The Behavior of Household and Business Investment over the Business Cycle

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The spillover effects associated with the decline in the housing market during 2007 and 2008 suggest the importance of this market for the overall economy. Yet the decision to purchase a house is only part of a broader plan of production and consumption of goods within the household. The residential services homeowners enjoy from their dwelling, the transportation services they enjoy from their automobiles, the meals prepared at home, the child/adult care services provided within the household, and the entertainment services derived from television and audio equipment are just a few examples of goods that are produced and consumed within the household, as opposed to goods that are purchased in the market. The size of this non-market output is quite significant: Benhabib, Wright, and Rogerson (1991) estimate that the output of the household sector in the United States is approximately half of the size of the output in the market sector. Furthermore, the production of non-market goods requires the use of capital. Greenwood and Hercowitz (1991) report that the stock of household capital is actually larger than the stock of capital in the market sector. Examples of household capital are the dwellings owned and occupied by the household, automobiles owned and used by the household’s members, home appliances, furniture, etc.

Given the size of the household sector, several studies have incorporated this sector into the real business cycle model with the goal of enhancing the understanding of aggregate fluctuations of economic activity. Even though...
the real business cycle model has proven to be a powerful tool for explaining basic patterns of business cycle fluctuations in the United States, it has faced several challenges when it has been utilized to account for the behavior of business and household investment. This article presents a summary of the literature that studies the behavior of household investment decisions over the business cycle.

Previous studies have emphasized three stylized facts about the cyclical behavior of household and business investment in the United States: (1) both investment components display a positive co-movement with output—as well as a positive co-movement with each other, (2) household investment is more volatile than business investment, and (3) household investment leads the cycle whereas business investment lags the cycle. With respect to the last finding, household investment is correlated more with future output than with current or past output, while business investment is correlated more with past output than with current or future output. This article discusses the performances of previous studies in terms of their ability to account for these stylized facts within a framework that is broadly consistent with the main properties of business cycles in the United States.

This article provides a summary of studies that have extended the real business cycle model in order to reach a better understanding of the facts described above. Alternative explanations for the positive co-movement and relative volatilities between the two investment components have relied on different degrees of complementarity between capital and labor in the production of home goods, the presence of alternative uses for labor and/or household capital, and the presence of a more costly adjustment in the stock of market capital compared with the stock of household capital. The leading behavior of household investment has been harder to explain. The two studies that have succeeded in accounting for this fact have relied on household capital as a factor that may enhance the quality of the labor force and on a multiple-sector model in which capital goods are produced in a separate sector. All the studies reviewed in this article rely on exogenous shocks to productivity levels as the driving force of cyclical fluctuations. This modeling strategy abstracts from explanations for cyclical fluctuations in which market imperfections lead to inefficiently low or high output levels. For example, none of the studies revisited in this article feature residential investment driven by house prices that may be misaligned with fundamentals. This implies that the studies surveyed in this article portray cyclical downturns as an efficient response of the economy to “bad shocks.”

The rest of the paper is organized as follows. Section 1 describes the main characteristics of the business cycle in the United States and the importance of household production. Sections 2 and 3 present a summary of the literature on the cyclical behavior of household and business investment. The conclusions are noted in Section 4.
1. DATA DESCRIPTION

The concept of business cycles refers to fluctuations of economic activity around its long-run growth path. The long-run growth path is commonly referred to as the trend of the time series of an economic variable. The cyclical component of the series is defined as the deviation from the trend. In real business cycle theory, economists study the behavior of the cyclical component. For example, studies of business cycles focus on notions of persistence in the detrended component of economic aggregates, co-movement among various detrended (cyclical) components and the leading or lagging behavior relative to the detrended component of output, and also the relative amplitudes of standard deviation or volatilities of various detrended series.

The remarkable feature about fluctuations of aggregate variables over time is that the cyclical components tend to move in a synchronized mode. There has been an extensive literature over the last 30 years aimed at reaching a coherent understanding of the regularities that characterize the business cycle in the U.S. economy. As was pointed out by Lucas (1977), the development of a theoretical explanation for these regularities constitutes a first step toward the design of sound policy measures.

This section does not provide an exhaustive description of the properties of business cycles in the United States. Instead, it focuses on the cyclical behavior of the aggregate variables that are studied in this article.

Table 1 presents the behavior of market output, market consumption, household and business investment, and total hours worked in the market sector. The moments are computed using data from the first quarter of 1964 to the second quarter of 2008. The second column reports the standard deviation of market output and ratios of the standard deviations of each variable relative to the standard deviation of market output. The remaining columns report the cross-time correlation between each variable and market output. In particular, the seventh column illustrates that there is a significant positive co-movement between all five variables. However, the highest magnitudes of the coefficients of correlations do not necessarily correspond to the contemporaneous correlations. Household investment is more closely correlated with market output one and two quarters ahead than with current market output:

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2 Market output consists of gross domestic product less consumption of housing services. Market consumption consists of personal consumption expenditures in nondurables and services less housing services. Household investment consists of residential fixed investment and expenditures in durable consumption goods. Business investment consists of nonresidential fixed investment. Market hours consists of total hours worked in the private sector. The Bureau of Economic Analysis is the primary source for the first four variables and the Bureau of Labor Statistics is the primary source for the last variable. The moments reported in the table correspond to deviations from the trend of the natural logarithm of each variable. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1,600.
### Table 1 Properties of Business Cycles in the United States, Selected Moments

<table>
<thead>
<tr>
<th></th>
<th>Std. Dev.</th>
<th>$x_{t-4}$</th>
<th>$x_{t-3}$</th>
<th>$x_{t-2}$</th>
<th>$x_{t-1}$</th>
<th>$x_t$</th>
<th>$x_{t+1}$</th>
<th>$x_{t+2}$</th>
<th>$x_{t+3}$</th>
<th>$x_{t+4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Output</td>
<td>1.66</td>
<td>0.26</td>
<td>0.47</td>
<td>0.68</td>
<td>0.86</td>
<td><strong>1.00</strong></td>
<td>0.86</td>
<td>0.68</td>
<td>0.47</td>
<td>0.26</td>
</tr>
<tr>
<td>Market Consumption</td>
<td>0.55</td>
<td>0.43</td>
<td>0.61</td>
<td>0.75</td>
<td>0.82</td>
<td><strong>0.79</strong></td>
<td>0.66</td>
<td>0.49</td>
<td>0.30</td>
<td>0.10</td>
</tr>
<tr>
<td>Business Investment</td>
<td>2.91</td>
<td>−0.06</td>
<td>0.13</td>
<td>0.37</td>
<td>0.59</td>
<td><strong>0.78</strong></td>
<td><strong>0.84</strong></td>
<td><strong>0.81</strong></td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>Household Investment</td>
<td>4.03</td>
<td>0.58</td>
<td>0.68</td>
<td><strong>0.78</strong></td>
<td><strong>0.81</strong></td>
<td><strong>0.73</strong></td>
<td>0.50</td>
<td>0.27</td>
<td>0.04</td>
<td>−0.15</td>
</tr>
<tr>
<td>Market Hours</td>
<td>1.11</td>
<td>0.02</td>
<td>0.22</td>
<td>0.46</td>
<td>0.69</td>
<td><strong>0.86</strong></td>
<td>0.89</td>
<td>0.82</td>
<td>0.69</td>
<td>0.51</td>
</tr>
</tbody>
</table>
corr\((x_{ht-2}, y_t)\) = 0.78 and corr\((x_{ht-1}, y_t)\) = 0.81, while corr\((x_{ht}, y_t)\) = 0.73.\(^3\)

On the contrary, business investment is correlated more with market output one and two quarters behind than with current market output: corr\((x_{mt+1}, y_t)\) = 0.84 and corr\((x_{mt+2}, y_t)\) = 0.81, while corr\((x_{mt}, y_t)\) = 0.78. In addition, both investment components are significantly more volatile than market output and consumption.

The leading behavior of household investment is also apparent in Figure 1. The graph illustrates the dynamics of household investment, business investment, and output before and after each of the last seven recessions. Except for the 2001 recession, household investment had already peaked and was in decline at the beginning of each recession. On the other hand, except for the recessions that started in 1969 and 2001, business investment peaked either at the beginning of the recession or after that.

Even though standard one-sector real business cycle models have been successful in accounting for the cyclical pattern of aggregate investment, the extensions to the one-sector model have been less successful. To some extent, this poses a challenge to the use of transitory shocks to aggregate productivity as the main source of aggregate business fluctuations. The next sections present a summary of the lessons that can be extracted from past work that has studied the cyclical behavior of household and business investment.

2. THE BASELINE NEOCLASSICAL GROWTH MODEL

Kydland and Prescott (1982) and Long and Plosser (1983) are the first studies to quantify the explanatory power of equilibrium theories to account for business cycle fluctuations. They consider different extensions of the stochastic growth model studied in Brock and Mirman (1972) and compare statistical properties of the data generated by their models with actual statistics. In Kydland and Prescott (1982) and Long and Plosser (1983), the only source of fluctuations in the economy is a shock to the aggregate factor productivity. Their work laid down the foundations of a vast literature that shows how equilibrium theories could provide a plausible explanation of aggregate fluctuations of economic activity. The rest of this section is devoted to elaborating on the structure of the one-sector real business cycle model and the different multi-sector models that have been used so far to explain the cyclical patterns of business and household investment.

As a simple case study, consider a closed economy with no government spending and complete markets. There is one good in the economy that can be either consumed or invested. Fluctuations in economic activity are driven by persistent shocks to total factor productivity. In the simple model, there is no

\(^3\)The leading behavior of household investment is shared by its two components: household purchases of durable goods and residential investment.
Figure 1 Real Investment and GDP Before and After Each of the Last Seven Recessions

Notes: The indexes take a value of 100 in the first quarter of each recession.

disutility of labor implying that the supply of labor is inelastic. Under a wide range of values for the parameters, a positive shock to productivity generates
higher output, consumption, and investment in the shock period, which can account for the positive co-movement of these three economic aggregates. In this economy, there are two effects through which a positive productivity shock may induce higher investment level in the shock period. First, agents become richer and may want to smooth out the current windfall of output. The only aggregate mechanism available to transfer current resources to future periods is capital accumulation. Secondly, if the shock is persistent enough, positive current productivity shocks predict a distribution biased toward positive shocks in the following period, which augments the marginal benefit to invest rather than to consume. Additionally, an agent’s ability to transfer resources across time by investing or disinvesting enables the model to account for the volatilities of consumption and investment relative to output.

What happens when investment is disaggregated between household and business investment? The answer is that the baseline model faces a hard time accounting for the cyclical pattern of these two components.

3. MODELS WITH HOME PRODUCTION

Greenwood and Hercowitz (1991) constitutes the first attempt to study the cyclical behavior of these two components of investment in a real business cycle model. They consider a two-sector model in which the representative household maximizes its expected lifetime utility, as given by

\[ E_0 \left[ \sum_{t=0}^{\infty} u(c_{Mt}, c_{Ht}) \right], \]

where \( c_{Mt} \) denotes the consumption of market goods, and \( c_{Ht} \) denotes the consumption of home-produced goods at time period \( t \). The consumption of market goods is identical to the purchases of consumption goods, \( c \), namely

\[ c_{Mt} = c_t, \]

while home goods, \( c_{Ht} \), are assumed to be a function of the stock of household capital, \( k_{Ht} \), and the number of hours allocated to produce home goods, \( h_{Ht} \),

\[ c_{Ht} = H(k_{Ht}, z_{Ht}h_{Ht}). \]

Market goods are produced using a technology that depends on the capital stock invested in the market sector, \( k_{Mt} \), and the number of hours supplied to the market sector, \( h_{Mt} \),

\[ y_t = F(k_{Mt}, z_{Mt}h_{Mt}). \]

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In choosing market consumption, $c_{Mt}$, and savings, the household faces the following budget constraint in period $t$:

$$\begin{align*}
    c_t + x_{Mt} + x_{Ht} &= (1 - \tau_k) r_t k_{Mt} + (1 - \tau_l) w_t h_{Mt} + \tau, \\
    \text{where } w_t \text{ is the wage rate in the market sector, } r_t \text{ is the rental price of capital in the market sector, } x_{Mt} \text{ and } x_{Ht} \text{ are the investment in household and market capital, respectively, } \tau_k \text{ is the tax rate on capital income, } \tau_l \text{ is the tax rate on labor income, and } \tau \text{ is a lump sum transfer.}
\end{align*}$$

The variables $z_{Mt}$ and $z_{Ht}$ represent labor-augmenting technological progress. In this study, an important assumption is that productivity shocks in the market and household sectors are perfectly correlated, i.e., $z_{Mt} = z_{Ht}$.

The endowment of hours in each period is normalized to 1 and it is assumed that all hours that are not used to produce market goods are used to produce home goods. That is,

$$h_{Mt} + h_{Ht} = 1.$$  \hfill (6)

Finally, the capital stocks in the market and household sector depreciate at the constant rates $\delta_M$ and $\delta_H$, respectively. This means that the capital stock in sector $i$ follows the law of motion

$$k_{it+1} = (1 - \delta_i) k_{it} + x_{it}, \quad \text{with } i \in \{M, H\}.$$  \hfill (7)

Similar investment motives to the ones described in the case of the one-sector model are also present in this environment. The difference is that now there is a tradeoff between the accumulation of business capital and that of household capital. In the baseline calibration of Greenwood and Hercowitz (1991), households respond to a positive productivity shock by increasing business investment and decreasing household investment in the shock period. This behavior explains why the simulated data sets obtained using their baseline calibration feature a strong negative co-movement between business and household investment.

The mechanism of this model is summarized by the following passage from Greenwood and Hercowitz (1991; 1,205):

\ldots The negative co-movement of the two investments, which stands in contrast with the positive one displayed by the actual data has to do with the basic asymmetry between the two types of capital. Business capital can be used to produce household capital, but not the other way around. When an innovation to technology occurs, say a positive one, the optimal levels for both capital stocks increase. Given the asymmetry in the nature of the two capital goods, the tendency for the benchmark model is to build business capital first, and only then household capital\ldots

Greenwood and Hercowitz (1991) show that a higher degree of complementarity between labor and capital in home technology helps in accounting for the co-movement between household and business capital accumulation. The Euler equation for household capital accumulation is given by

$$u_1(c_M, c_H) = \beta \int u_1(c'_M, c'_H) \left[ \frac{u_2(c'_M, c'_H)}{u_1(c'_M, c'_H)} H_1(k'_H, z'h_H) + 1 - \delta_H \right] dG(z'|z),$$

where $x'$ denotes the next-period value of variable $x$. The marginal value of household capital accumulation depends on the future shadow price of household consumption, $\frac{u_2(c'_M, c'_H)}{u_1(c'_M, c'_H)}$, and on the future marginal productivity of household capital, $H_1(k'_H, z'h_H)$.

The Euler equation takes a simple form for the parameterization used in Greenwood and Hercowitz (1991). They assume that the production function for the home good, $H(k_H, z h_H)$, is of the following form:

$$H(k_H, h_H) = \begin{cases} k_H^\eta (zh_H)^{1-\eta} & \text{if } \zeta = 0 \\ \eta k_H^{\zeta} + (1 - \eta) (zh_H)^\zeta & \text{if } \zeta \neq 0. \end{cases}$$

The value of $\zeta$ determines the elasticity of substitution between household capital and labor in the production of home goods. Both inputs are complements when $\zeta < 0$, and are substitutes when $0 < \zeta < 1$.

Greenwood and Hercowitz (1991) assume that the market technology is specified by a standard Cobb-Douglas production function with a labor-augmenting productivity shock. Firms seek to maximize profits given the rental rates for capital and labor.

The instantaneous utility function has the following form:

$$u(c_M, c_H) = \frac{C(c_M, c_H)^{1-\gamma} - 1}{1 - \gamma}.$$  

The consumption aggregator, $C(c_M, c_H)$, is given by

$$C(c_M, c_H) = c_M^{\theta} c_H^{1-\theta}.$$  

Under this parameterization, the Euler equation simplifies to

$$\frac{u_2(c'_M, c'_H)}{u_1(c'_M, c'_H)} H_1(k'_H, z'h_H) = \frac{1 - \theta}{\theta} c_M^\theta \frac{\eta k_H^{\zeta-1}}{\eta k_H^{\zeta} + (1 - \eta) (z'h_H)^\zeta}.$$  

In Greenwood and Hercowitz’s (1991) baseline calibration $\zeta = 0$, so the direct role of the future productivity shock, $z'$, on the future shadow price of household consumption and the future marginal productivity of household capital cancel each other out. However, when capital and labor are complements in the production of home goods ($\zeta < 0$), higher future productivity
shocks have a direct positive effect on the incentives to accumulate household capital. Thus, when \( \zeta < 0 \), a positive productivity shock in the current period increases the probability of observing higher shocks in the next period and generates a stronger desire to accumulate household capital in the period of the shock. The intuition is that when the ability to substitute capital for labor decreases, it becomes more costly for households to compensate a decrease in household capital with an increase in the number of hours devoted to the production of home goods. Greenwood and Hercowitz (1991) show that a value of \( \zeta = -1 \) suffices to generate a positive reaction of household investment to productivity shocks and hence, a positive co-movement between household and business investment. In addition, a value of \( \zeta = -1 \) also helps to account for the larger volatility of household investment relative to business investment.

**Modifications of the Baseline Model with Home Production**

*Differential capital adjustment costs in the market and household sector*

Gomme, Kydland, and Rupert (2001) point out that the alternative parameterization proposed by Greenwood and Hercowitz (1991) to account for the positive co-movement between the two investment components may be inconsistent with the presence of balanced growth.\(^5\) Gomme, Kydland, and Rupert (2001) extend the setup studied in Greenwood and Hercowitz (1991) by introducing a time-to-build technology for the production of market goods as well as utility from leisure.

In Gomme, Kydland, and Rupert (2001) the representative household lifetime utility is represented by

\[
E_0 \left[ \sum_{t=0}^{\infty} u \left( c_{M_t}, c_{H_t}, h_{L_t} \right) \right],
\]

where \( h_{L_t} \) denotes the number of hours devoted to leisure activities. The inputs required to produce market and home goods are the same as in equations (2)–(4).

\(^5\) If the model were extended to account for the decline in the price of durable goods, it would not be able to generate a constant fraction of expenditures in durable goods as observed empirically.
The assumption of time-to-build for market capital implies that an agent decides today the increase in the stock of business capital that will take place four periods ahead (a period refers to a quarter). In addition to that, the investment projects decided today entail a commitment of investment resources during four periods until the projects can become active. More precisely, when households decide at date \( t \) to increase their capital stock in the market sector at date \( t + 4 \) in one unit, they need to spend 0.25 units per period from date \( t \) until \( t + 3 \). This means that law of motion for capital in the market sector satisfies the following equation:

\[
k_{Mt+1} = (1 - \delta_M) k_{Mt} + p_{Mt-3},
\]

where \( p_{Mt} \) denotes the number of projects in the market sector started in period \( t \). Unlike in Greenwood and Hercowitz (1991), the investment in market capital in a given period depends on the number of projects started in that period as well as on the number of projects started over the last three periods, namely

\[
x_{Mt} = \frac{1}{4} \left[ p_{Mt} + p_{Mt-1} + p_{Mt-2} + p_{Mt-3} \right].
\]

However, Gomme, Kydland, and Rupert (2001) assume that it takes only one period to complete household investment projects. This means that equation (7) still applies for the stock of capital in the household sector.

Finally, Gomme, Kydland, and Rupert (2001) relax the strong assumption of perfect correlation between productivity shocks in the household and market sectors.

The main improvement over Greenwood and Hercowitz (1991) is that the model with time-to-build technology manages to replicate the positive co-movement between household and business investment and generates a stronger lag in the reaction of business investment to output. That result is obtained assuming a unitary elasticity of substitution between capital and labor in the home technology (\( \zeta = 0 \)). In order to assist the intuition, Figure 2 describes the impulse response on a one-time shock to the productivity level in the market sector (\( \epsilon_M \)).

Figure 2 shows that at the time of the shock, agents respond by starting more investment projects. This accounts for the increase in market investment at date 1 and at the dates that follow the shock. There are fewer investment projects started after date 1, which accounts for the decline in market investment observed after date 5. Even though the productivity level in the household sector remains unchanged throughout the period, the positive wealth effect because of the higher productivity in the market sector induces households to consume more homemade goods and thus to invest more in home capital. The upward pressure on wages triggered by the spike in market productivity induce
Figure 2 Impulse Responses to a One-Time Shock to Current Market Productivity

Notes: $x_M$ is business investment, $x_H$ is household investment, $y$ is market output, and $h_M$ is market hours. The deviations are expressed in percentage deviations from the steady-state values for each variable.

households to work more hours in the market sector. As a result of the higher supply of labor hours and the increase in factor productivity, market output increases upon the shock. The initial increase in output and labor hours tends to fade away until date 5. At that point, the investment projects started at date 1 become active and market output and hours worked in the market sector jump up again.

The results are symmetric in the case of a negative shock to market productivity. The simultaneous rise (fall) in household and business investment that tends to follow a rise (fall) in market productivity plays a key role in explaining the co-movement of both investment components.

As it is explained in Gomme, Kydland, and Rupert (2001; 1,127):
The effect of time to build is to mute the impact effect of the shock on market investment by drawing out the response over the four quarters it takes to build market capital. As a result, home investment need not take such a big hit in the initial period of the shock.

Chang (2000) explores a slightly different setup and provides an alternative mechanism that can explain the co-movement between market and household investment. The household’s objective is the same as the one specified in equation (1), with the difference that both consumption goods are produced within the household. That is, Chang (2000) replaces equation (2) with

\[ c_{Mt} = M(c_t, z_{Ct}h_{Ct}) , \]

where \( h_{Ct} \) denotes the number of hours allocated to the production of home goods that do not require nondurable inputs, and \( z_{Ct} \) is a labor-augmenting productivity shock. The production of home goods that require durable inputs satisfies equation (3). As in Greenwood and Hercowitz (1991), there is only one market sector in the economy. The market good can be used as a non-durable good, a durable good, or capital to be rented to firms in the market sector. These uses are nonreversible.

The household’s allocation of time must satisfy

\[ h_{Ct} + h_{Mt} + h_{Ht} = 1. \]

Chang (2000) assumes that the accumulation of durable goods and market capital are subject to an adjustment cost, \( \phi \), that is

\[ k_{it+1} = (1 - \delta_i) k_{it} + \phi \left( \prod_{i} \frac{x_{iit}}{k_{it}} \right) k_{it} \quad \text{for } i \in \{H, M\}. \]

The only source of uncertainty consists of a productivity shock in the market sector (\( z_H \) and \( z_C \) display a constant and deterministic growth rate).

Chang (2000) shows that when the household technology features a higher degree of substitutability between durable goods and labor than between nondurable goods and labor, a positive productive shock in the market sector generates a simultaneous increase in the investment of market capital and household stock of durable goods. The intuition is that a positive productivity shock induces households to increase their consumption while it increases their opportunity cost of time allocated to the production of consumption goods, given that the market wage increases. When the production of \( c_D \) displays a

\[ \footnote{Note that in Chang (2000) there are two types of household capital. One is composed of nondurable goods and fully depreciates at the end of each period. The other is composed of durable goods and is subject to partial depreciation.} \]
sufficiently higher degree of substitution compared to the production of $c_N$. Households find it optimal to increase their consumption of $c_D$ by using more capital (durable goods) and less labor. This accounts for the increase in the purchases of durable goods upon a positive productivity shock. In addition, Chang (2000) shows that it is the joint presence of a higher elasticity of substitution in the production of $c_D$ and the adjustment cost in the accumulation of durable goods and business capital that helps in generating a positive co-movement of purchases in durable goods and business investment. Once one of these two assumptions is relaxed, the model generates a negative co-movement between the accumulation of durable goods and business investment.

In contrast to Greenwood and Hercowitz (1991), the environment studied by Chang (2000) suggests that the positive co-movement between the two investment components can be explained by a high degree of substitutability in the production of the home good that requires durable goods. In addition, Chang (2000) estimates the elasticity of substitution between goods and time in different consumption activities and finds that durable goods seem to be a good substitute for time, a finding that is consistent with previous empirical studies.

**Home production as an input to market production**

Einarsson and Marquis (1997) are able to explain the co-movement of household and business investment in a setup in which households supply labor hours to the market sector and the non-market sector to accumulate human capital. In Einarsson and Marquis (1997), the household faces the same objective as in equation (1) and it has to satisfy the same restrictions defined in equations (2)–(5) with two differences. First, the term $h_{it}$ in equations (2)–(5) needs to be replaced by $E_i h_{it}$ for $i \in \{H, M\}$. The variable $E_i$ denotes the stock of human capital in period $t$. Second, there are no productivity shocks in the production of home goods.

Einarsson and Marquis (1997) assume that households can increase their stock of human capital using the following technology:

$$E_{t+1} = G(E_t, h_{E_t}),$$

where $h_{E_t}$ is the amount of time allocated in period $t$ to learning activities. That is, human capital has a few nonexclusive uses: it serves as an input in the production of human capital and it affects the quality of hours supplied to the market sector and allocated to the production of home goods. Thus,

$$h_{M_t} + h_{H_t} + h_{E_t} = 1.$$  

Finally, the law of motion for market and household capital satisfies equation (7).
In Einarsson and Marquis’s (1997) baseline calibration, a positive productivity shock in the market sector induces households to work more hours in the market and household sectors and decreases the number of hours devoted to accumulating human capital. In turn, the increase in hours worked in the household sector increases the marginal return on capital in that sector, which introduces an incentive to invest in household capital upon a positive productivity shock. Unlike Greenwood and Hercowitz (1991), Einarsson and Marquis (1997) do not rely on a high correlation of productivity shocks in the market and non-market sectors. In fact, they assume that only the production of market goods is hit with productivity shocks. Nonetheless as in Greenwood and Hercowitz (1991), they need to assume that capital and labor in the household sectors are complementary.

Even though the articles summarized in this section provide different tentative explanations for the positive co-movement of business and household investment, and the relative volatility of these two investment components, they cannot explain the leading behavior of household investment and the lagging behavior of business investment.

Fisher (2007) succeeds in this respect after introducing a direct role for household capital as an input in market production. Fisher (2007) extends Gomme, Kydland, and Rupert (2001) by introducing an additional use for household capital: Households can affect total effective hours supplied to business firms ($\tilde{h}_M$). The technology for determining $\tilde{h}_M$ is specified by

$$\tilde{h}_{Mt} = L (k_{HMt}, z_{Ht}h_{Mt}) = k_{Ht}^{\mu} (z_{Ht}h_{Mt})^{1-\mu},$$  \hspace{1cm} (22)

where $k_{MH}$ and $h_M$ denote the household capital and hours allocated to improve the quality of labor supply to business firms. As in Gomme, Kydland, and Rupert (2001), households produce a home good using household capital and labor:

$$c_{Ht} = H (k_{HHt}, z_{Ht}h_{Ht}),$$  \hspace{1cm} (23)

where $k_{HH}$ and $h_H$ denote the household capital and hours allocated to produce the home good. Note that unlike in Einarsson and Marquis (1997), households cannot affect the quality of the hours allocated to the production of home goods. The uses of household capital are constrained by the total stock of household capital in the period, namely

$$k_{Ht} = k_{Ht} + k_{HHt}.$$

(24)

In this setup, household capital is not only useful to produce home consumption goods, but it indirectly enhances the ability to produce market goods. In that context, Fisher (2007) shows that the model can replicate the leading behavior of household investment over business investment. When the share
of capital in the production of human capital ($\mu$) is below 0.25 (it is 0.19 in Fisher’s calibration), the optimal response of households to a positive productivity shock in the market sector is first to increase their investment in household capital. This allows households to increase their effective labor supply over periods following the shock, where higher productivity shocks would tend to push up wages. In turn, the higher labor supply will augment the production of market goods in future periods, which also helps to account for the leading behavior of household investment. The “strong” initial increase in household investment takes place at the expense of market investment, which displays a modest increase in the shock period. The household raises market investment in the periods following the positive shock.

**Models with Multiple-Market Sectors**

Finally, Davis and Heathcote (2005) and Hornstein and Praschnik (1997) study the cyclical behavior of residential investment and/or purchases of durable consumption goods without resorting to household production. These studies consider a structure in which all goods are produced in the market and in which households derive direct utility from the acquisition of durable goods. That is, in both setups the household maximizes the same objective function defined in equation (13), with the additional restrictions $c_{Mt} = c_t$ and $c_{Ht} = k_{Ht}$.

Unlike the articles surveyed above that study economies with only one market sector, Davis and Heathcote (2005) and Hornstein and Praschnik (1997) consider economies with multiple market sectors.

Davis and Heathcote (2005) consider a model with three intermediate inputs: construction ($b$), manufactures ($m$), and services ($s$) that are produced using labor and capital. Formally, let $y_{it}$ denote the production of intermediate good $i$:

$$y_{it} = F_i (k_{it}, z_{it}, h_{it}) , \quad \text{with } i \in \{b, m, s\} ,$$

(25)

where $k_{it}$ and $h_{it}$ denote the capital and labor hours used in the production of intermediate input $i$. These three goods are the only inputs in the production of two final goods: a consumption/capital good ($M$) and a residential good ($R$). Thus,

$$y_{jt} = F_j (b_{jt}, m_{jt}, s_{jt}) , \quad \text{with } j \in \{M, R\} ,$$

(26)

where $y_{jt}$ denotes the production of final good $j$, and $b_{jt}, m_{jt},$ and $s_{jt}$ denote the quantities of each of the three intermediate goods in the production of $j$. The residential good must be combined with land ($x_{Lt}$) to produce houses ($x_{Ht}$), namely

$$x_{Ht} = F_H (x_{Lt}, x_{Rt}) ,$$

(27)
where the stock of land is constant and equal to 1, i.e., $x_{Lt} \leq 1$. In their setup, houses are the only durable consumption good. In Davis and Heathcote (2005) there are three alternative uses for market capital and four alternative uses for the household’s endowment of hours, namely

$$k_{bt} + k_{mt} + k_{st} = k_{Mt}, \quad \text{and}$$

$$h_{bt} + h_{mt} + h_{st} + h_{Lt} = 1. \quad (29)$$

The law of motion for market capital, $k_{Mt}$, is the same as in equation (7), while the law of motion for the stock of houses is given by

$$k_{Ht+1} = (1 - \delta_H)^{1-\phi} k_{Ht} + x_{Ht}. \quad (30)$$

Finally, the resource constraint for final goods is given by

$$c_t + x_{Mt} + g_t = y_{Mt}, \quad (31)$$

where the government expenditures, $g_t$, are financed by labor and capital income taxes.

Davis and Heathcote (2005) show that the model can account for the co-movement between residential and nonresidential investment and the higher volatility of residential compared to nonresidential investment. The environment studied in Davis and Heathcote (2005) is quite different from the environment considered in previous studies. Davis and Heathcote (2005) carry on different experiments to identify the role of different features of the model. On page 753 they state that

First, although our Solow residual estimates suggest only moderate co-movement in productivity shocks across intermediate goods sectors, co-movement in effective productivity across final-goods sectors is amplified by the fact that both final-goods sectors use all three intermediate inputs, albeit in different proportions. Second, the production of new housing requires suitable new land, which is relatively expensive during construction booms. We find that land acts like an adjustment cost for residential investment, reducing residential investment volatility, and increasing co-movement. Third, construction and hence residential investment are relatively labor intensive. This increases the volatility of residential investment because following an increase in productivity less additional capital (which takes time to accumulate) is required to efficiently increase the scale of production in the construction sector. Fourth, the depreciation rate for housing is much slower than that for business capital. This increases the relative volatility of residential investment and increases co-movement, since it increases the incentive to concentrate production of new houses in periods of high productivity.

Hornstein and Praschnik (1997) propose a multi-sector economy in which the use of intermediate inputs helps to explain the co-movement of sectoral
employment and output. Their article also offers an explanation for the leading pattern of household investment. They consider a setup with two market sectors: one produces a durable good and the other produces a nondurable good. The durable good ($MX$) can be accumulated either as business capital or household capital. The nondurable good ($MC$) can be used either in consumption or as an input in the production of durable goods. Thus,

$$x_{MXt} + x_{MCt} + x_{Ht} = y_{MXt} = F_{MX} (k_{MXt}, z_{MXt}h_{MXt}, m_t)$$

and

$$c_M + m = F_{MC} (k_{MCt}, z_{MCt}h_{MCt}) ,$$

where $x_{it}$ denotes the investment in the stock of capital, $k_{it}$, $y_{MXt}$ denotes the production of durable goods, $k_{MXt}$ ($k_{MCt}$), $h_{MXt}$ ($h_{MCt}$) denotes the capital and labor hours used in the production of durable (nondurable) goods, $m_t$ denotes the amount of nondurable goods used as input in the production of durable goods, and $z_{MXt}$ ($z_{MCt}$) denotes a labor-augmenting productivity shock in the durable (nondurable sector).

The resource constraint for labor hours reads

$$h_{MXt} + h_{MCt} + h_{Lt} = 1,$$

while the law of motion for $k_{it}$ is the same as in equation (7), for $i \in \{MX, MC, H\}$. Note that in Hornstein and Praschnik (1997) investment decisions are nonreversible.

This setup not only explains the co-movement between household and business investment but it also explains the leading pattern of business investment. We quote Hornstein and Praschnik (1997, 589) below:

Following a productivity increase in either sector, capital becomes more productive and in order to increase the production of capital goods investment in the durable goods sector increases whereas investment in the nondurable goods sector is postponed for one period. The positive wealth effect of a productivity increase raises household consumption of capital services, and household sector investment increases contemporaneously with the productivity shock. Since investment in the nondurable goods sector represents the bulk of business investment, household investment leads business investment.

4. CONCLUSION

A substantial fraction of societal consumption is not purchased in markets but rather is produced and consumed within households. This article describes the main characteristics of the cyclical behavior of household and business investment over the cycle in the United States, and offers a summary of studies
that have tried to explain the dynamics of these two investment components. Even though we have reached a better understanding of what economic relationships may help in explaining the behavior of these two investment components, more research is needed. For example, changes in the relative prices of houses could be playing a significant role as a propagation mechanism or as a coordination device across households. However, most existing studies abstract from changes in the relative price of houses, and the ones that allow for that channel generate house price movements that are not aligned with the data.

REFERENCES


