Implications of the Capital-Embodiment Revolution for Directed R&D and Wage Inequality

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Wage inequality has increased dramatically in the United States since the late 1970s. In particular, we have witnessed growing wage differences between groups defined by observed skills such as education or experience. For example, the college premium—that is, the percentage difference between the average wages of college-educated and non-college-educated workers—increased by a factor of four. Since at the same time the relative supply of college-educated workers increased, we would have expected to see a fall of the college premium. The fact that a decrease did not occur suggests that something else changed too. A natural candidate is technical change that has been “biased” toward skilled labor over this time. If the nature of technical change makes skilled workers relatively more productive than unskilled workers, then the wage gap will widen, assuming that market wages reflect marginal productivities. But why should technical change be biased more toward skilled labor? In fact, technical change sometimes has been biased the other way. From a perspective of understanding the evolution of wage inequality, then, it is important to determine the possible bias of technical change.

In this article we investigate the long-term determinants of the bias of technical change using a dynamic model where R&D is endogenous and can be directed to specific inputs. One of the key determinants of the form of technical change...
change, then, is wage inequality itself: with a high value of skilled workers—a high skill premium—the value of new technologies directed for use with skilled workers will rise. Thus, in our theory, wage inequality and technology are simultaneously determined through a two-way feedback. We first study the long-run determination of wages and technologies by considering long-run outcomes: steady states.

Our ultimate aim, however, is to understand what causes changes to the equilibrium wage inequality. In particular, we want to evaluate the role of the IT revolution in shaping the last thirty years of wages and productivity. We think of the IT revolution as having been initiated in the mid-1970s; the defining event was that the relative price of new capital, which is complementary to skilled labor, fell significantly. We then consider two quantitative experiments. First, we consider a one-time fall in the relative price of new capital, which allows us to trace out the short-run dynamics of this model: In response to this impulse, how do wage inequality and the induced directed R&D react? We then consider a gradual and persistent fall in the relative price of new capital aimed at matching the actual behavior of this price series as measured by U.S. data. Now the question is quantitative: What is the possible role of the IT revolution, viewed this way, in accounting for the observed increase in wage inequality and associated changes in productivity?

Why Is Wage Inequality Relevant to Macroeconomists?

Our quantitative theory has joint implications for wage inequality and technology. Thus, not only can such a theory tell us how technical change influences relative wages, but it allows us to use wages to understand the nature of technical change. In particular, not only do wages reflect current marginal productivities, but they are also relevant for understanding where current R&D efforts are directed—both its composition and its effect on aggregate productivity—and thus for predicting future productivity movements. We therefore believe that, on a general methodological level, the development of quantitative theories of the joint determination of wage inequality and technology is important for furthering our understanding of aggregate economic performance.

Because of the connection between wages and technology, wage data are an interesting testing ground for different theories about what is going on in the aggregate economy. Namely, there has been widespread interest in what has happened to aggregate productivity, especially in light of the “IT revolution”: Has IT technology, and all the changes in the workplace it seems to have led to, also delivered higher productivity? In conducting stabilization policy especially, monetary or otherwise, information on the behavior of productivity is useful. Relatedly, is there unmeasured quality improvement in the goods and services produced by the new economy? This information is particularly
important in understanding how inflation really has influenced the purchasing power of our money: with significant unmeasured quality improvements, we are better off than the inflation figures indicate. To the extent that wage inequality speaks indirectly about productivity advances of different sorts, it is therefore arguably an important variable to follow.

Aside from the role wage inequality has as an indicator of what is happening—and what will happen—to aggregate economic performance, it is also relevant in itself and for understanding the political debate. Most obviously, wage inequality is often part of the distributional goals of policymakers (and voters), and indications of widening wage inequality may be taken as cause for some kind of action by these groups. As economists, we perhaps have instinctive reactions to caution against policies aimed at reducing wage inequality, since we think they may reduce workers’ efforts to work hard, accumulate human capital, and so on. The theory in this paper suggests that there are other reasons to react: reductions in wage inequality will certainly change the composition of R&D, and thus the nature of technology, and they are likely to change aggregate productivity growth as well.

To the extent that externalities in research and labor market frictions are not important, the market mechanism probably channels the R&D efforts to its different uses quite efficiently, and thus one should caution against policies leading to wage compression. However, with an imperfectly functioning market, the situation is more complicated. We do not characterize optimal policy in the environment we study, but one could. Do the market imperfections lead to too much or to too little wage inequality? The answer likely depends on details of the imperfections, including those in the labor markets (which we abstract from in this article). It is even a logical possibility that there is too much equilibrium wage inequality from the perspective of efficient R&D and that wage-compressing policies would be beneficial! However, it might also be the reverse: such policies might be even more harmful than indicated by our knee-jerk reactions. We hope to be able to address these important issues in future work.

**Capital-Embodied Technical Change and Wage Inequality**

A main purpose of our paper is the study of the short-, medium-, and long-run effects on the economy of an “IT revolution”: of a burst in capital-embodied technical change. In particular, we focus on its role in wage inequality between skilled and unskilled labor and subsequent R&D efforts. In the postwar U.S. economy, capital-embodied technical change seems to have been an important source of growth. As argued in Greenwood, Hercowitz, and Krusell (1997), to a first approximation, capital-embodied technical change is reflected in the decline of the price of new capital goods (such as computers and other
equipment) relative to the price of consumption goods. Since in the United States the relative price of new capital has been falling at an annual rate of close to 3 percent, this channel has been responsible for a sizable fraction of overall growth.

The implications of capital-embodied technology for wages alone have been studied previously. In earlier work, Krusell, Ohanian, Rios-Rull, and Violante (referred to hereafter as KORV) (2000) estimate features of the aggregate production function and use these features to argue that a higher capital stock, induced by the fall in the price of new capital, must have increased the relative productivity, and thus wage, of skilled labor, that is, the skill premium. The argument in KORV (2000) is based on a partial equilibrium analysis and takes relative factor productivities and relative factor supplies as given. In this paper we also take the latter as given; we take the view that whereas the relative supply of skilled labor can be expected to change, it is unlikely to be very elastic. For example, if we identify skilled labor with college graduates, then we might expect that, because of inherent ability-based differences, the supply of college graduates has an upper limit or, alternatively, that the average quality of college graduates would tend to fall as more students choose to go to college.

The contribution of this paper is the analysis of the equilibrium response of relative factor productivities to changes in the relative price of capital. Unlike changes in the relative supply of labor, there does not seem to be a natural upper limit to technology improvements, in particular to the relative improvements of different applications. In a number of recent papers, Acemoglu (1998, 2002a, 2002b, 2003) has argued forcefully and repeatedly that technical change is endogenous and is purposefully directed to different uses, that is, specialized for different kinds of workers/machines. We apply Acemoglu’s framework to the particular question of how changes in the relative price of capital affect the relative incentives for productivity improvements that are specific to capital, skilled labor, and unskilled labor.1

The argument in KORV (2000) that capital accumulation increases the skill premium is based on the different substitution possibilities between the inputs capital, skilled labor, and unskilled labor in the aggregate production function. For any pair of inputs, basic economic theory suggests that if firms minimize cost, then an input that becomes relatively more expensive is used relatively less, holding the output to be produced fixed. In other words, the relative input ratio falls as the relative price increases. The question is whether the relative

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1 Acemoglu (2002b) has studied how the interaction of directed R&D with a change in the relative supply of skilled labor affects wage inequality.
input ratio falls relatively more or less than the relative price increases. We say that two inputs are substitutes (complements) if following a 1 percent increase of the relative input price, the relative input use declines by more (less) than 1 percent.\textsuperscript{2} Alternatively, we can ask by how much relative input prices have to change such that input markets clear if the relative supply of inputs changes. Thus, if two inputs are substitutes (complements) and the relative supply of one input increases by 1 percent, then the relative price of that input has to fall by less (more) than 1 percent such that the input markets clear.

Based on a wide range of empirical work and on independent estimation, KORV (2000) argue that skilled labor is more complementary to capital, whereas unskilled labor is more substitutable for capital. A higher capital stock reduces the supply of skilled labor and unskilled labor relative to capital. Holding the labor endowments and productivities fixed, the price of skilled and unskilled labor relative to the price of capital thus increases in an equilibrium. Since skilled labor is complementary to capital, whereas unskilled labor is a substitute for capital, the price of skilled labor relative to capital has to increase more than the price of unskilled labor relative to capital. Therefore, the wage of skilled labor increases relative to the wage of unskilled labor.

Directed Technical Change and Factor Productivity

A major technological event such as the IT revolution will affect not only the accumulation of capital but also the way R&D is conducted. In general, we expect that R&D is purposefully directed toward improving the productivity of activities where it will receive the highest rewards. From our perspective, the important distinction is whether R&D is directed toward improving the productivity of skilled labor or unskilled labor, or whether it is used to further increase the productivity of existing equipment capital. Many recent technology developments seem skill-biased; for example, the development of advanced software is performed by skilled labor. However, there are many examples of how IT technology might also help unskilled labor improve its productivity; cash registers, for example, have become very easy to use and have drastically improved efficiency. Finally, general software development can be viewed as improving the productivity of existing computers. Since all these developments are the result of intentional research activities, and since they have very different implications for the relative productivity of different factors, understanding how these research activities respond to a fall in the price of capital seems potentially quite important.

\textsuperscript{2} With perfect complements the relative input use does not respond at all to a change in relative prices, and with perfect substitutes the relative input use may switch completely with a change in relative prices.
Acemoglu (1998, 2002b, 2003) describes a simple framework of endogenous technical change where R&D is purposefully directed toward the productivity improvement of different inputs. An important ingredient of this approach is that the returns to R&D that improve the productivity of an input are proportional to the total income of that input. This creates a “market size” effect of R&D: productivity-improving resources are allocated toward factor markets with large factor income. With endogenous technical change, it is quite possible that R&D resources are allocated to one factor at the expense of another factor if the market for the neglected factor is small. In the long run, the productivity of the neglected factor stagnates. Externalities in the R&D process—that is, productivity improvements to one factor that spill over to other factors—can overcome this effect such that in the long run productivity improvements proceed at the same rate for factors with small and big markets. We now describe how purposeful R&D affects the interaction of technical change and wage inequality.

We have already described how changes in the relative supply of capital together with different degrees of substitutability in production affect relative wages directly. More important, however, in an economy with directed R&D, relative supply changes also affect relative factor incomes, depending on the degree of substitutability. When factor productivities can change, the relevant factor supply is the product of factor endowment and factor productivity, that is, the number of available efficiency units. Now suppose that the effective supply of capital increases relative to the effective supply of skilled labor. Because capital and skilled labor are complements, in an equilibrium the wage of skilled labor relative to the price of capital has to increase by more than the supply of skilled labor relative to capital falls, and the total payments to skilled labor increase relative to payments to capital. Because of the market size effect, R&D is then redirected toward making skilled labor more productive relative to capital; that is, it increases the relative effective supply of skilled labor. This in turn lowers the relative income of skilled labor, and the R&D process is stable.

Now consider an increase of skilled labor productivity relative to unskilled labor productivity; that is, the effective relative supply of unskilled labor declines. Because unskilled labor is a substitute for skilled labor and capital, in an equilibrium the wage of unskilled labor relative to the wage of skilled labor has to increase by less than the relative supply of unskilled labor to skilled labor falls, and the total payments to unskilled labor decline relative to payments to skilled labor. Because of the market size effect, R&D is then redirected away from making unskilled labor more productive and the relative productivity of unskilled labor falls. This in turn again reduces the relative effective supply of unskilled labor, which in turn leads to even less R&D devoted to improve the productivity of unskilled labor, and so on. If this process is not stopped, the wage of unskilled labor will stagnate and over time will become negligible relative to the wage of skilled labor. This is a process that we have
not observed in the United States economic history. While there have been changes in the skill premium, these changes have remained bounded.

In order to prevent unskilled wages from losing out relative to skilled wages, we assume that there are research spillovers between skilled and unskilled labor. This does not seem entirely unreasonable a priori, besides helping ensure that the long-run shares of skilled and unskilled labor remain balanced. This mechanism is similar to Acemoglu (2002b), who studies the effects of directed R&D when the relative supply of skilled and unskilled labor is changing and skilled and unskilled labor are substitutes.

**Results**

We find that capital-embodied technical change together with induced factor-specific technical change due to directed R&D significantly raises the skill premium, that is, increases wage inequality. We limit our analysis to the study of balanced growth paths where each variable grows at a constant rate. On these balanced growth paths factor income and expenditure shares are constant. We find that a one-time increase of productivity in the capital-goods-producing sector generates a small but very persistent increase of the skill premium. In the long run, however, wage inequality is not affected. As we have pointed out, capital-embodied technical change is not a one-time event, but a process that has been ongoing for a long time. We therefore consider a sequence of repeated productivity improvements in the capital-goods-producing sector, and this sequence generates a significant increase in the skill premium that persists for a very long time, even after there is no more capital-embodied technical change.

The remainder of our paper is outlined as follows. In Section 1 we describe the model—that is, the environment and the market structure—and then characterize balanced growth paths of the model economy. In Section 2 we parameterize the model to match the long-run growth characteristics of the U.S. economy. In Section 3 we study the short- and medium-term dynamics of the economy when there is capital-embodied technical change; in particular, we study how the skill premium and labor income share respond. Section 4 concludes.

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3 Goldin and Katz (1999) argue that in the United States the skill premium declined (increased) in the first half (second half) of the twentieth century.

4 Research spillovers between labor of either sort and capital are not present in our model. Such spillovers would imply that there must be long-run technological change to augment the capital input, and this would make the capital-labor share unbalanced.

5 A similar mechanism is used by Goodfriend and McDermott (1998) to explain the determinants of relative national per capita products in world balanced growth.
1. THE MODEL

Preferences and Technology

Preferences

The model has the simplest possible consumer preference structure: preferences are linear in consumption streams over time, with a constant rate of discount:

$$\sum_{t=0}^{\infty} \beta^t C_t,$$

where $C_t$ is consumption at time $t$ and $\beta$ is the time discount factor. This preference specification implies that the goal of the consumer, or of any benevolent government planner, is simply to maximize present-value output using a constant interest rate that is equal to the consumer’s rate of discount.

Production of Final Output: Capital-Skill Complementarity

A final output good $Y$ (we omit time subscripts whenever there is no risk of confusion) is produced with three intermediate inputs, $Y_k$, $Y_s$, and $Y_u$, to an aggregate production function $F$. These intermediate inputs are in turn produced from the primary factors capital, skilled labor, and unskilled labor, respectively. We assume that $F$ is of the nested constant-elasticity-of-substitution (CES) form, as in KORV (2000):

$$Y = F(Y_k, Y_s, Y_u) = \left\{ \lambda \left[ \mu Y_k^\rho + (1 - \mu) Y_s^\rho \right]^{\frac{\sigma}{\rho}} + (1 - \lambda) Y_u^\sigma \right\}^{\frac{1}{\sigma}},$$

with $\rho, \sigma \leq 1$. The elasticity of substitution between skilled labor and capital is $1/(1 - \rho)$. This elasticity is less than one—that is, $\rho \leq 0$—since we assume that capital and skilled labor are complementary. On the other hand, the elasticity between unskilled labor and the aggregate of skilled labor and capital is $1/(1 - \sigma)$. This elasticity is greater than one—that is, $\sigma \geq 0$—since we assume that capital and unskilled labor are substitutes.

Production of Intermediate Goods

The production of intermediate goods is central to our model: it is where the “directed technical change” appears. Following a large part of the recent literature on endogenous growth, we assume that productivity increases via an expansion in the variety of inputs with which each intermediate good is produced.\footnote{See, for example, Romer (1990).} At any point in time, a type $j$ intermediate good $Y_j$, $j = k, s, u,$
is produced with a continuum of specialized inputs, $n_j$:

$$Y_j = \left[ \int_0^{n_j} Y_j(i) \frac{i}{\nu - 1} di \right]^{\frac{\nu}{\nu - 1}}, \nu \geq 1. \tag{3}$$

Each specialized input $Y_j(i), i \in [0, n_j]$, is produced from a primary factor with a distinct technology, which we discuss shortly. In a symmetric equilibrium, all specialized inputs to production of the same intermediate good are operated at the same level, $Y_j(i) = y_j$ for all $i$. This implies the following reduced form production function for an intermediate good:

$$Y_j = n_j^{\frac{\nu}{\nu - 1}} y_j. \tag{4}$$

**Production of Specialized Inputs**

Finally, the production of specialized inputs is closely tied to the three primary factors. A unit of capital produces one unit of any type of specialized input used in the production of capital-based intermediate goods:

$$Y_k(i) = K(i), \tag{5}$$

where $K(i)$ is the amount of capital used for specialized input $i$. Analogously, we have for skilled and unskilled labor

$$Y_s(i) = S(i) \text{ and } Y_u(i) = U(i), \tag{6}$$

where $S(i)$ and $U(i)$ are the amounts of skilled and unskilled labor, respectively, used for specialized input $i$.

The resource availability for each primary factor is as follows. At each point in time $t$ there is a fixed amount of capital $K_t$, and over time, $K_t$ can be increased by foregoing consumption. The other primary factors, skilled and unskilled labor, $S$ and $U$, are fixed. We can think of these as the amount of raw labor hours available in the two groups. We thus abstract from variations in the amount of hours supplied by each worker, in labor force participation, and in population growth. Perhaps more important, we abstract from education decisions; that is a topic worthy of further study.

In an equilibrium the demand and supply for primary factors is equalized, and in a symmetric equilibrium the total demand for a primary factor is equal to the product of the number and level of the specialized inputs using the factor

$$n_k Y_k = K, n_s Y_s = S, \text{ and } n_u Y_u = U. \tag{7}$$

Given the reduced form production function for a symmetric equilibrium, we can relate intermediate goods production to the primary factors as follows:

$$Y_k = A_k K, Y_s = A_s S, \text{ and } Y_u = A_u U, \tag{8}$$

where $A_j \equiv n_j^{1/(\nu - 1)}$. The variables $A_j$ will play the role of the productivity specific to factor $j = k, s, \text{ and } u$. Notice that the development of more specialized inputs, $n_j$, increases productivity, $A_j$, since $\nu > 1$. This development occurs through R&D and will be discussed below.
**Investment-Specific Technical Change**

The aggregate resource constraint in the economy is

\[ C_t + I_t / q_t = Y_t, \]

(9)

where the price of new capital goods—that is, investment \( I_t \)—in terms of consumption is \( 1/q_t \). Investment increases the capital stock

\[ K_{t+1} = (1 - \delta) K_t + I_t \]

(10)

after depreciation, \( 0 \leq \delta \leq 1 \). An increase in \( q_t \) is a form of technical progress, because it makes investment cheaper, and we call this form of technical progress “capital-embodied” or “investment-specific.” We will consider a gradual increase in \( q_t \) from an initial stable level to a new plateau, thus corresponding to a gradual fall in the price of new capital goods.

**R&D**

Finally, the development of new technologies occurs in a similar way for the three kinds of intermediate goods: there is a fixed amount of basic R&D input, \( R \), that can be divided into producing new varieties of specialized inputs of type \( j = k, s, \) and \( u \). One unit of research input produces \( b_j \bar{n}_j \) new specialized \( j \) inputs, where \( \bar{n}_j \) is a weighted average of existing research stocks (varieties). That is, researchers stand on the shoulders of past giants: with a larger available stock of past research in the form of many existing varieties, research productivity is higher. Besides R&D externalities from previously developed varieties to new varieties used in the production of the same intermediate input, there are also spillovers from R&D activities for one intermediate input type to other intermediate input types. In particular, we assume

\[ \bar{n}_s = n_s^{1+\phi} n_u^{1-\phi}, \]

(11)

\[ \bar{n}_u = n_u^{1+\phi} n_s^{1-\phi}, \]

(12)

\[ \bar{n}_k = n_k. \]

(13)

We assume that R&D spillovers are limited to skilled and unskilled labor research. These research spillovers between skilled and unskilled labor are symmetric and captured by the parameter \( \phi \in [-1, 1] \). Capital research does not lead to, nor does it receive, any spillovers.

Finally, the number of available specialized varieties depreciates at rates \( d_k, d_s, \) and \( d_u, \) respectively. Although we can interpret this assumption as exogenous obsolescence of ideas, it is essentially a technical requirement that is necessary to guarantee the local stability of balanced growth paths.
Summarizing the R&D sector, we have

\[ n_{k,t+1} = (1 - d_k)n_{k,t} + b_k \bar{n}_{k,t} R_{k,t}, \]  
\[ n_{s,t+1} = (1 - d_s)n_{s,t} + b_s \bar{n}_{s,t} R_{s,t}, \]  
\[ n_{u,t+1} = (1 - d_u)n_{u,t} + b_u \bar{n}_{u,t} R_{u,t}, \]

where \( R_{j,t} \) is the amount of R&D input devoted to type \( j \) product development. The market for R&D inputs clears

\[ R_{k,t} + R_{s,t} + R_{u,t} = R. \]

**Markets and Decentralized Equilibrium**

The market structure we consider is quite standard for this kind of model setup. There is perfect competition in the final goods market. Intermediate goods are bought and sold by perfectly competitive firms, too, but their inputs—the specialized inputs—are provided by monopolistically competitive firms. Each such monopolistic firm thus owns a right (infinitely-lived patent) to produce its good that it once bought from an R&D firm, and it controls the quantity supplied in every period—with knowledge of the demand curve—in order to maximize profits. With free entry into the monopolistic industry, the stream of profits is enough to just cover the cost of the patent. Researchers, or R&D labs, are perfect competitors, as are the providers of the primary factors capital and labor. The output of research that has market value is the patent; the effect on research productivity of future research efforts is an externality.

We will now look at profit maximization conditions for the different kinds of firms, starting with the final output sector.

**Final Output**

We normalized the price of the final output at one. The profit of a competitive final goods producer is

\[ F(Y_k, Y_s, Y_u) - P_k K - P_s S - P_u U. \]

A profit-maximizing final goods producer equates the marginal cost of a type \( j \) intermediate input—that is, its price, \( P_j \)—to the marginal value product of that input:

\[ P_j = F_j(Y_k, Y_s, Y_u) = F_j \left( \frac{Y_k}{Y_s}, 1, \frac{Y_u}{Y_s} \right). \]

For the last equality we have used the fact that if \( F \) has constant returns to scale, then its derivatives are homogeneous of degree zero.
Intermediate Goods

A competitive intermediate goods producer takes prices and technology, in particular the number of available specialized inputs, as given. The profit of a producer of type \( j = k, s, u \) intermediate goods is

\[
P_j Y_j - \int_0^{y_j} p_j(i) Y_j(i) di,
\]

(20)

where \( p_j(i) \) is the price for specialized input \( i \) for intermediate good \( j \). Again, a profit-maximizing choice equates the marginal value product of a specialized input with the marginal cost of the input, and we get

\[
P_j \left[ \frac{Y_j}{Y_j(i)} \right]^{\frac{1}{\nu}} = p_j(i).
\]

(21)

Conditional on the price of the intermediate good and the level of intermediate goods production, this equation defines the quantity demanded \( Y_j(i) \) as a function of its price, \( p_j(i) \). This demand function has a constant price elasticity, \( \nu \).

In a symmetric equilibrium, \( Y_j(i) \equiv y_j \) (it does not depend on \( i \)), \( p_j(i) = p_j \), and \( Y_j = y_j^{\nu/(\nu-1)} y_j = A_j y_j \), so that we have

\[
p_j = P_j \left( \frac{y_j A_j}{y_j} \right)^{\frac{1}{\nu}} = A_j P_j.
\]

(22)

Specialized Inputs

Specialized inputs are produced by monopolistically competitive firms; that is, they take into account the effect of the price they set on their sales while taking the level of demand and the price of the intermediate input for which they supply specialized inputs as given. For example, a firm that produces a specialized input for the capital-type intermediate good hires capital services \( K(i) \) at the rental rate \( w_k \) and maximizes profits:

\[
\max p_k(i) Y_k(i) - w_k K(i),
\]

(23)

subject to the inverse demand function for \( p_k(i) \) given in (21). Because of the demand function’s constant price elasticity, the profit-maximizing choice sets the specialized input price as a constant markup \( \mu \equiv \nu/(\nu-1) \) over marginal cost:

\[
p_k(i) = \mu w_k.
\]

(24)

Period profits are then

\[
\pi_k(i) = (\mu - 1) w_k K(i).
\]

(25)
Similarly, we obtain for firms using skilled and unskilled labor
\[ p_s(i) = \mu w_s \quad \text{and} \quad \pi_s(i) = (\mu - 1) w_s S(i), \]  
\[ p_u(i) = \mu w_u \quad \text{and} \quad \pi_u(i) = (\mu - 1) w_u U(i). \]  
(26) (27)

The capital value at time \( t \) of a specialized firm using factor \( j \) is
\[ V_{j,t} = \pi_{j,t} + \beta (1 - d_j) V_{j,t+1}, \]  
(28)

where we have used the fact that the firm dies randomly between one period and the next with probability \( d_j \) and firms discount future returns with the representative household’s discount factor, \( \beta \).

**The Research Sector**

Let \( w_R \) denote the price of the services provided by one unit of research. Each unit of research produces \( b_j \bar{n}_j \) new varieties that use the primary factor \( j \) in the next period. Alternatively, in order to obtain one specialized input \( i \), one needs to hire \( 1/b_j \bar{n}_j \) units of research services. Free entry in the research sector amounts to the requirement that the value of the patent to operate production of specialized input \( i \) using the basic input \( j \) from the next period on has to equal the cost of obtaining that patent. Thus,
\[ w_{R,t}/b_j \bar{n}_{j,t} = \beta V_{j,t+1} \]  
(29)
is the zero-profit condition for the research sector.

**Consumer Savings**

The intertemporal first-order condition for the consumer equates the marginal cost of a unit of investment good to the discounted value of its marginal value next period:
\[ \frac{1}{q_t} = \beta \left[ w_{k,t+1} + (1 - \delta) \frac{1}{q_{t+1}} \right]. \]  
(30)

**Balanced Growth**

In this economy, there will be long-run productivity and output growth provided that the research activity is potent enough. We will assume that this is the case. What is of more interest, however, is the form that this growth will take. We will focus attention on balanced growth paths (BGPs)—that is, paths where each variable of interest grows at a constant percentage rate—and all factors are used in production and account for positive and constant shares of total income. This economy also allows for asymptotic growth paths where some factors become unimportant in the long run and their income shares become arbitrarily small. We do not study these asymptotic growth paths but restrict attention to locally stable balanced growth paths, and we assume that
initial conditions are such that the economy is in a locally stable neighborhood of the balanced growth path.

For the analysis of the BGP, we also assume that there is no capital-embodied technical change; that is, the relative price of capital is constant. When the relative price of capital is not constant but declines at a constant rate, a BGP exists only if the elasticity of substitution between all primary factors in the production function (2) is unitary (Greenwood, Hercowitz, and Krusell 1997). Equal and unitary elasticities of substitution are, however, inconsistent with the observed differences in factor-substitution elasticities.

A BGP with Labor-Augmenting Technical Change

We first establish that the BGP of our economy with endogenous directed technical change has the same properties as the BGP of the neoclassical growth model with exogenous labor-augmenting technical change.7 Namely, output, capital, and the productivity of skilled and unskilled labor all grow at the same rate, and the productivity of capital is constant.

Prices for specialized inputs, $p_k$, $p_s$, and $p_u$ (where we have removed the index $i$ because of symmetry), are constant and equal markups on the prices of the associated primary factors $K$, $S$, and $U$ (equations (24), (26), and (27)). From equations (19) and (22), it then follows that the relative incomes of the three factors satisfy

$$\frac{w_k K}{w_s S} = \frac{p_k K}{p_s S} = \frac{P_k A_k K}{P_s A_s S} = \frac{F_k A_k K}{F_s A_s S},$$

$$\frac{w_u U}{w_s S} = \frac{F_u A_u U}{F_s A_s S}.$$  

(31)

(32)

The marginal products of intermediate inputs depend only on the intermediate input ratios, $Y_k/Y_s = (A_k K)/(A_s S)$ and $Y_u/Y_s = (A_u U)/(A_s S)$, because the production function $F$ is constant returns to scale. This in turn implies that the factor income ratios depend only on the intermediate input ratios. Since by assumption the two-factor income ratios are nontrivial constants on a BGP, the intermediate input ratios are then constant. Thus, on any BGP, (a) $A_s$ and $A_u$ grow at the same rate $g$, since $U$ and $S$ are constant; and (b) $A_k K$ grows at the same rate as $A_k$. This implies that intermediate inputs $Y_k$, $Y_s$, $Y_u$, and output $Y$ all grow at rate $g$. Furthermore, because we assume that $q$ is constant on a BGP, capital $K$ has to grow at the rate of final output; otherwise, the investment share goes to zero or one. Because output grows at the same rate as $A_s$, so does $K$. Hence, $A_k$ must be constant.

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7 For a similar environment, this was established by Acemoglu (2003).
The Equations that Characterize a BGP

The BGP is characterized by the constant (a) productivity growth rate $g$; (b) relative productivity of skilled and unskilled labor $\tilde{A}_u \equiv A_u/A_s$; (c) ratio of capital-skilled labor productivity $\tilde{K} \equiv K/A_s$; and (d) capital productivity $A_k$. We first turn to the R&D sector to derive two equations that determine the growth rate and relative productivity of unskilled and skilled labor. Capital productivity and the normalized level of capital then adjust to satisfy the optimal capital accumulation conditions.

Constant capital productivity $A_k$ together with (14) imply that

$$d_k = b_k R_k. \tag{33}$$

Since productivity growth rates are constant on the BGP and the total amount of resources, $R$, available for R&D purposes is constant, the R&D resources directed to the different uses are also constant. The restriction on the total amount of R&D input resources then delivers one equation in the unknowns $R_s + R_u = R - d_k/b_k$. From equations (15) and (16) equal growth in $A_s$ and $A_u$ now implies that

$$1 - d_s + b_s \tilde{A}_u \frac{(1-\phi)(\nu-1)}{2} R_s = 1 - d_u + b_u \tilde{A}_u \frac{-(1-\phi)(\nu-1)}{2} (R - d_k/b_k - R_s). \tag{34}$$

This equation determines R&D resources devoted to the improvement of skilled labor productivity $R_s$ as a function of the relative productivity of unskilled labor $\tilde{A}_u$. Together with the R&D equation for skilled labor, this determines aggregate growth:

$$g = -d_s + b_s \tilde{A}_u \frac{(1-\phi)(\nu-1)}{2} R_s. \tag{35}$$

The economic incentives that determine the direction of technical change are described by the free-entry conditions for R&D (equation (29)). These conditions imply that the marginal payoffs from R&D in each of the three basic uses are equalized to the marginal cost of R&D:

$$w_R = b j \tilde{n}_j V_j \text{ for } j = s, u, k. \tag{36}$$

The capital value of a firm that produces a specialized input is equal to the expected present value of current and future profits from production. For example, from equation (26) a firm that produces specialized inputs from skilled labor has profits $(\mu - 1) w_s S/n_s = w_s S A_s^{1-\nu}$, and profits decline at the gross rate $(1+g)^{2-\nu}$ since more and more firms have to share the available stock of skilled labor. On a BGP the capital value of such a firm is

$$V_{s,t} = \pi_{s,t} + \beta(1-d_s)\pi_{s,t+1} + \ldots = (w_s S/n_s) \left[ 1 + \beta(1-d_s)(1+g) + \ldots \right] w_s S/n_s \frac{1 - \beta(1-d_s)(1+g)^{-\nu}}{1 - \beta(1-d_s)(1+g)^{-\nu}}. \tag{37}$$
Notice that the capital value and therefore the return to R&D that improves the productivity of skilled labor is proportional to the total factor income of skilled labor. Similar expressions can be derived for the capital values of firms that use unskilled labor or capital.

Equalization of returns to R&D from productivity improvements for skilled and unskilled labor then implies the condition

$$\frac{b_s}{b_u} = \frac{w_s U}{w_s S} \tilde{A}_u^{(\phi - 1)(\phi - 1)} \frac{1 - \beta(1 - d_s)(1 + g)^{2-\nu}}{1 - \beta(1 - d_u)(1 + g)^{2-\nu}},$$

which involves the growth rate $g$, relative productivity $\tilde{A}_u$, and the normalized capital stock $\tilde{K}$ through the relative wages. Equalization of returns to R&D from productivity improvements for skilled labor and capital and manipulations similar to the ones above yield the condition

$$\frac{b_s}{b_k} = \frac{w_s K}{w_s S} \tilde{A}_u^{(\phi - 1)(\phi - 1)/2} \frac{1 - \beta(1 - d_s)(1 + g)^{2-\nu}}{1 - \beta(1 - d_k)(1 + g)}.$$

Note that the relative incentives to do R&D depend on the relative factor income shares.

Equations (34)–(39) involve four equations in four unknowns: $R_s$, $g$, $\tilde{A}_u$, and $\tilde{K}$. We will briefly discuss the solution to this system below. Having solved for these four variables, we find the remaining endogenous variables by using the BGP version of our equations. First, we determine the constant productivity of capital $\tilde{A}_k$. Given the exogenous price of new capital $q$, we get a constant value for the rental rate of capital $w_k$ from the optimal capital accumulation condition (30). Given markup pricing (24), the rental rate is equal to $p_k/\mu = P_k A_k/\mu = F_k A_k/\mu$, and since the marginal product of capital $F_k$ depends on known factor input ratios, this delivers $\tilde{A}_k$.

To find levels of variables at a point in time, we need to initialize our state variables at time 0. The state variables of the system are $K$, $A_k$, $A_s$, and $A_u$, of which we already know $A_k$. Thus, let $K (0) = 1$. Then $A_s (0)$ is implied by $A_k \tilde{K} = A_k K / A_s$. Finally, $A_u (0)$ follows from knowing $\tilde{A}_u$. Given the growth rates of all variables, we can now solve for the levels of quantities and prices at all points in time. Perhaps the last variable to solve for is the factor rental of the research input, $w_R$; it equals a present value of profits, where each profit flow is a fixed fraction of labor costs per product.

**Characteristics of Growth Paths**

In our economy, capital-embodied technical change—that is, technical progress in the investment goods sector—temporarily increases the growth rate and the skill premium, but it does not affect growth or the skill premium in the long run. The temporary effects of a once-and-for-all productivity increase in the investment goods sector are, however, extremely persistent. In our economy, deviations from the BGP path are persistent because induced technical
progress can be self-fulfilling, which makes the economy potentially unstable and introduces the possibility of multiple BGP. Counteracting this destabilizing force is a spillover between R&D activities devoted to productivity improvements of unskilled and skilled labor. In the next section we will show that for a calibrated version of the model economy, the research spillovers just overcome the self-fulfilling aspect of the growth process and the economy is just barely stable, which implies the high persistence of deviations from the BGP.

**The Role of Investment Technology for Growth and Wage Inequality**

The variable $q$ represents the relative productivity of the investment goods sector. One unit of final output can be transformed into one unit of consumption or $q$ units of new machines. Equivalently, $1/q$ is the relative price of new capital in terms of consumption goods. As we have just argued above, this technological parameter has no impact on long-run growth in this economy. Essentially, investment technology pins down the level of the marginal product of capital in production, but that is a level effect in this growing economy: it determines $A_k$, the productivity of installed capital that is constant over time. Growth is determined by R&D decisions, which respond to profits from innovation. Since profits are collected as a (constant) markup over costs, and costs are the expenditures on the primary factors, R&D decisions respond to factor income. The relative allocation of R&D resources toward factor-productivity improvements then depends on relative income shares. Finally, given the homogeneity of the production function, relative income shares depend on the relative input ratios ($Y_k/Y_s$ and $Y_u/Y_s$), but not on the productivity of capital per se.

This result also applies to an economy where consumers desire to smooth consumption, that is, where utility is not linear. The optimal capital-accumulation condition (30) then includes the long-run growth rate $g$, but this variable has already been determined in the R&D sector. The optimal capital-accumulation condition is still limited to the determination of $A_k$.

The skill premium of this economy is

$$\frac{w_s}{w_u} = \frac{A_u}{A_s} \frac{F_s}{F_u},$$

and it depends only on the relative input ratios. Since the relative input ratios are entirely determined in the R&D sector, the investment technology parameter $q$ does not have a long-run impact on wage inequality either. Again, a permanent increase in $q$ increases $A_k$, the productivity of installed capital, permanently; however, this variable does not influence $F_s/F_u$ in the long run. In other words, the variables $A_u/A_s$ and $K/A_s$ will adjust over time until $F_s/F_u$ returns to its initial value. Over the course of this adjustment, of course, there
are temporary effects on the skill premium, and the subject of the work below is to study these temporary effects.\footnote{Notice that the basic supplies of skilled and unskilled workers, $S$ and $U$, directly influence the long-run skill premium, even though they do not at all influence the relative total wage bills of the two groups. An interesting issue is how the endogenous accumulation of skills (e.g., education or on-the-job learning), which makes the relative supply of skilled labor endogenous, would interact with technological change to determine long-run wage inequality. We have argued before that there are limits to the extent that the relative skill endowment can be affected, and therefore we do not pursue this issue.}

Can Technology Growth Be Self-Fulfilling?

In our economy R&D decisions depend on scale: if the productivity of a primary factor is large—that is, if there are many specialized inputs using this factor—then this factor gets paid a high rental rate and receives a high income, which in turn increases the incentive to do more R&D for this factor. This argument, however, applies to all factors, and given the finite resources that can be used for R&D, what matters is the relative allocation of these resources among competing uses. Thus the behavior of relative factor incomes determines the relative allocation of R&D resources. As was pointed out by Acemoglu (2002b), the impact on relative factor incomes is connected to the substitutability features of the intermediate goods in final output production.

Consider the case of capital and skilled labor first. Suppose the productivity of skilled labor increases, that is, the relative supply of skilled-labor-based intermediate inputs increases. Since capital and labor are gross complements, the relative income of skilled labor falls, and resources are redirected toward capital accumulation. This in turn increases the relative supply of capital-based intermediate inputs, and the process is stable.

Alternatively, consider the case of skilled and unskilled labor, which are substitutes. Now an increase of the relative supply of skilled-labor-based intermediate inputs increases the income of skilled labor relative to unskilled labor, which leads to even more R&D resources devoted to the creation of skilled-labor-using specialized inputs, which in turn increases the relative supply of skilled-labor-based intermediate inputs. This productivity growth process feeds on itself and the relative productivity of skilled labor increases more and more, such that in the end the economy is effectively specialized in skilled-labor-based intermediate inputs. In order for the economy to remain stable, we need another mechanism that counteracts the scale effects: technology spillovers between the two kinds of labor. With spillovers, productivity improvements for skilled labor lower the R&D cost for unskilled labor, and if these spillovers are strong enough, they can stabilize the R&D process and prevent a complete specialization. The strength of spillovers is reflected in the parameter $\phi$: with $\phi = 1$, there are no spillovers and the strength of spillovers increases as $\phi$ declines.
The possibility of self-fulfilling productivity growth paths in our economy suggests that there might be multiple BGPs. To simplify the study of multiple BGPs, assume that the number of specialized inputs depreciates at the same rate in all sectors, $d \equiv d_u = d_i = d_k$. We can then solve equations (34) and (35) easily for the growth rate:

$$g = b_s \frac{\tilde{A}_u^{1-\phi(v-1)}}{1 + \frac{b_u}{b_n} \tilde{A}_u^{1-\phi(v-1)}(R - d_k/b_k) - d}.$$  

(41)

Note that with spillovers the growth rate is a non-monotone function of the relative productivity of unskilled labor $\tilde{A}_u$. Without spillovers ($\phi = 1$), the growth rate is a constant, independent of the relative productivity. Now use the nested CES aggregate production function (2) to derive explicit expressions for the factor income ratios:

$$\frac{w_u U}{w_s S} = \frac{\omega}{(1 - \omega)(1 - \lambda)} \left( \frac{\tilde{A}_u U}{S} \right)^\sigma \left( 1 - \lambda + \lambda \left( \frac{\tilde{A}_k \tilde{K}}{S} \right)^\rho \right)^{\frac{\rho - \sigma}{\rho}},$$  

(42)

$$\frac{w_K K}{w_s S} = \frac{\lambda}{1 - \lambda} \left( \frac{A K}{S} \right)^\rho.$$  

(43)

Inserting these two expressions in equations (38) and (39), we obtain

$$1 = \#1 \tilde{A}_u^{\sigma + (1-\phi)(v-1)} \left( 1 - \lambda + \lambda \left( \frac{\tilde{A}_k \tilde{K}}{S} \right)^\rho \right)^{\frac{\rho - \sigma}{\rho}},$$  

(44)

$$1 = \#2 \left( \frac{\tilde{A}_k \tilde{K}}{S} \right)^\rho \frac{\tilde{A}_u^{1-\phi(v-1)} 1 - \beta(1-d)(1+g)^{2-v}}{1 - \beta(1-d)(1+g)},$$  

(45)

where $\#1$ and $\#2$ are constants and $g$ depends on $\tilde{A}_u$. We now have two equations in two unknowns, $A_k \tilde{K}/S$ and $\tilde{A}_u$. They define two curves relating the two unknowns, and the balanced growth path is found as an intersection of the two curves. Is there a solution to this system, and if so, is there more than one? We will not go further here than to point out that both equations define upward-sloping curves so long as $\rho < 0 < \sigma$, which are the assumptions we use because of the data on cross elasticities between different inputs.\(^9\) And with two upward-sloping curves, multiple solutions are not only possible but, as we have verified numerically, hard to avoid in this framework. This is in contrast to the setups in Acemoglu (2002b, 2003), which deliver unique steady states. Because of our three-factor setup here, multiplicity is hard to avoid.

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\(^9\)To simplify the exposition, we treat the growth rate in equation (45) as a constant; that is, we ignore the feedback from equation (41). The dependence of $g$ on $\tilde{A}_u$ may cause non-monotonicities, but that is only a local property; globally, the equation defines an upward-sloping relation.
The interpretation is the one hinted at in several places above. On the right-hand side of equation (44) is the relative return on R&D with respect to productivity improvements of unskilled to skilled labor, and on the right-hand side of equation (45) is the relative R&D return with respect to capital and skilled labor. The two unknowns are the relative productivities of unskilled labor and capital (relative to that of skilled labor; in the case of capital, we measure the stock times the productivity). In equation (44), a higher productivity of unskilled labor raises the relative return on unskilled labor, because skilled and unskilled labor are substitutes ($\sigma > 0$) and because of the market size effect $((1 - \phi)(\nu - 1) > 0)$ if there are spillovers. To balance the increased relative return of unskilled labor, the productivity of capital has to increase. Because of capital-skill complementarity, $\sigma > 0 > \rho$, the higher capital productivity increases the return to skilled labor. In equation (45), an increase in the productivity of unskilled labor gives a reinforcing scale effect, because it can be viewed as a relative decrease in the productivity of capital, which is balanced in this case by an increase in the direct productivity of capital, since skilled labor and capital are complements ($\rho < 0$).

When there are multiple balanced growth paths, it is important to check “local stability” of each of these: do small deviations of the state variables from the balanced growth path lead back to the balanced path or do they lead away from it? In our numerical examples, we found one stable and one unstable path, the last of which is economically irrelevant (since no initial conditions would lead there). We also found cases where there is only one, unstable balanced growth path. In this case, the scale effects are simply too strong to admit convergence to a balanced outcome: any deviations from the balanced path would lead away from it. We tend to find at least one stable equilibrium when the spillovers are strong, i.e., when $\phi$ is low, and when knowledge depreciation is high, i.e., when $d$ is close to one.

2. CALIBRATION

Our intention is to provide a quantitative statement on how a decline of the relative price of capital affects wage inequality. Furthermore, our model is sufficiently complicated such that we cannot analytically characterize the stability properties of its balanced growth path. We therefore solve the model numerically, and in order to do this we have to decide what are empirically relevant values of the model's parameters. In the following we parameterize the economy such that its balanced growth path is consistent with observations on the U.S. economy in the latter part of the twentieth century.

We assume that a time period represents one year, and we choose the time discount factor $\beta$ such that the annual interest rate is 4 percent. The annual depreciation rate for equipment capital in the United States is $\delta = 0.125$. KORV (2000) estimate the elasticity parameters for the two-stage CES
production function (2) as $\rho = -0.5$ and $\sigma = 0.4$. We set the specialized input parameter $\nu = 11$ such that the equilibrium markup is 10 percent above marginal cost, $\mu = 1.1$. This choice is at the upper bound for estimates of profit rates in the U.S. economy. Acemoglu (2002a) provides various estimates of the factor income ratios of skilled to unskilled labor. We set the ratio $w_s S/w_u U = 0.5$, which corresponds to Acemoglu’s estimate of this ratio in the 1990s for a broad definition of skilled labor. We set the capital income share in final output to one-third, which roughly corresponds to the capital income share in the United States.

Estimates by the National Science Foundation (NSF) suggest that in the United States R&D expenditures are less than 3 percent of GDP. The NSF estimates include public and private expenditures on R&D. In the following we interpret the R&D input as a type of labor and include the value of R&D inputs in the model economy’s measure of GDP. Conditional on the factor income shares and assuming equal depreciation rates of knowledge, $d = d_j$, the R&D share in GDP determines the depreciation rate $d$. The R&D share in GDP is increasing in $d$, and with $d = 0$ the R&D share is 4.9 percent conditional on the other income shares. The BGP equilibrium is not stable for $d = 0$, but we obtain a stable BGP for $d = 0.01$, which implies a BGP R&D share of 5.9 percent. In the following we interpret the R&D input as another type of skilled labor.

No quantitative evidence is available on the R&D externality. We set the R&D externality parameter for skilled and unskilled labor to $\phi = 0.5$. Larger externalities, smaller $\phi$, have no appreciable impact on the medium-term to long-term dynamics. Smaller externalities, larger $\phi$, make the effects of shocks more persistent, but for $\phi$ approaching 0.9 we can no longer find a stable BGP.

Direct observations on $\tilde{A}_u$, $Y_u/Y_s$, or $Y_k/Y_s$, are also not available. For the calibration exercise the values of these variables and of $S/U$, $\omega$, and $\lambda$ are not determined. This is not a problem since, conditional on the calibration so far, the local dynamics around the BGP are independent of the choice for these variables. In the following we normalize $\tilde{A}_u = Y_u/Y_s = Y_k/Y_s = 1$.

3. RESULTS

In the previous discussion of the BGP we argue that a permanent change of the relative price of capital does not affect the economy’s long-run growth rate, factor income shares, or skill premium. We now want to argue that even though the effects of a permanent change in the relative price of capital are transitory, they are nevertheless very persistent. For this purpose we perform two experiments. First, we show that in response to a one-time permanent decline of the relative price of capital, the relative wage of skilled labor and the wage income share increases and these effects are extremely persistent. In
In the introduction we point out that capital-embodied technical change is not a one-time event, but an ongoing process. In a second experiment we therefore model ongoing embodied technical change through successive reductions of the relative price of capital and show that the skill premium and labor income share increase significantly over time and stay above their long-run values for a very long time.

We study a local approximation of the dynamic response of our economy to an exogenous shock. Since our economy is growing over time, we first have to transform the dynamic system such that all variables are stationary. This is possible since we study a BGP where all variables grow at constant rates. The state variables of the transformed system are \( (A_{k,t}, \tilde{K}_t, \tilde{A}_{u,t}) \).

A preliminary observation is worth making before going into the details of the experiment. If one computes the relative wages of skilled and unskilled workers in this economy treating productivity and investment levels as exogenous, it is apparent that an increase in \( q \), which automatically increases the capital stock, must increase the relative wage of skilled labor because \( \sigma > \rho \). This can easily be seen by taking the ratio of \( F_s \) to \( F_u \) and using \( \sigma > \rho \): this expression is increasing in \( K \). This essentially is the argument in KORV (2000) about why the skill premium has been increasing. Our main question below is, how does capital accumulation and endogenous directed technical change respond to the fall in the price of new capital goods?
**Experiment 1: A Permanent 1 Percent Decline of the Relative Price of Capital**

Figure 1 shows the response of the state variables to a 1 percent permanent decline of the relative price of capital. On impact, as investment in capital becomes more attractive and the economy starts to accumulate more capital and more resources are devoted to the improvement of capital productivity, both $\tilde{K} \equiv K/A_s$ and $A_k$ increase. After the initial impact, the economy devotes more resources to the improvement of skilled labor productivity since skilled labor and capital are complementary in production, and consequently the relative productivity of unskilled labor $\tilde{A}_u \equiv A_u/A_s$ declines. We have argued above that the BGP value of $\tilde{A}_u$ is independent of the relative price of capital and the relative productivity of unskilled labor returns to its long-run value over time. On the other hand, the BGP values of the normalized capital stock and the productivity of capital depend on the price of capital. In particular, the productivity of capital declines and the capital stock increases with the decline of the relative price of capital. From Figure 1 it is apparent that the shock has a very persistent impact on the state of the economy. Recall that one period represents a year. Even after 1,000 years the economy still has a long way to go to arrive at its new BGP.

The economy’s GDP growth, the labor income share, the skill premium, and the relative wage of R&D labor all increase following a decline of the relative price of capital (see Figure 2).\(^{10}\) As discussed above, the BGP growth rate, labor income share, and relative wages are independent of the relative price of capital. Whereas the impact on the growth rate dissipates very fast, the effect on relative wages and the labor income share is very persistent.\(^{11}\) The quantitative effect of a one-time 1 percent reduction of the price of capital is small; for example, the skill premium increases by less than 1 percent.

**Experiment 2: A Sequence of Relative Price of Capital Reductions**

In our economy a BGP does not exist if the relative price of capital declines at a constant rate. In order to model the effects of the observed secular decline of the relative price of capital, we therefore assume that this price declines at a constant rate for 100 years and then remains constant forever. We base our

\(^{10}\) The substantial volatility for the GDP growth rate can be attributed to the fact that preferences are linear in consumption. With concave utility in consumption, there would be an incentive to smooth consumption and we would not see the wild swings in the GDP growth rate.

\(^{11}\) It may appear odd that the relative wage of R&D labor is less than the wage of unskilled labor, but remember that we have said nothing about the units of R&D labor embodied in an R&D worker. Thus, the scale of the relative wage is arbitrary. The same can be said about the relative wage of skilled and unskilled workers.
study of the medium- to long-run effects of the price decline on a local approx-
imation of the economy’s dynamics. We therefore want to avoid deviating too
much from the BGP and limit ourselves to a 0.5 percent annual rate of price
decline. This is substantially less than the 3 percent annual rate of decline for
the relative price of equipment capital observed for the United States (Green-
wood, Hercowitz, and Krusell 1997). Our example should therefore only be
interpreted as a quantitative illustration of the effect of an ongoing decline of
the relative price of capital.

Figure 3 shows that as long as the relative price of capital declines, the
economy’s growth rate, labor income share, and relative wage of skilled labor
and R&D labor all increase. The impact of the capital price decline on relative
wages is quantitatively important. Over the 100-year decline of the relative
price of capital, the skill premium—that is, the price of skilled labor relative
to unskilled labor—increases by about 40 percent, and the relative price of
labor employed in the R&D sector increases by 15 percent. The change of
the GDP growth rate and the labor income share would not be noticeable in
the data. Changes of the magnitude implied by the model, one-tenth of a
percentage point for the growth rate and half a percentage point for the labor
income share, are dominated by other business-cycle-related fluctuations of these variables. Finally, all variables return to their initial BGP values once the relative price of capital no longer declines, but this process occurs at a very slow rate.

4. CONCLUSION

We find in this paper that a permanent decline in the relative price of capital has long-lasting, but not permanent, effects on wage inequality. In particular, we find that cheaper capital goods initially raise the relative wage of skilled workers due to capital-skill complementarity. In addition, cheaper capital goods also initially induce more technical change to augment the existing capital stock—a “scale effect” due to the incentives to do R&D—which works toward even larger wage inequality. However, in our model economy, the other factors of production eventually respond due to (a) complementarity in production with skilled labor and (b) spillovers from research into skilled-labor intensive industries to those mainly using unskilled labor. We do not have any way of knowing how strong such spillovers are; in the model we
assume that they are strong enough to counteract the initial impulse toward inequality. If they are in fact weaker than that, the long-run outcome likely would make the share of total income accruing to unskilled workers go to zero.

Our analysis focuses on how the initial impulse—the fall in the price of new capital—induces directed R&D. We have, however, abstracted from incentives to accumulate skill. In response to a higher wage premium to skill, one would expect more skill accumulation. How strong this effect is in reality is an open question. How it would interact with the other factors driving long-run inequality in our model is also an open question. We leave these interesting questions, as well as policy analysis, for future directed research.

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