

A MODEL OF MONEY STOCK DETERMINATION WITH LOAN DEMAND AND A BANKING SYSTEM BALANCE SHEET CONSTRAINT

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I. INTRODUCTION

Recent and proposed changes in the Federal Reserve's monetary control procedures include the shift from a funds rate instrument to a non-borrowed reserve instrument in October 1979, reserve requirement reform embodied in the Monetary Control Act of 1980, and the consideration that has been given to a move from lagged to contemporaneous reserve requirement regimes. Analysis of the impact of such changes requires a sufficiently general model of money stock determination. The "money multiplier" model of money stock determination, for example, is not wholly adequate for explaining and comparing money stock determination under different monetary control procedures. This article offers an alternative model. Although differences in money stock determination are illustrated here for several monetary control procedures, the intent is not to offer a comprehensive analysis or prescription for monetary control but merely to present a framework in which issues affecting money stock determination can be more adequately examined.

The model of money stock determination presented in this article takes explicit account of bank loan demand and the banking system balance sheet constraint. It explains money stock determination for alternative monetary control instruments, namely, funds rate, non-borrowed reserve, and total reserve instruments, and for lagged and contemporaneous reserve requirement regimes. Furthermore, the model explains determination of both "M1" and "M2" type monetary aggregates with the aid of a simple diagram.

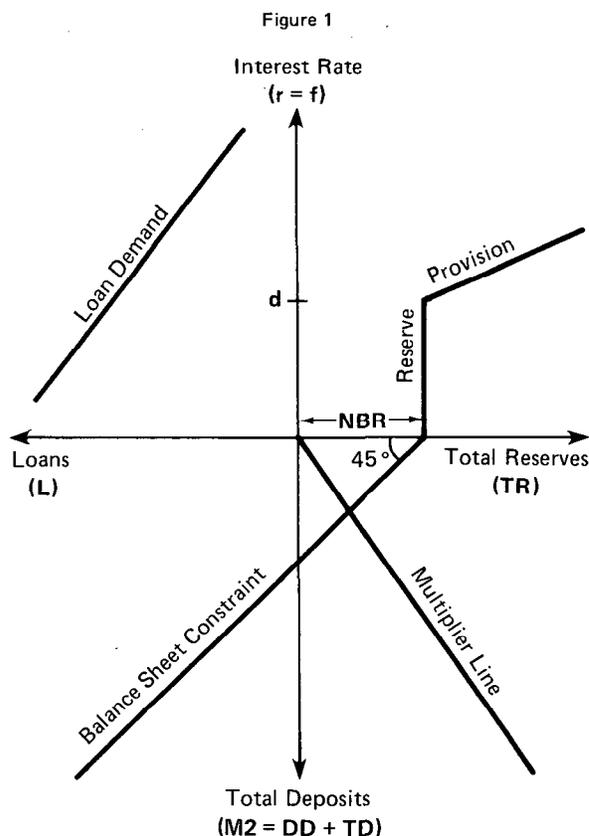
After the initial presentation of the model and its diagrammatic representation, the diagram is employed to illustrate money stock determination for various instrument-reserve requirement combinations. The role of the money multiplier in money stock

determination is highlighted throughout this discussion. The model is then employed to examine the effect of various disturbances on the monetary aggregates with a non-borrowed reserve instrument for both lagged and contemporaneous reserve requirement regimes. The analysis is summarized in the conclusion.

II. THE MODEL

A diagrammatic representation of the model of money stock determination is presented in this section. A complete diagram of the model is shown in Figure 1.¹

¹ The model is summarized in the appendix.



1. The Four Quadrants

Reserve Provision The northeast quadrant contains a reserve provision locus showing the relationship between total reserves in the banking system and the Federal funds rate.² The locus has a vertical and a nonvertical segment because reserves are provided to the banking system in two forms, as “non-borrowed” and as “borrowed” reserves. Non-borrowed reserves (NBR) are supplied by the Fed through open market operations, while borrowed reserves (BR) are provided through the Fed discount window.

The distance between the vertical segment of the reserve provision locus and the vertical axis is determined by the volume of non-borrowed reserves. The reserve provision locus is vertical up to the point where the funds rate (f) equals the discount rate (d) because when the funds rate is below the discount rate banks have no incentive to borrow at the discount window. Formally, if $f \leq d$, then $BR^D = 0$.

Conversely, when the funds rate is above the discount rate banks have an incentive to borrow at the discount window because they obtain a net saving on the explicit interest cost of reserves. This net saving consists of the differential ($f - d$) between the funds rate and the discount rate. Discount window administration imposes a nonpecuniary cost of borrowing that rises with volume; and banks tend to borrow up to the point where the nonpecuniary cost of borrowing just offsets the net interest saving. Consequently, borrowing is higher the greater the spread between the funds rate and the discount rate. That is why the reserve provision locus is positively sloped for funds rates above the discount rate. Formally, if $f > d$, then $BR^D(f - d) > 0$ and $BR^D'(f - d) > 0$.³

Loan Demand The nonbank public's net real demand for loans, L^D , is a decreasing function of the nominal rate of interest, i.e., $L^D(r)$, where $L^D'(r) < 0$.⁴ The nonbank public's net nominal demand for loans is therefore $P \cdot L^D(r)$, where P is the price

level. The loan market is assumed to clear so that the nonbank public's net nominal demand for loans $P \cdot L^D(r)$ equals the nominal volume of loans supplied by the banking system, L . Diagrammatically, the nonbank public's net nominal demand for loans appears in the northwest quadrant of Figure 1, for a given price level, as a decreasing function of the nominal rate of interest, r . The horizontal axis in the northwest quadrant is labeled L , since loan market equilibrium guarantees that $L = P \cdot L^D(r)$.

The loan demand function and the reserve provision schedule are drawn with a common vertical axis because bank arbitrage between Federal funds and bank loans is assumed to keep rates in the two markets aligned. Accordingly, the common interest rate axis is labeled “ $r = f$ ”, indicating the arbitrage activity which links the two quadrants.⁵

The Balance Sheet Constraint The line in the southwest quadrant represents the banking system's balance sheet constraint. In simple form, the banking system's balance sheet looks as follows:

CONSOLIDATED BANKING SYSTEM BALANCE SHEET	
Assets	Liabilities
Loans (L)	Demand Deposits (DD)
Non-borrowed Reserves (NBR)	Time Deposits (TD)
Borrowed Reserves (BR ^A)	Borrowed Reserves (BR ^L)

where

BR^A \equiv reserves obtained from the Fed discount window.

BR^L \equiv corresponding dollar for dollar promise to repay BR^A; BR^A = BR^L.

DD \equiv “checkable” type deposits whose rates of interest are fixed at a legal ceiling.

TD \equiv that portion of total deposits whose rates move with market interest rates.⁶

⁵ In fact, arbitrage does not keep the funds rate perfectly aligned with loan rates. The funds rate is a daily rate while loan rates and commitments in general are made for longer maturities. A loan rate is aligned with an average of anticipated future funds rates over the term of the loan, since the average anticipated funds rate is the anticipated opportunity cost of funding the loan. The funds rate-loan rate spread changes with movements of average anticipated future funds rates relative to the current funds rate.

⁶ Overnight repurchase agreements at banks are essentially “checkable” and pay a rate that moves with the market. Savings and small-time deposits are subject to legal ceilings below market rates. In other words, the distinction drawn between “DD” and “TD” type deposits in the model is blurred in practice.

² Banking system refers to depository institutions in general. Under the Monetary Control Act of 1980, all depository institutions subject to Fed reserve requirements have access to the Fed discount window.

³ See Goodfriend [4] for a detailed discussion of discount window borrowing.

⁴ In this model, portfolio equilibrium is characterized by a loan market equilibrium condition. Alternatively, portfolio equilibrium could have been characterized by a money market equilibrium condition. See Patinkin [7], Chapters IX:4 and XII:4, 5, 6.

In general, real income and real net wealth are arguments in the L^D function. They are ignored in the text.

The constraint implied by the T-account can be written as follows:

$$(1) \quad L + NBR \equiv DD + TD.$$

The balance sheet constraint has a slope of one, since for the banking system as a whole every dollar increase in deposits is matched by a corresponding dollar increase in loans. The intercept on the aggregate deposit axis is NBR because if banks had no loans outstanding deposits would match the volume of non-borrowed reserves.

The Multiplier Line Within the context of this model, aggregate deposits $DD + TD$ represent an "M2" type monetary aggregate, since the $DD + TD$ aggregate includes checkable and time deposits. DD represents an "M1" type monetary aggregate. Formally, for the purposes of this article

$$(2) \quad M1 \equiv DD \\ M2 \equiv DD + TD.^7$$

The multiplier line in the southeast quadrant relates M2 to total reserves (TR). The line passes through the origin because without reserves banks cannot legally hold deposits. The slope of the multiplier line, called the M2-TR multiplier, is

$$(3) \quad m_2 \equiv \frac{M2}{TR}$$

where $m_2 > 1$.

The M2-TR multiplier depends on (1) the nonbank public's portfolio preference for checkable deposits (DDs) relative to time deposits (TDs), (2) the Federal Reserve System's legal reserve requirements on DDs and TDs, and (3) the banking system's demand for excess reserves, i.e., reserves held above legal requirements.

The demand for TDs relative to DDs depends upon the spread between the TD rate and the DD rate. The DD rate is taken to be fixed, while the TD rate is assumed to be competitively determined and to move with market rates. Arbitrage is assumed to keep the interest rate on TDs aligned with the loan

⁷ Currency is ignored throughout, but technically currency in the hands of the public is in both M1 and M2. In addition, M2 includes components which are not liabilities of depository institutions located in the U. S., i.e., overnight Eurodollar deposits held by U. S. residents at Caribbean branches of U. S. banks; and M2 also includes money market mutual fund shares. Finally, not all net depository institution liabilities are in M2. For example, large-time deposits at all depository institutions and term RPs at commercial banks and savings and loan institutions are only in M3. See Simpson [9].

rate, which is assumed to move with the funds rate.⁸ The net effect of these arbitrage assumptions is to enable the model to operate as if there were one interest rate, r .

Given a legally fixed rate on DDs assumed to be below the TD rate, the ratio of TDs to DDs that the public desires to hold depends on r . Formally, the public's portfolio balance function is

$$(4) \quad \frac{TD}{DD} = \rho(r) \text{ or } \frac{M2}{M1} = 1 + \rho(r)$$

where $\rho'(r) > 0$.⁹

A higher r represents a higher opportunity cost of holding DDs relative to TDs, and so is associated with a higher ratio of TDs to DDs and M2 to M1 in the public's portfolio. Hence, $\rho'(r)$ is positive.

Banking system reserve demand equals the sum of required reserves and the demand for excess reserves. Let reserve requirement ratios on DDs and TDs be rr_1 and rr_2 , respectively, so required reserves (RR) can be written

$$(5) \quad RR \equiv rr_1 DD + rr_2 TD$$

where $rr_2 < rr_1 < 1$.¹⁰

Excess reserve demand (ER) is a function of respective deposit levels such that

$$(6) \quad ER \equiv k_1(r) DD + k_2(r) TD$$

where $k_2(r) < k_1(r)$ and $k_1'(r) < 0$, $k_2'(r) < 0$.

The presumption that $k_1(r)$ exceeds $k_2(r)$ implies that the precautionary need for excess reserves is

⁸ Even if loans and deposits were the same maturity, bank arbitrage would not drive loan and deposit rates into equality. Competition and profit maximization imply that the net marginal return on loans equals the net marginal cost of deposits. Formally, this arbitrage condition is written $r^L - c^L = \frac{1}{1-a}[r^D + c^D]$ where $r^L \equiv$ the loan rate, $c^L \equiv$ the marginal cost of loan production, $a \equiv$ the fractional reserve against deposits, $r^D \equiv$ the deposit rate, and $c^D \equiv$ the marginal cost of deposit production. Loan and deposit rates are parameters from the point of view of individual banks.

Note that even without legal restrictions on interest rates, if $a > 0$ then the $r^L - r^D$ spread is positively related to the level of interest rates.

⁹ If either (1) the legal ceiling on the payment of interest on DDs is ineffective or (2) the interest on DDs inclusive of the restricted explicit nominal rate and an implicit payment either through a gift or remittance of some of the cost of account management moves competitively with r , then the ratio of DDs to TDs that the public desires to hold may not be sensitive to r .

¹⁰ Actual reserve requirements are more complicated than those assumed here. See the **Federal Reserve Bulletin** for the current structure of reserve requirements.

greater for DDs than for TDs. Since excess reserves earn no interest, excess reserve demand for given DD and TD levels is negatively related to the interest rate, i.e., $k_1'(r) < 0$ and $k_2'(r) < 0$.

Assume that the reserve market clears, i.e., that total reserve provision equals total reserve demand, so that

$$(7) \quad TR = rr_1DD + rr_2TD + k_1(r)DD + k_2(r)TD.$$

Using equations (4) and (7) to substitute for DD, TD, and TR in (3), m_2 may be written

$$(8) \quad m_2(r) = \frac{1 + \rho(r)}{rr_1 + rr_2\rho(r) + k_1(r) + k_2(r)\rho(r)}$$

where $m_2'(r) > 0$.

The M2-TR multiplier *increases* with a rise in the interest rate. To see why m_2 interest sensitivity is positive, write $m_2 = 1 / [\frac{RR}{M2} + \frac{ER}{M2}]$. A rise in r induces the public to switch from DDs to TDs. Since $rr_1 > rr_2$ and $k_1 > k_2$, this portfolio switch lowers required reserves and excess reserves relative to M2. Therefore, $\frac{RR}{M2}$ and $\frac{ER}{M2}$ both fall with an interest rate rise. In addition, an interest rate rise lowers the k coefficients, i.e., the demand for excess reserves at given DD and TD levels, producing an additional reduction in $\frac{ER}{M2}$.¹¹

Before leaving this section, it can be pointed out that the "money multiplier" model of money stock determination is represented in this model by the M2-TR multiplier line in the southeast quadrant of

¹¹ The M1-TR multiplier is

$$m_1 \equiv \frac{M1}{TR}.$$

Using equations (4) and (7) in the text, m_1 may be written

$$m_1(r) = 1 / [rr_1 + rr_2\rho(r) + k_1(r) + k_2(r)\rho(r)].$$

The sign of the interest sensitivity of the M1-TR multiplier is ambiguous. To see why, suppose the interest rate rises. Both DD and TR could not remain unchanged because the increased demand for TDs relative to DDs would leave reserve demand in excess of reserve supply. Either DD must fall or TR must rise to clear the reserve market, causing the M1-TR multiplier to fall. However, the k coefficients are smaller at a higher interest rate and the reduced demand for reserves from this source may be sufficient to leave reserve supply in excess of reserve demand. In this case, either DD would have to rise or TR would have to fall to clear the reserve market, causing the M1-TR multiplier to rise. The net effect of an interest rate rise on the M1-TR multiplier is therefore ambiguous.

Figure 1.¹² However, in this model the M2-TR multiplier is merely a relation between total reserves and the M2 money stock. The discussion in Section III makes clear that the role of the money multiplier in money stock determination depends on the Fed's monetary control procedure. In particular, the discussion there shows that the money multiplier is not generally a complete model of money stock determination and is actually irrelevant to money stock determination for some monetary control procedures.

2. Federal Reserve Monetary Control Procedure

Determination of the monetary aggregates depends critically on the method that the Fed employs to control the money stock. The instrument of monetary control and the reserve requirement regime are the two most important components of the Fed's monetary control procedure.

Instruments of Monetary Control The instrument of monetary control is the variable the Fed predetermines on an ongoing basis in order to achieve its money stock target. Since October 6, 1979, the primary instrument of monetary control has been non-borrowed reserves. Two important alternative instruments are the Federal funds rate and total reserves.

With a non-borrowed reserve instrument the Fed supplies a predetermined volume of non-borrowed reserves and allows the volume of borrowed reserves and the funds rate to adjust to maintain reserve market equilibrium. With a funds rate instrument, the interest rate is predetermined in each reserve statement period. The Fed supplies whatever volume of non-borrowed reserves is required to maintain reserve market equilibrium at its chosen funds rate. To use a total reserve instrument the Fed could, for example, let the discount rate be a fixed penalty rate slightly above the funds rate. In this setup, discount window borrowing would be negligible, non-borrowed reserves would approximately equal total reserves, and the Fed could supply non-borrowed reserves to achieve a total reserve objective. The funds rate would adjust freely to maintain reserve market equilibrium with a total reserve instrument.

Reserve Requirement Regimes The reserve requirement regime refers to the set of rules imposed

¹² A well-known discussion and application of the "money multiplier" model is found in Friedman and Schwartz' **A Monetary History of the United States**. Appendix B of that volume contains the derivation of money multipliers for a variety of monetary standards. Those multipliers involve essentially the same types of relationships that are embodied in the multiplier line in this model.

on depository institutions under the Federal Reserve's Regulation D by which they are required to hold a fraction of their deposits as reserves. Reserve requirement rules specify the size of the reserve requirement according to deposit type, i.e., DD or TD, as well as the timing of reserve maintenance relative to the reserve statement period for which the required reserves are computed. Money stock determination is discussed in this article for two alternative reserve requirement regimes: lagged reserve requirements (LRR) and contemporaneous reserve requirements (CRR).

The Fed has been operating with LRR since September 1968 and is currently operating with LRR. The LRR rule is summarized as follows:

LRR Reserve requirements for the current reserve statement period are calculated on the basis of deposits held in a previous period.

The lag under LRR means that required reserves are predetermined as banks enter each reserve statement period.

The Fed operated with CRR prior to September 1968 and has been considering its re-implementation. The CRR rule is summarized as follows:

CRR Reserve requirements for the current reserve statement period are calculated on the basis of current deposit holdings.

III. MONEY STOCK DETERMINATION FOR ALTERNATIVE INSTRUMENT-RESERVE REQUIREMENT COMBINATIONS

In this section the model presented in Section II is employed together with various instrument-reserve requirement combinations to explain money stock determination under alternative Federal Reserve monetary control procedures. In general, it is seen that determination of the monetary aggregates differs significantly according to the method of monetary control.

1. A Non-Borrowed Reserve Instrument With Lagged Reserve Requirements Since October 1979 the Fed has primarily employed a non-borrowed reserve instrument with the lagged reserve requirement rules (LRR) currently in effect.¹³ With a non-borrowed reserve instrument and LRR, total reserve demand is essentially predetermined in each reserve

¹³ See Goodfriend [3] for a detailed appraisal of the NBR-LRR monetary control procedure.

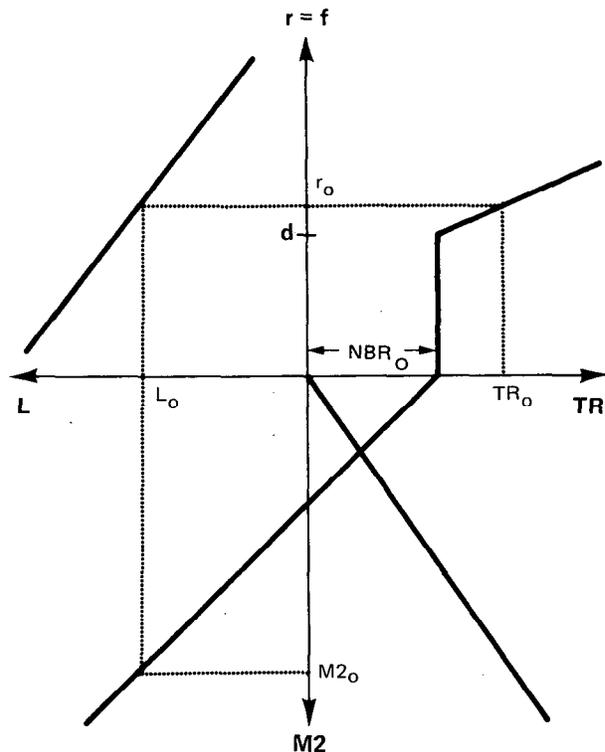
statement period. This is because required reserves are based on deposits in a previous statement period and because excess reserve demand is small and interest insensitive in this operating procedure.¹⁴

With this procedure, the Fed determines the funds rate required to hit its money stock target and then determines a discount window borrowing objective that will produce that funds rate. The Fed forces the banking system to borrow that quantity of reserves at the discount window by supplying only a portion of total reserves demanded as non-borrowed reserves. If BR_0 is the borrowing objective and TR_0 is predetermined total reserve demand, then the Fed supplies NBR_0 such that $BR_0(f - d) = TR_0 - NBR_0$.

This operating procedure is illustrated in Figure 2. Arbitrage brings the loan rate into equality with the funds rate at the desired interest rate, r_0 . Loan volume is determined by the public's demand for loans at the interest rate, r_0 . Finally, the balance sheet constraint indicates the volume of deposits

¹⁴ This is consistent with historical experience since October 1979. However, it should be noted that at very low interest rates, excess reserve demand could become larger and more interest sensitive.

Figure 2
NBR - LRR and $f - LRR$



associated with that volume of loans and the volume of non-borrowed reserves supplied by the Fed. The quantity of total deposits is denoted M_2 . The direction of causation in equilibrium determination is therefore counterclockwise, starting from the predetermined volume of total reserves demanded, TR_0 , moving through r_0 , to L_0 , and finally to M_2 .

Note that under LRR the current volume of deposits is not constrained by current reserves. Required reserves are held after the fact to support deposits held by the nonbank public in a previous period. In short, with a non-borrowed reserve instrument and LRR the M_2 - TR multiplier plays no role in M_2 determination.

Diagrammatically, the multiplier line is irrelevant to the determination of M_2 . M_2 is determined essentially by the demand for borrowed reserves and loan demand, together with the predetermined volume of total reserves demanded, TR_0 , and current non-borrowed reserve supply, NBR_0 .¹⁵

2. A Funds Rate Instrument With Lagged Reserve Requirements The Fed. operated exclusively with a funds rate instrument and lagged reserve requirements from September 1968 until October 1979. Since then, the Fed has continued to operate with a funds rate instrument (together with LRR) whenever it let the funds rate fall below the discount rate.¹⁶

With a funds rate instrument and LRR, total reserve demand is essentially predetermined in each reserve statement period as it is with a non-borrowed reserve instrument and LRR. However, in this case total reserve demand is accommodated by the Fed at a predetermined funds rate. In other words, the reserve provision locus is horizontal at the predetermined funds rate, not vertical and upward sloping above the discount rate as it is with a non-borrowed reserve instrument. Loan volume is determined along the loan demand function at the predetermined interest rate. The balance sheet constraint is anchored at $NBR = TR_0 - BR^D(f - d)$, where TR_0 is the predetermined demand for total reserves. If $f > d$, then borrowing is positive so $NBR < TR_0$; and if $f \leq d$, then borrowing is zero so $NBR = TR_0$. The M_2 money stock is determined by loan volume, the

volume of non-borrowed reserves supplied by the Fed, and the balance sheet constraint.¹⁷

Equilibrium determination with a funds rate instrument and LRR is illustrated in Figure 2. If r_0 is the interest rate predetermined by the chosen funds rate, then equilibrium determination may be traced along the dotted line through L_0 to M_2 as it is with a non-borrowed reserve instrument and LRR. The direction of causation in equilibrium determination is counterclockwise for a funds rate instrument and LRR as it is for a non-borrowed reserve instrument and LRR. Furthermore, the multiplier line is irrelevant to money stock determination with a funds rate instrument and LRR as it is with a non-borrowed reserve instrument and LRR.

3. A Non-Borrowed Reserve Instrument With Contemporaneous Reserve Requirements The Fed has been considering returning to contemporaneous reserve requirements (CRR). If it does return to CRR, the Fed seems likely to retain non-borrowed reserves as the primary instrument of monetary control at least initially. Therefore, it is useful to examine money stock determination with a non-borrowed reserve instrument and CRR.

Under CRR, total reserves are linked to total deposits within each reserve statement period through the M_2 - TR multiplier. This contrasts sharply with LRR where, regardless of the instrument, the M_2 - TR multiplier is irrelevant to money stock determination.

Furthermore, with a non-borrowed reserve instrument and CRR, not only can the interest rate affect M_2 volume through loan demand, but M_2 volume feeds back on the funds rate through the M_2 - TR multiplier and total reserve demand. In other words, the direction of causation in equilibrium determination is not simply counterclockwise as it is under LRR. Rather with a non-borrowed reserve instrument and CRR, loan volume, M_2 , total reserves, and the interest rate are all simultaneously determined.¹⁸ The dashed rectangle in Figure 3 illustrates an equilibrium for NBR_0 of non-borrowed reserves supplied by the Fed. Loan volume, M_2 , total reserves, and the interest rate are simultaneously determined at L_0 , M_2 , TR_0 , and r_0 , respectively.

¹⁵ With an NBR-LRR combination, M_1 is determined from M_2 and r by the portfolio balance function (4), $M_1/M_2 = 1/[1 + \rho(r)]$.

¹⁶ Evidence that this has been the case is presented in Goodfriend [3].

¹⁷ With an f -LRR combination, M_1 is determined from M_2 and r_0 by the portfolio balance function (4), $M_1/M_2 = 1/[1 + \rho(r_0)]$.

¹⁸ With an NBR-CRR combination, M_1 is determined from M_2 and r by the portfolio balance function (4), $M_1/M_2 = 1/[1 + \rho(r)]$.

Figure 3
NBR - CRR

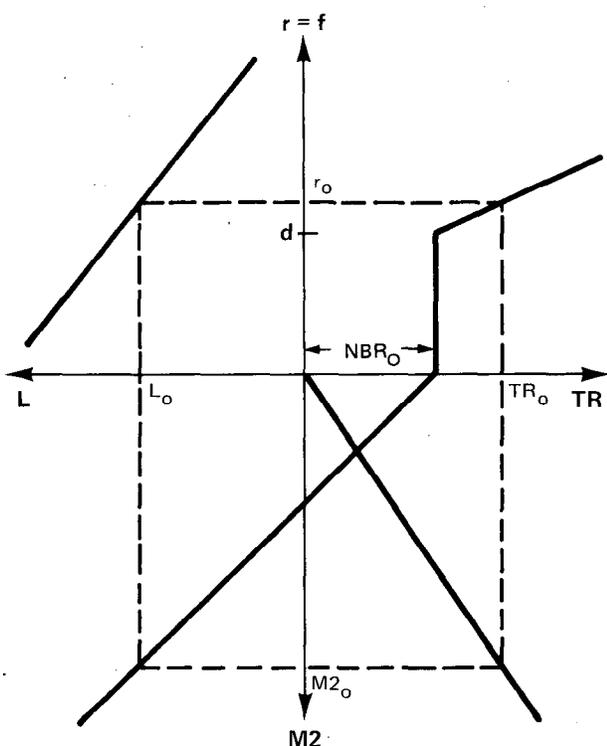
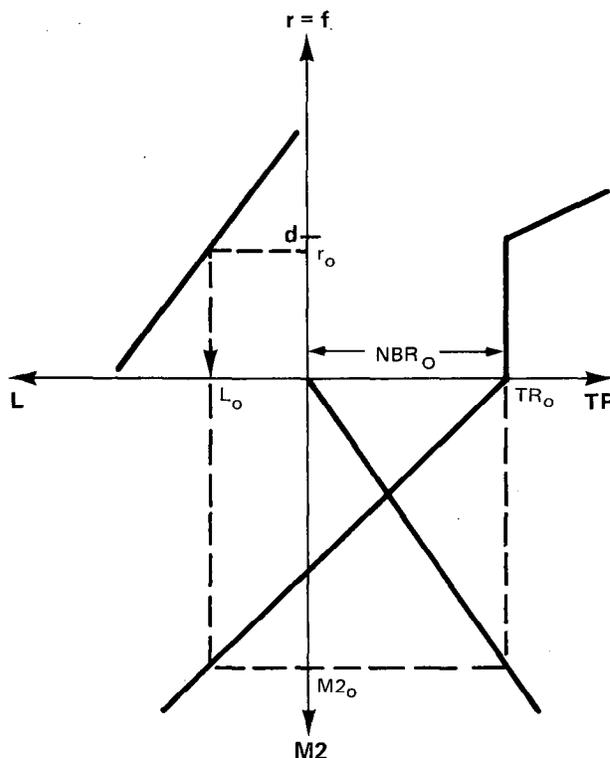


Figure 4
f - CRR
(f < d ⇒ BR = 0)



4. **A Funds Rate Instrument With Contemporaneous Reserve Requirements** Even though the Fed is likely to retain non-borrowed reserves as its primary instrument if it returns to CRR, barring discount window reform it is likely to continue to let the funds rate fall below the discount rate periodically as it has since October 1979 and to employ the funds rate as its instrument in such circumstances. It is therefore useful to examine money stock determination with a funds rate instrument and CRR.

Equilibrium determination with a funds rate instrument and CRR is illustrated in Figure 4. Equilibrium loan volume, L_0 , depends only on loan demand and the interest rate setting, r_0 . The volume of total deposits, $M2$, associated with L_0 depends on the position of the balance sheet constraint. If the funds rate is below the discount rate, as would presumably be the case if a funds rate instrument were employed with CRR, then borrowed reserves are essentially zero, i.e., $NBR = TR$. The balance sheet constraint is anchored at that volume of non-borrowed reserves that satisfies the demand for total reserves to support current deposits. In other words, $M2$ and NBR are simultaneously determined given r_0 and L_0 .

Formally, with CRR and an interest instrument set at r_0 , NBR and $M2$ are simultaneously determined by the balance sheet constraint

$$(9) \quad M2 = NBR + L(r_0)$$

and the $M2$ - TR multiplier relation

$$(10) \quad M2 = m_2(r_0)NBR.$$

The simultaneous solution of these equations yields NBR and $M2$ values

$$(11) \quad NBR = \frac{1}{m_2(r_0) - 1} L(r_0)$$

$$(12) \quad M2 = \frac{m_2(r_0)}{m_2(r_0) - 1} L(r_0)$$

where $m_2(r_0) > 1$.¹⁹

It is useful to contrast the f -CRR combination with $f < d$ to the f -LRR combination with $f < d$. For f -LRR and $f < d$, total reserve demand is accommodated entirely as non-borrowed reserves

¹⁹ With an f -CRR combination, $M1$ is determined from $M2$ and r_0 by the portfolio balance function (4), $M1/M2 = 1/[1 + \rho(r_0)]$.

$$(13) \quad NBR = RR_0 + ER$$

where RR_0 is the predetermined volume of required reserves. For f-LRR and $f < d$, M2 is determined from equation (13) together with equations (4) and (6) in conjunction with the balance sheet constraint (1) as

$$(14) \quad M2 = \frac{1}{1 - \frac{ER}{M2}(r_0)} [RR_0 + L(r_0)].$$

Two points are worth emphasizing in comparing the f-CRR and the f-LRR combinations. First, neither combination allows contemporaneous feedback from M2 to the interest rate through reserve demand.

Second, it is useful to compare how well M2 can be targeted under the f-CRR and f-LRR combinations. To start, rewrite equation (12) describing M2 determination for the f-CRR combination as

$$(15) \quad M2 = \frac{1}{1 - \frac{TR}{M2}(r_0)} [L(r_0)].$$

Now consider M2 determination for the f-LRR combination as described in equation (14). Although RR_0 , the predetermined volume of required reserves under LRR, is known at the beginning of each reserve statement period, the Fed cannot know the nominal volume of loans, $L(r_0)$, associated with a particular interest setting because $L(r_0)$ also depends on the price level which has to be estimated by the Fed when the interest instrument is set.

Suppose that price level estimation error is roughly the same for both instrument-reserve requirement combinations so that $L(r_0)$ is subject to roughly identical error in both cases. The relative precision in targeting M2 then depends on the coefficient preceding the bracketed terms in equations (14) and (15). But $\frac{ER}{M2}$ is smaller than $\frac{TR}{M2}$, so the coefficient in equation (14) is smaller than the coefficient in equation (15). This means that the effect of $L(r_0)$ error on M2 gets magnified for the f-CRR combination relative to the f-LRR combination.²⁰

²⁰ An analogous argument holds for relative M1 targeting error with an f-CRR combination and an f-LRR combination. This is seen by referring to footnotes 17 and 19.

Note that if a funds rate instrument is used with CRR, then both rr_1 and rr_2 should be set to zero to minimize M1 or M2 targeting error.

The comparison of targeting error is relevant to the Fed's possible return to CRR. Specifically, suppose the Fed were to return to CRR, but continue to let the funds rate fall below the discount rate periodically as it has since October 1979, reverting from a non-borrowed reserve to a funds rate instrument in those circumstances. The targeting error comparison indicates that monetary control with a funds rate instrument could be less precise with CRR than with LRR.

5. A Total Reserve Instrument With Contemporaneous Reserve Requirements The Fed moved to a non-borrowed reserve instrument in October 1979 after concluding that the funds rate was an unreliable instrument for controlling the money stock.²¹ However, as has been seen above, the funds rate continues to play a central role as an intermediate target in the monetary control procedure with a non-borrowed reserve instrument and lagged reserve requirements.

The main virtue of moving to contemporaneous reserve requirements is that it would allow the banking system to bring current required reserves into equilibrium with targeted total reserves. Borrowed reserves would no longer have to be made available to ensure adequate reserve market clearing. The Fed could keep the incentive to borrow at the discount window negative, for example, by making the discount rate a fixed penalty rate slightly above the funds rate. In other words, contemporaneous reserve requirements would make it easier for the Fed to control total reserves.

The major potential benefit of utilizing total reserves and contemporaneous reserve requirements is that such a combination could enable the Fed to target a money stock without concern for loan demand, borrowed reserves, or the interest rate. If the ratio of excess reserves to the targeted monetary aggregate were interest insensitive and reserve requirements were uniformly and solely applied to the targeted monetary aggregate, then there could be a direct and relatively stable link between total reserves and the targeted money stock.²²

²¹ See "The New Federal Reserve Technical Procedures for Controlling Money" [6].

²² A case for strict monetary control with a total reserve instrument and contemporaneous reserve requirements is made in Goodfriend [5].

Figure 7
A DISCOUNT RATE ADJUSTMENT

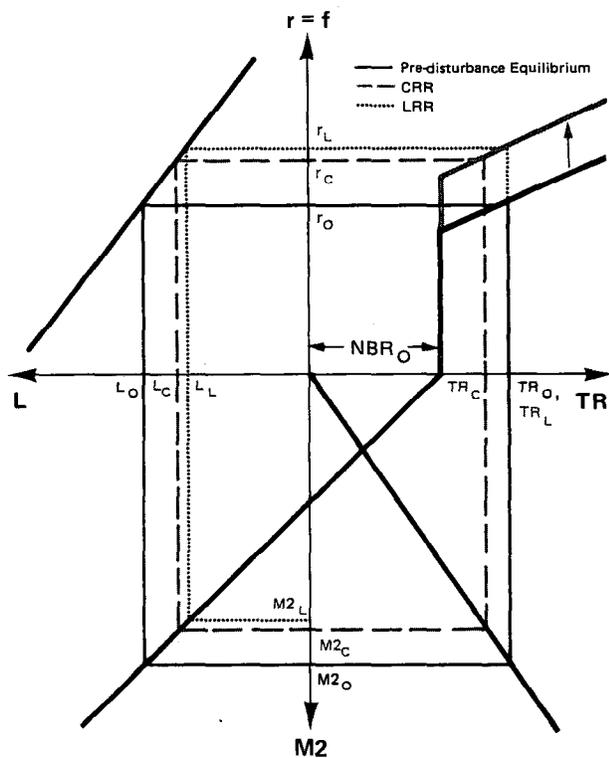
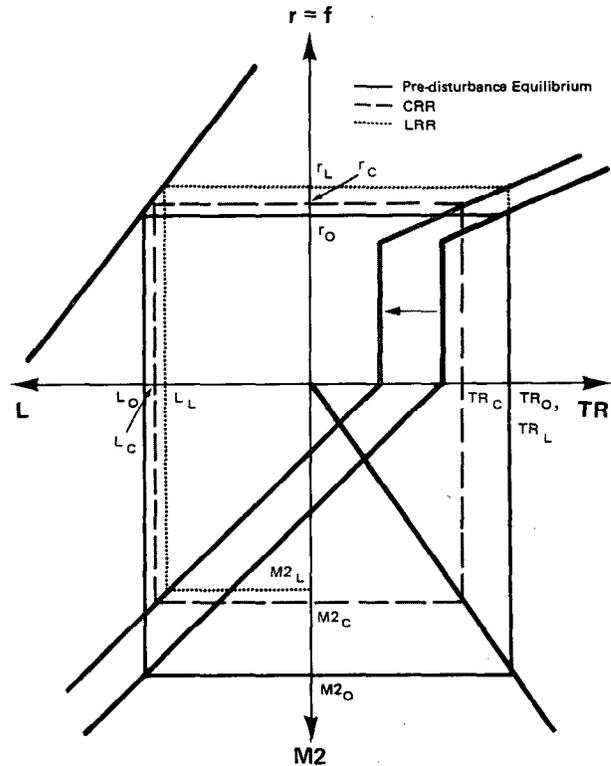


Figure 8
AN OPEN MARKET OPERATION



3. **An Open Market Operation** An open market sale of securities by the Fed, undertaken to reduce non-borrowed reserves and contract the money stock, is illustrated in Figure 8 as a leftward shift in the reserve provision schedule. As the public draws down its deposits to pay for securities purchased from the Fed, the banking system loses non-borrowed reserves. Diagrammatically, the balance sheet constraint line shifts upward by the amount of the non-borrowed reserve drain.

Under LRR, banks borrow in the funds market and at the discount window to replace the lost non-borrowed reserves in order to satisfy predetermined total reserve demand, TR_0 . The incipient excess demand for total reserves drives the funds rate up and thereby raises the $f - d$ spread. The reserve market comes into equilibrium at an $f - d$ spread high enough to raise discount window borrowing sufficiently to fully offset the initial non-borrowed reserve drain. The higher funds rate leads the banking system to contract loans until the interest rate on loans rises to the level of the funds rate. Banking system assets contract by the sum of the reduction in both loan volume and non-borrowed reserves.

Equilibrium is therefore reached at a higher interest rate and lower M2.

Under CRR, as illustrated in Figure 8, the interest rate rises and M2 falls but neither responds as much as under LRR.²⁷ This is because with CRR the M2 reduction also reduces total reserve demand so that a smaller funds rate rise clears the reserve market. The smaller interest rate rise produces a smaller contraction in both loans and M2 with CRR as well.

4. **An M2-TR Multiplier Shift** As can be seen in equation (8), the M2-TR multiplier can shift due to a revision of reserve requirements, a change in excess reserve demand, or a change in the demand for TDs relative to DDs.

For example, consider money market funds (MMFs). For the purpose of this discussion, MMFs may be assumed to sell shares to the public and purchase banking system TDs. MMFs essentially reduce the public's cost of holding TDs, so MMFs are presumed to raise the ratio of TDs to

²⁷ M1 also responds less under CRR than LRR.

DDs that the public wishes to hold at any interest rate.²⁸

The introduction of MMFs may therefore be examined in this model as an increase in the demand for TDs relative to DDs (and M2 relative to M1) at a given interest rate. As discussed following equation (8), a shift from DDs to TDs at a given interest rate raises the M2-TR multiplier. Diagrammatically, the M2-TR multiplier increase is illustrated in Figure 9 as a clockwise rotation of the multiplier line.

Under LRR, total reserve demand TR_0 is predetermined. Therefore, the M2-TR multiplier shift affects neither the funds rate, loan volume, or M2. By contrast, under CRR the increase in the M2-TR multiplier reduces total reserve demand at the initial total deposit volume, creating an incipient excess supply of reserves.

The reserve market is brought into equilibrium by a fall in the funds rate. The funds rate fall works to clear the reserve market through two channels. First, the funds rate fall reduces the $f - d$ spread and thereby reduces discount window borrowing and reserve supply. Second, the funds rate fall reduces the interest rate and thereby raises the volume of loans demanded. The increase in loan volume translates into an increase in total deposits, M2, through the balance sheet constraint; the M2 increase, in turn, raises reserve demand. As illustrated in Figure 9, equilibrium is reached at higher loan and M2 volume, lower total reserve volume, and a lower interest rate.²⁹

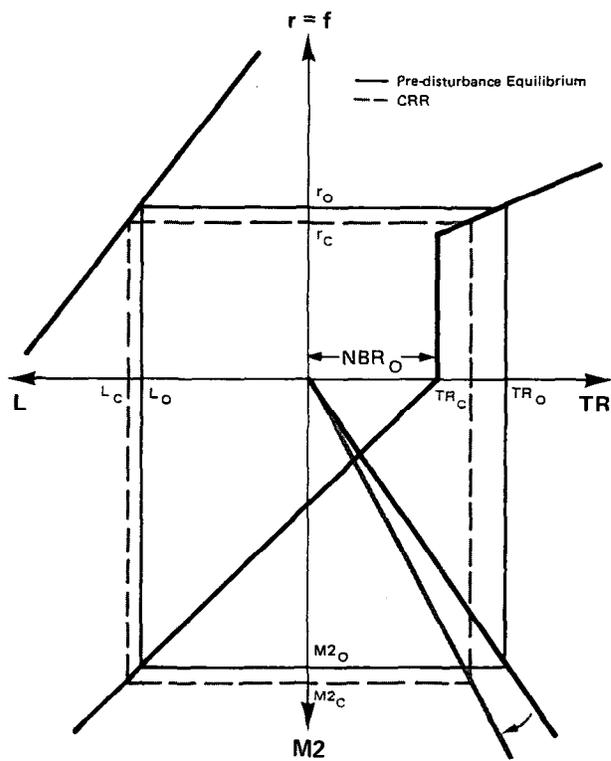
5. M2-TR Multiplier Interest Sensitivity To this point, the discussion in Section IV has been carried out under the assumption that the M2-TR multiplier is insensitive to interest rate changes. Now the effects of M2-TR multiplier interest sensitivity (with an NBR instrument) can be discussed.

As demonstrated following equation (8), the M2-TR multiplier varies positively with the interest rate. This means that the multiplier line in the southeast quadrant rotates clockwise with an increase in r . The M2-TR multiplier is irrelevant to the determination of loans, M2, total reserves, or the interest rate under LRR. But under CRR, as can be verified diagrammatically, taking interest sensitivity of the M2-TR multiplier into account reduces the impact of any disturbance on the interest rate. On the other hand, the impact on M2 can be reduced or magnified

²⁸ MMFs also raise $\rho'(r)$, the interest sensitivity of the demand for TDs relative to DDs and of M2 relative to M1.

²⁹ M1 volume is higher in the new equilibrium as well.

Figure 9
AN M2-TR MULTIPLIER SHIFT



depending on the source of the disturbance. M2-TR multiplier interest sensitivity *magnifies* the impact on M2 due to a loan demand shift, but *reduces* the impact on M2 of a discount rate adjustment, an open market operation, or a shift in the demand for DDs relative to TDs.³⁰

V. CONCLUSION

A model of money stock determination has been presented that takes explicit account of bank loan demand and the banking system balance sheet constraint. Money stock determination has been explained for alternative monetary control instruments, namely, funds rate, non-borrowed reserve, and total reserve instruments, and for lagged and contempo-

³⁰ The effect of interest sensitivity of the M2-TR multiplier on M1 under an NBR-CRR combination may be examined by looking at the portfolio balance condition $M1/M2 = 1/[1 + \rho(r)]$. For a discount rate adjustment, an open market operation, or a shift in the demand for DDs relative to TDs, M2 and r move in opposite directions and M2-TR multiplier interest sensitivity reduces both M2 and r response; so M1 response is reduced as well. A loan demand shift moves M2 and r in the same direction and M2-TR multiplier interest sensitivity reduces the r response but magnifies the M2 response; so in this case the effect on M1 is ambiguous.

aneous reserve requirements. Furthermore, determination of both "M1" and "M2" type monetary aggregates has been explained with the aid of a simple diagram.

Determination of the monetary aggregates has been shown to depend critically on the method of monetary control employed by the Fed. In particular, the discussion has shown that the money multiplier is not generally a complete model of money stock determination and is actually irrelevant to money stock determination for some monetary control procedures. Specifically, the money multiplier is irrelevant to determination of the monetary aggregates if lagged reserve requirements are in effect. On the other hand, the money multiplier can be a complete model of targeted money stock determination if contemporaneous reserve requirements are in effect, total reserves are the instrument of monetary control, required reserves are uniformly and solely applied to the targeted monetary aggregate, and the ratio of excess reserves to the targeted monetary aggregate is interest insensitive. With contemporaneous reserve requirements and either a funds rate or a non-borrowed reserve instrument, however, the money multiplier is necessary but not sufficient to explain determination of the monetary aggregates.

If the Fed does move to non-borrowed reserves with contemporaneous reserve requirements, it is likely to let the funds rate fall below the discount rate periodically and to employ the funds rate as the instrument of monetary control in such circumstances as it has since October 1979. A comparison of money stock targeting error for f-LRR and f-CRR combinations has indicated that monetary control with a funds rate instrument could be less precise with contemporaneous reserve requirements than with lagged reserve requirements.

The model has been employed to examine the impact of four disturbances on the monetary aggregates with a non-borrowed reserve instrument under lagged and contemporaneous reserve requirements. The NBR-LRR and NBR-CRR combinations have been examined in detail because the Fed is currently employing non-borrowed reserves with lagged reserve requirements and has given serious consideration to utilizing non-borrowed reserves with contemporaneous reserve requirements. The four disturbances examined were (1) a loan demand shift, (2) a discount rate adjustment, (3) an open market operation, and (4) an M2-TR multiplier shift. The M2 money stock was found to respond less under CRR than under LRR to the first three disturbances. The M1 money stock was found to respond less under

CRR than under LRR to a discount rate adjustment and an open market operation. However, relative M1 response to a loan demand shift was found to be ambiguous. Furthermore, whereas both monetary aggregates are insulated from a multiplier shift under LRR, neither is insulated from a multiplier shift under CRR.

Loan demand disturbances, multiplier disturbances, and problems associated with the funds rate falling below the discount rate could be reduced if appropriate reserve requirement and discount window reform were to accompany a move to contemporaneous reserve requirements. For example, if the discount rate were made a fixed penalty rate slightly above the funds rate, then borrowed reserves would be small and the Fed could supply non-borrowed reserves to achieve a total reserves objective. If, in addition, reserve requirements were uniformly and solely applied to the targeted monetary aggregate, and the ratio of excess reserves to the targeted monetary aggregate were interest insensitive, then there could be a direct and relatively stable link between total reserves and the targeted money stock. In short, with a total reserve instrument and contemporaneous reserve requirements the Fed's money stock targeting procedure could be well-insulated from loan demand, multiplier, borrowed reserve, and interest rate disturbances in general.

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APPENDIX

The model is summarized as follows:

Reserve Market Equilibrium

$$TR \equiv NBR + BR$$

$$BR = BR^D(r - d)$$

$$\begin{cases} r > d \Rightarrow BR^D(r - d) > 0 \text{ and } BR^{D'}(r - d) > 0 \\ r \leq d \Rightarrow BR^D = 0 \end{cases}$$

$$TR^D \equiv RR + ER$$

$$RR \equiv rr_1 DD + rr_2 TD$$

$$rr_2 < rr_1 < 1$$

$$ER \equiv k_1(r)DD + k_2(r)TD$$

$$k_2(r) < k_1(r) \text{ and } k_1'(r), k_2'(r) < 0$$

$$TR = TR^D$$

Loan Market Equilibrium and the Balance Sheet Constraint

$$L = P \cdot L^D(r)$$

$$L^{D'}(r) < 0$$

$$L + NBR \equiv DD + TD$$

Portfolio Balance and The Money Multiplier

$$M1 \equiv DD$$

$$M2 \equiv DD + TD$$

$$\frac{TD}{DD} = \rho(r)$$

$$\rho'(r) > 0$$

$$\text{or } \frac{M2}{M1} = 1 + \rho(r)$$

$$m_2(r) \equiv \frac{M2}{TR}$$

$$m_2(r) = \frac{1 + \rho(r)}{rr_1 + rr_2\rho(r) + k_1(r) + k_2(r)\rho(r)} \quad m_2'(r) > 0$$