

**PRECURSORS OF THE P-STAR MODEL**

Thomas M. Humphrey

Because the price level rises by a lesser extent than required for long-run equilibrium during the initial period of higher money growth, the price level has some catching up to do to reflect the higher rate of money growth. The rate of price change—the inflation rate—will therefore be above the long-run rate during this phase of the adjustment.

*William Poole, Money and the Economy: A Monetarist View, p. 95.*

**Introduction**

The preceding quotation states what economists have long observed: Prices, due to their initial lag in adjusting to unanticipated monetary changes, must temporarily rise at a faster-than-equilibrium pace if they are to reach their new equilibrium level consistent with the monetary change. Recently, analysts at the Board of Governors of the Federal Reserve System have embodied this insight into their latest inflation forecasting schema, the P-Star model—so-called because it refers to the equilibrium level of prices $P^*$ to which actual prices $P$ tend to adjust. From the gap or discrepancy between equilibrium and actual prices, $P^* - P$, the P-Star model predicts the direction of movement of the inflation rate. More precisely, it predicts that inflation will rise, fall, or stay unchanged as actual prices are below, above, or at their equilibrium level. Here is an inherently plausible inflation forecasting model that anyone can understand. Small wonder that it has caught the attention of the popular press, including the *New York Times, Business Week, The Wall Street Journal,* and the *American Banker.*

The *New York Times* has hailed P-Star as a “new theory.” In fact, however, P-Star is neither new nor is it solely a theory. Instead it combines the quantity theory of money with the observed empirical fact of lagged price adjustment into a predictive model whose major components have existed for at least 230 years. Every economist who ever believed (1) that the long-term trend of prices is roughly determined by money per unit of full-capacity real output, (2) that actual prices adjust to this trend with a lag, and (3) that during the process of adjustment such prices will be rising faster (or slower) than their trend rate of change qualifies as a P-Star proponent. Besides giving these ideas a catchy name, the Board has condensed them into an econometric equation that tracks inflation well. But the ideas themselves have a long tradition in mainstream monetary theory. To provide some needed historical perspective, this article documents the preceding assertions by showing that quantity theorists from David Hume to Milton Friedman would have recognized the P-Star model as a formalization of their own views on inflation. Before doing so, however, it is necessary to spell out the essentials of the P-Star model itself so that one can specify what earlier monetary theorists had to say about it.

**The P-Star Model**

The P-Star model is simple and straightforward. From the quantity theory equation of exchange $MV = PQ$, it defines equilibrium prices $P^*$ as the product of the relevant money stock $M_2$ per unit of potential output $(M_2/Q^*)$ and $M_2$'s equilibrium circulation velocity $V^*$. In symbols, $P^* = M_2V^*/Q^*$ with the asterisks referring to equilibrium magnitudes of the attached variables. Then the model predicts that inflation (1) will rise when actual prices $P$ fall short of equilibrium prices $P^*$, (2) will fall when $P$ exceeds $P^*$, and (3) will remain unchanged at a steady rate determined by the differential growth rates of money and real output when $P$ equals $P^*$. In short,

---

1 Jeffrey J. Hallman, Richard D. Porter and David H. Small, “$M_2$ per Unit of Potential GNP as an Anchor for the Price Level,” April 1989, Staff Study 157, Board of Governors of the Federal Reserve System.


\[ \Delta I = f(P^* - P) \]
where \( \Delta I \) denotes a change in the inflation rate and \( f \) is an empirical function relating that inflation change to the price gap which causes it. In other words, if no price gap exists then inflation remains unchanged at its given inherited rate sufficient to keep actual and equilibrium prices growing along the same path. But if actual prices fall short of equilibrium prices, inflation must rise to bring \( P \) up to \( P^* \) thus restoring equality between the two. That is, actual prices must temporarily climb a steeper path to reach equilibrium. Conversely, if actual prices exceed equilibrium prices, inflation must fall to bring actual into line with equilibrium prices. In short, the price gap \( P^* - P \) predicts the direction of movement of the inflation rate.

Figure 1 illustrates how a price gap produces a rise in the inflation rate that lasts until the gap disappears. Depicted on the diagram are hypothetical time paths of equilibrium (dashed line) and actual (solid line) prices. The slopes or rates of change of these lines represent inflation rates. Up to time \( t_0 \) actual and equilibrium prices are the same. At time \( t_0 \), however, a presumed rise in the rate of monetary expansion raises the equilibrium rate of inflation and opens a gap between equilibrium and actual prices. After a lag, actual prices start to rise. To reach equilibrium, however, they must grow at a rate in excess of the equilibrium inflation rate (slope \( CD > \text{slope BE} \)) before stabilizing at that latter rate. During the interval of adjustment, the actual inflation rate temporarily rises above the equilibrium or steady-state rate. It also rises above its initial (preexisting) rate (slope \( CD > \text{slope AC} \)). Here is the model's prediction that an emerging price gap augurs a temporary rise in inflation.

The price gap also implies the existence of corresponding output and velocity gaps—a result dictated by the equation-of-exchange identity \( MV = PQ \). To preserve the identity, changes in the money stock \( M \) not absorbed by changes in prices \( P \) must be absorbed by changes in real output \( Q \) and velocity \( V \). Accordingly, one or both of those variables must deviate from their long-run equilibrium values until \( P \) converges on its equilibrium level \( P^* \) dictated by the monetary change. In this way, an emerging price gap presaging changes in inflation \( \Delta I \) also presages temporary movements in output and velocity about their long-run equilibrium levels. Thus for a given realized equilibrium money stock the price gap is equivalent to the sum of the velocity and output gaps.

**Historical Roots**

The ideas underlying the P-Star model have a rich heritage. They were endorsed by virtually every quantity theorist who ever believed that prices respond to money with a lag. To these analysts such a lagged price response implied at least two things. First, a money-induced jump in the equilibrium price level (or a steepening of its time path) would open a gap between equilibrium and actual prices that would last until the latter fully adjusted to the former. Correspondingly, real output would exhibit a temporary rise and velocity perhaps a temporary fall before returning to their pre-existing equilibrium levels. Second, during the interval of adjustment, actual prices would have to rise at a faster-than-
equilibrium rate if they were to reach their equilibrium level. In this way an emerging price gap \( P^* > P \) would herald a rise in the inflation rate just as a reverse gap \( P^* < P \) would herald its fall.

David Hume

Among early quantity theorists, David Hume (1711-76) expressed the foregoing view most succinctly. In his 1752 essays "Of Interest" and "Of Money" he argued four points. First, increases in the money stock produce equiproportional increases in the equilibrium price level consistent with output and velocity being at their long-run equilibrium levels. That is, \( \Delta P^* = (V^*/Q^*)\Delta M \). Second, owing to initial distribution effects, imperfect information, sluggish nominal wages, and price perception errors, actual prices temporarily lag behind equilibrium prices. Third, their incomplete adjustment is, assuming no velocity gap, completely offset by compensating transitory rises in real economic activity above its equilibrium level. Fourth, the same process works for falls in the equilibrium price level. Such falls induce a temporary depression of real activity during the deflationary convergence of actual to equilibrium prices.

In Hume's own words:

Augmentation [in the quantity of money] has no other effect than to heighten the price of labour and commodities... In the progress toward these changes, the augmentation may have some influence, by exciting industry; but after the prices are settled, suitably to the new abundance of gold and silver, it has no manner of influence. 4

Though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the increasing quantity of gold and silver is favourable to industry. When any quantity of money is imported into a nation, it is not at first dispersed into many hands, but is confined to the coffers of a few persons, who immediately seek to employ it to advantage... They are thereby enabled to employ more workmen than formerly, who never dream of demanding higher wages, but are glad of employment from such good paymasters. If workmen become scarce, the manufacturer gives higher wages, but at first requires an encrease of labour; and this is willingly submitted to by the artisan, who can now eat and drink better, to compensate his additional toil and fatigue... It is easy to trace the money in its progress through the whole commonwealth:


where we shall find, that it must first quicken the diligence of every individual, before it encrease the price of labour... There is always an interval before matters be adjusted to their new situation; and this interval is as pernicious to industry, when gold and silver are diminishing, as it is advantageous when those metals are encreasing. 5

Here then, is Hume’s anticipation of the P-Star model: During transition periods actual prices deviate from equilibrium ones as do output and employment from their normal levels. Those deviations are corrected either by inflation (when \( P^* > P \)) or by deflation (when \( P^* < P \)). Either case requires a temporary change in the inflation rate from its zero steady-state level assumed by Hume. The price gap predicts the direction, positive or negative, of changes in the inflation rate.

Hume's Case Diagrammed

As the prototypical P-Star model, Hume’s analysis warrants diagrammatic treatment (see Figure 2). The top diagram shows the time paths of actual (solid line) and equilibrium (dashed line) prices. The bottom line shows the time path of the inflation rate which is the same as the slope or rate of change of the path of actual prices. Up to time \( t_0 \) actual and equilibrium prices coincide at a zero rate of inflation. At time \( t_0 \) a one-time jump in the money stock raises equilibrium prices as shown by the corresponding jump or step increase of the \( P^* \) path from B to C. Were actual prices \( P \) to adjust instantaneously they too would rise from B to C without, however, affecting the inflation rate (slope of \( BD \)). Instead, they adjust with a lag over the path BD. During the adjustment period (from \( t_0 \) to \( t_1 \)) the rate of inflation (slope of BD) will be higher than its steady-state rate of zero. Consistent with the P-Star model, the price gap produces a rise in the inflation rate. This rise, however, vanishes when the price gap disappears. Using this same technique, one can also show how a step fall in \( P^* \), by creating a negative price gap, produces a temporary fall in the inflation rate below its zero steady state level as actual prices traverse a descending path to equilibrium.

Henry Thornton

After Hume, Henry Thornton (1760-1815) also enunciated the essentials of the P-Star model. In his classic The Paper Credit of Great Britain (1802), Thornton explicitly stated (1) that changes in the money stock produce equiproportional changes in equi-

5 Ibid., pp. 37-8, 40.
Their natural equilibrium levels, and (4) that such deviations eventually vanish when actual prices converge on equilibrium ones. Since actual prices must rise (or fall) at a faster pace if they are to reach equilibrium, these propositions imply that the price gap \( P^* - P \) is a good indicator of forthcoming changes in the inflation rate.

**Thomas Attwood**

Thornton focused on how \( P^* \)-Star price gaps presage changes in inflation. By contrast, Thomas Attwood (1783-1856), an inflationist proponent of full employment at any cost and leader of the so-called Birmingham School, focused on the output and employment implications of the price gaps. Like Thornton, Attwood acknowledged (1) that prices lag behind monetary changes, (2) that the resulting gap between actual and equilibrium prices requires a rise in inflation to bring the former into line with the latter, and (3) that the logic of the equation of exchange dictates that price gaps must be matched by compensating gaps in output and employment as those variables deviate from their equilibrium magnitudes. But whereas Thornton held that such gaps were of temporary duration only and could not be exploited for policy purposes, Attwood thought that they could be sustained by a continuous succession of monetary injections. According to him, such injections would keep equilibrium prices forever marching ahead of actual prices, perpetually frustrating the latter’s attempts to catch up. Output and employment would in this way be given a permanent stimulus. To this end he advocated inflationary monetary policy to achieve absolute full employment. Said he,

I beg to be understood as . . . recommending that the Bank . . . be obligated or otherwise be induced, to increase the circulation of their notes as far as the national interests may require, that is to say, until all the labourers in the kingdom are again in full employment at ample wages.

An increased money stock, wrote Thornton, will “be occupied in carrying on the sales of the same, or nearly the same, quantity of articles as before, at an advanced price; the cost of goods being made to bear the same, or nearly the same, proportion to their former cost, which the total quantity of paper at the one period bears to the total quantity at the other.” Henry Thornton, *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain* (1802), ed. F.A. von Hayek (New York: Rinehart and Company, Inc., 1939), p. 241.
Elsewhere he reiterated this sentiment when he declared that prosperity has occurred whenever the government has filled the Country with what is called Money; and this plenty of Money has necessarily produced a general elevation of prices; and this general elevation of prices has necessarily produced a general increase of profit in all occupations; and this general increase of profits has, as a matter of course, given activity to every trade in the kingdom; and whilst the workmen, in one branch of trade, are producing one set of articles, they are inevitably consuming an equal amount of all other articles. This is the prosperity of the Country, and there is no other prosperity which ever has been enjoyed, or ever can be enjoyed.  

In short, Attwood’s version of the P-Star model allowed monetary changes to exert permanent non-neutral effects on real variables. By contrast, Thorton’s version allowed for temporary non-neutrality only. So too did the version enunciated by John Stuart Mill (1806-73) in his celebrated 1833 critique of Attwood’s inflationist schemes. There Mill argued that, once information and price-perception lags are inevitably overcome, equilibrium prices dictated by money per unit of capacity output are realized promptly such that output and employment gaps disappear. It follows, said Mill, that such gaps are at best transitory phenomena that cannot be sustained by expansionary policy.

Other Classical Economists

Of course, not all classical quantity theorists accepted the P-Star model with its presumption of the short-run nonneutrality of money. David Ricardo (1772-1823) and John Wheatley (1772-1830), for example, believed that prices and inflation rates were always at their equilibrium levels such that no price gaps or associated nonneutralities could emerge. While agreeing that money determines the price level and money’s growth the rate of inflation, these theorists rejected the notion of lagged price adjustment and the resulting transitory output and employment effects stemming from it. But their view was not universally held. For the most part, classical and neoclassical quantity theorists took the position of Hume, Thorton, Attwood, and Mill; they believed in lagged price adjustment and the temporary non-neutrality of monetary shocks. Except for Attwood and his fellow Birmingham inflationists, however, all agreed that price gaps eventually vanished and hence money was neutral in the long run.

Irving Fisher

Irving Fisher (1867-1947) endorsed the elements of the P-Star model in his Purchasing Power of Money (1912). On the distinction between equilibrium prices that vary equiproportionally with money according to the P-Star equation $\Delta P^* = \frac{(V^*/Q^*)\Delta M}{V^*Q^*}$ and those actually prevailing during transition adjustment periods, Fisher remarked: “The strictly proportional effect on prices of an increase in $M$ is only the normal or ultimate effect after transition periods are over.”  

Regarding the delayed response of prices to monetary shocks such that other variables in the equation of exchange absorb part of the shocks, he used a train analogy. “Normally the caboose [P] keeps exact pace with the locomotive [M], but when the train is starting or stopping this relationship is modified by the gradual transmission of effects through the intervening cars [V and Q].”

Again, on the transition period when the economy adjusts to monetary shocks partly through temporary movements in velocity and output and partly through prices, he observed: “As to the periods of transition, we have seen that an increase in $M$ produces effects not only on the p’s, but on all the magnitudes in the equation of exchange. . . . We have seen, for instance, that a sudden change in the quantity of money and deposits will temporarily affect the velocities of circulation and the volume of trade.” These velocity and output gaps, he noted, vanish once actual prices $P$ converge on equilibrium prices $P^*$.

Finally, on actual prices rising or falling to equilibrium at a faster-than-steady-state pace (assumed zero by Fisher), he wrote: “Rising [actual] prices must be clearly distinguished from high [equilibrium] prices, and falling from low. . . . Rising prices mark the transition between a low and a high [equilibrium] level of prices, just as a hill marks the transition between flat lowlands and flat highlands.” Fisher’s analysis, like the Board’s P-Star model, predicts a rise in inflation when equilibrium prices $P^*$ exceed actual prices $P$. Conversely it predicts a fall in inflation when $P$ exceeds $P^*$.

Holbrook Working

The 1920s saw variants of the P-Star model employed in statistical tests of the quantity theory

---

10 Thomas Attwood, The Late Prosperity, and the Present Adversity of the Country Explained (1826), pp. 11-12, in Selected Economic Writings of Thomas Attwood.


---


13 Ibid., p. 161.

14 Ibid., p. 159.

15 Ibid., p. 56.
of money. For example, Holbrook Working (1895-1985) in his 1923 *Quarterly Journal of Economics* article "Prices and the Quantity of Circulating Medium, 1890-1921" used the model to show that money stock changes cause price level changes, albeit with a lag. Similar to Fisher and his predecessors, Working argued (1) that equilibrium prices are determined by velocity-adjusted money per unit of real transactions with the transactions and velocity variables being at their long-run normal or trend levels, (2) that actual prices adjust to equilibrium prices with a lag, (3) that during the adjustment period velocity and real transactions deviate from their long-run normal values, and (4) that such deviations vanish when actual prices converge on equilibrium ones.

To demonstrate these points, Working presented time series charts comparing actual and equilibrium prices for the period 1890-1921. The equilibrium price series he derived by multiplying each year's average money stock by the normal or trend value of the velocity-to-real-transactions ratio. By substituting in the equation $MV/T = P$ the values of the index number of circulating medium ($M$) and of the index number of normal values of $V/T$ it is possible to obtain what may be called normal values of $P$.16

Upon comparing these computed equilibrium prices with the corresponding series of actual prices Working found that the latter tended to adjust to the former with a six- to nine-month lag. This lag itself affected the time path of prices. To reach equilibrium, actual prices had to rise at a faster- or slower-than-equilibrium pace depending upon whether they were initially below or above their equilibrium level. In other words, inflation temporarily rose or fell to eliminate price gaps and the associated deviations from equilibrium of the velocity-to-transactions ratio. For just as

the departure of the actual price level from the [normal] price level ... is associated with a corresponding departure of the actual value of $V/T$ from the normal value of $V/T$ it follows that the actual value of $V/T$ being thus reduced below its normal value, as determined by the habits, business custom and productive power of the population, $V/T$ tends to be restored to normal by an increase in $P$ [as the latter variable eventually fully adjusts to the prior monetary change].18

In short, velocity and output gaps disappear when actual prices converge to their equilibrium level. Here are all the components of the P-Star model, the statistical findings on which Working interpreted as confirming the quantity theory hypothesis of money-to-price causality.

**Carl Snyder**

Working was not the only analyst in the 1920s to publish time series charts comparing actual and equilibrium prices. Carl Snyder (1869-1943), an economist with the Federal Reserve Bank of New York, also published such charts.19 Like Working, Snyder derived his equilibrium price series by multiplying each period's average level of bank deposits ($M$) by the normal or trend value of the ratio of velocity to trade, and substituting the results into the equation $P = MV/T$. For actual prices he computed his own index of the general price level. Unlike Working, however, he attributed differences in the two price series to measurement error and not to temporary deviations in the velocity-to-trade ratio from its long-run normal or trend value. He argued that velocity and trade moved in unison in the short run, exhibiting identical and offsetting deviations from equilibrium such that the ratio $V/T$ never varied from its trend value.20 Thus, except for measurement error, equilibrium and actual prices should always be the same. Strictly speaking, Snyder's denial of the existence of price gaps $P^* - P$ and of the resulting need for inflation changes $\Delta P$ to close them disqualifies him as a P-Star proponent. Nevertheless, he did anticipate the empirical charts employed by Board P-Star economists today. His charts and their use by the New York Fed in the 1920s invalidate the *New York Times* statement that "The Fed has never had such a tool."21

**Milton Friedman**

Among modern quantity theorists, Milton Friedman stands as the foremost anticipator of the P-Star model. All the model's elements—long-run prices determined by velocity-adjusted M2 per unit of output, lagged price response, transitory velocity and output gaps, price gaps and the resulting changes in inflation—are to be found in his monetarist analysis of the inflationary process.

---

17 Ibid., p. 250.
18 Ibid., p. 233.
20 He further argued that velocity was trendless such that it played no role in price level movements.
According to Friedman, a permanent increase in growth rate of money per unit of output steepens the associated time path of equilibrium prices. Actual prices, however, do not immediately adjust. Instead, velocity at first falls and then output rises to absorb the burden of adjusting to the monetary change. Prices eventually respond, converging on their long-run natural magnitudes. Prices, because they lagged behind, must for a time grow at a faster-than-equilibrium pace if they are to reach their higher equilibrium level. Thus the emergence of a gap between equilibrium and actual prices signifies a temporary rise of inflation above its steady-state rate. In this connection, Friedman presents in his 1969 essay "The Optimum Quantity of Money" a diagram showing the inflation rising above its steady-state rate before stabilizing at that rate as prices P move to their equilibrium level P*.22

Monetarist Textbooks

The preceding has focused on the scholarly literature. But the essentials of the P-Star model also found expression more than ten years ago in at least two monetarist textbooks. Thus Michael Darby on page 160 of his 1976 text Macroeconomics presents a diagram showing (1) lagged adjustment of actual to equilibrium prices, (2) the resulting price gap P* - P, (3) the corresponding output gap Q - Q*, and (4) the resulting changes in the inflation rate.

Darby assumes a permanent steepening of the time path of equilibrium prices owing to a permanent increase in the growth rate of money. Sticky nominal wages, however, cause actual prices to lag equilibrium prices, opening a gap between P* and P. Output immediately adjusts in lieu of prices by rising temporarily above its equilibrium path. Inflation then rises to bring P into line with P* as output returns to its natural equilibrium level. With the closing of the price and output gaps, inflation stabilizes at its long-run steady-state rate. This much is consistent with the P-Star model.

Similarly, William Poole on page 96 of his 1978 text Money and the Economy: A Monetarist View presents a diagram depicting hypothetical time paths of the price level, the unemployment rate (a proxy for real output), and the money stock. Poole's diagram shows how a rise in the growth rate of the money stock steepens the steady-state path of equilibrium prices and implicitly opens a gap between equilibrium and actual prices. To reach equilibrium, actual prices must for a time rise at a faster-than-equilibrium pace. That is, inflation must rise before stabilizing at its steady-state rate. During the adjustment process, unemployment moves in lieu of prices before returning to its long-run natural level. Here again is the essence of the P-Star model.

Conclusion

The P-Star model is hardly new; its essential components have been in use for a long time. Employing these components the Board has estimated the short-run dynamics of the relationship between the price gap P* - P and changes in inflation ΔI and has shown that the relationship accurately predicts inflation's behavior over the last decade or so.

To be sure, the model is based on an observed empirical regularity—namely prices' lagged response to monetary shocks—that may not hold across all policy regimes. Nevertheless, as a practical forecasting tool the model has much to recommend it. Besides tracking recent inflation well, it has over a longer period survived the test of time, having been used successfully by quantity theorists as far back as David Hume. Equally important, it focuses on the price level, the one variable with which the Fed should be most concerned.