Monetary Policy and the Term Structure of Interest Rates

Bennett T. McCallum

A major puzzle in financial economics is the apparent drastic inconsistency of U.S. data with the expectations theory of the term structure of interest rates.\footnote{This article is a slightly revised version of NBER Working Paper 4938, issued in November 1994, which has been cited and utilized by a number of authors but not previously published. A few expositional changes have been made and Section 5 has been added to fill crucial gaps in the argument and to include a few references to subsequent work.} As documented extensively by Campbell and Shiller (1991), both short changes in long rates and long changes in short rates fail to be related to existing long-short spreads in even approximately the manner implied by the expectations theory together with rational expectations; a convenient summary of the evidence is provided by Campbell (1995, Table 2). This failure is analogous, however, to the apparent drastic failure of uncovered interest parity in foreign exchange, which can be rationalized—it is argued by McCallum (1994)—as a consequence of monetary policy behavior that is ignored in the usual regression tests. In the present article it is shown that a similar result is applicable to the term-structure puzzle. In particular, the above-mentioned failure is shown to be a plausible consequence of monetary policy behavior that features interest rate smoothing in combination with policy responses to movements in the long-short spread.\footnote{General aspects of the failure are discussed by Cook and Hahn (1990), Campbell and Shiller (1991), Fama (1984), Mankiw and Summers (1984), and Evans and Lewis (1994), among others.} This explanation is...
entirely consistent with, but more general and more fully developed than, the one proposed in a notable study by Mankiw and Miron (1986). The article’s organization is as follows. In Section 1, the term-structure puzzle is reviewed and the article’s rationalization is developed for the simplest two-period case. Then in Section 2, the analysis is extended to long rates of greater maturity. Additional evidence is developed in Section 3 after which the article’s original conclusion appears as Section 4. Then a short review of more recent developments is included in Section 5, where an important difficulty neglected in the original version is described together with a resolution due to Romhányi (2002). Important subsequent work by Kugler (1997), Hsu and Kugler (1997), Dai and Singleton (2002), Gallmeyer, Hollifield, and Zin (2005), and others is briefly discussed.

1. TWO-PERIOD CASE

We begin by considering the basic issue and our proposed explanation for the two-period case, i.e., for the relationship between yields on one-period and two-period bonds, denoted \( r_t \) and \( R_t \) respectively. Assuming that the securities in question are pure discount bonds, the expectations theory of the term structure posits that the “long” rate \( R_t \) is related to \( r_t \) and the expected future short rate \( E_t r_{t+1} \) as follows:

\[
R_t = 0.5(r_t + E_t r_{t+1}) + \xi_t. \tag{1}
\]

Here \( E_t r_{t+1} = E(r_{t+1} \mid \Omega_t) \) with \( \Omega_t = \{r_t, r_{t-1}, ..., R_t, R_{t-1}, ... \} \) so we are assuming rational expectations. The random variable \( \xi_t \) is a “term premium” that is often assumed constant. Defining the expectational error \( \epsilon_{t+1} = r_{t+1} - E_t r_{t+1} \), equation (1) implies

\[
0.5(r_{t+1} - r_t) = (R_t - r_t) - \xi_t + 0.5\epsilon_{t+1}. \tag{2}
\]

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3 After first drafting the article I became aware of a study with a rather similar objective by Rudebusch (1994), which is also intended to provide a generalization of the Mankiw-Miron hypothesis. The type of policy behavior assumed there is quite different, however, as instrument settings are responsive to current conditions in my setup but are determined exogenously in his. Most significantly, Rudebusch’s analysis does not offer an explanation for the empirical phenomena rationalized below at the end of Sections 2 and 3.

4 The relationship is exact, if the interest rates are based on continuous compounding, or an approximation otherwise: see Shiller (1990).

5 Terminologically, many writers define the expectations hypothesis in a manner that requires that \( \xi_t \) is a constant. Campbell (1995), for example, does so and also defines the “pure expectations theory” as implying that the constant is zero. The definition used in this article permits a time-varying \( \xi_t \) but requires that (in the present case) \( R_t \) must move point for point with \( 0.5(r_t + E_t r_{t+1}) \) for any given value of \( \xi_t \).
Then if $\xi_t$ is assumed constant, $\xi_t = \xi$, the orthogonality of $\epsilon_{t+1}$ with $R_t$ and $r_t$ implies that the slope coefficient $\beta$ in a regression of the form

$$0.5(r_t - r_{t-1}) = \alpha + \beta(R_{t-1} - r_{t-1}) + \text{disturbance} \tag{3}$$

should have a probability limit of 1.0. An estimated value significantly different from 1.0 is inconsistent with either the expectations theory or one of the maintained hypotheses.

In fact, it has been documented by many researchers that slope coefficients tend to be well below 1.0 in post-1914 data for the United States, often significantly so in terms of estimated standard errors. Point estimates obtained in a number of studies are reported in Table 1. There we see that the slope coefficient values are all well below 1.0, with the exception of Mankiw and Miron’s value for 1890-1914 and Campbell and Shiller’s final value.\(^6\) The former, which pertains to observations taken before the founding of the Federal Reserve, will be discussed in Section 3. The latter is accompanied by a rather large asymptotic standard error that, according to Campbell and Shiller (1991, 510), seriously understates “the true uncertainty about the regression coefficients” due to finite-sample bias.\(^7\)

One possible explanation for these findings is, of course, that the expectations theory is simply untrue—but the quantitative extent of the discrepancy seems implausibly large. Another possibility is invalidity of the rational expectations (RE) hypothesis,\(^8\) but it seems unlikely that the same general type of systematic expectational error would prevail over different sample periods. Also, it would again appear that the magnitude of the discrepancy is too large to be explained by a departure from expectational rationality.\(^9\) In any event, my proposed explanation is that $\xi_t$ is not constant—i.e., that there is a variable term premium—and that monetary policy is conducted in a manner to be explained momentarily. The process generating $\xi_t$ is assumed to be covariance stationary but not necessarily white noise. For specificity, the $\xi_t$ process will be taken to be autoregressive of order one [AR (1)]:

$$\xi_t = \rho \xi_{t-1} + u_t. \tag{4}$$

Here $u_t$ is white noise and $| \rho | < 1.0$. To this writer it seems implausible that there would not be some period-to-period variability in the discrepancy $\xi_t$.

\(^6\) An analogous result holds for the case of three-month and one-month rates; see Kugler (1988, 1990).

\(^7\) The Roberti, Runkle, and Whiteman (1993) results are for Treasury bills. This study also reports results using federal funds and repo securities and finds one slope coefficient close to 1.0 for the former, using the sample period 1979.10–1982.10.

\(^8\) This possibility has been explored, using survey data on expectations, by Froot (1989).

\(^9\) This point has also been made by Dotsey and Otrok (1995).
Table 1 Empirical Results, Two-Period Case

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample Period</th>
<th>Short Rate</th>
<th>Slope Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mankiw &amp; Miron (1986)</td>
<td>1959–1979</td>
<td>3 mo.</td>
<td>0.23</td>
</tr>
<tr>
<td>Mankiw &amp; Miron (1986)</td>
<td>1951–1958</td>
<td>3 mo.</td>
<td>-0.33</td>
</tr>
<tr>
<td>Mankiw &amp; Miron (1986)</td>
<td>1934–1951</td>
<td>3 mo.</td>
<td>-0.25</td>
</tr>
<tr>
<td>Mankiw &amp; Miron (1986)</td>
<td>1915–1933</td>
<td>3 mo.</td>
<td>0.42</td>
</tr>
<tr>
<td>Mankiw &amp; Miron (1986)</td>
<td>1890–1914</td>
<td>3 mo.</td>
<td>0.76</td>
</tr>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>0.42</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>0.50</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>3 mo.</td>
<td>-0.15</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>12 mo.</td>
<td>-0.02</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>60 mo.</td>
<td>2.79</td>
</tr>
<tr>
<td>Fama (1984)</td>
<td>1959–1982</td>
<td>1 mo.</td>
<td>0.46</td>
</tr>
<tr>
<td>Roberds, Runkle &amp; Whiteman (1993)</td>
<td>1984–1991</td>
<td>3 mo.</td>
<td>-0.01</td>
</tr>
<tr>
<td>Roberds, Runkle &amp; Whiteman (1993)</td>
<td>1975–1979</td>
<td>3 mo.</td>
<td>0.43</td>
</tr>
</tbody>
</table>

in (1), a random component that reflects changes in tastes regarding the need for financial flexibility or any of a myriad of other disturbing influences, none major enough to justify separate recognition. In any event, it is not the case that the inclusion of a random $\xi_t$ disturbance in (1) converts the expectations theory into a tautology. That would be true if $\xi_t$ were related to $r_t$, $E_r r_{t+1}$, and $R_t$ as in (1) without restriction. But instead the present assumption is that $\xi_t$ is exogenous with respect to $r_t$ and $R_t$. This reflects the idea that the expected one-period holding yields on one-period and two-period bonds are equal up to a constant plus a random disturbance, i.e., that these yields differ from that constant only randomly. This is, for the case at hand, the essence of the expectations theory.

Regarding monetary policy, our hypothesis begins with the observation that actual policy behavior in the United States (and many other nations) involves manipulation of a short-term interest rate “instrument” or “operating target.” Specifically, we assume that

$$r_t = \sigma r_{t-1} + \lambda (R_t - r_t) + \xi_t,$$

(5)

10 For values of $\sigma$ less than 1.0, a constant term should also be included in (5) if $E_{\xi_t} = 0$. We have not shown it here, however, because the case with $\sigma = 1$ will be featured below and because little interest attaches to the constant term in any case.
where $\sigma \geq 0$ is presumed to be close to 1.0 and $\lambda \geq 0$ to be smaller than 2.11 Thus there is a considerable element of interest rate “smoothing”—keeping $r_t$ close to $r_{t-1}$—and also a tendency to tighten policy (by raising $r_t$) whenever the spread $R_t - r_t$ is larger than normal. Whether this reaction to $R_t - r_t$ occurs because the central bank views it as a good predictor of future output growth or as a good indicator of recent policy laxity does not matter for current purposes. The final term $\zeta_t$ reflects other components of policy behavior. It would not impair our analysis to let $\zeta_t$ be autocorrelated, but it would not help, either. Accordingly, we shall assume that $\zeta_t$ is white noise.

It may be helpful to briefly consider the rationale for the specification of policy behavior in (5). Regarding the $r_{t-1}$ term, there exists some controversy regarding the reason behind central banks’ proclivity for interest rate smoothing—and, indeed, for their use of interest rate instruments. But there is virtually no disagreement with the proposition that the Fed—and other major central banks—have in fact employed such practices during most (if not all) of the last 50 years.12 (For some useful discussion, see Goodfriend [1991] and Poole [1991]). In addition (5) reflects the assumption that the central bank tends to tighten policy when the spread $R_t - r_t$ is large. One possible rationalization is that the spread is an indicator of monetary policy expansiveness, as suggested by Laurent (1988), so that an unusually high value indicates the need for corrective action. A different idea is that the spread provides an indicator of the state of the economy from a cyclical perspective. Various investigators, including Estrella and Hardouvelis (1991) and Hu (1993), have documented that spread measures have predictive value for future real GNP growth rates. Also, Mishkin (1990) has shown that a spread variable has some predictive content for future inflation rates. Thus an attempt by the central bank to conduct a forward-looking countercyclical policy would call for a response of the type indicated in (5), i.e., a tightening when $R_t - r_t$ is high.13 Admittedly,
in actual practice the Fed has used other predictor variables in addition to or instead of the spread. But to the extent that these and the spread are useful predictors, the policy response would be much the same as implied by (5).

Relations (1) and (5) constitute only a portion, of course, of a macroeconomic system. But if we assume that the disturbances $\xi_t$ and $\zeta_t$ are independent of those in the remaining relations, the system will be recursive and the subsystem (1)(5) will determine $r_t$ and $R_t$ without reference to the other variables or shocks. Whether the remainder of the model does or does not feature relations of the IS-LM type is irrelevant, for example, as is the extent to which prices of goods are flexible. Let us consider, then, a rational expectations solution to the system (1)(5).14

Presuming that attention is to be focused on the fundamental or bubble-free solution yielded by the minimal-state-variable (MSV) criterion discussed by McCallum (1983), we combine (1) and (5) to yield

$$\left(1 + \lambda\right)r_t = \sigma r_{t-1} + \lambda\left[0.5\left(r_t + E_t r_{t+1}\right) + \xi_t\right] + \zeta_t \quad (6)$$

and seek values of the undetermined coefficients $\phi_0$, $\phi_1$, $\phi_2$, and $\phi_3$ that will provide a $r_t$ solution of the form

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t. \quad (7)$$

Clearly, the latter implies that $E_t r_{t+1} = \phi_0 + \phi_1 \phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t + \phi_2 \rho \xi_t$, so we substitute these into (6) to obtain

$$\left(1 + \lambda\right)[\phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t] = \sigma r_{t-1} + \lambda\left[0.5\left(\phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t\right) + 0.5\left(\phi_0 + \phi_1 \phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t\right) + \phi_2 \rho \xi_t + \phi_2 \rho \xi_t\right] + \xi_t + \zeta_t. \quad (8)$$

Thus for (7) to be a solution—i.e., to hold for all $\xi_t$, $\zeta_t$ realizations—it must be true that:

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14 Students of the price level determinacy literature—e.g., McCallum (1981)(1986), Dotsey and King (1983), Canzoneri, Henderson, and Rogoff (1983)—will wonder about the absence of nominal variables in the system (1)(5). But the price level can be brought in by adding (e.g.) an IS-type relation in which a real rate such as $r_t - (E_t p_{t+1} - p_t)$ appears, $p_t$ being the log of the price level. Then determinacy of $p_t$ will require the presence of an additional term in the policy rule (5), one that includes a nominal variable such as $p_t$ or $E_t p_{t+1}$. Algebraic analysis becomes much more difficult because the counterpart of (10) below will be a cubic in many such cases. But a cubic must have at least one real root, so in principle determinacy can be investigated. My examination of a case with $p_t$ included in (5) indicates that determinacy would be guaranteed unless $\sigma = 1.0$ exactly. Thus for $\sigma$ close to 1.0, the results would be approximately the same as those emphasized below.
\begin{align}
(1 + \lambda) \phi_0 &= \lambda \phi_0 + 0.5 \lambda \phi_1 \phi_0 \\
(1 + \lambda) \phi_1 &= \sigma + 0.5 \lambda \phi_1 + 0.5 \lambda \phi_1^2 \\
(1 + \lambda) \phi_2 &= 0.5 \lambda \phi_2 + 0.5 \lambda \phi_1 \phi_2 + 0.5 \lambda \rho \phi_2 + \lambda \\
(1 + \lambda) \phi_3 &= 0.5 \lambda \phi_3 + 0.5 \lambda \phi_1 \phi_3 + 1.
\end{align}

The second of these is satisfied by two values of \( \phi_1 \), namely,

\[ \phi_1 = \frac{(1 + 0.5 \lambda) \pm [(1 + 0.5 \lambda)^2 - 2 \lambda \sigma]^{1/2}}{\lambda}, \]

but the MSV criterion implies that the one with the minus sign is relevant.\(^{15}\)

Then the remaining coefficients are straightforwardly given by the other three equalities in (9).

In analyzing the implications of this solution it will be useful to emphasize the important special case involving \( \sigma = 1 \), which is the value suggested by interest rate smoothing behavior. When \( \sigma = 1 \), the MSV solution for \( \phi_1 \) becomes \( [(1 + 0.5 \lambda) - (1 - 0.5 \lambda)]/\lambda = \lambda/\lambda = 1 \) and the other three equalities in (9) are simplified considerably. They yield \( \phi_0 = 0, \phi_2 = \lambda/(1 - 0.5 \rho \lambda), \) and \( \phi_3 = 1 \) so the solution for \( r_t \) is

\[ r_t = r_{t-1} + \frac{\lambda}{(1 - 0.5 \rho \lambda)} \xi_t + \xi_t \] (11)

Furthermore, \( E_t r_{t+1} - r_t = \phi_2 \rho \xi_t \), so we find that the spread obeys

\[ R_t - r_t = 0.5(E_t r_{t+1} - r_t) + \xi_t = (1 - 0.5 \rho \lambda)^{-1} \xi_t, \] (12)

Finally, equations (11) and (4) imply

\[ r_t - r_{t-1} = \frac{\lambda \rho}{1 - \lambda \rho/2} \xi_{t-1} - 1 - \lambda \rho/2 u_t + \xi_t, \] (13)

so we can combine (12) and (13) to obtain

\[ 0.5(r_t - r_{t-1}) \frac{\lambda \rho}{2} (R_{t-1} - r_{t-1}) + \frac{\lambda}{1 - \rho \lambda/2} u_t + 0.5 \xi_t. \] (14)

But here \( u_t \) and \( \xi_t \) are uncorrelated with \( R_{t-1} - r_{t-1} \), so (14) represents a population version of the regression described in (3). Thus the slope coefficient in (3) is a consistent estimator of \( \rho \lambda/2 \), so the analyst should anticipate a slope

\(^{15}\) This is the root that yields \( \phi_1 = 0 \) when \( \sigma = 0 \), a special case in which it is clear that \( r_{t-1} \) would be an extraneous state variable (as discussed in McCallum [1983]).
well below 1.0. Indeed, if $\xi_t$ were white noise, with $\rho = 0$, a slope coefficient of zero would be implied—even though relation (1) is the main behavioral relation of the system. That result demonstrates, I would suggest, not only that the usual regression test is inappropriate but also that it is misleading to think of the expectations theory in terms of the “predictive content” of the spread for future changes of the short rate.\textsuperscript{16} Such predictive content is not a necessary implication of that theory.

In addition, a zero slope coefficient would be implied if $\lambda = 0$, i.e., if the central bank did not respond to the current value of the spread but simply set $r_t$ equal to $r_{t-1}$ (plus, perhaps, $\zeta_t$). This special case, of the special case with $\sigma = 1$, represents the hypothesis of Mankiw and Miron (1986)—that the Federal Reserve has practiced extreme interest rate smoothing and thereby induced short rates to approximate a random walk process in their behavior. Our result strongly supports the general idea of the Mankiw and Miron hypothesis, but shows that it holds much more generally (i.e., even if $r_t$ behavior is not that of a random walk).\textsuperscript{17}

A few readers have remarked that (14) appears to be inconsistent with the fact that a regression of form (3) should yield a slope coefficient of 1.0 in the special case in which the term premium $\xi_t$ is a constant. But with $\sigma = 1.0$ in (5), a constant $\xi_t$ implies that $R_t - r_t$ is also constant—see equation (12). Thus there is a degenerate regressor, in this case, so the regression cannot be conducted. And in the case with $\sigma < 1.0$, (14) does not apply, so again there is actually no inconsistency.

Let us now briefly consider the situation with $\sigma < 1.0$. In such cases we would need to include a non-zero constant term in (5) to permit a stationary equilibrium with $E\zeta_t = 0$. The solution in this case yields a relationship analogous to (14) that is less tidy than the latter and includes additional predetermined variables. But it remains true that the probability limit of the slope coefficient in a regression of $r_t - r_{t-1}$ on $R_{t-1} - r_{t-1}$ is not in general equal to 1.0 and is most likely to be smaller than 1.0; a demonstration is provided in the Appendix. Accordingly, the same general message applies as in the more tractable case with $\sigma = 1.0$. That message is that the realization of (say) a positive value of $\xi_t$ will drive up $R_t$ relative to $r_t$ via (1). But then $R_t - r_t$ will be negatively correlated with the composite disturbance $-\xi_{t-1} + 0.5\epsilon_{t+1}$ in

\textsuperscript{16}The claim here is not that it is inappropriate to estimate a relation of the form (3), but only that it is inappropriate to view a test of $\beta = 1$ as a test of the expectations theory.

\textsuperscript{17}One reader has pointed out correctly that the formal analysis based on (14) presumes that policy response is to the current-period spread, not a lagged value. The argument of the present paragraph suggests that the downward-bias effect would be present, nevertheless, if response was to the lagged spread. In any case, the timing assumed in (5) is consistent with that in much of the recent literature such as Rudebusch and Wu (2004) or Bekaert, Cho, and Moreno (2005) when periods are interpreted as months or six-week intervals.
(3), implying that least-squares estimation of (4) will yield a slope coefficient that has a probability limit not equal to 1.0.

2. N-PERIOD CASE

Now we turn to the more interesting case in which the long rate, $R_t$, is for a bond with a maturity of more than two periods. In this case an approximation to the expectations-hypothesis relationship between $R_t$ and $r_t$ can be written as

$$R_t - NE_t(R_{t+1} - R_t) = r_t + \xi_t,$$

(15)

where $N+1$ is a measure of the duration of the long rate. In (5) the left-hand side is an approximation to the one-period holding return on the long-rate bond since $N(E_tR_{t+1} - R_t)$ is the (approximate) expected capital loss on the long bond. (The inexactness arises because the term $R_{t+1}$ should pertain to a maturity one period less than that for $R_t$.) Thus for bonds with a distant maturity date, the approximation should be adequate.

In this case the apparent empirical failure to be explained arises from writing (15) as

$$N(R_{t+1} - R_t) = (R_t - r_t) - \xi_t + N\epsilon_{t+1},$$

(16)

where $\epsilon_{t+1} = R_{t+1} - E_tR_{t+1}$ is an expectational error that with RE is uncorrelated with $R_t$ and $r_t$. Thus if $\xi_t$ were constant, the slope coefficient in a regression of $N(R_{t+1} - R_t)$ on $R_t - r_t$ should have a probability limit of 1.0, according to the expectations theory. But such regressions again actually yield slopes well below 1.0 with U.S. data. Indeed, the values reported by Evans and Lewis (1994) and Campbell and Shiller (1991) are predominantly negative, as is documented in Table 2, and increase in absolute value with $N$.  

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18 For pure discount bonds, $N+1$ is the maturity.

19 Equation (15) is based on the expression $R_t = (1 - \delta) \sum^{\infty} E_t r_{t+k} + \text{term premium}$, with the summation from 0 to $\infty$, i.e., an infinite-maturity version of the linearization developed by Shiller (1979), with $N = \delta/(1 - \delta)$. An exposition is provided by Mankiw and Summers (1984, pp. 226-7). This approximation has also been used by Shiller, Campbell, and Schoenholtz (1983), Campbell and Shiller (1991), Fuhrer and Moore (1993), and Hardouvelis (1994). The reason this approximation is adopted here is so that only two maturities—two endogenous variables—will be involved in the model.
Table 2 Empirical Results, N-Period Case

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample Period</th>
<th>Short Rate</th>
<th>N+1</th>
<th>Slope Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>2</td>
<td>-0.17</td>
</tr>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>4</td>
<td>-0.70</td>
</tr>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>6</td>
<td>-1.27</td>
</tr>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>8</td>
<td>-1.52</td>
</tr>
<tr>
<td>Evans &amp; Lewis (1994)</td>
<td>1964–1988</td>
<td>1 mo.</td>
<td>10</td>
<td>-1.89</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>4</td>
<td>-0.44</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>6</td>
<td>-1.03</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>24</td>
<td>-1.81</td>
</tr>
<tr>
<td>Campbell &amp; Shiller (1991)</td>
<td>1952–1987</td>
<td>1 mo.</td>
<td>120</td>
<td>-5.02</td>
</tr>
<tr>
<td>Hardouvelis (1994)</td>
<td>1954–1992</td>
<td>3 mo.</td>
<td>120</td>
<td>-2.90</td>
</tr>
</tbody>
</table>

As in the last section, we assume that the policy reaction equation (5) obtains with \( \lambda < 1/N \) and that \( \xi_t = \rho \xi_{t-1} + u_t \).\(^{20}\) Then one can combine (5) and (15) to obtain

\[
(1 + N)R_t = NE_tR_{t+1} + (1 + \lambda)^{-1}[\sigma r_{t-1} + \lambda R_t + \zeta_t] + \xi_t. \tag{17}
\]

The MSV solution will be of the form

\[
R_t = \pi_1 r_{t-1} + \pi_2 \xi_t + \pi_3 \zeta_t, \tag{18}
\]

implying \( E_tR_{t+1} = \pi_1(1 + \lambda)^{-1}[\sigma r_{t-1} + \lambda(\pi_1 r_{t-1} + \pi_2 \xi_t + \pi_3 \zeta_t) + \zeta_t] + \pi_2 \rho \xi_t \), which can be substituted with (18) into (17) to give

\[
(1 + N)[\pi_1 r_{t-1} + \pi_2 \xi_t + \pi_3 \zeta_t] =
N\pi_1(1 + \lambda)^{-1}[\sigma r_{t-1} + \lambda(\pi_1 r_{t-1} + \pi_2 \xi_t + \pi_3 \zeta_t) + \zeta_t] +
N\pi_2 \rho \xi_t + (1 + \lambda)^{-1}[\sigma r_{t-1} + \lambda(\pi_1 r_{t-1} + \pi_2 \xi_t + \pi_3 \zeta_t) + \zeta_t] + \xi_t. \tag{19}
\]

For (18) to be a solution, then, we must have

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\(^{20}\)The condition \( \lambda < 1/N \) is the condition to prevent infinite discontinuities in \( \pi_2 \). It is analogous to, although different than, the condition \( \lambda < 2 \) for the two-period case (presumably because of the approximation in (15)) and is again assumed but not strictly required. That the larger is \( N \), the smaller should be \( \lambda \), is intuitive because the response in (5) is now to only one of the various long rates that could be considered.
\[(1 + N)\pi_1 = N\pi_1(1 + \lambda)^{-1}(\sigma + \lambda \pi_1) + (1 + \lambda)^{-1}(\sigma + \lambda \pi_1)\]
\[(1 + N)\pi_2 = N\pi_1(1 + \lambda)^{-1}\lambda \pi_2 + N\pi_2 \rho + (1 + \lambda)^{-1}\lambda \pi_2 + 1\]
\[(1 + N)\pi_3 = N\pi_1(1 + \lambda)^{-1}(\lambda \pi_3 + 1) + (1 + \lambda)^{-1}(\lambda \pi_3 + 1).\]

The first of these amounts to \((1 + \lambda)(1 + N)\pi_1 = (N\pi_1 + 1)(\sigma + \lambda \pi_1)\), so we have
\[\pi_1 = \frac{[(1+\lambda)(1+N) - \lambda - N\sigma] \pm \sqrt{[(1+\lambda)(1+N) - \lambda - N\sigma]^2 - 4N\lambda \sigma}}{2N\lambda}.\] 

The term in square brackets will be positive, so the MSV solution for \(\pi_1\) is the expression in (21) with the minus sign.\(^{21}\) Given this value, the second and third of equations (20) determine \(\pi_2\) and \(\pi_3\).

To facilitate analysis, let us again focus attention on the case with \(\sigma = 1\). Then we have \([(1 + \lambda)(1 + N) - (\lambda + N))^2 = (1 + \lambda)^2(1 + N)^2 - 2(1 + \lambda)(1 + N)(\lambda + N) + (\lambda + N)^2 = 1 + 2N\lambda + N^2\lambda^2\), and the term inside curly brackets in (21) becomes \(1 - 2N\lambda + N^2\lambda^2 = (1 - N\lambda)^2\). Consequently, we have \(\pi_1 = [(1 + N\lambda) - (1 - N\lambda)]/(2N\lambda) = 1\). Then with \(\pi_1 = 1\), the final equation in (20) implies \(\pi_3 = 1\) and \(\pi_2 = (1 + \lambda)/(1 + N - N\rho(1 + \lambda))\).

Because \(1 > N\lambda\), \(\pi_2\) is strictly positive. Given these values, we readily see that
\[R_t = r_{t-1} + \frac{1 + \lambda}{1 + N - N\rho(1 + \lambda)} \xi_t + \zeta_t\] 

and
\[r_t = r_{t-1} + \frac{\lambda}{1 + N - N\rho(1 + \lambda)} \xi_t + \zeta_t.\] 

Accordingly, the spread variable obeys
\[R_t = r_t + \frac{1}{1 + N - N\rho(1 + \lambda)} \xi_t\] 

and using (22) and (4) we also have

\(^{21}\)Again this is because with \(\sigma = 0\), \(r_{t-1}\) should not appear in the solution for \(R_t\).
\[ R_t - R_{t-1} = \frac{\lambda + 1}{1 + N - N\rho(1 + \lambda)} \xi_t - \frac{1}{1 + N - N\rho(1 + \lambda)} \xi_{t-1} + \zeta_t \]

\[ = \frac{(\lambda\rho + \rho - 1)\xi_{t-1} + (1 + \lambda)u_t}{1 + N(1 - \rho(1 + \lambda))} + \xi_t \]

\[ = (\lambda + \rho - 1)(R_{t-1} - r_{t-1}) + \frac{(1 + \lambda)}{1 + N(1 - \rho(1 + \lambda))} u_t + \zeta_t. \] (25)

Consequently, we see that a regression of \( N(R_t - R_{t-1}) \) on \( R_{t-1} - r_{t-1} \) will have a slope coefficient whose probability limit is \( N(\lambda\rho + \rho - 1) \) or \(-N(1 - \rho(1 + \lambda))\). Clearly, the latter will be negative except for very large values of \( \rho \) and/or \( \lambda \), and will be larger in absolute value (for a given \( \rho \)) with longer maturities (larger \( N \)).\(^{22}\) In qualitative terms, both of these characteristics match the results of Evans and Lewis (1994) and Campbell and Shiller (1991) reported above in Table 2.

3. ADDITIONAL EVIDENCE

The article by Campbell and Shiller (1991) concludes with an attempt to provide a summary characterization of term structure behavior that would be consistent with their battery of empirical findings, which include many more than those reported here. In their words, “The explanations we will consider are not finance-theoretic models of time-varying risk premia, but simply econometric descriptions of ways in which the expectations theory might fail” (1991, 510). In terms of the notation of the present article, the two summary characterizations considered are (for the two-period case)

\[ R_t - r_t = 0.5E_t(r_{t+1} - r_t) + c + v_t, \] (26)

where \( v_t \) is added noise that is orthogonal to \( E_t(r_{t+1} - r_t) \), and

\[ R_t - r_t = k0.5E_t(r_{t+1} - r_t) + c \] (27)

where \( k > 1 \). The latter “could be described as an overreaction model of the yield spread,” according to Campbell and Shiller (1991, 513). They explore the implications of these two summary characterizations of ways in which the expectations theory might fail and conclude that (27) is consistent with the data but that (26) is not.

Let us consider how these characterizations compare with the explanation of the present article. Looking back at Section 1, we see that equation (12)
is of a similar form to that of (26), but with the crucial difference that $\xi_t$ in (12) is not orthogonal to $E_{t-1}r_{t+1} - r_t$. Thus the inadequacy of (26) does not serve to discredit the model of Section 2. Furthermore, using the expression $E_{t-1}r_{t+1} - r_t = \phi_2 \rho \xi_t$ to eliminate $\xi_t$ from (12) results in

$$R_t - r_t = \frac{1}{\rho \lambda} E_t(r_{t+1} - r_t)$$

for the model of Section 2. But with $0 < \lambda < 2$ and $| \rho | < 1$, (28) implies that $k > 1$ in (27) if $\rho$ is positive. So Campbell and Shiller’s summary characterization is consistent with the present article’s rationalization.23

It was mentioned above that the slope coefficient reported in Table 1 for the years 1890–1914 was closer (than for more recent periods) to the value of 1.0 that has been focused on in previous investigations. As Mankiw and Miron (1986) emphasize, those years precede the founding of the Federal Reserve System and therefore pertain to a period during which interest rate smoothing behavior would be absent. In a similar vein, Kugler (1988) finds that slope coefficients are closer to 1.0 for Germany and Switzerland than for the United States during recent years. This result he attributes to a smaller degree of interest smoothing behavior by the Bundesbank and the Swiss National Bank, in comparison with the Fed, a hypothesized behavioral difference that is consistent with the beliefs of many students of central banking behavior. Since the model in Sections 1 and 2 presumes a substantial degree of interest rate smoothing, this article’s explanation is consistent with both of these findings.24

4. REMARKS

The discussion of the foregoing paragraph suggests that one possible way of conducting additional tests of this article’s hypothesis would be to consider different monetary policy regimes corresponding to different time periods for the United States and to different nations. Reaction functions corresponding to (5) would be estimated and the implications of their parameter values for the crucial slope coefficients then compared with values of the coefficients obtained for these different regimes. Now, it may prove possible to make some progress toward execution of such a study. There is, however, a substantial difficulty that needs to be mentioned. Specifically, it is the case that actual central banks do not respond only to term spreads in deciding upon changes in $r_t$. Thus equation (5) represents a simplification relative to actual behavior of the Fed, which almost certainly responds to recent inflation and output or employment movements as well as the spread. So, if one were to attempt to

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23 The foregoing discussion implies, incidentally, that there is actually nothing bizarre or irrational about a finding expressible as $k > 1$ in (27).
24 For additional discussion of the Mankiw-Miron hypothesis, see Cook and Hahn (1990).
econometrically estimate actual reaction functions, then measures of inflation and output gaps would need to be included. But in that case, values of these variables would need to be explained endogenously, so the system of equations in the model would have to be expanded. Furthermore, the dynamic behavior of inflation and output would need to be modeled “correctly,” which is an exceedingly difficult task given the absence of professional agreement about short-run macroeconomic dynamics. In short, this type of study would require specification and estimation of a complete dynamic macroeconomic model.25

In light of the foregoing discussion it will be seen that, because of the simplified nature of our policy equation (5), this article’s proposed explanation might be regarded as more of a *parable* than a fully worked-out quantitative model. I would argue, however, that this is not a source of embarrassment, for most knowledge in economics is actually of the parable type.26 The relevant issue is whether a proposed parable is fruitful in understanding important economic phenomena. In this particular case the proposed parable suggests that slope estimates in regressions of the form (3) or (16) differ from 1.0 despite the validity of a version of the expectations theory of the term structure. This version permits the holding-period yields on securities of various maturities to differ by a random discrepancy that is exogenous but perhaps serially correlated. The basic idea of the parable is that the estimated slope coefficient is a composite parameter reflecting policy behavior as well as the behavior of market participants, with the type of policy postulated involving interest rate smoothing and response to the long-short spread, the latter reflecting important aspects of the state of the economy. The fact that essentially the same parable can rationalize a major anomaly in foreign exchange markets must be regarded as a significant mark in its favor.

5. ADDITIONAL DISCUSSION

Since the article consisting of the foregoing sections was written, there have been several directly relevant developments. First, Kugler (1997) and Hsu and Kugler (1997) have conducted empirical studies based on the article’s framework. In both of these studies, the results are described as supportive of the “policy reaction” hypothesis. In the process of conducting these empirical investigations, Kugler (1997) developed significant extensions of the article’s theoretical analysis, one example being an application for the case

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25 Recently, Cochrane and Piazzesi (2002) have developed a “high frequency” empirical strategy that yields results for the United States that are basically consistent with policy behavior of the type hypothesized above.

26 Consider the usual depiction of a production function as $y_t = f(n_t, k_t)$, where the symbols should not require definition. Can this depiction be considered anything more than a parable?
in which there are available observations for shorter time periods than those corresponding to the short rate (itself assumed to match the central bank’s decision period). This extension is quite useful for econometric analysis of the model linking term-structure and monetary policy behavior.

A more fundamental development concerns a basic problem with the foregoing analysis of the N-period cases in Section 2. Since (15) pertains to different long maturities \( N + 1 \), it should be written more completely as

\[
R^{(N+1)}_t - N E_t (R^{(N)}_{t+1} - R^{(N+1)}_t) = r_t + \xi^{(N+1)}_t
\]

for \( N = 1, 2, 3, \ldots \), where we do not retain the approximation \( R^{(N)}_{t+1} = R^{(N+1)}_{t+1} \) used in (15). A crucial question, then, is how are the term premia \( \xi^{(N+1)}_t \) related to each other? Also, which of the long rates is it that appears in the monetary policy rule? Evidently, the solution equations (22)–(25) cannot be correct for all \( N \) since each of them implicitly assumes that the particular long rate being analyzed is the one that appears in the central bank’s policy rule.

Both of these flaws in the Section 2 analysis have been addressed by Romhányi (2002), who assumes that \( \xi^{(N+1)}_t = N \Psi_t \), with \( \Psi_t \) being the same for all \( N \) and obeying the first-order AR process \( \Psi_t = \rho \Psi_{t-1} + u_t \). Then we have

\[
\frac{1 + N}{N} R^{(N+1)}_t = r_t + \frac{N E_t R^{(N)}_{t+1}}{N} + \Psi_t,
\]

which implies that for every maturity the average holding-period yield discrepancy, between the bond of duration \( N + 1 \) (on the one hand) and the one-period bond plus \( N \) periods with the \( N \)-period bond (on the other hand), is the same. This equality is evidently necessary to rule out arbitrage possibilities.

With respect to the central bank’s choice of a long rate for definition of the spread that is used in its policy rule, Romhányi (2002) shows that the crucial solution equation (24) becomes

\[
R^{(N+1)}_t = r_t + \gamma_q [1 - \frac{1}{N+1} - \frac{1}{1-\rho}] \Psi_t,
\]

where \( q \) is the maturity chosen for the policy rule and where \( \gamma_q \) depends upon \( q \) as well as \( \lambda \) and \( \rho \) but is the same (given \( q \)) for all \( N \)—see Romhányi (2002). For plausible values of \( \lambda, \rho, \) and \( q \) the coefficient \( \gamma_q \) will be positive and decreasing in \( q \). Romhányi’s modification therefore eliminates the logical inconsistencies in the argument of Section 2 above.

Over the past decade, 1995–2005, analysis of term-structure relationships has been dominated by no-arbitrage affine factor models, in which (zero-coupon) bond prices are given by a pricing equation that specifies the pricing kernel process as an affine (linear with intercept terms permitted) function
of unobservable factors (state variables). Then the prices of bonds of all maturities, which satisfy no-arbitrage conditions, are also affine functions of the state variables. The substantive content of such models lies in the specification of the number of factors and the process generating the state variables. Dai and Singleton (2002) have shown that empirical features of the U.S. term structure data can be well explained by a three-factor affine model in which the “price of risk” is linearly related to the state variables. In this context, Dai and Singleton (2002, 436) report that “it turns out that McCallum’s (1994) resolution of the expectations puzzle based on the behavior of a monetary authority is substantively equivalent to our [single-factor] affine parameterization of the market price of risk.”

Very recently, Gallmeyer, Hollifield, and Zin (2005) have more extensively explored the relation of this article’s model to “endogenous” models of the term premium, i.e., models in which the term premia are constrained to obey no-arbitrage constraints. In addition, they study the way in which this article’s policy rule is related to the policy rule of Taylor (1993), which has been extremely influential in both positive and normative analyses of monetary policy over the past two decades. Quantitative analysis indicates that the two parameters of this article’s policy rule are plausible for a stochastic volatility specification of state variable behavior but not with a stochastic price-of-risk specification. The latter, however, is shown by Dai and Singleton (2002) to provide a superior match to actual U.S. yield-curve properties. In combination with Romhányi’s results, this suggests that this article’s policy rule should not be taken literally, a conclusion that is consistent with the discussion in Section 4 above.

APPENDIX

Here the concern is with the model of Section 1 when \( \sigma < 1.0 \). From (9), we find that

\[
rt = \phi_0 + \phi_1 r_{t-1} + \frac{\lambda}{\delta - \rho \lambda/2} \xi_t + \frac{1}{\delta} \xi_t
\]

(A-1)

where \( \delta = 1 - (\phi_1 - 1) \lambda / 2 \). Then from (A-1) it follows that

\[
E_t r_{t+1} - rt = \phi_0 + (\phi_1 - 1) r_t + \lambda \rho / (\delta - \rho \lambda/2) \xi_t
\]

(A-2)

27 Other notable papers that integrate monetary policy and term-structure analyses include Rudebusch and Wu (2004) and Bekaert, Cho, and Moreno (2005).
and thus using (12) that

\[ R_t - r_t = \frac{1}{2} [\phi_0 + (\phi_1 - 1) r_t + (\rho \lambda / (\delta - \rho \lambda/2)) \xi_t] + \xi_t. \]  

(A-3)

Now, equation (2) indicates that the plim of the slope coefficient on \( R_t - r_t \) in the regression (3) will equal 1.0 minus plim \( T^{-1} \xi_t (R_t - r_t) / \text{plim} T^{-1} (R_t - r_t)^2 \).

Its value will be smaller than 1.0, then, if \( E\xi_t (R_t - r_t) \) is positive.

From (A-3) it is clear that there are two components to \( E\xi_t (R_t - r_t) \). One of these is

\[ \frac{\rho \lambda/2}{\delta - \rho \lambda/2} + 1) \sigma_\xi^2, \]  

(A-4)

which is necessarily positive since the term in parentheses equals

\[ \frac{1 + (1 - \phi_1) \lambda/2}{1 + (1 - \phi_1) \lambda/2 - \rho \lambda/2}. \]  

(A-5)

Here \( (1 - \phi_1) \lambda/2 \) is positive, since \( \phi_1 < 1 \) when \( \sigma < 1 \) (see below), and \( |\rho \lambda/2| < 1 \). Thus expression (A-5) is unambiguously positive. The second component is

\[ (1/2)(\phi_1 - 1) E_r \xi_t, \]  

(A-6)

in which the term \( \phi_1 - 1 \) is negative but will be small for \( \sigma \) (and \( \phi_1 \)) close to 1.0. To sign \( E_r \xi_t \), we use (A-1) and (4) as follows, assuming \( E\xi_t \zeta_t = 0 \):

\[ E_r \xi_t = E[\phi_0 + \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t] \xi_t \]

\[ = \phi_1 E_r \xi_{t-1} + \phi_2 \sigma_\xi^2 = \phi_1 E_r \xi_{t-1} \rho \xi_{t-1} + \phi_2 \sigma_\xi^2. \]  

(A-7)

Then since \( E_r \xi_t = E_r \xi_{t-1} \), we have

\[ E_r \xi_t = \frac{\phi_2 \sigma_\xi^2}{1 - \phi_1 \rho}. \]  

(A-8)

The latter is unambiguously positive since \( \phi_2 < 0 \) and \( |\phi_1 \rho| < 1 \). Thus the second component is negative but will tend to be small relative to the first.

It remains to demonstrate that \( \phi_1 < 1 \) when \( \sigma < 1 \). But we have found that

\[ \phi_1 = \frac{(1 + \lambda/2) - [(1 + \lambda/2)^2 - 2 \lambda \sigma]^{1/2}}{\lambda}. \]  

(A-9)

With \( 0 < \sigma < 1 \), we have \( 2 \lambda > 2 \lambda \sigma > 0 \) so the term in square brackets is positive and larger than \( (1 - \lambda/2)^2 \). Thus the value of \( \phi_1 \) is smaller than when
this term equals \((1 - \lambda/2)^2\), i.e., when \(\sigma = 1\). But \(\phi_1\) remains non-negative because the term in brackets is smaller than \((1 + \lambda/2)^2\).

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Items marked (*) have been added in 2005.


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