.

Appendix A shows how the business cycle and irregular components of a variable represent the frequency components of that same variable that fall within a particular frequency band and where each frequency receives the same weight. Calculating changes in a variable, that is, first differences, is another data transformation that includes all frequencies but gives more weight to higher frequencies (Baxter and King 1995). Thus first differences emphasize components with short periodicity relative to components with long periodicity, and therefore the properties of a first-differenced variable are more closely related to the properties of the irregular component than to the properties of the business cycle component of that variable.

Disaggregating Inventory Investment

Most of the results from the study of aggregate variables also apply when aggregate production, sales, and inventory investment are disaggregated into their sectoral components: manufacturing and trade. It is useful to study the sectoral components of inventory investment because the production-smoothing and the (S, s) models seem to be more or less appropriate for different types of inventories. For example, the production-smoothing model appears to be more appropriate for finished goods inventories in the manufacturing sector, whereas the fixed order cost model appears to be more appropriate for material inventories in the manufacturing sector and inventories in the wholesale and retail trade sector. This suggests that one should focus attention on the theory of inventory investment that is most appropriate for the sector that contributes the most to aggregate inventory investment volatility. Unfortunately, it turns out to be difficult to attribute aggregate inventory investment volatility to individual sectors.

Table 3 shows the variance of the components of total inventory investment: manufacturing and trade inventories. Over the business cycle, finished goods inventories in the manufacturing sector account for only about 10 percent of the total variance of inventory investment. On the other hand, inventories in the trade sectors and materials in the manufacturing sector account for about a quarter of total inventory volatility. Note that, although inventory investment in the trade sector accounts on average for more than half of total inventory investment, it accounts for less than 20 percent of the volatility of inventory investment over the business cycle. Any attempt to attribute the variance of total inventory investment to particular components, however, meets with limited success because more than half of total inventory volatility is due to the comovement of inventory investment components. In particular, within the manufacturing sector about 50 percent of total volatility is due to the comovement of finished goods, goods-in-process, and materials inventories. For total inventory investment volatility, about 40 percent of total volatility is due to the comovement of the individual components: manufacturing, retail, and wholesale trade. This observation is the main difference between our results and those of Blinder and Maccini (1991). They find that, of manufacturing inventory volatility, only 25 percent is due to the comovement of finished goods, materials, and goods-in-process inventories. And for total inventory investment, only 20 percent is due to covariance terms.

Blinder and Maccini (1991) define the business cycle component of a time series as fluctuations around a linear trend. In effect, their definition of the business cycle eliminates long-run growth components but not the irregular component, or short periodicity movements, from consideration. Table 3 reveals as much. It shows that these high-frequency movements are not highly correlated across sectors; that is, the results of Blinder and Maccini (1991) represent

Table 3 Variance Decomposition of Inventory Investment

Inventory Component	Percent of Total Investment	Business Cycle Component Variance	Business Cycle Component Percent	Irregular Component Variance	Irregular Component Percent
Manufacturing and trade		2.491		4.753	
Manufacturing	43.5	1.035	41.6	1.538	32.4
Finished goods	15.2	0.107	10.3	0.443	28.4
Goods-in-process	14.7	0.249	23.9	0.487	31.3
Materials and supplies	13.6	0.150	14.4	0.556	35.7
Covariance terms		0.535	51.4	0.072	4.6
Wholesale trade	26.5	0.172	6.9	0.924	19.4
Retail trade	30.0	0.312	12.5	1.990	41.9
Covariance terms		0.973	39.1	0.301	6.3

a mixture of the properties of business cycle and irregular components. Also, for the irregular component, inventory investment in the trade sector accounts for a much bigger share of overall inventory investment variance.

Production, Sales, and Inventory Investment at the Sectoral Level

The behavior of production, sales, and inventory investment is remarkably similar in the different sectors. In all sectors, production tends to be more volatile than sales, substantially so in the retail and wholesale trade sector. This is true for both the business cycle components and the irregular components (see Table 4), thus confirming Blinder and Maccini's (1991) results. Note also that over the business cycle, the durable goods sectors are much more volatile than the nondurable goods sectors. This is consistent with other work on sectorally disaggregated data (Hornstein and Praschnik 1997).

Table 5a documents the pattern of comovement between inventory investment and sales for the business cycle components. Over the business cycle, inventory investment and sales are positively correlated. What is of interest is that there are different patterns of lead-lag relationships between inventory investment and sales in the various sectors. For example, in the manufacturing sector, inventory investment in nondurable manufacturing is essentially uncorrelated with sales, but such investment in durable manufacturing bears a strong contemporaneous correlation with sales. In the wholesale trade sector, inventory investment in the durable sector is also contemporaneous with sales, but inventory investment in the nondurable sector leads sales by three months. Furthermore, in retail trade, inventory investment leads sales by three months,

Table 4 Relative Variance of Production Y and Sales X

	Business Cycle Component			Irregular Component		
Sector	**Var (Y)**	**Var (X)**	$\frac{Var(Y)}{Var(X)}$	**Var (Y)**	**Var (X)**	$\frac{Var(Y)}{Var(X)}$
Manufacturing	43.2	41.9	1.03	6.84	6.66	1.03
	47.9		1.14	7.36		1.11
Durables	20.0	19.2	1.04	3.59	3.37	1.07
	23.2		1.21	3.92		1.16
Nondurables	4.9	4.8	1.02	1.31	1.18	1.11
	4.9		1.01	1.38		1.17
Wholesale trade	11.49	10.3	1.12	1.48	1.74	1.11
Durables	6.0	5.0	1.19	2.14	0.50	1.18
Nondurables	1.6	1.6	1.03	1.45	0.90	1.01
Retail trade	9.9	8.1	1.23	1.64	1.78	1.21
Durables	3.9	3.1	1.28	1.43	1.08	1.26
Nondurables	1.5	1.2	1.18	2.35	0.27	1.17

Note: For the manufacturing sector, the first row refers to the narrow inventory definition (finished goods inventories only) and the second row refers to the broad inventory definition (finished goods and goods-in-process inventories).

Table 5a Comovement of Inventory Investment ΔN and Sales X for Business Cycle Components

	Correlation coefficient for X_t and ΔN_{t+s}, where $s =$								
	−4	**−3**	**−2**	**−1**	**0**	**1**	**2**	**3**	**4**
Manufacturing	0.09	0.15	0.20	0.25	0.30	0.33	0.36	0.38	0.38
	0.47	0.51	0.54	0.57	0.60	0.61	0.61	0.60	0.58
Durables	0.23	0.28	0.32	0.36	0.39	0.41	0.43	0.43	0.43
	0.59	0.62	0.65	0.66	0.67	0.67	0.66	0.64	0.61
Nondurables	−0.13	−0.08	−0.03	0.01	0.05	0.08	0.10	0.12	0.12
	−0.11	−0.08	−0.04	−0.01	0.01	0.03	0.04	0.04	0.03
Wholesale trade	0.43	0.44	0.44	0.43	0.39	0.34	0.29	0.23	0.16
Durables	0.49	0.51	0.53	0.53	0.52	0.50	0.47	0.43	0.37
Nondurables	0.19	0.17	0.14	0.09	0.03	−0.04	−0.12	−0.20	−0.27
Retail trade	0.55	0.54	0.53	0.50	0.47	0.43	0.37	0.31	0.24
Durables	0.54	0.54	0.53	0.51	0.48	0.44	0.38	0.32	0.25
Nondurables	0.35	0.34	0.32	0.30	0.28	0.25	0.22	0.18	0.14

Note: See Note to Table 4.

Table 5b Comovement of Sales X and Inventory Investment ΔN for Irregular Components

	Correlation coefficient for X_t and ΔN_{t+s}, where $s =$								
	−4	**−3**	**−2**	**−1**	**0**	**1**	**2**	**3**	**4**
Manufacturing	−0.14	−0.23	−0.15	−0.06	−0.08	0.18	0.13	0.15	0.20
	−0.11	−0.14	−0.06	0.04	−0.06	0.18	0.15	0.09	0.11
Durables	−0.11	−0.13	−0.07	0.02	0.04	0.18	0.09	0.03	−0.02
	−0.06	−0.07	−0.02	0.09	−0.02	0.17	0.14	0.02	0.01
Nondurables	−0.16	−0.17	−0.09	−0.05	−0.10	0.10	0.10	0.14	0.21
	−0.17	−0.13	−0.04	0.01	−0.08	0.10	0.11	0.10	0.16
Wholesale trade	−0.01	0.08	0.01	0.11	−0.04	−0.06	−0.06	−0.09	−0.00
Durables	−0.00	0.04	−0.13	0.04	−0.09	−0.03	0.08	0.08	0.01
Nondurables	−0.02	0.11	0.11	0.11	0.02	−0.18	−0.13	−0.05	−0.06
Retail trade	−0.01	−0.05	−0.03	−0.04	−0.23	0.05	0.18	0.07	0.06
Durables	−0.02	−0.03	0.06	−0.07	−0.34	0.01	0.15	0.05	0.07
Nondurables	−0.02	0.07	−0.15	0.02	−0.10	0.02	0.11	−0.05	0.04

Note: See Note to Table 4.

both for durable and nondurable goods.[13] This observation might be useful to differentiate between models of inventory investment across sectors.

For the irregular component, inventory investment and sales are essentially uncorrelated, with a tendency towards negative correlations (see Table 5b). In particular, for the retail trade sector, sales and inventory investment are somewhat negatively correlated. As for aggregate data, it appears as if inventory stocks are used to buffer unforeseen short-term fluctuations in sales.

The Cyclical Behavior of the Retail-Price Markup

One last variable, the retail-price markup, is of interest because extensions of simple (S, s) inventory models suggest that sellers have some control over the prices they set. Such control means that decisions on inventory investment, sales, and prices are interrelated. Fisher and Hornstein (1997) describe such an (S, s) inventory model for the retail sector. They argue that the cyclical behavior of the retail-price markup depends on whether supply or demand shocks are more important for a market. In particular, their model predicts that if productivity shocks to the suppliers of the retailers are predominant, then the retail-price markup should be positively correlated with sales. On the other

[13] The fact that inventory investment leads sales over the business cycle does not mean that inventory investment can be used to predict future sales. The reason is simply that the business cycle component of a variable represents a moving average of past and future values of the variable.

Table 6 Comovement of Sales *X* and Retail Markups *M* at Business Cycle Frequencies

	Correlation coefficient for X_t and M_{t+s}, where $s =$								
	−4	−3	−2	−1	0	1	2	3	4
All retail	0.61	0.60	0.58	0.56	0.54	0.51	0.48	0.45	0.41
Durable goods	0.28	0.27	0.25	0.23	0.21	0.18	0.15	0.11	0.07
Autos	0.03	0.01	−0.00	−0.02	−0.03	−0.05	−0.06	−0.06	−0.07
Furniture	0.13	0.09	0.04	−0.02	−0.07	−0.12	−0.18	−0.23	−0.27
Building mat.	−0.66	−0.70	−0.73	−0.75	−0.76	−0.76	−0.75	−0.74	−0.71
Nondurable	0.50	0.53	0.56	0.58	0.60	0.61	0.62	0.62	0.61
Food	0.37	0.34	0.30	0.26	0.21	0.16	0.11	0.07	0.04
Apparel	0.17	0.09	0.02	−0.06	−0.14	−0.21	−0.27	−0.33	−0.38
Others	0.48	0.42	0.35	0.28	0.21	0.15	0.09	0.03	−0.03

hand, if shocks to demand for the retailer's product are predominant, then the retail-price markup should be negatively correlated with sales.

The comovement over the business cycle between retail-price markups and sales for a selected number of products is documented in Table 6. Apparently there is no strong consistent pattern in the data. For nondurable goods, with the exception of apparels, the markup tends to be positively correlated with sales, and for durable goods the markup tends to be negatively correlated with sales. Of interest is the absence of any strong comovement for cars. One note of caution: the lack of correlation between markup and sales should not be taken as evidence for inflexible prices. The markup is defined as the ratio of retail prices to producer prices, both of which tend to be strongly correlated with sales over the business cycle. In particular, the retail price is negatively correlated with sales for all goods, and, with the exception of building materials, the producer price indexes are negatively correlated with sales.

Finally, we have not presented results for the irregular component because for these frequencies the markup is essentially uncorrelated with sales. In this case the markup is uncorrelated with sales, because both the retail price and the producer price index are uncorrelated with sales.

4. CONCLUSION

The description of the data above suggests that it is important to distinguish between the irregular and the business cycle components of inventory investment, production, and sales. Bearing this in mind, the findings can be summarized as follows. First, inventory investment fluctuations are not important for output fluctuations over the business cycle, but they are important for short-term output fluctuations. Second, over the business cycle, we cannot attribute total inventory investment volatility to its individual components because all components are highly correlated. Third, inventory investment is positively correlated with sales over the business cycle but tends to be uncorrelated or negatively correlated with sales for short-term fluctuations. Fourth, production tends to be more volatile than sales; this feature is common to all sectors, and it applies to business cycle and short-term fluctuations.

How well do the existing models of inventory investment match these stylized facts? The production-smoothing model is in principle consistent with the finding that production is more volatile than sales in the particular case where cost shocks are assumed to be more important than demand shocks. Essentially the production-smoothing model is used in order to say something about the relative importance of unobserved demand and supply shocks in the economy. Unfortunately, with direct observations on cost and demand shocks, the production-smoothing model often is no longer consistent with the stylized facts, given the observed relative volatility of shocks (for some recent work, see Durlauf and Maccini [1995]). Furthermore, the production-smoothing model has problems accounting for the comovement of sales and inventory investment, even if cost shocks are more volatile than demand shocks.

Less can be said about how well the (S, s) inventory framework conforms to the stylized facts because only recently has work begun that tries to incorporate this framework in quantitative general equilibrium models. For a simple general equilibrium model with (S, s) inventory policies, Fisher and Hornstein (1997) have shown that the model's quantitative implications are consistent with the stylized facts. But more work needs to be done.

Let me conclude with a remark on whether inventory investment can destabilize the economy.[14] Obviously the stylized facts reviewed in this article by themselves have nothing to say about this issue. Potential destabilization can only be addressed within some theory of inventory investment. For example, the fact that production appears to be more volatile than sales does not mean that, because of inventory investment, production is excessively volatile. If one believes that the production-smoothing model is a useful representation of the economy, then at least for a firm, this outcome is optimal if marginal cost varies over time.

Most inventory investment models, with few exceptions, are partial equilibrium in nature; that is, they describe the behavior of a firm/industry and take the behavior of the rest of the economy as given. A complete analysis of the role of inventory investment requires that the particular inventory investment model is embedded in a general equilibrium model in order to study how inventory investment affects the rest of the economy and vice versa. It is not clear that

[14] This is a well-known property of inventories in the traditional inventory-accelerator models (Metzler 1941).

inventory investment will be destabilizing in such a general equilibrium model or even what such destabilization means. One possible interpretation is that, with inventories, the equilibrium of an economy is no longer determinate. In this case, one could construct particular equilibria where output fluctuates even though the fundamentals of the economy do not change at all; however, such work remains to be done.[15]

APPENDIX A: THE CYCLICAL COMPONENTS OF A TIME SERIES

The decomposition of a time series into business cycle and irregular components using a bandpass filter is a statistical method based on the frequency domain analysis of time series.[16] Essentially, this method interprets a time series as the sum of a very large number of sine and cosine waves, and it isolates groups of waves within particular frequency bands. Rather than describing in detail this technique and the underlying statistical theory, I simply want to provide some insight on how it works. For an introduction to the analysis of time series in the frequency domain, see Harvey (1993) or Hamilton (1994). For a description of bandpass filters, see Baxter and King (1995).

Extracting Periodic Components from Deterministic Time Series . . .

In order to illustrate the problem, consider the following example. Define a variable Y_t as the sum of sine and cosine functions

$$Y_t = \sum_{i=1}^{3} [\alpha_i \cos(\omega_i t) + \beta_i \sin(\omega_i t)].$$

A sine (cosine) function has amplitude one and periodicity $T = 2\pi$. A function is periodic with period T if the function repeats itself every T periods.[17] For a periodic function, its frequency $1/T$ denotes how many cycles are completed within a unit of time. The transformation of the sine (cosine) function $\alpha \cos(\omega t)$ $[\beta \sin(\omega t)]$ has amplitude α (β), periodicity $T = 2\pi/\omega$, and frequency $\omega = 2\pi/T$.

[15] See Benhabib and Farmer (1997) for a survey on endogenous business cycle models.

[16] Other methods have been used to identify the business cycle components of time series, for example, stochastic trends or linear trends. One close relative of a bandpass filter for the business cycle is the Hodrick-Prescott filter, also described in Baxter and King (1995).

[17] The sine function satisfies $\sin(t + 2\pi j) = \sin(t)$ for all t and $j = \ldots, -1, 0, +1, \ldots$.

In the example, Y_t is the sum of three periodic functions and is itself periodic. Assume that the unit of time is a month and that the first component has a periodicity of 50 years ($\omega_1 = 2\pi/(12 \cdot 50)$); the second component has a periodicity of five years ($\omega_2 = 2\pi/(12 \cdot 5)$); and the third component has a periodicity of one year ($\omega_3 = 2\pi/(12 \cdot 1)$). For this example, the first component represents long-run trends (low-frequency movements); the second component represents a business cycle (medium-frequency movements); and the third component represents short-run fluctuations (high-frequency movements), like seasonal fluctuations. Suppose there is a finite number of observations on Y_t as shown in Figure A1: How can the three different components be extracted from Y_t?[18]

An ideal bandpass filter extracts the components of a time series whose frequencies are within a given frequency band (Baxter and King 1995). This filter assigns a weight of one to all frequencies that fall within the specified band and zero weight to all frequencies outside the specified band. The ideal bandpass filter is represented by a moving average with infinitely many leads and lags, $\hat{Y}_t = \sum_{s=-\infty,...,\infty} a_s Y_{t+s}$, and the filter is defined by its weights, $\{a_s\}_{s=-\infty,...,+\infty}$, which depend on the frequency band to be extracted from the time series. Since only a finite number of observations is available, the bandpass filter has to be approximated. It turns out that an approximate bandpass filter has the same moving average representation except that the weights are truncated, $\hat{Y}_t^S = \sum_{s=-S,...,S} a_s Y_{t+s}$, and the number of leads/lags S determines the approximation quality. Because the bandpass filter is approximate, it will pass some components with frequencies outside the specified frequency band, and it will not assign all frequencies within the specified frequency band the same weight. The approximation improves with the number of leads and lags included in the moving average term.

I follow Baxter and King (1995) and identify the business cycle with periodicities between one and one-half years and six years, and for monthly (quarterly) data I use an approximation involving 36 (12) leads and lags. The irregular component (trend component) is identified with the periodicities of less than one and one-half years (more than six years). When this procedure is applied to the time series of Y_t in Figure A1, the approximate bandpass filter extracts its low-, medium-, and high-frequency components quite well.[19]

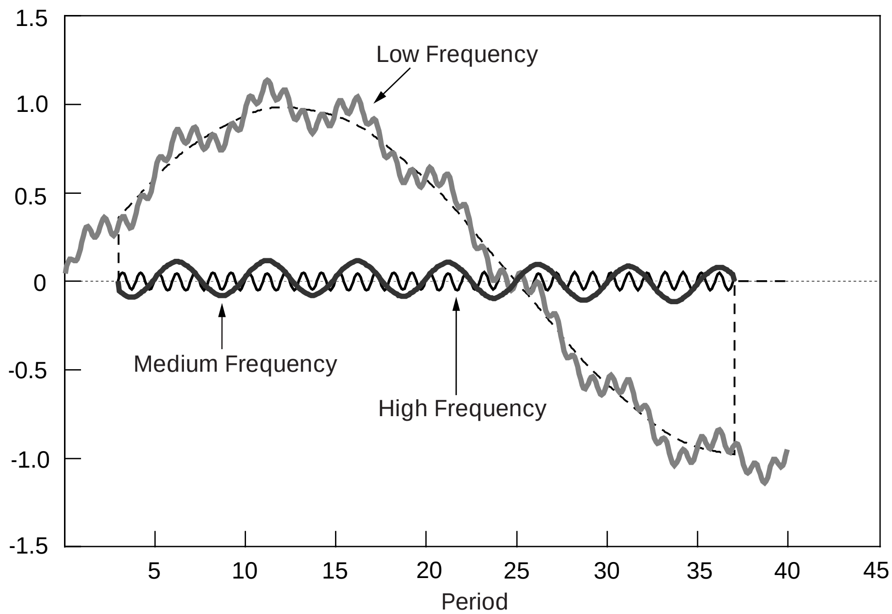
. . . and Stochastic Processes

The business cycle is not a deterministic process, which should be apparent from the graphs of GDP growth. Definitions of the business cycle, such as Burns and Mitchell's (1946, p. 3) above, recognize this fact and refer to ". . .

[18] I have set $\alpha_1 = 1$, $\alpha_2 = 0.1$, $\alpha_3 = 0.05$, and $\beta_i = 0$ for $i = 1, 2, 3$.

[19] The filtered series is not defined for the first and last S observations since the filter uses S leads and lags.

Figure A1 A Sine Function and its Cyclical Components



recurrent but not periodic . . .” movements. In particular, Lucas ([1977], 1989, p. 217) states that “. . . movements about trend . . . can be well described by a stochastically disturbed difference equation of very low order.” Yet, I have discussed the bandpass filter as a way to extract periodic components from a time series that is the sum of deterministic cycles.

This approach remains valid for the study of covariance stationary stochastic time series, because of the spectral representation theorem.[20] The theorem states that any covariance stationary time series can be written as the integral of randomly weighted sine and cosine functions

$$Y_t = \mu + \int_0^{\pi} [\alpha(\omega)\cos(\omega t + \beta(\omega)\sin(\omega t)]d\omega, \tag{1}$$

where the random variables $\alpha(\omega)$ and $\beta(\omega)$ are in a sense “mutually uncorrelated” with mean zero. The property that α and β are uncorrelated is useful because it allows us to attribute the variance of Y_t to its various components. Let

[20] A stochastic process Y_t is covariance stationary if the first and second moments of the process are time independent; that is, expected values are $E[Y_t] = \mu$ and $E[Y_t Y_{t-s}] = \rho_s$ for all t, s.

$0 \leq \omega^L < \omega^H \leq \pi$ and write the interval $[0, \pi]$ as the union of a low-frequency trend component $I^{TR} = [0, \omega^L]$, a medium-frequency business cycle component $I^{BC} = [\omega^L, \omega^H]$, and a high-frequency irregular component $I^{IR} = [\omega^H, \pi]$. The bandpass filter can be applied to the stochastic process Y_t and extract the components associated with each frequency band $I = [\omega_0, \omega_1]$. Because the sine and cosine functions at different frequencies are uncorrelated, the variance of Y_t is the sum of the variances of the nonoverlapping frequency components.

To get some idea of how a bandpass filter works for a stochastic process, consider the following example. Suppose the stochastic process Y_t is described by a low-order stochastic difference equation, in particular, Y_t is first-order autoregressive (AR(1)),

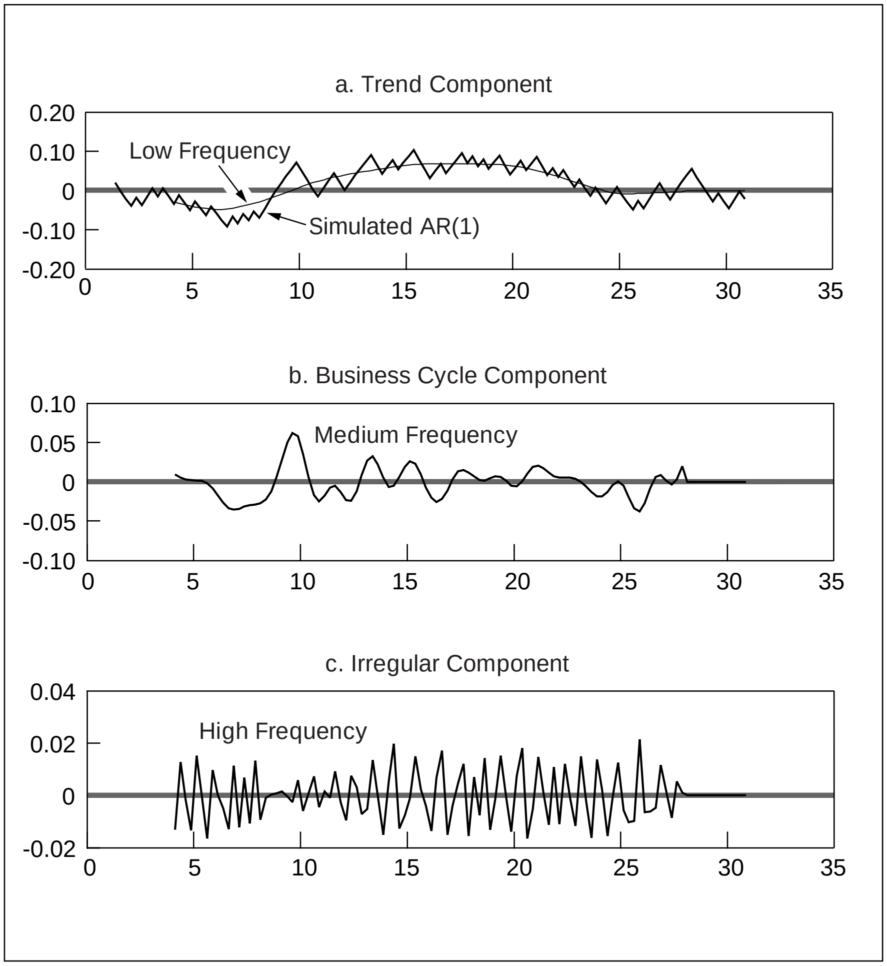
$$Y_t = 0.95Y_{t-1} + \varepsilon_t \text{ for } t = 1, 2, \ \ldots,$$

where Y_0 is given, and ε_t is an identically and independently distributed random variable that takes on values -0.1 or $+0.1$ with a probability of one-half each. Suppose that each time period represents a quarter. Make 120 independent draws of the random variable ε_t and construct a particular 30-year time path of this process $\{y_t\}_{t=0,\ldots,30 \cdot 4}$. The result is something like Figure A2a. Recurrent but not periodic movements are clearly recognizable in the time path. As a next step apply a bandpass filter to this time path and extract the trend, business cycle, and irregular components of the time series, shown in Figure A2a-c. As is apparent from this figure, the business cycle component is quite smooth, with cycles between two and six years, whereas the irregular component has somewhat less amplitude and no particular cyclical pattern.

Finally, note that the spectral decomposition theorem applies to covariance stationary stochastic processes. In particular, this means that there should be no trend in the stochastic process; that is, the mean of the random variable Y_t should not change over time. As pointed out above, most of the economic time series do have a trend. In this context it is useful to know that a bandpass filter which excludes components with zero frequency, that is, infinite periodicity, also removes any linear and quadratic trend from a time series (Baxter and King 1995).[21] Thus a bandpass filter that isolates the business cycle and irregular components also eliminates linear and quadratic trends.

[21] It also removes any components that are integrated of order one or two.

Figure A2 AR(1) Stochastic Process and its Cyclical Components



APPENDIX B: THE DATA

The data used in this article are taken from DRI U.S. Central Database. For the study of aggregate inventory investment, I use quarterly data from 1960:1 to 1995:4 on GDP and the change in business inventories. Both series are in billions of chained (1992) dollars, seasonally adjusted. For the study of disaggregated inventory investment, I use monthly data from January 1960 to December 1996 for manufacturing and trade sales and inventories. All series are billions of chained (1992) dollars, seasonally adjusted. Inventories are end of period.

Production is defined as sales plus inventory investment. For the manufacturing sector I follow Blinder and Maccini (1991) and consider two definitions of output. For the narrow definition of output, I use only inventory investment in finished goods, and for the broad definition of output, I include inventory investment in goods-in-process as well.

The quantity index for a variable is usually obtained by deflating the nominal values with a price index. The quantity indexes for sales and inventories are not directly comparable because they are measured in different units. In particular, nominal sales are deflated with a "market" price index, while inventories are deflated with a "cost" price index. Since production in a sector is defined as the sum of sales and inventory investment, either inventories or sales have to be adjusted. For constant dollar quantity indexes, West (1983) suggests rescaling the inventory series using the base-period ratio of (business receipts)/(costs of goods sold) from corporate income tax returns. I follow West even though his procedure is not quite appropriate for my data set: I use a chain-type quantity index rather than the constant dollar quantity index that West uses. It does not appear as if my decision to follow West significantly affects the results. Since the scale factor is constant, the effects of a particular choice for the scale factor are limited to the properties of production relative to other variables. However, here I get similar results as Blinder and Maccini (1991).

For the study of the relative prices of retail goods to producer goods, I use monthly data from January 1967 to December 1996 for retail sales, implicit price deflators for retail sales, and the producer price index. All series are seasonally adjusted; retail sales are in billions of chained (1992) dollars. The commodity categories for retail data and producer price data are not the same, and I follow Blinder (1981) in the way the categories are linked:

Commodity	**Retail Sales/Prices**	**Producer Price Index**
Durable goods	Total durable goods	Durable goods
Cars	Automotive dealers	Passenger cars
Furniture	Furniture and audio video group	Furniture and household durables
Building materials	Building materials group	Lumber and wood products
Nondurable goods	Total nondurable goods	Consumer nondurables (less food)
Food	Food group	Processed foods and feeds
Apparel	Apparel group	Textile products and apparel
Other nondurable goods	Other nondurable goods	Consumer nondurables (less food)

The business cycle (irregular) component of a time series x_t is calculated as follows. Because production, sales, and inventory stocks are characterized by geometric growth, that is, a log-linear trend, I start out with the log transformation of the variables. First, we extract the business cycle (irregular) component $\ln \tilde{x}_t$ from the log of the time series, then we define the business cycle (irregular) component as $\hat{x}_t = x_t - \exp(\ln x_t - \ln \tilde{x}_t)$. For inventory investment the business cycle (irregular) component is defined as the first difference of the corresponding component of inventory stocks.

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