A MONETARIST MONEY DEMAND: F U N C T I O N

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Introduction

In the first part of this article, inflation as a monetary phenomenon is discussed. The discussion is from the perspective of the modern formulation of the quantity theory. (See, in particular, Chapter 2 in Friedman and Schwartz, *Monetary Trends in the United States and the United Kingdom* [2]). In the second part of the article, empirical estimation of the relationship between money growth and inflation is discussed. The article that accompanies this one; "The Behavior of the M1 Demand Function in the Early 1980s" contains the results of estimating this relationship for the post-Korean War period in the United States.

The Quantity Theory Framework

The modern formulation of the quantity theory places the determination of the price level within the analytical framework of supply and demand. The price level, or more appropriately its inverse, is the goods price of money. The price level is determined "by the interaction of the supply and demand for money.

A discussion of the determination of the price level must begin with the distinction between real and nominal variables. Nominal variables are either measured directly in current dollars or in a way that depends upon the use of dollars as a measure of value: Examples of nominal variables are the number of dollars in circulation and the dollar expenditure of the public on final goods and services. Because market, rates of interest, at least in principle, vary directly in response to changes in the future price level anticipated by the public, they are also examples of nominal variables. Real variables, in contrast, are measured in ways that do not make use of current dollars as the unit of account. Examples of real variables are the exchange rate between two commodities and final physical output. The quantity of money expressed in terms of its purchasing power over goods and services is a real variable.

The public cares about the real quantity of money it holds, while the actions of the central bank determine the nominal quantity of money available for the public to hold. The price level translates the real quantity of' money demanded, by the, public into the corresponding nominal quantity demanded. The price level varies in order to equate this nominal quantity demanded to the given nominal quantity supplied.

The quantity equation serves as a useful summary of the supply and demand relationship that determines the price level. In expression (1), the nominal money stock, M, equals the product of a factor k and nominal expenditure, where nominal expenditure is expressed as the product of the price level, P; and real expenditure, Y. By definition, k is the fraction of nominal expenditure held as nominal money balances.

$$(1) \quad M = k [P * Y]$$

The definitional relationship shown in (1) can be transformed into a substantive economic hypothesis by interpreting it within the framework of a supply and demand relationship.

$$(2) \quad M = P [k \cdot Y]$$

For this purpose, (1) is rewritten as (2). Now, M is interpreted as the nominal quantity of, money supplied. The product in brackets, k * Y, is interpreted as the real quantity of money demanded. Finally, the price level, P, adjusts to equate money demanded to money supplied. Below, expression (2) is given substance as a theory of the determination of the price level through specific hypotheses about the behavior of its components and their interaction. Interpretation of (2) within the framework of supply and demand, however, already entails the substantive economic hypothesis that the determinants of the supply of money can generally be considered as con ceptually distinct from the determinants of the demand for money.

The Money Supply

From the perspective of the quantity theory, the nominal quantity of money is assumed to be determined largely independently of the public's demand for money. The public adjusts to the nominal money stock by varying the rate of its nominal expenditure. Conversely, the nominal money stock does not adjust to the rate of expenditure of the public: Each individual economic entity can proportion its money holdings to its expenditure by varying its money holdings, but collectively this behavior is not possible. Collectively, the public proportions its money holdings to its expenditures through a change in expenditures. From the quantity theory perspective, major changes in the nominal expenditure of the public reflect the public's adjustment to changes in the money stock.

Money Demand

Holders of money care about the real quantity of money they hold, that is, what they can purchase in terms of goods and services with their money holdings. The real quantity of money demanded is expressed in (2) by the terms k * Y. Y is considered here to be real annual expenditure on final output. The demand for real money holdings is, then, expressed as a fraction k of the annual real expenditure of the public. The term k * Y expresses the fraction of a year's real expenditures that the desired real money stock would finance. The inverse of k is the expenditure velocity of money, that is, the number of times that a dollar on average is used in a year to effect transactions involving the sale of final goods and services. The term k is considered to be a predictable function of a small number of variables, for example, the nominal rate of interest and real income or wealth.

The statement that inflation is a monetary phenomenon is true in a trivial sense in that the price level is determined by the interaction of the demand for and the supply of money. This statement, however, refers to the empirical generalization that changes in the demand for money proceed in a fairly predictable, moderate fashion, while changes in the supply of money frequently occur that are large relative to changes in the demand for money.

Adjustment of the Price Level to Money

The price level is the rate of exchange between real output and dollars. It translates the real quantity of money desired by the public into a corresponding desired nominal quantity. As discussed below, the quantity theory assumes that ultimately the price level adjusts when, at the pre-existing price level, changes in the nominal quantity of money supplied produce excess supply or demand for money.

Changes in nominal money are not offset, except over short periods, by changes in k, the factor expressing the way in which the public's demand for real money balances depends upon variables such as real income and the market rate of interest. On the contrary, changes in k may reinforce the effect of prior changes in money growth. For example, an increase in the rate of growth of the money supply will at some point result in a higher anticipated inflation rate, which will increase market rates of interest. As money becomes more expensive to hold, the public will reduce its real money holdings. This reduction, a reduction in k, will drive the price level up beyond what was implied by the increased money growth.

Historically, changes in money growth have affected real expenditure before affecting the inflation rate. The effect of money on real expenditure and output is referred to as the nonneutrality of money and is not well understood theoretically. According to the quantity theory tradition, however, this effect causes movements in the money supply to be the major source of the business cycle. As explained below, the quantity theory assumes that the ultimate impact of changes in money is on changes in the price level, not on real economic activity.

Nothing about the number of dollars in circulation affects in any fundamental way the real resource endowments of an economy, the technological capacity of an economy to transform endowments into output, or the preferences of individuals with respect to consumption of this output. For these reasons, a change in the nominal quantity of money cannot exert a permanent effect on real economic activity.

The hypotheses of this section can be summarized by reference to expression (2). A change in M will not be absorbed by an offsetting change in k, nor will a change in M affect Y in a lasting way. A change in M must ultimately affect P. The sections that follow constitute a discussion of one way of quantifying the

empirical relationship existing between money and price level.

Empirical Association of Money Growth and Inflation

A functional form is suggested in this section for examining the empirical evidence on the ability of the, growth rate of the money supply to predict inflation. (See Hetzel [3].) The following section displays the associated algebra.

Initially, a functional form is posited to explain the public's demand for real money holdings. This form is assumed to depend upon time, a nominal interest rate, and real expenditure. In order to estimate this functional form, dependent and independent variables must be chosen and a lag structure imposed on the latter. This task is effected through choice of an "adjustment equation" in the spirit of (2) in which the public adjusts its real money holdings through variation in the price level. The price level varies so as to translate the real money holdings desired by the public into a desired nominal quantity equal to the nominal quantity supplied by the central bank. Combining the basic functional form expressing the public's demand for real money holdings with this adjustment equation yields an expression relating the price level to contemporaneous and lagged values' of the nominal interest rate, real expenditure, and the nominal money stock. This expression can be estimated as a regression equation.

Algebra

A standard money demand function is as follows:

$$(3) \quad m_t^* = f(X_t) = e^k e^{-at} R_t^{-b} Y_t^c .$$

Real money holdings, the ratio of nominal money holdings to the price level, M/P, are denoted by m. Real money holdings demanded by the public; m*, depend upon time (t), a nominal interest rate (R), and real expenditure (Y). Also, k is a constant; a is the trend rate of growth in the demand for money; and b and c are the elasticity of the demand for real money balances with respect to the nominal rate of interest and real expenditure, respectively. The symbol e denotes the base of the system of natural logarithms.

An adjustment equation in the quantity theory spirit is (4), where ln is the natural logarithm. (A difference in the logarithms of variables, multiplied by 100, can be interpreted as the percentage difference in the variables.)

$$(4) \quad \ln P_t - \ln P_{t-1} = \lambda[\ln M_t - \ln m_t^* P_{t-1}].$$

The percentage change in the price level is assumed to be a constant fraction λ of the percentage discrepancy between the nominal money stock determined by the central bank and the nominal money stock desired by the public at the inherited price level. (The desired nominal money stock is the product of the desired real money stock and the price level.) Combining (3) and (4) yields

$$(5) \quad \ln P_t = -\frac{\lambda}{1-(1-\lambda)L} \ln f(X_t) + \frac{\lambda}{1-(1-\lambda)L} \ln M_t.$$

I, is a shift operator, $L^k X_t = X_{t-k}$, that is, it shifts the date of a variable into the past. Also,

$$\frac{\lambda}{1-(1-\lambda)L} = [1 + (1-\lambda)L + (1-\lambda)^2L^2 + \ldots .].$$

The immediately preceding term indicates that the variables it multiplies, ln f (X_t) and ln M_t in (5), enter as a sum of contemporaneous and past values with weights that decline geometrically, that is, by some power of $(1 - \lambda)$. In (6) below, ln M_t and the variables that comprise ln f (X_t) are allowed to enter with a simple distributed lag pattern, rather than with a geometrically declining lag pattern, that is, the coefficients on the affected variables are not constrained to follow a particular pattern. (The geometrically declining lag pattern was assumed only for expositional purposes.)

Below, in (6) ln f(X_t) is written out as follows :

$$\ln f(X_t) = k - at - b \ln R_t + c \ln Y_t.$$

(The use of first differences, indicated by the A, causes the k and the t in the at term to drop out.)

Estimation

The functional form (5) is shown below in first difference form as regression equation

$$(6)\quad \Delta \ln P = a + \sum_{i=0}^{-n1} b_i \Delta \ln R_i - \sum_{i=0}^{-n2} c_i \Delta \ln (Y/N)_i + \sum_{i=0}^{-n3} d_i \Delta \ln (M/N)_i + u.$$

The 8 notation indicates the use of simple distributed lags. The trend rate of growth of the demand for money is a, while the respective sums of the b_i and c_i coefficients are the. elasticity, of the demand for real money balances with respect to the nominal rate of interest and real income, respectively. The error term is u. N is population. Dividing real expenditures and nominal money by population means that a change in population that leaves per capita real expenditure and per capita nominal money holdings unchanged will not affect the price level.

Regression equation (6) is in principle amenable to estimation as a money demand function. The price level, rather than the real money holdings of the public, is the dependent variable. The derivation of (6), however, was performed under the quantity theory assumption that nominal money holdings are given to the public. The behavior of the price level, therefore, determines the behavior of real money holdings.

The estimation of (6) as a money demand function presents a number of difficulties. The specification of the adjustment equation (4) should in principle determine the specification of the functional forms (5) and (6). The theory needed to specify (4) in a satisfactory way is, however, largely lacking. What is needed is a theory explaining the way in which a change in nominal money produces changes over time in the price level. The way in which a change in nominal money breaks down in the short run into changes in real expenditure and in the price level is, however, one of the major unresolved issues in economics. Satisfactory estimation of money demand functions requires a better understanding of the dynamics of the process whereby the public eliminates discrepancies between actual and desired real money holdings.

Failure to account satisfactorily for these dynamics could cause the behavior of the central bank to affect the apparent structural stability of the estimated money demand regression. Estimation of (6) could be affected by supply side, rather than solely demand side, behavior. For example, if the central bank were to cause the behavior of nominal money to become more predictable, some theories would predict a reduction in the time required for a change in money to change the price level. The mean lag associated with the estimated distributed lag coefficients on money in (6) would fall.

The above comments need to be qualified, however, by noting that estimation of (6) can still offer evidence on shifts in the public's money demand function. Over a period of time long enough for the price level to adjust fully, the quantity theory implies that real money holdings are demand determined by the public. A persistent underprediction of the price level, say, with (6), and thus overprediction of the level of real money holdings, must be explained by the behavior of the demand for money, not the supply of money. This hypothetical underprediction of the price level would reflect a leftward shift in the public's money demand function.

A problem related to those discussed above is that over periods of time short enough for money to affect real economic activity, the price level, the dependent variable in (6), and the rate of interest and the rate of real expenditure, independent variables in (6), are simultaneously determined. A single equation estimation procedure applied to (6) will not capture this mutual interaction and, consequently, will yield biased estimates of the true parameters of the public's money demand function.

Finally, the true functional form of the money demand function might not be (3), but might be an alternative functional form such as the Cagan [1] money demand function. With this latter form, real money holdings depend upon the inflation rate expected by the public. The price level, consequently, depends upon the money supply expected to obtain in the future. Estimation of the parameters of the money demand function requires knowledge of the process generating the money supply. (See Sargent [4].) Ordinarily, estimation of (6) would be expected to produce a sum of coefficients on the contemporaneous and lagged money terms close to one so that a one percentage point change in money growth would lead to a one percentage point change in the inflation rate. If the Cagan money demand function were the appropriate function, however, the sum of estimated coefficients on money might differ from one in that the money terms are serving as a proxy for expected money growth.

Despite, the problems discussed above, estimation of (6) remains a useful way of organizing an overview of the data relevant for assessing the stability of

the public's money demand function. Problems of specification and simultaneous equations bias do not alter the fact that (6) depends upon the public's money demand function in an essential way. Stability over time of an estimated regression equation like (6) constitutes empirical evidence in favor of a stable money demand function. Estimation of (6) permits, in particular, an assessment of the degree to which the empirically regular association between money growth and inflation, predicted by the quantity theory, exists. This estimation is performed in the following article within a context of a discussion of whether the recent deregulation of the financial system has altered the character of M1.

References

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