A MONETARIST MODEL
OF THE INFLATIONARY PROCESS

Thomas M. Humphrey

Given the inherent complexity of the current inflation problem and the tendency of individuals to differ in their interpretation of events, it is not surprising that a number of competing theories of inflation exist today. This article seeks to explain one of these theories—namely, the monetarist view—with the aid of a simple dynamic macroeconomic model developed by the British economist Professor David Laidler. Laidler's model is enlightening for reasons quite apart from its monetarist orientation. Although exceedingly simple, it nevertheless effectively conveys all the essentials of dynamic process analysis—steady-state solutions, disequilibrium dynamics, stability conditions, etc. It is representative of a whole class of models that deal not with levels but rather rates of change of economic variables. These models are gradually supplanting the once-popular standard textbook or diagrammatical version of the Hicks-Hansen IS-LM model, whose static equilibrium format is not ideally suited to deal with the phenomenon of continuing inflation or with the dynamics of disequilibrium processes wherein economic variables evolve and interact over time. Therefore, regardless of the particular theory being expounded, Laidler's model can be viewed as an introduction to a distinctive form of macroeconomic analysis that attempts to specify the time paths of the inflation rate and related variables.

A word should be said at the outset about the article's position on rival theories of inflation. Regarding the merits of alternative views, this article takes a deliberately neutral stance. Neither monetarism nor any other theory is advocated as being the most nearly correct. No claims are made for the superiority or indeed even the validity of the monetarist view. The sole aim is to articulate the monetarist interpretation within the framework of a mathematical model whose exposition constitutes a useful exercise in its own right. It should be strongly emphasized, however, that the model constitutes a severe oversimplification of a complex process and thus would probably fit the statistical data poorly. As used in this article, the model is intended solely as an expository device and therefore purposely abstracts from many of the variables and behavior relationships that a well-specified empirical model would contain.

Monetarist Propositions Any mathematical model that purports to convey the essence of monetarism must embody certain key propositions or postulates that characterize the monetarist position. Not all of these propositions, however, can be regarded as exclusively monetarist. Some would be accepted to a greater or lesser degree by nonmonetarists. It is therefore desirable to divide these propositions into two groups, namely, those that are distinctively monetarist and those that are not. A partial listing of the uniquely monetarist propositions would include the following:

1. MONETARY THEORY OF INFLATION. Monetarists hold that inflation is a purely monetary phenomenon that can only be produced by expanding the money supply at a faster rate than the growth of capacity output. Thus at any given time the actual rate of inflation is seen as reflecting current and past rates of monetary expansion. Monetarists reject nonmonetary explanations of inflation—i.e., those that attribute rising prices to such alleged causes as shifts in autonomous private expenditures, government fiscal policies, cost-push influences, food and fuel shortages, etc.—on the grounds that an increased stock of money per unit of output is required in all cases and therefore constitutes the true cause of inflation. In short, the sole necessary and sufficient condition for the generation of inflation is said to be excessive monetary growth.

2. LONG-RUN STABILITY (NEAR-CONSTANCY) OF VELOCITY. The proposition of a near-constant circulation velocity or rate of turnover

---

1. Laidler presents his model in two papers: "The 1974 Report of the President's Council of Economic Advisers: The Control of Inflation and the Future of the International Monetary System," American Economic Review, 64 (September 1974), pp. 535-49, and "The Influence of Money on Real Income and Inflation: A Simple Model with some Empirical Tests for the United States 1965-72," Manchester School of Economic and Social Studies, 41 (December 1973), pp. 367-95. The version of the model contained in the present paper differs from Laidler's in at least five respects. First, it is simpler and employs a different notation. Second, its numerous close linkages with monetarism are identified. Third, it is employed solely to explain the monetarist view of inflation. Fourth, an explicit derivation of the equations is provided. Fifth, the model and its components are expounded in considerably greater detail than in Laidler's rather terse treatment.

2. Monetarists readily admit that nonmonetary influences—e.g., union wage pressure, monopoly (administered) pricing policies, OPEC cartels, oil embargoes, crop failures, commodity shortages, and the like—can directly affect particular prices. But they argue that without excessive monetary growth such nonmonetary-induced rises in the prices of some commodities eventually would be offset by declines in the prices of others, leaving the average price level unchanged.
of money follows logically from the monetarist view that inflation stems solely or largely from excessive monetary growth. For if velocity were not a constant it would exhibit a non-zero rate of change that would supplement monetary growth as a separate and independent determinant of inflation. It follows, therefore, that monetarists must assume that velocity is at least a quasi-constant if they are to assert that inflation stems solely or primarily from changes in the stock of money per unit of output.

3. EXOGENEITY OF THE NOMINAL STOCK OF MONEY. Monetarists treat the quantity of money and its rate of growth as variables whose magnitudes are fixed outside the system. This theory contrasts sharply with the nonmonetarist treatment of money as an endogenous variable determined within the system by the level of economic activity and by the public's preferences for money and for liquid-asset money substitutes. The exogeneity postulate implies that monetary growth enters the system as a datum to determine the growth rates of spending, prices, and nominal income. The postulate is therefore consistent with the monetarist view of monetary growth as the independent causal factor governing the rate of inflation.

4. ABSENCE OF REVERSE CAUSALITY RUNNING FROM INCOME TO MONEY. Implied by the exogeneity condition, this proposition rejects the notion of passive income-determined monetary growth and asserts the monetarist view of the unidirectional channel of influence of monetary perturbations originating in money and running from money to spending to income to prices. Monetary growth is seen as entering this sequence not as a dependent or accommodative variable responding passively to prior income growth but rather as the active independent variable that precedes and causes inflation. It is true that monetarists, in their asides and qualifications, acknowledge that income may influence money indirectly through the policymakers' reactions to changes in the economy. But for the most part they have not incorporated such policy response functions into their formal models, and they continue to treat monetary policy as largely exogenous.

The preceding constitutes the group of uniquely monetarist tenets. As for the remaining key propositions, i.e., those that monetarists share with at least some nonmonetarists, they can be listed briefly. They include the following: (5) the non-neutrality of money in the short run (i.e., the tendency for changes in monetary growth to have substantial effects on real output and employment in the short run); (6) the long-run neutrality of money (i.e., the tendency for changes in monetary growth to have no lasting impact on real output and employment but only on the rate of inflation); (7) the view of erratic and volatile monetary growth as the prime cause of business cycles; (8) the inherent stability of the economy (i.e., the view of the system as a self-regulating mechanism, perturbations of which tend to generate only damped cycles about full-employment equilibrium); (9) the existence of long lags in the response of inflation to changes in the rate of monetary growth; and finally (10) the importance of inflationary expectations in determining market wage- and price-setting behavior. As shown below, Laidler's model is capable of accommodating all these propositions.

The Model and Its Components. The model itself is composed of three equations, the first being the monetary growth equation. A dynamic version of the static Cambridge cash-balance formula, this equation relates the rate of growth of real (price-deflated) cash balances to the growth rate of real output. The second relation in the model is a price-adjustment equation that explains the determination of the current rate of inflation. The third component is an expectations-formation equation that embodies a particular hypothesis about how people formulate their expectations of the future rate of inflation. Using these three equations one can solve for the three endogenous variables of the model, namely, (1) the current rate of inflation, (2) the expected rate of inflation, and (3) an excess demand variable represented by the gap between actual and capacity real income. In addition to these endogenous variables there is one exogenous variable, the growth rate of the nominal money stock, and one exogenous constant, the growth rate of full-capacity real income. This treatment of the monetary variable reflects the monetarist view of the money stock and its growth rate as largely exogenous magnitudes determined by an autonomous central bank via its control over a base of so-called high-powered money, consisting of currency and bank reserves. It also effectively rules out any reverse-causation feedbacks running from income to money. The assumption of a fixed capacity growth rate also squares with monetarist doctrine, which holds that the long-run path of potential output is independently determined by fundamental real economic conditions including technological progress and labor force growth.

Three other features of the model should be mentioned at the outset. First, all relations are linear and are expressed in logarithmic form. There is a specific reason for this formulation. Modern monetarist analysis is usually stated in terms of percentage rates of change of the relevant variables. And since the percentage change of any variable over a given interval of time can be represented mathematically by the first time difference of its logarithm, it follows that a log-linear formulation facilitates the analysis.

A second feature of the model is the introduction of time delays in the form of lagged relationships among the variables. These lags reflect the mone-
monetarist view of the many delays or frictions inherent in the inflationary process. Their inclusion also permits the analyst to describe the time paths taken by output and prices following a monetary disturbance.

The third feature of the model is its extreme simplicity, as manifested by the minimal number of variables it contains. In particular, the model possesses neither an interest-rate variable nor a variable to represent a discrepancy between actual and desired real cash balances. As a result, the model ignores two potentially important elements in the inflationary process, namely, (1) changes in the rate of interest and (2) the transitory rise in real cash balances (or the temporary fall in the velocity of money) that occurs at the beginning of inflationary periods immediately following a rise in the growth rate of money. These elements could of course be explained in a more complex model, but such a model would lose in simplicity, manageability, and ease of comprehension what it gains in completeness. Moreover, Laidler's model, despite its simplicity, is capable of explaining a large part of the inflationary process, namely, how variations in the growth rate of the money stock are divided between changes in real output and prices both in the short and the long run.

As for notation, the model employs the following symbols. Let \( m \) be the money stock, \( y \) actual real income, \( y_c \) standard or normal capacity real income, \( x \) the excess demand variable represented by the difference between actual and capacity income, i.e., \( x = y - y_c \), and \( p \) the price level—with all variables expressed as logarithms. Actual real income, \( y \), can exceed capacity, \( y_c \), because the latter is defined not as the absolute physical limit or maximum ceiling level of output but rather as the output associated with the economy's normal or standard level of operation. This concept of capacity or potential output corresponds roughly to the monetarist notion of the natural rate of unemployment, i.e., the unemployment rate that, given the inevitable frictions, rigidities, and market imperfections existing in the economy, is just consistent with equilibrium between demand and supply in the labor market. The superscript \( e \) denotes the expected value of a variable, and the subscripts \(-1\) and \(-2\) denote time lags of one and two periods, each defined as being a year in length.\(^1\) The symbols \( \Delta \) and \( \Delta^2 \) appearing before a variable denote first and second time differences, respectively, so that the model is effectively expressed in terms of proportional rates of change and rates of acceleration of
deceleration of those rates of change. Finally, a bar over a variable indicates that it is exogenous, i.e., determined outside the system.

**The Monetary Growth Equation** The first equation of the model is the monetary growth equation:

\[
(1) \quad \bar{\Delta} m - \Delta p = \Delta y = \Delta x + \Delta y_c.
\]

This equation states that the rate of growth of the real money stock—i.e., the percentage rate of nominal money growth, \( \Delta m \)—less the percentage rate of price inflation, \( \Delta p \)—determines the percentage change in real expenditure and hence real income, \( \Delta y \), that occurs during the given period. More precisely, a rate of growth of the real money stock, \( \Delta m - \Delta p \), in excess of the growth rate of capacity output, \( \Delta y_c \), causes the growth rate of actual output, \( \Delta y \), to deviate from the capacity growth rate, where the deviation is represented by the variable \( \Delta x \), i.e., \( \Delta x = \Delta y - \Delta y_c \).

Equation (1) implies a constant unitary income elasticity of demand for real (price-deflated) money balances. This condition follows from the notion—associated with the old Cambridge cash-balance version of monetarism—that people desire to maintain a stable (constant) proportional relationship between their real cash balances and real income. If the ratio of real balances to real income is to remain fixed, then both elements of the ratio must grow at the same percentage rate, as in equation (1). The monetary growth equation also expresses the strong monetarist view of a stable equiproportional relationship between changes in nominal money and nominal income and likewise between changes in nominal money per unit of output and the price level. The equation predicts that a given percentage change in nominal money will be matched by an identical percentage change of nominal income. The same holds for percentage changes of nominal money per unit of output and of prices. Note, however, that the equation, by itself, is incapable of expressing a stable predictable short-run relationship between nominal monetary growth and the inflation rate. This is because, in the short run, monetary growth may stimulate output as well as prices. And one cannot determine from equation (1) alone the proportions in which the stimulus will be divided between price changes and output changes. One has to supplement equation (1) with the price-adjustment equation to explain this division.

Equation (1) may also be interpreted as embodying a crude monetarist view of the direct expenditure mechanism whereby monetary impulses are transmitted directly to income via a prior effect on the

---

\(^1\)The time period in Laidler's model is not specified but is defined here as one year to conform to the monetarist interpretation that this article is developing.
demand (spending) for goods. The direct mechanism should be contrasted with the indirect interest-rate mechanism—often stressed by nonmonetarists—in which monetary changes influence income indirectly via a prior effect on the rate of interest.\(^5\) As shown in Appendix A, the money growth equation is derived from the celebrated Cambridge cash-balance equation and assumes that the velocity of money (or the Cambridge \(K\)) is constant and that the money market clears with sufficient rapidity to maintain equality between money demand and supply. The constant-velocity assumption is what insures that given rates of monetary growth, real and nominal, will be matched in equation (1) by corresponding identical rates of income growth, real and nominal.

The Price-Adjustment Equation The second equation of the model explains how the current rate of inflation is determined, i.e., the rate at which businessmen mark up their product prices. The price-adjustment equation is written in the following way:

\[
\Delta p = ax^{-1} + \Delta p^{-1}
\]

where \(\Delta p\) is the current rate of inflation, \(x^{-1}\) is excess demand lagged one period, and \(\Delta p^{-1}\) is the rate of inflation expected to prevail in the present period as of the preceding period. The price-adjustment equation expresses a short-run relationship between the rate of inflation, \(\Delta p\), and excess demand, \(x\), the latter measured by the gap between actual and potential (i.e., normal capacity) output. The existence of a gap implies that businessmen are straining productive capacity in an effort to meet demand. Spare plant and equipment are being drawn into use and increasing resort is being had to overtime and marginal labor. In brief, resources become increasingly scarce relative to demand as production approaches and then surpasses standard capacity output. The size of the gap measures the pressure of resource scarcity on prices. The larger the gap, the greater the pressure. As the gap expands, wages are bid up, labor-lou productivity falls, unit costs rise, bottlenecks develop, and the backlog of unfilled orders mounts. All these forces combine to cause prices to rise at an increasingly rapid rate. Thus inflation accelerates as the gap expands.

From the preceding discussion, it is evident that the price-adjustment equation is similar to so-called Phillips-curve equations that state a trade-off rela-

\(^5\) Modern monetarists acknowledge that interest rate effects are always present. They view the direct mechanism merely as an empirical proxy for the indirect mechanism in which many specific interest rate effects cannot be captured statistically either because they are implicit and hence unobservable or because they are too weak and too brief to be measured.
Embodied in the equation is a particular theory—the so-called adaptive-expectations or error-learning hypothesis—of how inflationary expectations are formed. According to the error-learning hypothesis, people formulate expectations about the inflation rate, observe the discrepancy between the actual and anticipated rates, and then revise the anticipated rate by some fraction of the error between the actual and anticipated rates. It can also be shown that the adaptive-expectations hypothesis is equivalent to the theory that people formulate price-expectations by looking at a geometrically-weighted average of current and past rates of inflation with the weights diminishing exponentially as time recedes. This weighting scheme implies that people assign higher weights to more recent phenomena when forming expectations.

How realistic is the error-learning hypothesis? Some economists claim that it is not an accurate description of how anticipations are formed. These analysts argue that expectations are as likely to be generated from direct forecasts of the future as from mere projections of the past. Moreover, they assert that people probably base anticipations at least as much on current information about a variety of developments as on old data pertaining solely to past price changes. There is undoubtedly much truth in these observations. Nevertheless, the error-learning formulation will be retained in this article subject to the caveat that purely extrapolative price forecasts may be modified by additional information.

The Complete System Taken together, the money growth, price-adjustment, and expectations-formation equations form a simple three-equation system that embodies a monetarist view of the inflationary process. To recapitulate, the complete system is written as follows:

1. \[ \Delta m - \Delta p = \Delta x + \Delta y_e = \Delta y \]
2. \[ \Delta p = ax_{-1} + \Delta p^e_{-1} \quad a > 0 \]
3. \[ \Delta p^e = b \Delta p + (1-b) \Delta p^e_{-1} \quad 0 < b < 1 \]

The variables in this system of equations interact to determine the rates of expected and actual inflation and the short-run growth rate of real income. The logic of the system implies that variations in the money growth rate initially affect excess demand, thereby inducing real income to deviate from its full-employment path. Lagged excess demand interacts with lagged price-expectations in equation (2) to determine the current rate of inflation. The current rate of inflation enters equation (3) to influence the expected rate, which in turn feeds back into equation (2) to become a determinant of next period’s inflation rate. Finally, in equation (1) the current rate of inflation interacts with the given rate of monetary growth to determine the growth rate of real income. In this manner the system and its constituent elements determine the division of monetary growth, \( \Delta m \), between price and output growth, \( \Delta p \) and \( \Delta y \).

Less formally, the model implies the following causal chain.

1. Inflation is determined by excess demand and inflationary expectations.
2. Inflationary expectations are generated by previous inflationary experience and hence by previous excess demand.
3. Excess demand is created by excessive monetary growth.
4. Therefore, excessive monetary growth—past and present—is the root cause of inflation.

The Long Run and the Short It is useful at this point to distinguish between the long-run and the short-run properties of the system of equations. This dichotomy, of course, corresponds to the two main stages or phases of the inflationary process, i.e., the temporary or transition phase in which changes in monetary growth affect real output and employment and the final or permanent stage in which the sole impact is on the rate of inflation. It also corresponds to the monetarist distinction between the long-run neutrality and the short-run non-neutrality of money. In the context of the model, the long run refers to the equilibrium or steady-state solution of the system after it has completely adjusted to a monetary disturbance. By contrast, the short run refers to the disequilibrium transitional adjustment period between successive long-run equilibria. Regarding the long run, the relevant question is whether a monetary shock has any lasting impact on real variables, i.e., is there a permanent trade-off between inflation and output. As for the short run, one should focus on the type of monetary shocks that initially disturb the system and upon the subsequent reaction of the system to those shocks. Does a monetary disturbance affect output as well as prices in the short run? What types of time paths do the variables describe in disequilibrium? How do the variables interact to produce these paths? Finally and most important, do these paths tend to converge on the long-run equilibrium, i.e., is the system stable?

Long-Run Steady-State Solution of the System According to monetarist doctrine, long-run monetary equilibrium is characterized by the following conditions:
1. Equality between actual and expected rates of price change, reflecting the long-run tendency of people to correctly anticipate inflation and fully adjust to it;

2. The absence of any trade-off between inflation and output, reflecting the tendency of monetary shocks to have no lasting impact on real variables but only on prices;

3. A constant steady-state (non-accelerating, non-decelerating) rate of inflation equal to the difference between the growth rate of the money stock and the growth rate of capacity output;

4. Attainment of full-capacity real income reflecting the long-run tendency of actual output to adhere to its full-employment growth path.

Does the model yield these conditions? Only a look at its steady-state properties will tell, i.e., the model must be analyzed at its long-run equilibrium position. The concept of equilibrium, of course, implies equality between aggregate demand and supply, i.e., a state of zero excess demand. Setting the excess demand variable, $x$, equal to zero in the price-adjustment equation yields $\Delta P = \Delta p_{-1}$. Thus, actual and expected inflation are equal, as required. Moreover, the zero numerical value of the excess demand variable (an index of real economic activity) in the price equation signifies the absence of long-run inflation-output trade-offs, as required. Money growth has a neutral long-run impact on real variables, at least in the model.

The next step is to set the first difference of excess demand, $\Delta x$, at zero in the money growth equation. Doing so enables one to solve for the steady-state rate of inflation, which is $\Delta P = \Delta m - \Delta y_c$. In brief, the model does yield the monetarist conclusion that the equilibrium rate of inflation is the difference between the respective growth rates of the money stock and full-capacity income. The final step is to recognize that when excess demand goes to zero, actual output growth converges on its full-capacity path, consistent with the fourth condition of monetary equilibrium. Therefore, the model contains all the equilibrium conditions required by monetarist doctrine.

Disequilibrium Dynamics of the System in the Short Run So much for equilibrium analysis, which is a relatively simple and straightforward exercise. The next stage is disequilibrium dynamic analysis. Unfortunately, the analytics of the short-run disequilibrium behavior of the system are somewhat more complex and involved. For one thing, the excess demand variable does not drop out of the short-run analysis as it does in the long-run equilibrium case; nor is the current rate of inflation stationary and identical to the expected rate.

The short-run analysis involves at least two steps. First, because interest centers on the time-paths of (1) inflation and (2) the excess demand gap between actual and capacity income, one must derive expressions for the dynamic behavior of these two variables. This derivation is accomplished in Appendix B. Second, the resulting expressions must be analyzed to determine whether the system is dynamically stable, i.e., whether the variables will eventually converge on their long-run equilibrium values.

Disequilibrium Dynamical Equations As shown in Appendix B, the expressions for the respective short-run time paths of the inflation rate and excess demand are:

$$\Delta p = ax_{-1} - a(1-b)x_{-2} + \Delta p_{-1}$$

and

$$x = \Delta^2 m - \Delta^2 y_c + (2-a)x_{-1} - [1-a(1-b)]x_{-2}$$

Two monetarist features are immediately apparent even from the most casual inspection of these equations. First is the appearance of the second time difference, $\Delta^2$, of the money stock variable in the excess demand equation. This second difference, of course, measures the rate of change (i.e., acceleration or deceleration) of the money stock growth rate. Its role in the equation as an active independent variable and determinant of the excess-demand gap is consistent with several monetarist propositions. It squares with the monetarist view of variation in the growth rate of money as the prime initiating cause of business cycles. It corresponds with the monetarist argument that sharp changes in money growth can disturb real income in the short run. In general, it is consistent with the monetarist focus on changes in the growth rate rather than the level of money as a key indicator of recent policy shifts and future price movements.

The second conspicuous monetarist feature is the appearance of lagged values of excess demand in the price-change equation. The equation states that demand leads inflation by as much as two periods, each defined as a year—another manifestation of the monetarist view of the tendency for shifts in demand to influence quantities first, prices only later. This lead-lag relationship corresponds to the monetarist notion of long and complex lags in the monetary transmission mechanism.

The lag structure of the model carries some important policy implications. Given the long lag in the response of prices to changes in demand—not to
mention additional delays in the influence of money on demand—inf lation will be slow to respond to contractionary policy. This is especially true if inflationary expectations have become firmly embedded in behavior patterns. It is a generally-accepted principle that an inflation rate that comes to be anticipated will resist a period of deficient demand much longer than a rate that is not anticipated. To reduce the actual rate of inflation one must reduce the expected rate, since the latter is a determinant of the former. This requires a recession during which the actual rate falls below the expected rate, inducing a gradual downward revision of the latter. According to the adaptive expectations hypothesis, however, expectations are based on a weighted average of current and past rates of inflation. And it may take a long time before the decelerating current rate begins to outweigh the lagged influence on expectations of accelerating past rates. During this time there exists the danger that the authorities, observing the failure of their actions to achieve quick results, may be tempted to abandon monetary restraint as ineffect ve. Monetarists, however, would counsel perseverance, believing that contractionary policy, if adhered to long enough, would eventually bring down the rate of inflation. Monetarists would argue, moreover, that there is no other option—continued monetary restraint is the only way to reduce inflation permanently.

Stability Analysis of the System The last step in the analysis of the model is to examine the dynamic stability of the system. Here the term stability means the tendency of the system when in disequilibrium to converge on its long-run steady-state equilibrium. The concept of stability is central to the rules versus discretion debate between monetarists and nonmonetarists. Some of the latter group claim that the economic system may be inherently unstable such that once disturbed it tends either to oscillate ceaselessly about equilibrium in cycles of regular or increasing amplitude, or alternatively, to move steadily away from equilibrium via a divergent monotonic path. Other nonmonetarists believe that, while the system is stable, the adjustment process takes too long to be left to itself. These views lead to the advocacy of discretionary stabilization policy to counter or smooth the cycle. By contrast, the monetarist group views the economy as an inherently stable self-regulating mechanism capable of restoring equilibrium without the intervention of discretionary policy. In fact, monetarists contend that due to the existence of long, variable, and unpredictable lags in the monetary transmission mechanism, discretionary stabilization policy has a capricious and often destabilizing impact on the economy, amplifying rather than dampening cyclical swings. This argument forms the basis of the monetarist advocacy of a rigid policy rule fixing the growth rate of the money stock.

What about the stability of the model? Will output converge on its capacity growth path and will the excess demand gap vanish as the monetarists predict? To answer these questions one must analyze the excess demand equation:

\[ (5) \quad x = \Delta^m - \Delta^y_c + (2-a)x_{-1} - \{1-a(1-b)\}x_{-2}. \]

It is assumed that the initial monetary disturbance has ended and that, consequently, money is now growing smoothly at a constant rate. In other words, the rate of change of the money growth rate—\( \Delta^m \)—is zero. Moreover, it is also assumed that the growth rate of capacity output is a constant, i.e., that the rate of change of the capacity growth rate—\( \Delta^y_c \)—is also zero. Setting these first two terms on the right-hand side of the equation at zero leaves the second-order difference equation:

\[ (6) \quad x = (2-a)x_{-1} - [1-a(1-b)]x_{-2}. \]

Specialists in dynamic models have worked out a set of stability conditions for this type of equation. These conditions are listed in Appendix C. By referring to the stability criteria, it can be shown that, given plausible values of the coefficients \( a \) and \( b \), the system will be stable. Depending upon the specific magnitudes of the coefficients, the system may approach long-run equilibrium either monotonically or cyclically, but it will always converge upon it. Hence the model conforms to the monetarist specification of an inherently stable system.

Monetarist View of the Inflationary Process The foregoing section completes the analysis of the steady-state and disequilibrium dynamical properties of the model. These properties were shown to be consistent with the basic postulates of monetarist doctrine. It remains to compare Laidler’s formal model with a leading monetarist’s verbal description of the inflationary process to see if the two agree with regard to treatment of timing, direction of causation, and pattern of interaction of key variables.

---

*Only the excess demand equation is examined here. Exactly the same type of analysis can be performed on the difference equation expressing the behavior of the inflation rate following a step increase in the monetary growth rate. Such an analysis reveals that the rate of inflation eventually stabilizes at a level equal to the difference between the new monetary growth rate and the growth rate of capacity output.*
Professor Milton Friedman, perhaps America's foremost monetarist, summarizes the inflationary process as a stylized sequence of events.

Start from a hypothetical, reasonably balanced situation when monetary growth has been proceeding for some time at a constant rate so that the public in general has adjusted to that rate. GNP in nominal terms will then be growing at about the same percentage rate as \( M_2 \), prices at about 2.0 to 4.0 percentage points less. Let the growth rate of \( M_2 \) accelerate. For something like six months, the main effect will be that actual balances will exceed desired balances, which may temporarily depress short-term interest rates but will have little other effect.

After about six to nine months, the rate of growth of nominal GNP will accelerate, as holders of the excess cash seek to dispose of it. The increased spending . . . will 'excite industry,' as producers facing unexpectedly high nominal demands treat the increase as special to them and so seek to expand output. For a time they can do so, because their suppliers too, including laborers, take the increase in demand as special and temporary and do not alter their anticipations. This, if you will, is the temporary Keynesian phase, where output responds more quickly than prices. In its course, prices do respond, rising more rapidly than before, and interest rates stop falling and start to rise. But it takes about eighteen months after output starts to quicken—or two years after money accelerates—for the main effect to have shifted from output to prices. During this period, anticipations are changing, reflected most sensitively perhaps in interest rates, but even after prices have started to absorb the bulk of the acceleration in money, anticipations have not fully caught up. In the next year or so they will, which will force a decline in the rate of growth of output back to or below the 'natural level,' producing the stagflation stage.7

Friedman's description clearly implies a chain of causation running from money to spending to output to prices to inflationary expectations, with deviations between actual and expected rates of inflation feeding back into the process to determine the division of the increase in spending between price and output growth. Moreover, there are substantial time lags operating in each link of the chain or stage of the inflationary process. Together, these feedbacks and time lags produce growth cycles, i.e., oscillations of output growth about the equilibrium or full-capacity growth rate.

How does the formal model compare with Friedman's description? Two differences are immediately apparent. The first relates to the initial (money-spending) link. Friedman asserts the existence of a six-to-nine month lag in the response of spending to monetary stimuli. During this interval, the total impact of the monetary shock is absorbed by a passive rise in undesired cash balances; none of the shock is transmitted to spending. By contrast, the model implies an instantaneous first-round response of spending to changes in money growth. The difference stems from the model's simplifying assumption that actual and desired real cash-balances are always identical, implying the absence of an adjustment lag for real balances. As a second departure from Friedman's version, the model—again for purposes of simplicity—contains no interest rate variables and therefore cannot describe the impact of inflation on interest rates. In brief, Friedman's description implies the existence of one additional time-lag and one additional variable absent from the model.8

As for (1) direction of causation and (2) pattern of interaction of variables, however, the model is quite similar to Friedman's description. Causation runs from money to output to prices to expected inflation and back again to real income. Specifically, in the model the sequence is as follows.

(1) Accelerated money growth generates excess demand, thus causing real output to rise above its full-capacity growth path. See equation (5).

(2) After a lag, excess demand begins to influence the current rate of inflation, causing it to rise above the expected rate. See equation (2).

(3) The rise in the actual inflation rate in turn influences the expected rate, which will feed back into next period's actual rate. See equations (3) and (2).

(4) The rate of inflation interacts with the given rate of money growth to determine the growth rate of output. See equation (1). Moreover, since the rate of inflation itself is determined by the level of excess demand and by expected inflation, these two variables may be regarded as determining the division of monetary growth between output and price level growth.

(5) Finally, current output growth as determined in equation (1) feeds back into equation (2) to become

---


8 It should be noted that Friedman's explanation of the expectations-formation mechanism is consistent with the so-called rational expectations hypothesis and thus may differ from the adaptive expectations model employed by Laidler. According to the rational expectations hypothesis, the inflationary expectations that individuals formulate represent the most-accurate (unbiased) forecasts given the available market information on the stochastic process generating the inflation. By contrast, the adaptive expectations hypothesis may imply nonrational forecasting behavior. That is, it can be shown that under certain conditions, the adaptive expectations mechanism will produce forecasts that are systematically wrong. For example, suppose the monetary authority follows a policy rule of continually accelerating the rate of inflation. In this case the backward-looking adaptive expectations model would yield a predicted rate of inflation that lags consistently behind the actual rate. That is, inflation will be systematically underestimated, adherence to the adaptive expectations model despite persistent forecasting errors implies nonrational behavior. Rational individuals would revise their forecasting model to produce unbiased predictions. Under other very restrictive conditions, however, the adaptive expectations model is consistent with rational behavior, at least within the context of monetarist models.
a determinant of next period's inflation rate, etc. As mentioned in the preceding section, this iterative process is capable of producing oscillations much like those mentioned by Professor Friedman.

To summarize, both Friedman and Laidler agree that, owing to the operation of lags in the monetary transmission mechanism, the effect of money growth on the rate of inflation is spread over substantial periods of time. During the interim, quantities as well as prices are affected, i.e., variations in money growth can produce business fluctuations. But changes in money growth have no lasting impact on output. Ultimately, the entire effect is on the rate of inflation.

Policy Implications of the Model Since much of the monetarist discussion of inflation tends to be strongly policy-oriented, it is appropriate to close the article with a brief mention of some of the policy implications of Laidler's model. From the point of view of the policymaker, two features of the model are of particular interest. The first feature is the time it takes for changes in the rate of money growth to work through to the rate of inflation. The second feature is the marked short-run impact of changes in money growth on real output. These features combine to produce in the model dissimilar patterns of response of output and prices to the monetary change. These response patterns have important implications for monetary stabilization policy.

First, owing to the slow response of inflation to a monetary change, it necessarily takes a long time for anti-inflationary monetary policy to work. Quick monetary remedies for inflation do not exist. Moreover, since the first effect of a change in the growth rate of money is on output and employment rather than on prices, monetary restraint would almost surely entail a recession or at least a marked retardation in the expansion of the economy. In sum, a temporary but protracted period of high unemployment and sluggish growth would have to be tolerated if monetary policy were to be successful in permanently lowering the rate of inflation.

Second, due to the difference in timing of the responses of output and prices to a monetary change, anti-inflationary monetary policy may appear impotent or, even worse, counter-productive and perverse. Because inflationary movements tend to subside so slowly, prices may continue to rise long after output and employment have turned down. Thus inflation can persist even in slack conditions—a condition variously known as inflationary recession, stagflation, or slumpflation. During such periods, monetary restraint may be wrongly blamed for causing both the slump and the accompanying inflation, and the temp-

FEDERAL RESERVE BANK OF RICHMOND
elimination of excess demand is the \textit{sine quae non} for the success of a controls program. And the excess demand problem may be compounded by the inevitable shortages created by controls.

**Summary** This article has expounded the principal postulates of monetarist doctrine within the context of Professor David Laidler's three-equation macroeconomic model. This model can account for the phenomena of stagflation (i.e., the persistence of inflation long after aggregate demand has slackened), for the entrenchment of inflationary expectations, for the intractability or resistance of inflation to anti-inflationary monetary policy, and, finally, for the output and employment effects of such a policy. Since the model embodies virtually all of the monetarist predictions relating to the long-run neutrality and short-run non-neutrality of monetary disturbances, it can be interpreted as capturing the essence of the monetarist view of the inflationary process. Moreover, the very simplicity of the model renders it a pedagogically useful introduction to the economics of long-run steady-state equilibrium and of short-run dynamic disequilibrium processes in which economic variables interact and evolve over time. It also provides a framework for stating clearly the public-policy issues involved in the monetarist-nonmonetarist controversy.

**APPENDIX A**

**Derivation of the Monetary Growth Equation from the Cambridge Cash-Balance Equation**

Let $M$ be the money stock, $P$ the price level, $Y$ the level of real national income, and $K$ the desired ratio of real cash balances to real income. This Cambridge $K$, the reciprocal of the income velocity of money, is treated as a fixed constant. These elements comprise the Cambridge cash-balance equation, $M/P = KY$. This equation is interpreted as the equilibrium solution of a three-equation demand-supply system. Specifically, the Cambridge formulation implies: (1) a relation expressing the demand for real balances as a function of income, $M_d/P = KY$; (2) an exogenously-determined nominal money supply, $M_s = M$; and (3) an equilibrium (market-clearing) condition stating that nominal money supply must equal nominal money demand, $M_s = M_d$, resulting in the Cambridge cash-balance formula, $M/P = KY$.

To transform the Cambridge formula into the money growth equation of the text, simply take the logarithm of both sides of the formula. Remembering (1) that the logarithm of a ratio is equivalent to the logarithm of its numerator minus the logarithm of its denominator, and (2) that the logarithm of the product of two terms is equal to the sum of their respective logarithms, one obtains $\log M - \log P = \log K + \log Y$. Expressing the logarithms of the variables as lower-case letters allows the preceding relation to be expressed more simply as $m - p = k + y$. Taking the first difference of this equation yields $\Delta m - \Delta p = \Delta y$, the money growth equation of the text. The first difference of $k$ is of course zero and thus drops out of the money growth equation, i.e., $\Delta k = 0$, since $k$ is a constant by definition.

**APPENDIX B**

**Derivation of the Expressions for the Disequilibrium Time Paths of the Inflation Rate ($\Delta p$) and Excess Demand ($x$)**

(1) **Derivation of the expression for $\Delta p$.**

The model in the text is

1. $\Delta m = \Delta x + \Delta y + \Delta p$
2. $\Delta p = ax_{-1} + \Delta p_{-1}$
3. $\Delta p = b\Delta p + (1-b)\Delta p_{-1}$

First, lag equation (3) one time period to get

4. $\Delta p_{-1} = \Delta p_{-1} + (1-b)\Delta p_{-2}$

Next, substitute (4) into (2) to get

5. $\Delta p_{-1} = ax_{-1} + b\Delta p_{-1} + (1-b)\Delta p_{-2}$

Then, rewrite (2) as

6. $\Delta p_{-1} = \Delta p_{-1} - ax_{-1}$
Next, lag (6) one time period to obtain

\[ \Delta \rho^{a-2} = \Delta \rho_{-1} - ax_{-2}. \]

Next, substitute (7) into (5) to get

\[ \Delta p = ax_{-1} + b \Delta p_{-1} + (1-b) (\Delta p_{-1} - ax_{-2}). \]

Finally, expand (8) and simplify to obtain

\[ \Delta p = ax_{-1} - a(1-b)x_{-2} + \Delta p_{-1}. \]

Equation (9) is the expression for the disequilibrium time path of the inflation rate that appears in the text. Recognizing that \( \Delta p - \Delta p_{-1} = \Delta^2 p \), one can also express (9) as

\[ \Delta^2 p = ax_{-1} - a(1-b)x_{-2}. \]

(II) Derivation of the expression for \( x \).

First, start with equation (1) again, i.e.,

\[ \Delta m = \Delta x + \Delta y_c + \Delta p. \]

Then, take the first difference of (1) to get

\[ \Delta^2 m = \Delta^2 x + \Delta^2 y_c + \Delta^2 p. \]

Next, expand \( \Delta^2 x \) to obtain

\[ \Delta^2 x = \Delta x - \Delta x_{-1} = (x-x_{-1}) - (x_{-1}-x_{-2}) = x-2x_{-1} + x_{-2} \]

or

\[ \Delta^2 x = x - 2x_{-1} + x_{-2}. \]

Now, substitute (12) and (10) into (11) to get

\[ \Delta^2 m = [x-2x_{-1}+x_{-2}] + \Delta^2 y_c + [ax_{-1}-a(1-b)x_{-2}]. \]

Finally, solve (13) for \( x \) and simplify to obtain

\[ x = \Delta^2 m - \Delta^2 y_c + (2-a)x_{-1} - [1-a(1-b)]x_{-2}. \]

Equation (14) is the expression for the disequilibrium time path of the excess demand variable, as stated in the text.

APPENDIX C

Stability Conditions for Second-Order Homogeneous Difference Equations

The general homogeneous second-order difference equation \( x + a_1x_{-1} + a_2x_{-2} = 0 \) has two solutions or roots (\( r \)) which can be found by solving the quadratic characteristic equation \( r^2 + a_1r + a_2 = 0 \) corresponding to the difference equation. Depending on the numerical values of the roots, the time path of \( x \) will move toward, away from, or around equilibrium. It is not, however, necessary to solve for the roots of the equation to determine if the system is dynamically stable, i.e., tends to converge on equilibrium either via damped-oscillatory or monotonic paths. One needs only to refer to the stability conditions pertaining to the difference equation. For stability, all of the following conditions must be met:\(^3\):

\[ 1 + a_1 + a_2 > 0 \]
\[ 1 - a_2 > 0 \]
\[ 1 - a_1 + a_2 > 0. \]

In the excess demand equation of the text, the term \(-(2-a)\) corresponds to the coefficient \( a_1 \) of the stability conditions and the term \( [1-a(1-b)] \) corresponds to coefficient \( a_2 \). Substitution of these terms for \( a_1 \) and \( a_2 \) in the stability conditions quickly reveals that the first and second conditions are automatically satisfied as long as \( a > 0 \) and \( 0 < b < 1 \), the range of values specified in the model of the text. The third stability condition will be satisfied if \( a > \left[ \frac{4}{(2-b)} \right] \).

To determine whether the stable path is oscillatory or monotonic, one must analyze the characteristic roots of the system. The roots of the characteristic equation \( r^2 + a_1r + a_2 = 0 \) are

\[ r_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} \]

where \( a_1 = -(2-a) \) and \( a_2 = [1-a(1-b)] \). The system will exhibit oscillatory behavior if the roots are complex, i.e., if \( 4a_2 > a_1^2 \), or in terms of the model, if \( [4 - 4a(1-b)] > -(2-a)^2 \). The latter inequality reduces to \( a^2 < 4ab \), hence oscillatory behavior is obviously possible for \( a > 0 \) and \( 0 < b < 1 \).

---