Economic Quarterly—Volume 96, Number 1—First Quarter 2010—Pages 59–82

Inside-Money Theory after Diamond and Dybvig

Ricardo de O. Cavalcanti

This article argues that the model in Diamond and Dybvig (1983, DD hereafter) was a significant conceptual and methodological advance in studying banking arrangements. Its methodological contribution was the use of mechanism-design theory rather than the old strategy, still prevalent in textbooks and some of macro, of tacking a banking sector onto a model of market exchange. A great deal of attention has been given to the model's multiple equilibria and interpreting them as financial fragility. This attention is warranted, but there are other less recognized implications of the model. I provide examples in which the model is used to address banker incentives and means of payment. I also show how its methodology is related to recent work that uses monetary models to consider money, credit, and imperfect monitoring.

Recent events provide a good opportunity to put into perspective progress in the field of money and banking. The idea that banks are inherently unstable is as old as the field itself. The recent economic crisis in the United States and around the world was met by a new generation of central bankers familiar with the notion of financial fragility in DD. The dramatic increase in the balance sheet of the Fed, for instance, led by the purchasing of private securities whose markets had virtually disappeared, seems to indicate that financial meltdowns may not be restricted to unique conditions like those of the Great Depression.[1]

I am grateful to Huberto Ennis, Juan Sánchez, and Ned Prescott for helpful comments. I also thank graduate students Jefferson Bertolai, Artur Carvalho, Murilo Ferreira, and Bruno Teixeira, who agreed to present several of the papers discussed in this article in our money and banking course at FGV, as well as the *EQ* managing editor, Amanda Kramer, for her invaluable help. The views expressed in this article are not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System. E-mail: ricardo.cavalcanti@fgv.br.

[1] In this sense, macroeconomics is far from a "solved problem." Even though bank runs, as traditionally described, were not significant, the fact that returns on short-term Treasury bonds approached zero indicates a generalized lack of confidence in the strategy of depositing funds at private institutions.

Overall, the DD framework is a model of intertemporal trade with relatively few variables. It allows for a sharp description of frictions that private information and sequentiality of transactions imposed on the provision of insurance against preference shocks. This tractability emphasizes the question of whether or not optimal allocations are implemented uniquely, that is, whether or not the optimum is fragile to runs. A less appreciated issue, and my focus here, is how the DD framework suits developments in monetary theory that emphasize imperfect monitoring.

Micro and Macro Mechanisms

Findings about the multiplicity of equilibrium outcomes are common in the field of money and banking. Expectations about future behavior are important for the current value of money and its substitutes. While this property tends to raise the possibility of multiple outcomes, precise conclusions depend heavily on assumptions about what is traded and how markets are organized. There is an old habit in macroeconomics of building models around institutions seen empirically as important devices with which to organize savings: deposits, bonds, capital, alternative currencies, etc. DD represent a new modeling approach with their use of a very compact model of the pooling and intertemporal redistribution of resources in a single resource-constraint world. A bonus of this minimalist approach is the speed at which the literature identifies key elements that drive the role for liquidity provision and the possibility of financial fragility in their theory. These elements, taken as immutable primitives, rule out remedies like deposit insurance that even DD point to in their title.

Reviewing the DD contribution on the basis of this sort of "micro" problem alone is attractive because of the sharp results achieved by follow-up work. This can be appreciated by the material I present in Section 3, which follows an informal summary of model choices in Section 2. In addition to the emphasis on the issue of multiplicity guided by the model of Green and Lin (2003), I make considerations about imperfect monitoring that are inspired by the model of Prescott and Weinberg (2003). In Section 4, those considerations provide a bridge to identify monitoring in models where banks are people, which is different from the original DD setup. Examples include the models of Calomiris and Kahn (1991) and Deviatov and Wallace (2009). The latter, a monetary model, leads us to much larger mechanisms in macroeconomics.

I find it important, however, to also offer some history of thought perspective by starting in Section 1 with brief discussions of the models of Shubik and Wilson (1977) and Bryant (1980). These early models illustrate the difficulty of applying general-equilibrium theory to money and banking. These attempts were too early to benefit from the mechanism-design approach, and imposing banking exogenously meant they could not answer some important fundamental questions about the role of imperfect monitoring and sequential service in the provision of insurance. In their defense, the mechanism-design approach of today is more abstract and takes the model further away from the data than does the traditional approach.[2]

These early general-equilibrium models of money and banking address a rich set of questions that the DD methodology can only partially answer. I provide below a very short description of a planner problem that I find useful and serves the purpose of introducing the liquidity problem set by DD.

A Benchmark

Let us first consider a static endowment economy with two goods, grapes and wine, and a finite population (as I note below, a more precise analogy to the DD model would have the supply of these goods competing for the same resources). Individuals are initially identical in their preferences and endowments, but an exogenous stochastic process generates a distribution of marginal utilities across people. A benevolent social planner is called to organize the best allocation of goods, constrained by the fact that marginal utilities are private information.

This can be called a liquidity problem to the extent that the designed system first acquires control of all endowments and then makes transfers of goods based on individual announcements about preferences, the way actual banking systems seem to operate when we interpret withdrawals as announcements of impatience. Incidentally, in monetary theory, people are subject to numerous periods of shocks and transfers. When the history of transfers to particular individuals becomes difficult to monitor, money becomes necessary as a way to introduce recordkeeping. In our benchmark economy, in contrast, individuals are identified and their announcements are perfectly recorded. If the planner can collect all announcements before making the transfers of goods, contract theory can be used to design constrained-efficient allocations, as in the literature on adverse selection, but without a convincing notion of financial fragility. DD, and the literature that follows, identify a key aspect of banking that is capable of tying together liquidity and fragility. This aspect is the sequential service constraint, which is the counterpart in banking to anonymity in monetary theory.

Sequentiality is an assumption about the physical environment of grapes and wine that makes transfers more difficult to organize. The exogenous stochastic process now includes the assembly of a queue and forces the allocation of grapes to be made sequentially, that is, according to the flow of announcements. The allocation of wine is made after all transfers of grapes

[2] This explains why the agenda of pursuing general-equilibrium theory with limited forms of institutional design is experiencing a revival.

take place. As a consequence, consider an individual with high marginal utility for wine relative to grapes. This individual would not be concerned if people ahead in the queue state a preference for grapes, unless this behavior has implications for the aggregate endowment of wine. The DD model introduces this concern by replacing grapes and wine for, respectively, date-1 and date-2 goods that compete for the same resource constraint. Now an above-average desire for grapes creates a concern about the scarcity of wine.

Special assumptions about preferences and the stochastic process governing the queue and the distribution of marginal utilities give rise to a tractable model. In this case, the optimum is identified by a separating equilibria of the mechanism chosen by the planner. A bank run is identified by a pooling equilibria (for a population subset) in which the planner fails to observe the true types in sequence because individuals misrepresent their preferences by announcing a high desire to consume good 1, the grapes. This is an outcome of low welfare.

Notice that the value of truthful reporting is twofold. By declaring a true type, an individual not only helps with the allocation of scarce resources today, but he also provides invaluable information about the aggregate state of nature when preferences are correlated across people. Truth-telling behavior thus gives rise to a positive externality by helping the planner estimate the distribution of marginal utilities for the whole population, improving future transfers. A question emerges about whether or not individuals find incentives for providing this externality. DD give us a significant conceptual and methodological advance in studying banking arrangements because questions like this can now be studied formally.

A final note may help the reader to evaluate model choices in monetary models. Money models usually assume the existence of a continuum of individuals because deviation payoffs become computable by simple reference to equilibrium outcomes. But there is no reason for the money literature to insist on total anonymity. Hybrid models can allow for imperfect monitoring of money traders, bringing the analysis closer to banking. Some recent models achieve a co-existence of money and credit by restricting monitoring to cover only a subset of the population. Like in DD, the mechanism-design approach has an interesting implication: Optimal allocations have subtle features that are difficult to anticipate in terms of standard market outcomes.[3]

1. BUILDING DICHOTOMIES

A review linking DD to inside-money theory requires a perspective on the field of money and banking and the early tradition that they replace. The

[3] Future research could also consider the option of introducing the friction of sequential service in monetary models, raising the issue of financial fragility in the provision of inside money.

models proposed by Shubik and Wilson (1977) and Bryant (1980) are early and influential attempts to give money and banking important roles in the allocation of resources in endowment economies. There is, however, a kind of dichotomy that is common to both approaches. The authors build on basic models for which price theory works well, adding an exogenous financial sector, similar in spirit to the way cash-in-advance constraints have been used to give money a role in dynamic general-equilibrium economies. These models illustrate how DD break away from the dichotomy tradition of imposing a trade game to an otherwise well-organized economy.

The Shubik-Wilson Model

Shubik and Wilson (1977, SW hereafter) pioneered a formulation of optimal bankruptcy rules in a monetary setting. Although the trading games proposed by Shubik in his extensive work can be criticized as having been chosen arbitrarily, it is clear that penalty devices used by SW to constrain credit reappear in a variety of borrowing constraints in modern general-equilibrium models.[4]

In the SW model, individuals inhabit a pure endowment economy with a single date and two consumption goods. The population is equally divided into two types. The marginal utility of their Cobb-Douglas preferences and the initial endowments vary with types so there is a strong incentive to trade. It is assumed that individuals allocate a fraction of their endowments for sale and bid units of bank money for goods in two trading posts, one for each good. The corresponding prices are given by the ratio of total monetary bids to quantities of goods put up for sale in each market.

An "outside bank" appears as a mechanism to supply bank money in exchange for payment promises issued by individuals, since individuals are not endowed with money.[5] By introducing a sequence of events to the process for borrowing from the bank, making bids, receiving proceeds from sales, and repaying bank loans, SW give strategic choices a sequential interpretation. The implicit assumption is that individuals cannot commit to future actions. The mechanism or bank fixes the per capita amount of money it issues. Then individuals bid quantities of personal notes in this third trading post set by the bank and called the money market. Likewise, the money price is computed as the ratio of total bids to money supply. This ratio can also be identified as the gross rate of interest on loanable funds, a fee intended to "protect" the bank against default.

---

[4] See, for instance, Dubey, Geanakoplos, and Shubik (2005) and the extensive literature on endogenous borrowing constraints.

[5] Bank money, being part of an allocation, can be viewed as inside money.

Bankruptcy is allowed in the sense that an individual who is unable to pay his debt in full receives a utility penalty, which takes the form of a linear function and maps the unpaid fraction of promises into a utility loss. Utility penalties are exogenous and are allowed to vary with preference types. Money leftovers provide no utility so that individuals plan bankruptcy to different degrees as these parametric penalties change.

SW use their model to compare symmetric (across types) equilibrium allocations with those that would attain in the absence of intermediation (allocations in the contract curve of the Edgeworth box) for alternative assumptions about the information structure on which strategies are made contingent. In one specification, individuals receive some information about prices and bids in the money market prior to selecting bids and sales in the markets for goods. In another specification, SW assume that all bids and sales quantities are chosen simultaneously in noncooperative fashion and where deviations cannot affect prices because the population size is taken to infinity. They find combinations of penalties for which individuals of different types alternate in being solvent, as well as a region with bankruptcy for both types in which trade and prices are the same as the competitive equilibrium. If penalties are high enough there is no bankruptcy and equilibrium is again competitive.

SW defend their approach as a complete and consistent microeconomic foundation for monetary economics (the reason to build on the basic general-equilibrium model) where there are rules that describe all moves and cover all contingencies.[6] The role of money is to finance the float in trading posts requiring, by construction, bids and offers be made simultaneously. Yet, the role of credit is a passive one: The bank auctions enough universally accepted means of payment so that competitive-equilibrium optimality is restored. Because penalties are exogenous, the model is not meant to provide a theory of inside money in its true sense: a prediction of how much credit society is able to accommodate and how much money is needed to organize trade.

The Bryant Model

Compared with SW, the approach presented by Bryant (1980) also gives bank firms special abilities in channeling intertemporal savings. While written in less formal terms, Bryant's article is also more ambitious because it introduces bank fragility, and it does so using a model featuring monetary equilibria derived from first principles. Bryant builds on the overlapping-generations model of money, introducing a role for bank deposits, fractional reserves, and

[6] That aspect is also cited by Amir et al. (2009), in a special issue of *Games and Economic Behavior* in honor of Shubik, as the reason for a "natural" emergence of financial institutions such as money, credit, and bankruptcy rules.

anticipation of withdrawals. The analysis of his model is simplified by the emphasis on steady states without inflation.

There are two parts in Bryant's analysis. In the first part, a basic model without uncertainty is presented. It assumes that young individuals can acquire money by selling goods to old individuals, but that credit transactions must be done through an intermediary, a bank firm. The bank is valuable because two types of individuals are born in a given period and one of the types is composed of people who are only endowed with goods when old. They must thus borrow to finance consumption when young. The bank maximizes profits but its linear lending technology yields zero profits in the steady-state competitive equilibrium. This part of the model delivers predictions about consumption of the two types of individuals, holdings of money and deposits, and debt positions. It also discusses how lump-sum taxation of those receiving positive endowments when young can be used to finance payments of interest on government bonds that can only be held by the bank.

A good sense of Bryant's model can be captured in a version with two-period lived generations; no uncertainty; a continuum of newborns who are equally divided into two subgroups of unity-measure population, types 1 and 2; and where there is one consumption good per date. The per capita endowment for a type 1 is k units of goods when young and zero when old, while the reverse holds for a type 2. There is also a quantity of m units of fiat money evenly distributed among the initial old. In addition to the option of acquiring money from the old by selling goods at price $1/p$, the type-1 individuals use part of the money acquired to hold deposits with the bank. The variables indicating their final holdings of money and deposits are m^1 and d, respectively. The bank can then lend money to type-2 individuals, charging an interest, r, to be paid in money at the next date. The per capita debt of type-2 individuals is m^2.

Because the creation of deposits is costly, with costs in proportion $g \in (0, 1)$ to the goods value of dollars deposited, type-2 individuals only borrow what they need to consume when young and hold no money in equilibrium. In addition to the creation of deposits and loans, the bank can buy b units of government bond: a promise of a dollar next period, at price s. The per capita tax levied on type-1 individuals is, in equilibrium, $p(1 - s)b$.

Individual type 1 chooses (m^1, d) in order to maximize his utility in the budget constraint set defined by $c_1^1 \leq k - pm^1 - pd - p(1 - s)b$ and $c_2^1 \leq pm^1 + pd$. Individual type 2 sets m^2 so as to maximize her utility in the budget given by $c_1^2 \leq pm^2$ and $c_2^2 \leq k - (1 + r)pm^2$. In each period, the bank chooses levels of deposits, loans, and bond holdings so as to maximize profits. Its maximization problem is linear and the necessary conditions for zero profits with positive intermediation and bond holdings can be shown to be $(1 + r)^{-1} = 1 - g$ and $s = 1 - g$. In equilibrium, all goods are consumed or used up in intermediation, all the supply, m, of money is held by the type-1 young ($m^1 = m$), and all the supply, b, of bonds is held by the bank. As the government chooses different levels of b, keeping $m + b$ constant, more of the social cost of intermediation is transferred from borrowers to lenders (the taxpayers).

This structure guides a discussion of how to finance excess withdrawals in a (not fully detailed) stochastic version of the model in the second part of the article. The new ingredients are as follows. Each period is divided into two subperiods, early and late. Type-1 individuals receive a privately observed preference shock that mandates consumption early in their second period of life with probability α, and learn about the realization of the shock after all markets close when young. Type-2 individuals receive their endowment of goods only late in their second period of life. Young type-1 individuals are the only ones who can supply goods early to the old who receive the liquidity shock. The next assumptions are that banks only receive proceeds from bond investments late and that trading deposit claims is prohibitively costly. As a result, the old who must consume early need money to buy goods from the young.

Because claims to deposits have no use for the old with liquidity needs, the need appears for the bank to keep money reserves in order to accommodate early withdrawals. Without aggregate uncertainty, the mass of such withdrawals is α and the framework is well-suited for predicting equilibrium outcomes. A new problem arises, however, when the (late) endowment of type-2 individuals becomes random. An excess redemption can now take place under the assumption that an additional fraction, β, of type-1 individuals does not draw the realization of early consumption but receives instead a signal about the quantity of the random endowment. In events when an early withdrawal is advantageous to the bank, a bank that has planned money reserves for α withdrawal requests will face withdrawals from a measure of $\alpha + \beta$ individuals. These events are called bank runs.

This model illustrates several important issues debated by the field. For instance, Bryant discusses how a random taxation scheme could ensure the real value of deposits against variations in loan repayments, given that type-2 individuals cannot share risk with type-1 individuals of the next generation. There is also the possibility of government money being created to cover excess redemptions in a form of deposit insurance that can dominate, for the impatient, a partial-suspension scheme. Another issue discussed is that, given high payoffs accruing to deviations, coordination of behavior around particular no-run outcomes is difficult to implement. Early redemption of bonds is yet another possible intervention to inject liquidity in the system.[7] One conclusion

[7] The kind of insurances discussed may not prevent runs. Another difficulty is the need to model how uninformed individuals learn about runs and, in particular, if early prices reveal runs for all individuals alive at the moment or just those holding money and making early purchases.

is that reactions by the private sector may partially offset redistributive effects of interventions. Another is that it may be desirable to occasionally let the government print money instead of inducing banks to store costly reserves.

2. THE NEW TRADITION

The transition from the models of SW and Bryant to that of DD outlined in the introduction can be motivated in many ways. A great deal of effort goes into simplifying the liquidity problem but maintaining a coherent description of the physical environment. Instead of focusing on the infinite horizon of Bryant or the static setting of SW, a natural choice becomes focusing on the two-period economy with a single resource constraint. This choice reveals an interest in the provision of insurance against privately observed shocks, as described by Bryant, but without his emphasis on monetary allocations. As I point out below, Green and Lin (2003) propose simplifying the setting even further by adopting a finite trader specification with important consequences. In some aspects, the bank in SW is a technology: Individuals in their economy do not have access to some goods without the assistance of the bank. This feature is less present in DD, where the central question becomes the level of insurance that the bank can provide. But the bank in SW, like the one in DD, is operated by a benevolent social planner. In contrast, bank deposits in Bryant have to compete in rate of return with outside money as alternative assets.

Bryant's point is that runs are distortions that undermine the desired insurance protection against preference shocks and are triggered by the attempts of informed individuals to gain on the uninformed through early purchases when prices are likely to be low. His discussion of how efficiency can be restored, perhaps partially, with the help of government interventions is limited by the class of contracts banks are offering in the first place. His view that withdrawals must be paid on a first-come, first-served basis is important to the concept of fragility. He also notes the second-best aspect of regulation: Suspension of payments or conversion of deposits to currency at a much-reduced rate would transfer too much of the burden to those with real liquidity needs.

In what has become a key feature of the DD model, the true state of nature is revealed partially and sequentially according to the volume of withdrawals. This property lends support to the now-accepted view that bank contracts have a second-best nature. Because optimality is not discussed formally in Bryant's model, it is unclear to what extent desirable allocations incorporate some financial fragility or if what is called a run is compensation for having access to early information.

Another distinguishing feature of the literature that starts with DD is the limited reference to devices that can regulate bank fragility. This is explained by the emphasis on efficient allocations and the consequent limits to the range of admissible policies and interventions. Although the discussion of deposit insurance and partial suspension pointed out by Bryant made its way to the original DD setup, extensions focusing on aggregate uncertainty and sequential service skip considerations to such devices. Wallace (1988) was pivotal in stressing the importance of sequential service for the notion of fragility, as well as the need to study the contract that best deals with the imperfect flow of information and, thus, when deposit insurance is not feasible. Green and Lin (2003) successfully implement that agenda in a tractable version with finite traders. They rule out bank runs with specific assumptions about preferences and independency of shocks. Peck and Shell (2003) reinstate multiplicity by resorting to limited information about positions in the queue formed at the bank and to preferences leading to active truth-telling constraints.

In summary, the generality of key results about financial fragility have been revisited and is the subject of ongoing research. In order to provide some perspective on this research, I present below a particular formalization of the DD model that I find suitably short. Before doing that, let me stress some common elements of follow-up articles.

Four points on model choices can be highlighted. First and foremost, markets are no longer a primitive in the setup. The premise is that, if we are to find financial fragility as a result of asymmetric information, we should do so in the best way society can find to process information and to organize transfers while respecting individuals' incentives to truthfully reveal information. Imposing particular market organizations may hide better ways to organize exchanges.

Second, for reasons of tractability, it becomes important to abstract from payment instruments, like the use of money, the existence of which may depend on additional assumptions that would complicate the analysis. As a result, all the action in the model is restricted to a two-period structure in which a "bank" is a programmable technology that can commit to making transfers of real goods in these two periods according to announcements of preference shocks. In this sense, the bank becomes an aggregation of the intermediary, the productive sector, and the government.

Third, since it is not reasonable to assume that the government has more information about preferences than the individuals themselves, the bank machine is restricted to making transfers that are contingent on the flow of information provided by the requests to withdraw—the first-come, first-served structure of Bryant (1980). A combined bank-government, making payments in real goods, will then find it impossible to promote deposit insurance unless it can bypass the sequentiality of consumption (soliciting announcements first and then making payments only after collecting all answers). But if it can bypass sequentiality then a bank is never fragile. Thus, it becomes evident that sequential service must be taken as part of the physical environment. As Wallace (1988) discusses further, this observation moves the analysis definitively away from initial attempts of mixing banks and markets.

Fourth, once banking, production, and the benevolent government are aggregated into a mechanism that must respect sequential service, it follows that the best strategy for an individual is a function of the previous announcements made by those already serviced according to their position in line. If there is aggregate uncertainty about these requests then consumption is a random variable. One trivial case appears in the absence of aggregate uncertainty (as in Bryant [1980], in case there is no risk about future endowments and thus no inside information). The bank does not need to make payments contingent on line position and can suspend payments after the known fraction of impatient people has withdrawn. This scheme rules out any meaningful fragility.

3. SOME EXTENSIONS

In this section I provide a more formal description of the benchmark planner problem outlined in the introduction. This kind of specification has paved the way for very sharp results on the existence of multiple equilibria for the optimal deposit contract. Apart from the problem of runs, I shall also discuss how an element of imperfect monitoring, taking the form of delayed communication, sheds new light on the means of payment required to implement the optimum.

The Green-Lin Diamond-Dybvig Model

In the benchmark economy, bank runs can appear in the form of an early withdrawal by a patient individual concerned about the behavior of other patient individuals in the presence of aggregate uncertainty. Green and Lin (2003) demonstrate, however, that, with a finite number of individuals who receive shock realizations independently from each other, the solution of the optimal problem proposed by DD defines a deposit game that has the optimum as the unique equilibria. Green and Lin's demonstration relies on a class of preferences for which the optimum does not feature active truth-telling constraints. Their specification assumes simultaneous play: Individuals know their position in the queue but cannot choose strategies contingent on previous announcements.

Follow-up work produced at least three important results. First, Peck and Shell (2003) restored multiplicity with preferences that imply active truth-telling constraints when individuals are not informed about their position in line. Second, Ennis and Keister (2009b) return to the Green-Lin preferences and are able to provide a construction of the optimum explicitly even when shocks are correlated in a particular fashion. They shed light on the incentives for providing the informational externality alluded to in the introduction. In a run, the first individual in the queue announces his desire for early consumption even when truly patient. He thus fails to "inform" the planner about an important signal concerning the overall distribution of tastes when shocks are correlated. The second individual in the queue, when patient, is concerned about the likelihood that many other individuals are impatient since the optimum was designed with truth-telling, that is, under the assumption of the best provision of the informational externality. The second individual thus cannot use the best conditional distribution for the preferences of others. As Ennis and Keister (2009b) find, the concern is justified as a bank-run equilibrium attains in some numerical examples.

Third, Andolfato, Nosal, and Wallace (2007) provide clarification of several points. They start by modifying the Green-Lin specification to include more general preferences and to let individuals know the announcements of others holding previous positions in the queue. They present a different demonstration of the result that, with independent shocks, individuals do not need the externality to predict the distribution of preferences of those ahead. The conditional distribution is the same regardless of whether or not initial traders have chosen to misrepresent their types. Thus, that distribution is the same as the one used to define truth-telling constraints under the assumption that all individuals reveal their type. Since the optimum is constructed so as to respect those constraints, telling the truth is a maximizing choice even when initial players choose differently. Since the population is finite, a backward induction argument can be used to demonstrate uniqueness.

Andolfato, Nosal, and Wallace (2007) also raise questions about signalling in this setup. Although the formulation adopted by Green-Lin and Ennis-Keister has simultaneous play, it is not clear whether or not this specification is necessarily mandated by the DD environment. This issue is relevant since sequentiality of plays creates the opportunity for individuals to signal their true type. In addition, if beliefs are required to satisfy the intuitive criterion of Cho and Kreps (1987), among others, then it can be argued that the run equilibria do not survive the refinement: Given reasonable beliefs, a patient individual has no conflict of interest in providing the informational externality because his announcement reinforces truth-telling, which tends to save resources for the future (relative to runs). In summary, if the planner can inform individuals about announcements made by others, and if doing so eliminates fragility, then there are grounds for taking the sequential formulation of Andolfato, Nosal, and Wallace (2007) as a welcome improvement.[8]

Based on the arguments above, including the footnote, I take Andolfato, Nosal, and Wallace (2007) as a reasonable formulation of what is called the Green-Lin Diamond-Dybvig model, presented below. It is important to keep in mind, however, that a great deal about runs and their relationship to the

[8] Truth-telling constraints change when play is simultaneous. While the examples constructed in Ennis and Keister (2009b) feature inactive truth-telling constraints, a separating equilibrium may not satisfy constraints in the version with sequential play. But if it does satisfy constraints, say for some parameters, then the models would be observationally equivalent.

provisions of information and shock structures can be learned from the models of Peck-Shell and Ennis-Keister.

The problem considered by Andolfato, Nosal, and Wallace is more easily presented by referencing their deterministic case, ignoring possible welfare improvements associated with the use of lotteries. There is one consumption good for each of two dates and the number of individuals is N. There is an aggregate endowment of Y units of good 1 and a linear technology that transforms x units of good 1 into R units of good 2 ("rate of return," $R-1$). Each individual becomes type $t \in T$, where $T = \{$ i (impatient), p (patient)$\}$, and is assigned utility $u(c_1, c_2, t)$ according to a stochastic process and the mechanism assigning announcements to consumption bundles (c_1, c_2). Each individual maximizes expected utility. The environment also includes a random process that determines the vector, $\mathbf{t}$, representing the queue $(t_1, t_2, .., t_N)$, with the understanding that types are private information and t_i is the type of the jth individual in line. The stochastic process for $\mathbf{t}$ is specified by the probability measure $\pi = (\pi_0, \pi_1, ..., \pi_N)$ that describes the distribution of realizations of the total number of patient people, considering that all permutations determining place in line are equally likely. The draw of k patient individuals occurs with probability π_k.

Allocations are allowed to depend on positions in line and announcements, but are otherwise symmetric regarding identities. The mechanism reveals to each individual earlier announcements (so that the quantity of resources left is easily inferred). A strategy for an individual with place n is a function, $\mathbf{s}_n$, mapping $T^{n-1} \times T$ into announcements of types in T. The second argument is his or her true type, while the first is the vector of earlier announcements. A mechanism is a mapping (c_1^n, c_2^n) for each n , where c_1^n maps an announcement list of size n in T^n into good-1 consumption, and c_2^n maps each list, $\mathbf{t}$, into good-2 consumption, when all announcements become known.

Associated with mechanism $\mathbf{c}$, representing (c_1^n, c_2^n) for each n, and a strategy profile $\mathbf{s} = (\mathbf{s}_1, ..., \mathbf{s}_N)$, there corresponds an ex-ante expected utility

$$w(\mathbf{c}, \mathbf{s}) = \sum_{k,\mathbf{t},n} \pi_k \binom{N}{k}^{-1} u(c_1^n(\mathbf{s}_n), c_2^n(\mathbf{s}_N), t_n). \tag{1}$$

The planner's problem is to choose $\mathbf{c}$ in order to maximize ex-ante utility (1) under truth-telling, $w(\mathbf{c}, \mathbf{t})$, subject to feasibility,

$$R(Y - \sum_n c_1^n) \geq \sum_n c_2^n, \tag{2}$$

and truth-telling constraints. The latter are written according to beliefs about $\mathbf{t}_{n+1}$, the vector of types of those in line after n. For an individual, n, with type t_n, the probability of outcome $\mathbf{t}_{n+1}$ conditional on his or her type, as well as on earlier announcements, defines belief $\phi(\mathbf{t}_{n+1}; \mathbf{s}_{n-1}, t_n)$. The truth-telling constraint for individual n experiencing line history $\mathbf{t}_n$, including own type t, is

$$\sum_{\mathbf{t}_{n+1}} \phi(\mathbf{t}_{n+1}; \mathbf{t}_n) u(c_1^n(\mathbf{t}_n), c_2^n(\mathbf{t}_n, \mathbf{t}_{n+1}), t) \geq$$
$$\sum_{\mathbf{t}_{n+1}} \phi(\mathbf{t}_{n+1}; \mathbf{t}_n) u(c_1^n(\mathbf{t}_{n-1}, s), c_2^n(\mathbf{t}_{n-1}, s, \mathbf{t}_{n+1}), t) \quad (3)$$

for all $s \in T$.

It is required that ϕ be consistent with Bayes' rule. When shocks are independent across individuals, $\phi(\mathbf{t}_{n+1}; \mathbf{t}_n)$ is constant in $\mathbf{t}_n$. This allows the induction argument that rules out multiplicity.

Green and Lin (2003) also present a dynamic programming problem that corresponds to a relaxed planning problem that follows from ignoring truth-telling constraints. Green and Lin (2000) explicitly solve this problem for a class of preferences with linear indifference curves and three traders. The class of preferences is such that the constraint (3) does not bind when other individuals follow truth-telling strategies and the solution of the problem is the optimal allocation. As hinted above, Ennis and Keister (2009b) generalize their programming problem for an arbitrary number of traders and a particular structure of correlated shocks: The probability that a person in position n is patient is a function of the number of previous patient draws, not of their order among the $n - 1$ people. They are able to construct a solution recursively and to show, by means of examples, that the associated mechanism can also implement an equilibrium with misrepresentation (run).

Imperfect Monitoring

The formal emphasis on optimality pursued by DD on the issue of bank illiquidity has of course initiated many developments in the field that cannot be covered here. Alternative formulations of aggregate and extrinsic uncertainty have been proposed.[9] Weakening of the ability of the bank to commit has been pursued in fruitful ways.[10]

Prescott and Weinberg (2003, PW hereafter), for instance, propose a new extension. They compare two payment instruments, in a version of DD without aggregate uncertainty, where the use of payment devices can be distorted by opportunistic behavior. If we ignore the initial planning period, there are two dates and one consumption good per date in their model. The counterparts in their model for the DD consumers ("buyers" in their language) have preferences $tu(c_1) + v(c_2)$, where c_1 is consumption of date-1 good, c_2 is

[9] See Hellwigh (1994) for the case of stochastic last-period endowments.

[10] See Ennis and Keister (2009a) for a re-examination of suspension schemes when the planner cannot commit.

consumption of date-2 good, $v(0) = 0$, and t is a preference shock defining types and takes values in a finite set.

PW depart from the direct contact between consumers and the DD bank machine. Their basic goal is to compare the performance of bank drafts and checks as communication devices between consumers and the machine (the planner). The typical DD allocation could be implemented with the use of drafts, that is, pieces of paper that communicate that the buyer has funds available to make a purchase. If enough multiple drafts, with pre-set amounts are initially distributed to consumers, then no noise can occur in the communication with the machine. A problem appears because drafts are assumed to be costly to produce.

Checks are an alternative communication device that can be produced and distributed at zero cost. Check technology is, however, subject to fraud. There are two ways to describe the communication problem produced by checks. In the version detailed by PW, some consumers are randomly able to use checks with many sellers in a way that multiple purchases temporarily become private information to chosen individuals.

Because sellers are agents with trivial choices in their model, there is another description of the communication inefficiency. In this alternative version, consumers contact the bank machine directly and write checks according to their privately observed type, t. Consumers, however, also draw an opportunity to re-enter the bank line and make new withdrawal requests. The fraud of entering in line multiple times is only detected in period 2. Because $v(0) = 0$, there is limited punishment that can be imposed on period 2. As a result, optimality requires the imposition of an upper bound on the values that consumers can write on their checks.

PW restrict attention to symmetric allocations in the sense that consumers with different realizations of fraud opportunities are treated equally. A similar outcome could attain under the assumption that even a small fraud is too costly for society. Hence, the model is used to predict allocations under the *threat* of fraud (or “bingeing”).[11]

The framework allows for differentiated initial individual wealth, w. In the simpler case, w is public information and a bank is formed to deal exclusively with “population w” in isolation. That is, w becomes a parameter for the comparative statics predicting the use of drafts or checks in that population.

Assuming the existence of a continuum of consumers, an allocation is a pair of functions $(c_1(t, w), c_2(t, w))$ defined on the Cartesian product of the set of types and the set of initial endowments. Feasibility requires that aggregate

[11] Cavalcanti and Nosal (2010) endow to a subset of pairwise traders a particularly low cost of falsifying money, finding optima with a positive mass of counterfeits. A monetary model with closer links to the PW idea would, however, be the model of counterfeiting threats of Nosal and Wallace (2007).

endowment finances expect total consumption $c_1(t, w) + c_2(t, w)$ on a linear basis, net of the costs of using drafts.

When w is public information, the truth-telling constraints are common to both draft and check economies,

$$tu(c_1(t, w)) + v(c_2(t, w)) \geq tu(c_1(t', w)) + v(c_2(t', w))$$

for all (t, t'), but checks require a new constraint, because of the upper bound referred to above,

$$tu(c_1(t, w)) + v(c_2(t, w)) \geq tu(d(w)).$$

The right-hand side is the deviation payoff that follows from the worst fraud of appearing in line N times (consuming from N sellers, instead of one) and announcing the highest discount factor at each time, so that the property, d, of an allocation is defined by

$$d(w) = N \max_t c_1(t, w).$$

Two distortions implied by the upper-bound constraint are easily seen. First-period consumption in a check economy cannot vary with t as much as in a draft economy, otherwise an undesirable increase in payoff d is produced. Also, by moving from a draft economy (d identically zero) to a check economy, the upper-bound constraint becomes tighter and more consumption on date 2 needs to be allocated, further reducing the bank's ability to insure risk. PW also deliver results about the choice of payments as individuals become wealthier. Since, in their setup, wealthy individuals plan higher levels of date-2 consumption, there is a sufficient increase of punishment to fraud in date-2 ($c_2 = 0$) so as to increase check limits as w grows. Thus, the tendency is to shift from drafts to checks with increases in wealth. It is clear that many more payment questions can be asked in this line of research that managed to stay so close to the original DD model.

4. BANKS AS PEOPLE

There is no need to limit attention to banks that can be easily programmed to perform intermediation duties. Important progress has been made by models that give banks incentive constraints. As we shall see, this has become a useful device for introducing banking in less centralized environments where money plays a role of medium of exchange.

Endogenous Sequential Service

Calomiris and Kahn (1991) propose treating banks as individuals with commitment difficulties and who can hide resources. Fraud outcomes can be mitigated by investments on information acquisition by depositors, as well as by the employment of an additional technology that removes control of the bank's assets at a cost. Under the interpretation that the use of this technology corresponds to an early withdrawal or liquidation, they conclude that optima require bank liabilities in the form of demand deposits.[12]

The approach in Calomiris and Kahn (1991) leads to models in which sequential service is not an ingredient for explanations of fragility as in DD but is instead an equilibrium feature that exists to discipline banks. One is led to the conclusion that there appears to exist a choice between studying the DD view of fragility when the emphasis is on the behavior of other depositors and the costly state-verification model that applies not only to banking but also to optimal contracts in abstract principal-agent problems.[13]

While there are many ways to introduce elements of fraud in economic models, it is also important to work with abstractions that capture the essence of what financial markets do. SW and Bryant have pointed to basic issues: attempts to understand the co-existence between money and credit or between banks and payment instruments. One can return to these fundamental issues knowing that future developments can always add further considerations about fraud.

Money and Credit

The agenda of making explicit the role of money with endogenous supply borrows a great deal from mechanism design. The influence of DD on this agenda can be illustrated by the importance given in the monetary literature to the concept of the essentiality of money. The analogous question in the DD model is whether or not banking arrangements are indeed fragile. It is important to rule out other arrangements, such as deposit insurance or partial suspension, that can provide the same levels of utility as in the optimum but without being exposed to multiplicity. By looking for a physical environment with such properties, one is identifying primitives that give rise to the notion of financial fragility. As discussed above, sequential service has been identified as a necessary friction, although not a sufficient one, for the fragility feature. When runs are present, sequential service is one of the conditions that make fragility *essential* in the DD model (without fragile allocations welfare would not be maximized).

Likewise, in monetary theory, one is always looking for conditions that make money essential. It has been shown that imperfect monitoring and the

---

[12] Another advantage of fragility, pursued by Diamond and Rajan (2001), is the notion that when creditors can commit to a run, the bank is encouraged to monitor borrowers. There is also a link in their work to the human capital interpretation of collection technologies emphasized by Kiyotaki and Moore (2000).

[13] See Townsend (1979), Diamond (1984), and Dewatripont and Tirole (1994), including references in the latter to models of entrepreneurial banks.

absence of commitment to future actions are necessary conditions to rule out trigger strategies and other credit arrangements that can substitute for money. Like sequential service, those conditions are frictions that impede smooth operations of markets and call into question the validity of dichotomies alluded to in Section 1. In this sense, money and banking have a lot in common and should, perhaps, be studied in a unified framework.

Notice, however, that the strategy in early work on models of medium of exchange, influenced by this essentiality reasoning, was to adopt a strong form of imperfect monitoring: total anonymity. Although anonymity preserves money, it rules out all forms of credit. A natural step in this literature was to look for weaker primitives that make the use of money essential in trade but facilitate credit in ways that do not eliminate money from the optimum. That strategy has the chance of producing banks according to their interaction with money-creation mechanisms.[14]

Such a link between money and banking appears in Deviatov and Wallace (2009, DW hereafter), provided that we interpret their monitored individuals as banks. Their mechanism is a representation of a central bank that can inject and destroy money in a particular fashion over seasons. Individuals are prohibited from creating money (notes) in their computed examples, but extending the model to an inside-money version is conceptually straightforward.

The model neither assumes that all individuals accomplish intertemporal trade based on announcements nor that they are all anonymous (like Kiyotaki and Wright [1989] or Levine [1991]).[15] It assumes instead that an exogenous fraction of the population is perfectly monitored (the m people) and that the remainder is not monitored at all (the n people). Only the planner can commit to future actions.[16]

In other respects, DW build on a typical random-matching model of money with seasons (it is useful to think of random matching as a restriction on the physical movement of goods between people).[17] The continuum of people rules out aggregate uncertainty, the horizon is infinite, the common discount factor is $\beta \in (0, 1)$, and goods are perishable. There is a symmetric division of people according to the goods they like and produce. There are two stages

[14] Another avenue is the study of how sequential service would change established *real* models of intertemporal trade. The problem solved by Green (1987), for instance, predicts transfers across a continuum of individuals facing privately observed endowment shocks. The optimal allocation could be considered an illiquid one and it is not known how sequential service would change the predictions.

[15] The environments compared in Kocherlakota (1998) are also extreme cases.

[16] The setup follows from Cavalcanti and Wallace (1999a, 1999b, 2008). Cavalcanti (2004) considers a version in which banks store capital and take announcements from the nonbank public. Cavalcanti, Erosa, and Temzelides (1999, 2005) study an equilibrium version in which banks are monitored by the creation and redemption of notes against reserves.

[17] See Cavalcanti and Nosal (2009) for a random-matching model with seasons in which the optimum requires ongoing interventions in the money supply because money can get "stuck in the wrong hands."

at each date. The first stage has pairwise meetings and the second stage has a centralized meeting. In the first stage an individual meets randomly a producer with probability $1/K$ or a consumer of the good he can produce with probability $1/K$ (no relevant meeting with probability $1 - 2/K$). The period utility from consuming q units of the desired good is $u(q)$. The period utility from producing q units for a capable producer is $-q/\delta_t$, where the productivity parameter moves seasonally: $\delta_t = \delta_l$ at odd dates (low aggregate productivity) and $\delta_t = \delta_h$ at even dates (high aggregate productivity). The monitored status and the consumer-producer status in meetings are common knowledge. Holdings of money are private for n people and observable for m people.

An n person must receive money in order to consume. For simplicity, money holdings are assumed indivisible and restricted to $\{0, 1\}$ at stage 1 and to $\{0, 1, 2\}$ at stage 2. An n person with money is thus so rich that he cannot be induced to produce, creating a nontrivial problem of liquidity distribution for this economy (he cannot lend his money in period 2 because the lending action of a person n cannot be recorded). There is, however, a need to arrange borrowing and lending among the m people, which can be done at stage 2.

DW study allocations that are two-date periodic (stationary) and that treat the same people in the same state: a point in $\{m, n\} \times \{0, 1\}$ at the beginning of stage 1 and a point in $\{m, n\} \times \{0, 1, 2\}$ at the beginning of stage 2. An allocation describes trade in output and money in stage 1, according to states and season. It also describes transfers of money in stage 2 and the fraction of each type who has money at the start of a season.

DW use lotteries to model transfers of money because of indivisibilities and consider that m people can be punished with banishment to the set of n people, although that never happens in equilibrium. Only individual defection is allowed in stage 2 but deviation by the pair is also allowed in stage 1. Since the holdings of n people are private information, an allocation must propose a menu of trades to which they self-select. The objective of the planner is average expected utility, assuming that the initial date has low productivity.

The optimum is computed numerically for an arbitrary example with high discount factor. DW find that the optimum has injection of money (stage 2) at dates of high productivity. There are no stage-2 transfers of money to n people (in order to preserve incentives for them to acquire money). Put another way, m people spend more than they earn at high dates (the opposite for low dates). DW also find that m people always start a date with money, and thus the threat of punishment is important to force them to produce. Computed welfare indicates that the main beneficiaries of the money interventions are the n people. Type-m people can be instructed to give gifts, that is, to produce to n people without money.

The seasonal monetary policy provides smoother output in meetings where the producer is type n (and has no money) and the consumer is type m (and has money). The computed lotteries in those meetings predict higher spending during the high season compensated by injections of money that increase the fractions holding money. The injection must be offset by destruction during low seasons in order to preserve stationarity.

Although the model is formulated so that strong stationarity assumptions deliver a small state space, it documents that general principles about the nature of the optimum are difficult to anticipate, in contrast, for instance, to old-style monetary models for which one can guess that the Friedman rule is optimal from the start. DW conjecture that this difficulty is here to stay, at least if one wants to preserve a model capable of addressing the circulation of private bank notes.

5. FINAL REMARKS

In this article I compare the approach of Diamond and Dybvig with some other influential work in the field of money and banking. It is natural for reviews of this topic to mention previous attempts to mix general-equilibrium theory with banking. Having presented reasons for avoiding the old strategy of mixing banks and markets, one also has to explain why banking models seem so distant from monetary theory. I provide a unifying explanation. The DD approach has been successful in its choice for mechanism design. Without it, conclusions about fragility would be controversial. One has to give serious consideration to the possibility that financial fragility is an intrinsic consequence of trying to extend to individuals the best liquidity arrangements possible. This favors mechanism design over general-equilibrium theory. In addition, banking models become closer to monetary theory when monitoring becomes weaker.

There are, of course, negative aspects to consider. By posing a well-specified model of liquidity provision in which mechanism design and new formalizations of sequential service could be easily adopted, DD depart from the macroeconomic tradition of staying close to the data in the way it is traditionally presented, that is, with a great deal of reference to market statistics like rates of returns, and instead focus on payoffs accruing to individuals.

The dichotomy-tradition alternative is reflected in the way most textbooks in money and banking are organized.[18] Empirical observations about the functions performed by financial systems motivate "extensions" of the competitive-equilibrium model so that they incorporate payment services, risk management, collateral arrangements, etc. In this tradition, the DD model falls in the chapter of liquidity risk, which is to say it is supposed to provide guidelines

[18] See, for instance, Niehans (1978) and Freixas and Rochet (1997).

about deposit insurance or suspension of bank payments in contingencies of instability.[19] I have discussed concerns about this tradition on many grounds.

It is, however, difficult to predict how macroeconomics will incorporate lessons provided by the Diamond-Dybvig structure and other models of inside money. Presently, a variety of approaches are currently being considered. The validity of building market games with weak commitment assumptions, an approach dating back to the pioneering work of Shubik, is as debatable now as it was 30 years ago. For the moment, much can be learned from understanding that the assumptions that make banks important in the Diamond-Dybvig model are not very far from those that make inside money important in exchange models.

REFERENCES

Amir, Rabah, Robert J. Aumann, James Peck, and Myrna Wooders. 2009. "Introduction to the Special Issue of *Games and Economic Behavior* in Honor of Martin Shubik." *Games and Economic Behavior* 65 (January): 1–6.

Andolfato, David, Ed Nosal, and Neil Wallace. 2007. "The Role of Independence in the Green-Lin Diamond-Dybvig Model." *Journal of Economic Theory* 137 (November): 709–15.

Bryant, John. 1980. "A Model of Reserves, Bank Runs and Deposit Insurance." *Journal of Banking and Finance* 4 (December): 335–44.

Calomiris, Charles W., and Charles M. Kahn. 1991. "The Role of Demandable Debt in Structuring Optimal Banking Arrangements." *American Economic Review* 81 (June): 497–513.

Cavalcanti, Ricardo. 2004. "A Monetary Mechanism for Sharing Capital: Diamond and Dybvig Meet Kiyotaki and Wright." *Economic Theory* 24 (November): 769–88.

Cavalcanti, Ricardo, Andrés Erosa, and Ted Temzelides. 1999. "Private Money and Reserve Management in a Random-Matching Model." *Journal of Political Economy* 107 (October): 929–45.

[19] Some authors believe that an "asymmetric paradigm" can handle banks just as well as it can handle managers and firms. These partial-equilibrium models would be very difficult to blend with monetary models.

Cavalcanti, Ricardo, Andrés Erosa, and Ted Temzelides. 2005. "Liquidity, Money Creation and Destruction, and the Returns to Banking." *International Economic Review* 46: 675–706.

Cavalcanti, Ricardo, and Ed Nosal. 2009. "Some Benefits of Cyclical Monetary Policy." *Economic Theory* 39 (May): 195–216.

Cavalcanti, Ricardo, and Ed Nosal. 2010. "Counterfeiting as Private Money in Mechanism Design." JMCB conference mimeo.

Cavalcanti, Ricardo, and Neil Wallace. 1999a. "Inside and Outside Money as Alternative Media of Exchange." *Journal of Money, Credit and Banking* 31 (August): 443–57.

Cavalcanti, Ricardo, and Neil Wallace. 1999b. "A Model of Private Bank-Note Issue." *Review of Economic Dynamics* 2 (January): 104–36.

Cavalcanti, Ricardo, and Neil Wallace. 2008. "New Models of Old(?) Payment Questions." In *The Future of Payment Systems*, edited by Andrew G. Haldane, Stephen Millard, and Victoria Saporta. New York: Routledge, 75–86.

Cho, In-Koo, and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics* 102 (May): 179–221.

Deviatov, Alexei, and Neil Wallace. 2009. "A Model in which Monetary Policy is about Money." *Journal of Monetary Economics* 56 (April): 283–8.

Dewatripont, Mathias, and Jean Tirole. 1994. *The Prudential Regulation of Banks*. London: The MIT Press.

Diamond, Douglas W. 1984. "Financial Intermediation and Delegated Monitoring." *Review of Economic Studies* 51 (July): 393–414.

Diamond, Douglas W., and Philip H. Dybvig. 1983. "Bank Runs, Deposit Insurance and Liquidity." *Journal of Political Economy* 91 (June): 401–19.

Diamond, Douglas W., and Raghuram G. Rajan. 2001. "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking." *Journal of Political Economy* 109 (April): 287–327.

Dubey, Pradeep, John Geanakoplos, and Martin Shubik. 2005. "Default and Punishment in General Equilibrium." *Econometrica* 73: 1–37.

Ennis, Huberto, and Todd Keister. 2009a. "Bank Runs and Institutions: The Perils of Intervention." *American Economic Review* 99 (September): 1,588–607.

Ennis, Huberto, and Todd Keister. 2009b. "Run Equilibria in the Green-Lin Model of Financial Intermediation." *Journal of Economic Theory* 144 (September): 1,996–2,020.

Freixas, Xavier, and Jean-Charles Rochet. 1997. *Microeconomics of Banking*. London: The MIT Press.

Green, Edward J. 1987. "Lending and the Smoothing of Uninsurable Income." In *Contractual Arrangements for Intertemporal Trade*, edited by Edward C. Prescott and Neil Wallace. Minneapolis: University of Minnesota Press, 3–25.

Green, Edward J., and Ping Lin. 2000. "Diamond and Dybvig's Classical Theory of Financial Intermediation: What's Missing?" Federal Reserve Bank of Minneapolis *Quarterly Review* 24: 3–13.

Green, Edward J., and Ping Lin. 2003. "Implementing Efficient Allocations in a Model of Financial Intermediation." *Journal of Economic Theory* 109 (March): 1–23.

Hellwig, Martin. 1994. "Liquidity Provision, Banking, and the Allocation of Interest Rate Risk." *European Economic Review* 38 (August): 1,363–89.

Kiyotaki, Nobuhiro, and John Moore. 2000. "Inside Money and Liquidity." London School of Economics mimeo.

Kiyotaki, Nobuhiro, and Randall Wright. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy* 97 (August): 927–54.

Kocherlakota, Narayana. 1998. "Money is Memory." *Journal of Economic Theory* 81 (August): 232–51.

Levine, David K. 1991. "Asset Trading Mechanisms and Expansionary Policy." *Journal of Economic Theory* 54 (June): 148–64.

Niehans, Jurg. 1978. *The Theory of Money*. Baltimore, Md.: The John Hopkins University Press.

Nosal, Ed, and Neil Wallace. 2007. "A Model of (the Threat of) Counterfeiting." *Journal of Monetary Economics* 54 (May): 994–1,001.

Peck, James, and Karl Shell. 2003. "Equilibrium Bank Runs." *Journal of Political Economy* 111: 103–23.

Prescott, Edward S., and John A. Weinberg. 2003. "Incentives, Communication, and Payment Instruments." *Journal of Monetary Economics* 50 (March): 433–54.

Shubik, Martin, and Charles Wilson. 1977. "The Optimal Bankruptcy Rule in a Trading Economy using Fiat Money." *Journal of Economics* 37 (September): 337–54.

Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory* 21 (October): 265–93.

Wallace, Neil. 1988. "Another Attempt to Explain an Illiquid Banking System: The Diamond and Dybvig Model with Sequential Service Taken Seriously." Federal Reserve Bank of Minneapolis *Quarterly Review* 12 (Fall): 3–16.