What determines the relationship between yield and maturity (the yield curve) in the money market? A resurgence of interest in this question in recent years has resulted in a substantial body of new research. The focus of much of the research has been on tests of the “expectations theory.” According to the theory, changes in the slope of the yield curve should depend on interest rate expectations: the more market participants expect rates to rise, the more positive should be the slope of the current yield curve. The expectations theory suggests that variation in the slope of the yield curve should be systematically related to the subsequent movement in interest rates. Much of the recent research has focused on whether this prediction of the theory is supported by the data. A surprising finding is that parts of the yield curve have been useful in forecasting interest rates while other parts have not.

A novel and interesting aspect of some of the recent literature is its emphasis on the possible role of monetary policy in explaining the behavior of the yield curve. This literature views the Federal Reserve’s policy instrument as the federal funds rate, and it posits that money markets rates at different maturities are strongly influenced by current and expected levels of the funds rate. In this view, explaining the behavior of the yield curve requires understanding how the Federal Reserve moves the funds rate over time. A key paper in this area (Mankiw and Miron [1986]), for example, argues that the persistence of changes in the federal funds rate engineered by the Federal Reserve helps explain why the yield curve from three to six months has had negligible forecasting power.

This paper surveys the recent literature on the determinants of the yield curve. It begins by reviewing the expectations theory and recent empirical tests of the theory. It discusses two general explanations for the lack of support for the theory from these tests. Finally, the paper discusses in more detail the behavior of market participants that might influence the yield curve, and the role that monetary policy might play in explaining this behavior.

I. THE EXPECTATIONS THEORY

Concepts

Two concepts central to the tests of the expectations theory reviewed below are the “forward rate premium” and the “term premium.” Suppose an investor can purchase a six-month Treasury bill now or purchase a three-month bill now and reinvest his funds three months from now in another three-month bill. The forward rate is the hypothetical rate on the three-month bill three months in the future that equalizes the rate of return from the two options, given the current three- and six-month rates. The forward rate calculated from the current six-month rate (R6) and the current three-month rate (R3), which we denote F(6,3), is defined as:

\[(1 + R6) = (1 + R3)(1 + F(6,3)),\]  
\[F(6,3) = \frac{(1 + R6)}{(1 + R3)} - 1\]  

where the yields are simple unannualized yields.

Virtually all of the studies surveyed in this paper use continuously compounded yields, which enable the forward rate to be expressed as an additive (rather than a multiplicative) function of the current six- and three-month rates. Using continuously...
compounded annualized yields (denoted here by lower case letters) the forward rate becomes:

$$f(6,3) = 2r6 - r3$$  \tag{2}

The "forward rate premium" is defined as the difference between the forward rate and the current short-term spot rate:

$$f(6,3) - r3 = (2r6 - r3) - r3 = 2(r6 - r3)$$  \tag{3}

When the maturity of the long-term rate is twice the maturity of the short-term rate, as in this case, the forward rate premium is simply twice the spread between the long- and short-term rates.

The "term premium ($\theta$)" is generally defined as the difference between the forward rate and the corresponding expected spot rate:

$$\theta = f(6,3) - Er(3:t+3),$$  \tag{4}

where $r(3:t + 3)$ denotes the three-month rate three months in the future and $E$ denotes the current expectation of that rate. Spot and forward rates not followed by a colon are measured as of time "t". Equation (4) can be rewritten in terms of the forward rate premium by rearranging terms and subtracting $r3$ from both sides:

$$f(6,3) - r3 = [Er(3:t + 3) - r3] + \theta$$  \tag{5}

This expression now decomposes the forward rate premium into the expected change in interest rates and a term premium.

To illustrate these concepts, suppose the current three-month rate is 6 percent, the current six-month rate is 7 percent, and the expected three-month rate three months in the future is 7 1/2 percent. Then the implied forward rate on a three-month security three months in the future is 8 percent and the forward rate premium is 2 percentage points. The forward rate premium can be decomposed into an expected change in the three-month rate of 1 1/2 percentage points and an expected term premium of 1/2 percentage point.

An equivalent decomposition of the forward rate premium used in some papers employs the concept of "holding period yield," which is the return earned on a security sold prior to maturity. The forward rate premium can be divided into (1) the expected change in the three-month rate and (2) the difference between the expected holding period yield earned by investing in a six-month bill and selling it when it is a three-month bill three months in the future, $Eh(6,3:t+3)$, and the return from investing in a three-month bill:

$$f(6,3) - r3 = [Er(3:t + 3) - r3] + [Eh(6,3:t+3) - r3]$$  \tag{6}

In the above example, the forward rate premium of 2 percentage points can be decomposed into an expected change in the short-term rate of 1 1/2 percentage points and an expected excess return of 1/2 percentage point for holding six-month bills for three months rather than investing in three-month bills.

Assumptions

The "expectations theory" is based on two assumptions about the behavior of participants in the money market. The first is that the term premium that market participants demand for investing in one maturity rather than another (and issuers are willing to pay to issue that maturity) is constant over time. Under this assumption equation (5) becomes:

$$Er(3:t + 3) - r3 = -c + [f(6,3) - r3]$$  \tag{7}

where $c$ is now a constant term premium. Note that equation (7) can be rewritten using equation (3) as:

$$r6 = \frac{1}{2}c + \frac{1}{2}[r3 + Er(3:t + 3)],$$  \tag{8}

which says that under the expectations hypothesis the long-term rate is equal to an average of the current and expected short-term rates plus a constant which reflects the term premium.

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2 The relationship between a simple yield (R) and the corresponding continuously compounded yield (r) is:

$$(1 + R) = \exp(r).$$

Hence, using continuously compounded yields, equation (1) in the text can be rewritten:

$$\exp(r6) = \exp(r3)\exp(f(6,3)),$$

which taking logarithms of both sides becomes:

$$f(6,3) = r6 - r3.$$  

If we now let the lower case letters stand for annualized continuously compounded yields, the expression for the forward rate becomes:

$$\frac{1}{4}f(6,3) = \frac{1}{4}r6 - \frac{1}{4}r3,$$

or

$$f(6,3) = 2r6 - r3.$$
Equation (7) is the focus of most of the recent empirical work testing the expectations hypothesis. Researchers using equation (7) to test the expectations hypothesis do not know the values of Er(3:t + 3). The procedure generally used to get these values is to assume that interest rate expectations are formed "rationally," so that:

\[ r(3:t + 3) = Er(3:t + 3) + e:t + 3, \]  

(9)

where e:t + 3 is a forecast error that has an expected value of zero and is assumed to be uncorrelated with any information available at time t. The ideas behind the rational expectations assumption are that (1) there is a stable economic environment, (2) market participants understand this environment, (3) therefore, they should not systematically over- or under-forecast future interest rates, and (4) they should not ignore any readily available information that could improve their forecasts. This assumption specifically requires that forecast errors are not correlated with the forward rate premium at time t or its two components, the expected change in interest rates and the expected term premium. Substituting (9) into (7) yields the following regression equation:

\[ r(3:t + 3) - r3 = a + b[f(6,3) - r3] + u:t + 3 \]  

(10)

Under the rational expectations assumption the error term in equation (10) is uncorrelated with the right-hand side variable so that the coefficient b can be estimated consistently. The theory predicts that b should not differ significantly from one. A significantly different value would contradict either the assumption of a fixed term premium or the rational expectations assumption.5 An estimated coefficient of zero would be evidence that the forward rate premium has no forecasting power for the subsequent behavior of the three-month rate.

While equation (10) is the most common regression estimated in this literature, a number of other specifications have also been used.6 An alternative specification used by Fama [1984a, 1986] replaces the change in the three-month rate in equation (10) with the holding period premium:

\[ h(6.3:t + 3) - r3 = a1 + b1[f(6.3) - r3] + u:t + 3 \]  

(11)

The estimates of the coefficients of equation (11) provide the same information as the estimates of equation (10) because the dependent variables in the two equations sum to the common independent variable (as indicated by equation 6). Hence, b plus b1 equal one, and the sum of the constants in the two equations equals zero.7 A value of b1 greater than zero is evidence that the current yield curve has forecasting power for the excess return earned by investing in six-month bills for three months over the return from investing in three-month bills. Given the rational expectations assumption, a value of b1 equal to one would indicate that all variation in the yield curve is due to variation in expected excess returns (i.e. the term premium) and none due to variation in the expected change in rates.

II. REGRESSION ESTIMATES

Three major sets of postwar monthly interest rate data have been used by the studies surveyed in this article to estimate equations (10) and (11): (1) Treasury yields from the Center for Research in Security Prices (CRSP) at the University of Chicago, (2) Yield series constructed from Treasury rate data by a cubic spline curve-fitting technique by McCulloch [1987] and (3) Yields for Treasury and private sector securities from Salomon Brothers' An Analytical Record of Yields and Yield Spreads. In addition, Hardouvelis [1989] uses weekly data on Treasury bills obtained from the quotation sheets of the Federal Reserve Bank of New York, and Mankiw and Miron [1986] construct a quarterly series on three- and six-month loan rates at New York banks from 1890 through 1958. The regression results we report in this paper use the McCulloch data for Treasury rates and the Salomon Brothers data for private sector rates. We also used Treasury rates from the CRSP data and the Salomon Brothers data and found little difference in the results. All interest rates used in the paper are converted to continuously compounded annual rates as described in the Appendix I.

5 We discuss in detail in Sections III and IV and in Appendix II the expected effect on the estimate of "b" if either of these assumptions is not valid.

6 Campbell and Shiller [1989] derive and estimate two other specifications to test the expectations theory using a short m-period rate and a longer n-period rate. In the first the difference between the yield on an n−m period bond m periods ahead and the current yield on an n-period is regressed on the spread between the current n-period and m-period rates, where the spread is weighted by m/(n−m). In the second a weighted average of the m-period rate over (n−m)/m periods is regressed on the current spread between the n-period and m-period rates.

7 This statement is correct if the long maturity (n) is equal to twice the short maturity (m). If n is not equal to 2m, then the statement is still true if the dependent variable is multiplied by an appropriate constant.
Estimates of the Standard Regression

The standard test of the expectations theory uses a long-term rate with a maturity equal to twice that of the short rate. Numerous studies have used the three-and-six-month rates to calculate a three-month forward rate three months in the future and estimate the coefficients of equation (10) or a comparable equation using data over the postwar period. These include Hamburger and Platt [1975], Mankiw and Miron [1986], Mankiw and Summers [1984], and Shiller, Campbell and Schoenholtz [1983]. All these studies report coefficients for the forward rate premium that are not significantly different from zero, indicating that the yield curve from three to six months has had negligible power to forecast the changes in the three-month rate. Fama [1986] finds that the Treasury bill yield curve from six to twelve months has had no forecasting power for the subsequent six-month rate, although he does find some forecasting power for the CD yield curve from six to twelve months.8

The lack of support for the expectations theory using postwar Treasury bill rates at the three-, six-, and twelve-month maturities is shown in the top of Table I, which reports regression results using the McCullough data.9 Table I also shows little support for the theory using private security rates. The coefficients in these regressions all are positive, but only one is significant at the five percent level, and the explanatory power of the regressions is negligible. The results for the private rates are similar to those reported by Fama [1986], except that his dependent variable is the holding period premium so that his coefficients are roughly 1 minus the coefficients reported in Table I.

Mankiw and Miron [1986] estimate equation (10) from 1890 to 1914, prior to the founding of the Federal Reserve, and over four subperiods from 1914 through 1979. They find that the spread between the six- and three-month rates had substantial forecasting power for the three-month rate only in the period prior to 1914. In fact, the estimated slope coefficient in this period is only slightly below the value predicted by the expectations theory. We discuss this interesting result in more detail below.

Estimates of Non-Standard Regressions

A number of recent studies also report regression results for sections of the yield curve over which the maturity of the long-term rate is not equal to twice that of the short rate. One type of regression measures the "cumulative" predictive power of the slope of the yield curve between a one-period rate and longer-term rates at various maturities. For example, we can estimate the predictive power of the yield curve from one to six months with the regression:

\[ r(1:t + 5) - r(1:t + 4) = a + b[f(6,5) - f(5,4)] + u_{t+5} \]  

(12)

The dependent variable in this regression is the change in the one-month rate over the following five months. The independent variable is the difference between the forward rate for a one-month bill five months in the future and the current one-month spot rate. The forward rate on a one-month bill five months in the future can be calculated from the current five- and six-month yields — hence, the notation \( f(6,5) \).10 A coefficient of 1 for \( b \) in this regression supports the expectations hypothesis, and a coefficient less than 1 but significantly greater than zero provides evidence that the yield curve over this range has forecasting power for the subsequent movement in rates.

A second type of non-standard regression estimates the "marginal" ability of small sections of the yield curve to forecast the subsequent movements in rates over a corresponding future period. For example, the predictive power of the yield curve for the change in rates from four to five months in the future can be estimated with the regression:

\[ r(1:t + 5) - r(1:t + 4) = a + b[f(6,5) - f(5,4)] + u_{t+5} \]  

(13)

where the dependent variable is the change in the one-month rate from four to five months in the future.

---

8 Also, Hendershott [1984] finds forecasting power for the bill yield curve from six to twelve months after adding unexpected changes in inflation and unexpected changes in other variables to his estimated equation.

9 The forecast horizon in these regressions is generally longer than the monthly period between observations. As a result there will likely be serial correlation in the error term of the regressions. For example, a regression of the three-month change in the three-month rate on the forward rate premium using monthly three- and six-month rates will likely generate a moving average error term of order 2 because the forecasts in months two and three are made before the error from month one's forecast is known. The standard errors provided in the tables are calculated using the consistent variance-covariance estimate from Hansen [1982] with the modification by Newey and West [1987]. For discussion of this procedure see Mishkin [1988, pp. 307-309].

10 The formula used to calculate the forward rate on an \( n - m \) month bill \( m \) months in the future is:

\[ f(n,m) = \frac{1}{1/(n-m)}\ln \left( \frac{r(n)}{r(m)} \right) \]
and the independent variable is the difference between the one-month forward rate five months in the future (calculated from the current five- and six-month rates) and the one-month forward rate four months in the future (calculated from the current four- and five-month rates). A coefficient not significantly different from one supports the expectations hypothesis, and a coefficient less than one but significantly greater than zero provides evidence that the yield curve from four to six months has predictive power for the movement in the one-month rate four to five months in the future.

Estimates for the non-standard regressions using the McCulloch data are shown in Table II. The results of the marginal predictive power regressions show that virtually all of the forecasting power of the bill yield curve is in the spread between the one-month ahead one-month forward rate and the current one-month spot rate.

Fama [1984a] estimates cumulative and marginal predictive power regressions using Treasury bill rates with maturities up to six months from the CRSP data from 1959 through 1982, and Mishkin [1988] repeats Fama’s regressions using the same data set extended through 1986. Both studies report full sample results for the cumulative predictive power regressions roughly similar to those reported in the top of Table II. One difference is that Fama finds coefficients significant at the five percent level in his regressions covering the cumulative change in rates one, two, and three months in the future, and Mishkin finds significant coefficients in regressions covering the cumulative change in rates one and two months.
Table II
ESTIMATES OF NON-STANDARD REGRESSIONS*

A. Cumulative Regressions: \( r_{t+n-1} - r_1 = a + b[f(n,n-1) - r_{t+1}] + u_{t+n-1} \)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( a )</th>
<th>( b )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{t+1} - r_1  )</td>
<td>-0.18</td>
<td>0.50</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+2} - r_1  )</td>
<td>-0.19</td>
<td>0.36</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+3} - r_1  )</td>
<td>-0.21</td>
<td>0.33</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+4} - r_1  )</td>
<td>-0.04</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+5} - r_1  )</td>
<td>0.03</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.14)</td>
<td></td>
</tr>
</tbody>
</table>

B. Marginal Regressions: \( r_{t+n-1} - r_{t+n-2} = a + b[f(n,n-1) - f(n-1,n-2)] + u_{t+n-1} \)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( a )</th>
<th>( b )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{t+1} - r_1  )</td>
<td>-0.18</td>
<td>0.50</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+2} - r_{t+1} )</td>
<td>-0.01</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+3} - r_{t+2} )</td>
<td>0.02</td>
<td>-0.07</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+4} - r_{t+3} )</td>
<td>-0.00</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>( r_{t+5} - r_{t+4} )</td>
<td>-0.03</td>
<td>0.62</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.34)</td>
<td></td>
</tr>
</tbody>
</table>

*Standard errors are in parentheses. Interest rates are continuously compounded annual rates in percentage points. "n" refers to maturity in months. Estimation period is 1952:1 to 1986:8.

in the future. As with the McCulloch data, however, the full sample marginal predictive power regressions reported by Fama and Mishkin have significant coefficients only in the regression for the change in rates one month ahead, \( r_{t+1} - r_1 \), confirming that virtually all of the forecasting power of the bill yield curve is in the shortest maturities.

Fama estimates subperiod regressions for 1959 to 1964, 1964 to 1969, and 1969 to 1982, and Mishkin reports regressions for these subperiods and also for 1982 to 1986. They find that in each subperiod the difference between the one-month ahead one-month forward rate and the current spot rate had forecasting power for the movement in the one-month rate over the following month. They also find that in some subperiods—notably those in the 1960s—the difference between the two-month ahead forward rate and the one-month ahead forward rate had significant forecasting power for the change in rates one to two months in the future.

Hardouvelis [1988] uses weekly data on Treasury bill rates from 1972 through 1985 to calculate two-week forward rates at one week intervals from one to twenty-four weeks in the future. Hardouvelis estimates coefficients for cumulative and marginal forecasting regression equations over three periods corresponding to three Federal Reserve policy regimes from 1972 through October 1979, October 1979 through October 1982, and October 1982 through November 1985. In the first period the yield
The Forecasting Power of the Yield Curve from One to Five Years

A final set of regression results that we briefly review relate to the forecasting power of the yield curve from one to five years. Fama and Bliss [1987] find that the yield curve from one to five years has had substantial forecasting power for the change in rates over the following three or four years. For example, they find that the difference between the forward rate on a one-year Treasury security four years in the future (calculated from the current four- and five-year rates) and the current one-year rate explains 48 percent of the variance of the 4-year change in the one-year rate. Table III reports these regressions using the McCulloch data. The results are generally similar to those reported by Fama, although the explanatory power of the four-year rate change regressions is smaller.

Campbell and Shiller [1989] use the McCulloch data to test a different specification of the expectations theory in which the current spread between an n-period maturity rate (such as a five-year rate) and a shorter m-period maturity (one-year) rate forecasts a weighted average change of the m-period rate over the next n − 1 periods (4 years). They regress the weighted average change of the m-period rate on the current spread and get results similar to those of Fama and Bliss. Specifically, they find that the spread between the 4-year and 1-year rates and the spread between the 5-year and 1-year rates have significant forecasting power for the weighted average change in the one-year rate over the next 3 or 4 years.

Table III

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>a</th>
<th>b</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>r(1:t+1) − r1</td>
<td>0.15</td>
<td>0.38</td>
<td>0.02</td>
</tr>
<tr>
<td>(0.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r(1:t+2) − r1</td>
<td>0.25</td>
<td>0.73</td>
<td>0.08</td>
</tr>
<tr>
<td>(0.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r(1:t+3) − r1</td>
<td>0.17</td>
<td>1.28</td>
<td>0.23</td>
</tr>
<tr>
<td>(0.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r(1:t+4) − r1</td>
<td>0.10</td>
<td>1.53</td>
<td>0.29</td>
</tr>
<tr>
<td>(0.51)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. Interest rates are continuously compounded annual rates in percentage points. "n" refers to maturity in years. Estimation period is 1952:1 to 1983:2.

III. Evidence of a Variable Term Premium

The studies surveyed in the previous section strongly reject the expectations theory, especially when the theory is tested with the standard regression using three- and six-month or six- and twelve-month rates. The rejection of the theory implies that either (1) the term premium is not constant, (2) the rational expectations assumption is not valid, or (3) both. We discuss evidence regarding the variable term premium in this section and evidence regarding the rational expectations assumption in the following section.

Most explanations of the lack of empirical support for the expectations theory have focused on the possibility that the expected term premium is not constant, as assumed by the theory, but varies substantially over time. If the term premium is variable, the estimate of b in equation (10) will differ from the value of one predicted by the expectations theory. A number of papers have discussed the determinants of the estimated coefficient and derived expressions for the probability limit of the coefficient when the variance of the term premium is positive. (See Hardouvelis [1988, pp. 342-343] and Mankiw and Miron [1986, pp. 218-220].) The derivation of one of these expressions is shown in Appendix II. One conclusion of these papers is that, generally, the greater the fraction of the variance in the spread between the forward and spot rates due to the variance in the expected term premium—and the smaller the fraction due to the variance of the...
expected change in rates—the lower will be the coefficient below the value of one predicted by the expectations theory.\textsuperscript{12} If the variance of the expected change in rates is equal to the variance of the expected term premium, then the estimate of the coefficient converges to one-half.

From this perspective the relevant questions are (1) does the expected term premium vary and (2) how much does it vary relative to the expected change in rates. Evidence from a variety of sources suggests that the expected term premium does vary substantially over time and, moreover, that the magnitude of the variance is comparable to the variance in the expected change in rates.

Evidence from Holding Period Premium Regressions

As discussed in Section I, an alternative and complementary way to estimate the standard regression is to make the dependent variable the holding period premium rather than the expected change in rates:

\[ h(6,3:t+3) - r_3 = a_1 + b_1[f(6,3) - r_3] + u:t+3 \]  \hspace{1cm} (14)

A value of \( b_1 \) greater than zero is evidence that the forward rate premium has had forecasting power for the excess return from holding six-month versus three-month securities over a three-month period. A value of \( b_1 \) equal to one would be evidence that virtually all variation in the yield curve is due to variation in expected returns. (This conclusion, of course, depends on the rational expectations assumption.)

Fama \cite{1986} estimates equation (14) using one- and three-month rates, three- and six-month rates, and six- and twelve-month rates. He reports values of \( b_1 \) that are close to one for bills and average a little over one-half for CDs and commercial paper. These results indicate that in the yield curve provides systematic information about expected excess returns. As Fama \cite{1984a, 512] emphasizes, this is evidence that the current slope of the money market yield curve is influenced by expected term premiums that change over time.

Evidence from Lower Bound Estimates

A few papers have tried to measure the variance of the term premium by estimating interest rate forecasting equations using data that was available to market participants at the time of their forecasts. Startz \cite{1982} regresses the current interest rate, \( r \), on lagged values of spot and forward rates. He then uses the standard error of this equation as a maximum estimate or “upper bound” of the standard deviation of the market’s forecast error, assuming that the set of variables used in the regression represents a minimum set of information available to market participants to forecast rates.

Startz then decomposes the spread between the forward rate and the subsequent matching spot rate (which he labels the “forward deviation”) into the expected term premium (\( P \)) and the forecast error (\( e \)):

\[ f-r:t+3 = (f-Er:t+3) + (Er:t+3-r:t+3) = P + e \] \hspace{1cm} (15)

The variance of \( f-r:t+3 \) is:

\[ \text{var}(f-r:t+3) = \text{var}(P) + \text{var}(e) + 2\text{cov}(P,e) \] \hspace{1cm} (16)

The covariance of \( P \) and \( e \) is zero under the rational expectations assumption, however, because \( P \) is known at the time of the forecast and should not be correlated with forecast errors. Hence,

\[ \text{var}(P) = \text{var}(f-r:t+3) - \text{var}(e) \] \hspace{1cm} (17)

From equation (17) we can see that if \( \text{var}(e) \)—the standard error of the regression squared—is an upper bound estimate for the true variance of the market’s forecast error, then \( \text{var}(f-r:t+3) - \text{var}(e) \) is a lower bound estimate of the true variance of the term premium.

Startz calculates lower bound estimates over the period from 1953 through 1971 of the proportion of the variance of the spread between the forward rate for a one-month bill and the subsequent matching spot rate that was due to variation in the term premium. These estimates range from one-third to two-thirds over horizons from one to twelve months. This conclusion implies that lower bound estimates of the ratio of the variance of the premium to the variance of the forecast error ranged from one-half to two. Of course, this is a lower bound estimate of the ratio of the variance of the premium to the variance of the forecast error, not to the variance of the expected change in rates. Nevertheless, these results suggest that the variation in the premium is substantial.\textsuperscript{13}

\textsuperscript{12} Specifically, these papers find that if the correlation coefficient between the expected change in rates and the expected term premium is zero or greater than zero, the probability limit of the estimated coefficient in equation (10) is a strictly increasing function of the ratio of the variance of the expected change in rates to the variance of the expected term premium.

\textsuperscript{13} Moreover, in the interest rate survey data discussed in the following section, the variance of the expected change in rates is less than the variance of interest errors, in which case one-half to two would also be a lower bound estimate for the ratio of the variance of the premium to the variance of the expected change in rates.
DeGennaro and Moser [1989] employ essentially the same procedure as Startz to calculate lower bound estimates over the period from 1970 through 1982 of the proportion of the variance of the spreads between the forward rates for four- and eight-week bills and the subsequent matching spot rates that was due to variation in the term premium. Their estimates range from one-fifth to three-fifths for horizons from one to 49 weeks.

IV.
EVIDENCE ON THE RATIONAL EXPECTATIONS ASSUMPTION

The previous section presented evidence that a variable term premium contributes to the rejection of the expectations hypothesis in the tests reported in Section II. The remaining question is whether violation of the rational expectations assumption also contributes to the regression results. A way to evaluate this question is to use survey data to get an estimate of the market's interest rate expectations. For instance, suppose ESr(3:t + 3) is the mean response from survey participants of the expected level of the three-month rate three months in the future. Then the coefficients of the standard equation can be estimated with the regression:

\[
ESr(3:t + 3) - r_3 = a + b'[f(6,3) - r_3] + u \quad (18)
\]

The variables in equation (18) are all measured at time t, the time of the survey. Consequently, the expected coefficient estimates do not depend on the rational expectations assumption. That is, if the term premium is constant, then the estimated coefficient of the forward rate premium in equation (18) should be close to 1 regardless of how expectations are formed.

A small number of studies, including Friedman [1979] and Froot [1989], have used the "Goldsmith-Nagan" survey data to estimate versions of equation (18). The survey data are based on a quarterly survey of 25 to 45 market participants on the interest rates they expect three and six months in the future. The survey was originally conducted by the Goldsmith-Nagan Bond and Money Market Letter and is now published in the newsletter "Washington Bond & Money Market Report. The survey collects forecasts of the three-month bill rate, the twelve-month bill rate, and a private sector three-month rate, along with forecasts of a number of long-term interest rates. Through the March 1978 survey the private rate was the three-month Eurodollar rate. Since then, the private rate has been the three-month commercial paper rate.

There is typically about a two-week period between the time the survey forms are mailed to the respondents and the latest market close prior to publication of the responses. The average timing of the latest close prior to publication is the end of the quarter, and in estimating equation (18) we matched the survey data with the end-of-quarter data on Treasury bill rates from McCulloch and the end-of-quarter data on Eurodollar and commercial paper rates from Salomon Brothers.\(^\text{14}\) We also used the six- and nine-month Treasury bill rates from the McCulloch data to calculate the six-month ahead forward rate for a three-month bill, f(9,6), and estimated the coefficients of an equation with the survey expected change in the three-month bill rate six months in the future as the dependent variable:

\[
ESr(3:t + 6) - r_3 = a + b'[f(9,6) - r_3] + u \quad (19)
\]

Equation (19) can not be estimated for private sector rates because Salomon Brothers does not have the nine-month rates needed to calculate f(9,6).

The top part of Table IV shows the regression results for equations (18) and (19) using the Goldsmith-Nagan survey data. The coefficients of the forward rate premium in these regressions are all positive and significant. The low Durbin-Watson statistics, however, suggest that serial correlation is a serious problem, and inspection of the regression residuals indicated that they fall sharply in recessions. Consequently, we reestimated the regressions with a dummy variable set equal to one for all the survey dates that occurred in recessions.\(^\text{15}\) The coefficients of the forward rate premium in these regressions range from 0.45 to 0.59 and are significant at the one percent level in the Treasury bill rate and

\(^{14}\) On average over the period covered by the survey regressions the latest market close prior to publication of the survey results falls on the last day of the quarter. The average absolute difference between the latest close and the last day of the quarter is four days. We know of no reason to expect that the differences between the timing of the survey and the timing of the calculation of the forward rate premium would bias the estimate of b in equations (18) and (19). Froot [1989, p. 285, footnote 9] experiments with data sets one week and two weeks before the end of the quarter and finds that the regression results are the same as with end-of-quarter data.

\(^{15}\) The dummy variable equals 1 from the fourth quarter of 1969 through the third quarter of 1970, the fourth quarter of 1973 through the fourth quarter of 1974, and the first quarter of 1980 through the third quarter of 1982. The latter period covers two recessions that are separated by three quarters.
Table IV
TEST OF THE EXPECTATIONS THEORY USING SURVEY DATA*

A. Dependent Variable: Survey Expected Change in Rates
\[ \text{ES}_r(m:t+n-m) - r_m = a + b[\text{f}(n,n-m) - r_m] + cD + u \]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>( R^2 )</th>
<th>DW</th>
<th>Estimation Period (quarter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury Bills</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>-0.33</td>
<td>0.44</td>
<td></td>
<td>0.19</td>
<td>0.43</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>-0.11</td>
<td>0.54</td>
<td>-0.96</td>
<td>0.70</td>
<td>1.24</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.11)</td>
<td>(0.10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+6) - r_3 )</td>
<td>-0.43</td>
<td>0.53</td>
<td></td>
<td>0.31</td>
<td>0.59</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+6) - r_3 )</td>
<td>-0.08</td>
<td>0.50</td>
<td>-1.09</td>
<td>0.68</td>
<td>1.27</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.08)</td>
<td>(0.15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eurodollars</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>-0.67</td>
<td>0.75</td>
<td></td>
<td>0.42</td>
<td>0.67</td>
<td>69:3-78:1</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>-0.37</td>
<td>0.45</td>
<td>-0.70</td>
<td>0.56</td>
<td>1.17</td>
<td>69:3-78:1</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.18)</td>
<td>(0.23)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial Paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>-0.12</td>
<td>0.90</td>
<td></td>
<td>0.35</td>
<td>1.35</td>
<td>78:2-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ES}_r(3:t+3) - r_3 )</td>
<td>0.14</td>
<td>0.59</td>
<td>-0.86</td>
<td>0.58</td>
<td>2.11</td>
<td>78:2-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.15)</td>
<td>(0.23)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. Interest rates are continuously compounded annual rates in percentage points. "n" and "m" refer to maturity in months. D is a dummy variable set equal to 1 from the fourth quarter of 1969 through the third quarter of 1970, the fourth quarter of 1973 through the fourth quarter of 1974, and the first quarter of 1980 through the third quarter of 1982.

B. Dependent Variable: Actual Change in Rates
\[ r(m:t+n-m) - r_m = a + b[\text{f}(n,n-m) - r_m] + u:t+m \]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>( R^2 )</th>
<th>DW</th>
<th>Estimation Period (quarter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury Bills</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r(3:t+3) - r_3 )</td>
<td>0.20</td>
<td>-0.35</td>
<td>0.02</td>
<td>2.56</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.42)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r(3:t+6) - r_3 )</td>
<td>-0.16</td>
<td>0.14</td>
<td>0.00</td>
<td>1.61</td>
<td>69:3-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eurodollars</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r(3:t+3) - r_3 )</td>
<td>-0.31</td>
<td>0.62</td>
<td>0.07</td>
<td>1.57</td>
<td>69:3-78:1</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td>(0.36)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial Paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r(3:t+3) - r_3 )</td>
<td>-0.04</td>
<td>0.23</td>
<td>0.00</td>
<td>2.65</td>
<td>78:2-86:2</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
<td>(0.82)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
commercial paper rate regressions. The coefficients of the dummy variable are all negative and significantly different from zero. Moreover, the Durbin-Watson statistics for these regressions rise sharply, although they still indicate some serial correlation in three of the four regressions. A plausible explanation for the significance of the dummy variable coefficient is that the term premium rises in recessions. We discuss this possibility in Section VI.

For comparison with the survey regression results, the bottom part of Table IV shows estimates of the regressions over the same sample period and the same quarterly observations but with the actual change in rates as the dependent variable. The negligible explanatory power of these regressions is in sharp contrast to the survey regressions.

We can derive an estimate of the term premium implied by the survey results by subtracting the expected change in rates from the forward rate premium at the time of the survey. This estimate can be used to calculate an estimate of the relative magnitude of the variances of the premium and the expected change in rates. These variances are summarized in Table V for both Treasury bills and private securities at the three-month forecast horizon. The ratio of the variance of the premium to the variance of the expected change in rates is 1.11 for bills and about 0.65 for private securities. These numbers appear roughly consistent with the evidence from the studies reviewed in Section III that used forecasting equations to generate lower bound estimates of the variance of the premium.

The survey regression results suggest that the rational expectations assumption used in the studies surveyed in Section II is not valid for the time period covered by the survey data. To see this, note that the actual change in the three-month rate used as the dependent variable in these studies can be decomposed into the expected change in the rate plus a forecast error. If the actual change in interest rates is uncorrelated with the forward rate premium—as indicated by the regression results reported in Section II—but the expected change is positively correlated with the forward rate premium, then the survey forecast error must be negatively correlated with the forward rate premium. This is a violation of the rational expectations assumption specified by equation (9).

As shown in Appendix II, a negative correlation between forecast errors and the forward rate premium reduces the coefficient of the forward rate premium in tests of the expectations theory (estimated with actual changes in rates) below the value of 1 predicted by the theory. Following Froot [1989, pp. 290-92], we can use the survey data to get estimates of the effects of the variable term premium and forecast errors on the coefficient of the forward rate premium. The probability limit of the coefficient of the forward rate premium can be written as one minus a deviation due to the variable term premium plus a deviation due to systematic expectational errors:

\[
b = 1.0 - \frac{\text{cov}(\theta, FP)}{\text{var}(FP)} + \frac{\text{cov}(e, FP)}{\text{var}(FP)}
\]

where FP refers to the forward rate premium, \(\theta\) refers to the term premium, and \(e\) refers to expectational errors. The survey data can be used to derive estimates of the terms on the right-hand side of equation (20). According to these estimates, shown in Table VI, roughly half the deviation from 1.0 of the

| Table V |
|-----------------|-----------------|
| **VARIANCE OF SURVEY EXPECTED CHANGE IN RATES AND SURVEY PREMIUM** |
| **Treasury Bills (69:3-86:2)** |
| Variance of Premium | 0.42 |
| Variance of Expected Change in Rates | 0.38 |
| Ratio | 1.11 |
| **Eurodollars (69:3-78:1)** |
| Variance of Premium | 0.29 |
| Variance of Expected Change in Rates | 0.46 |
| Ratio | 0.63 |
| **Commercial Paper (78:2-86:2)** |
| Variance of Premium | 0.41 |
| Variance of Expected Change in Rates | 0.62 |
| Ratio | 0.66 |
DECOMPOSITION OF THE COEFFICIENT OF THE FORWARD RATE PREMIUM IN TESTS OF THE EXPECTATIONS THEORY

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Forecast Horizon</th>
<th>Regression Coefficient</th>
<th>Component Attributable to Term Premium</th>
<th>Component Attributable to Forecast Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-Month Treasury Bill</td>
<td>3 Months</td>
<td>-0.35</td>
<td>0.56</td>
<td>-0.79</td>
</tr>
<tr>
<td>3-Month Treasury Bill</td>
<td>6 Months</td>
<td>0.14</td>
<td>0.47</td>
<td>-0.39</td>
</tr>
<tr>
<td>3-Month Eurodollar</td>
<td>3 Months</td>
<td>0.62</td>
<td>0.25</td>
<td>-0.13</td>
</tr>
<tr>
<td>3-Month Commercial Paper</td>
<td>3 Months</td>
<td>0.23</td>
<td>0.10</td>
<td>-0.67</td>
</tr>
</tbody>
</table>

* The construction of this table follows Froot (1989, p. 291, Table III). The regression coefficients are from Part B of Table IV. Columns (2) and (3) are calculated using the Goldsmith-Nagan survey data from the third quarter of 1969 through the second quarter of 1986. Column (1) equals 1.0 minus column (2) plus column (3).

The decomposition of the coefficient of the forward rate premium is due to a variable term premium and half results from the correlation of forecast errors with the forward rate premium.

The survey regression results suggest that market participants build their expectations into the yield curve, but their forecasts have been so poor at the three- and six-month horizons that the yield curve has had negligible forecasting power for the subsequent three-month and six-month rates. Of course, this interpretation depends critically on the assumption that the mean of the survey forecasts is an unbiased estimate of the market forecast, and that the survey expectations can be interpreted as determining the current slope of the yield curve. One can imagine circumstances under which this might not be true. For example, the forecasters used in the survey might be influenced by the current shape of the yield curve in determining their interest rate forecasts, in which case the regression results would be spurious. Or they might systematically differ in their forecasts from the market in general for other reasons, perhaps because their forecasts are made public or because they are not actively involved in buying and selling securities. We know of no evidence that either of these possibilities is true.

A final point to make here is that there is a distinction between the specialized form of the rational expectations hypothesis used in the literature surveyed in this article—indicated by equation (9)—and the general principle of rational expectations, which is that market participants use available information efficiently in forming their expectations. Webb (1987) discusses a number of reasons why rational market participants might not behave over a given time period according to the specialized form of the hypothesis. A general point is that it is difficult to say anything definite about whether market participants have formed expectations rationally without a clear understanding of the process determining interest rate movements.

17 The poor forecasting of market participants at the three- and six-month horizons is documented by Hafer and Hein (1989, p. 37, Table 1), who evaluate the forecasting power of both the Goldsmith-Nagan survey data and the Treasury bill futures market. They find that naive forecasts of no change in the three-month bill rate over the following three and six months do about as well as the changes forecast by the Goldsmith-Nagan survey or by the futures market. Similarly, Belongia (1987, p. 13, Table 1) finds that a forecast of no change in rates over six months does as well as the consensus forecast of a group of economists surveyed regularly by the Wall Street Journal.

18 Kane (1983, pp. 117-118) emphasizes that survey respondents should be decision-makers with the authority and willingness to commit funds in support of their forecasts. He finds (p. 119) that in his survey the response of "bosses" (i.e., decision-makers) differs from the response of non-bosses. Many of the respondents in the Goldsmith-Nagan survey are senior officials in their respective organizations and would seem to fit the label of "decision-maker." We are not aware, however, of any general classification of the Goldsmith-Nagan respondents along these lines.

To illustrate this point, consider the behavior of the Goldsmith-Nagan forecasts over the period from late 1979 through mid-1982. In reaction to rising inflation, the Federal Reserve at the beginning of the period unexpectedly raised short-term interest rates sharply and then kept them at an unusually high level for most of the following 2 1/2 years. The Fed’s policy over this period was generally not anticipated by market participants. As a result, following the initial increase in rates the survey participants forecasted large declines in rates for several quarters in a row. (See Chart 1 in the following section.) It seems reasonable that in this episode the expectations of market participants at the three- and six-month horizons would be influenced by their judgment that monetary policy actions had driven short-term rates to a level that could not be sustained. Moreover, the expectation that rates were going to fall sharply eventually proved correct. Yet ex post the expected declines in rates at the three- and six-month horizons over this period were accompanied by large positive forecast errors. This contributed to a negative correlation over the estimation period between the expected change in rates and forecast errors, but it does not seem accurate to say that market participants formed expectations irrationally over this period.

V. FEDERAL FUNDS RATE EXPECTATIONS AND THE BEHAVIOR OF INTEREST RATES

The regression results reported in Section II indicate that the slope of the yield curve from three to twelve months has had no forecasting power for three- and six-month rates. A puzzling aspect of the strong rejection of the expectations theory from this type of test is that it seems at odds with the standard view among money market participants that money market rates are largely determined by the current and expected levels of the shortest-term rate, the federal funds rate. A second puzzling aspect of the regression results is that the yield curve from one week to three months and from one year to five years has had forecasting power, even though the yield curve from three to twelve months has had no forecasting power. This section discusses possible explanations for these two puzzles.

Federal Funds Rate Expectations and the Mankiw-Miron Hypothesis

Market participants view the federal funds rate as the instrument used by the Federal Reserve to carry out its policy decisions. In forming expectations of the future level of the funds rate they attempt to identify Federal Reserve actions signaling changes in the funds rate target, and they attempt to forecast values of macroeconomic variables they believe influence the Fed’s decisions. Many studies over the past decade have found that Treasury bill rates respond to monetary policy announcements or actions that influence funds rate expectations. Similarly, many studies have found that bill rates respond to incoming news on variables—such as the money supply—that market participants believe are likely to influence policy actions. If money market rates are so sensitive to funds rate expectations, as these studies suggest, why do tests of the expectations theory using rates from three to twelve months fail so badly?

A possible answer focuses on the way market participants form expectations of the future behavior of the federal funds rate. Mankiw and Miron [1986, p. 225] suggest that at each point in time the Federal Reserve sets the short rate (i.e., the federal funds rate) at a level that it expects to maintain given the information affecting its policy decisions. They hypothesize that market participants understand this behavior and therefore expect changes in the short rate at any point in time to be zero: “Under this characterization of policy, while the Fed might change the short rate in response to new information, it always (rationally) expected to maintain the short rate at its current level.” If this view is correct, then the whole spectrum of money market rates would adjust up and down in response to changes in the funds rate targeted by the Fed, but the slope of the yield curve would be unchanged. Hence, expected changes in interest rates would be negligible, and the variance of expected changes in rates would be small. This expectations behavior coupled with a variable term premium could explain the regression results in Section II. The paradox according to this explanation is that tests of the expectations theory using three- and six-month rates provide little support for the theory, even though rates at these maturities are, in fact, responding strongly to funds rate expectations.

Mankiw and Miron provide support for this argument by testing the expectations theory using three- and six-month interest rates over the 25-year period prior to the founding of the Fed and over four periods...
since. They find virtually no support for the expectations theory in any of the latter periods. In the period prior to the founding of the Fed, however, they find that the yield curve from three to six months had substantial forecasting power and that the slope coefficient for this time period is only modestly below the value predicted by the expectations theory. Mankiw and Miron also present evidence that after the founding of the Fed there was a sharp deterioration in the ability of time series forecasting equations to forecast changes in interest rates three months in the future. In light of this evidence they conclude that the ability of market participants to forecast changes in short-term rates fell sharply after the founding of the Fed, resulting in a sharp rise in the ratio of the variance of the premium to the variance of the expected change in interest rates.

Cook and Hahn [1989, p. 345] catalogue the reactions of the three-month, six-month, and twelve-month Treasury bill rates to events changing federal funds rate expectations and find these reactions to be broadly consistent with the Mankiw-Miron hypothesis. These reactions are summarized in Table VII, which shows the estimated coefficients of regression equations of the form:

$$\Delta R T B_i = a + b\Delta X_j + u,$$  \hspace{1cm} (21)

Where $\Delta R T B_i$ is the change in the Treasury bill rate at maturity $i$ and $\Delta X_j$ is the change in a variable $j$ that influences the market’s funds rate expectations. The top of the table shows the reaction of bill rates to changes in the Federal funds rate target over the period from September 1974 to September 1979. The middle shows the reaction of bill rates to discount rate announcements with policy content in the 1973-1985 period (i.e., announcements indicating the discount rate is being changed for reasons other than to simply realign it with market rates). The bottom of the table shows the reaction of bill rates to announcements of unexpected changes in the money supply. Under the “policy anticipations hypothesis”—which is the most widely accepted explanation for this phenomenon—this reaction occurs because the Fed is expected to raise or lower the funds rate in response to deviations of money from its target path.

A striking aspect of the regression coefficients in Table VII is the relative stability from the three-month through the twelve-month maturities. This suggests that new information influencing expectations of the future level of the funds rate—even though it has a strong effect on bill rates at all maturities—has little effect on the slope of the Treasury yield curve from three to twelve months. In light of this evidence it seems plausible that the variance of the yield curve over this range has been dominated by movement in the term premium, as suggested by Mankiw and Miron.

**A More General Monetary Policy Explanation for the Regression Results**

While the Mankiw-Miron hypothesis can help explain the absence of forecasting power of the yield curve from three to twelve months, it is inconsistent with the evidence that the yield curve up to three months and from one to five years has had forecasting power. One can pose a more general version of the monetary policy explanation that is consistent with this evidence, and, we believe, more in line with the way market participants actually view monetary policy.

The Mankiw-Miron hypothesis assumes that the Fed reacts continuously to new information affecting its policy decisions, whereas in practice Fed policy changes are of a more discontinuous nature. That is, changes in the Fed’s target for the funds rate typically occur infrequently after they are triggered by the cumulative weight of new information on economic activity and inflation. Consequently, at times there is a gap between the release of new information influencing policy expectations and when policy actually changes. This information could take the form of a policy announcement—such as a discount rate announcement—which signals an upcoming change in the funds rate target. Or it could take the form of news on an economic variable—such as the money supply or employment—that is viewed by market participants as likely to influence the Fed’s target for the funds rate.

If policy and news announcements affect expectations of changes in the funds rate over a relatively short term, then the slope of the bill yield curve out to three months will vary more in response to changing interest rate expectations than will the slope from three to twelve months. In this case the reaction of market participants to such announcements could generate a pattern of funds rate expectations that is consistent with the regression results. For example,
suppose a discount rate announcement generates expectations of a 50 basis point change in the funds rate the following week, after which no further change in the rate is expected. Under the expectations theory the effect on the slope of the yield curve out to one or two months would be considerable, but the effect on the slope from three to six months and six to twelve months would be negligible.

Hegde and McDonald [1986] find that Treasury bill futures rates have substantially outperformed a no-change forecast from one to four weeks prior to delivery, even though they have not been superior to a no-change forecast from five to thirteen weeks prior to delivery. This evidence is consistent with the hypothesis that market participants are at times able to forecast rate changes over the near-term and build these expectations into the yield curve.

A second modification one could make to the Mankiw-Miron hypothesis notes that funds rate target changes are persistent (i.e., not quickly reversed) but not permanent. If so—and if market participants expected this type of funds rate behavior then increases in the funds rate target would be associated with decreases in the slope of the yield curve between short-term rates and rates on longer maturities of five to ten years, and changes in this slope would have some forecasting accuracy.

A number of recent papers have suggested that the forecasting power of the spread between long- and short-term rates is at least partially a reflection of monetary policy. (See Bernanke and Blinder [1989], Laurent [1989], and Stock and Watson [1990, pp. 25-26].) The basic reasoning is that monetary policy has a strong influence over short-term rates but that

23 Fama and Bliss [1987] find that the one year Treasury rate is highly autocorrelated but slowly mean-reverting, which is consistent with the view that changes in the funds rate are highly persistent but not permanent.

Table VII
THE REACTION OF TREASURY BILL RATES BY MATURITY TO EVENTS CHANGING FEDERAL FUNDS RATE EXPECTATIONS*
(Coefficients of Treasury Bill Rate Regressions)

<table>
<thead>
<tr>
<th></th>
<th>3-month</th>
<th>6-month</th>
<th>12-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal funds rate target changes:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 1974-Oct. 1979</td>
<td>0.554</td>
<td>0.541</td>
<td>0.500</td>
</tr>
<tr>
<td>(8.10)</td>
<td>(10.25)</td>
<td>(9.61)</td>
<td></td>
</tr>
<tr>
<td>Discount rate announcements:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 1973-Oct. 1979</td>
<td>0.26</td>
<td>0.32</td>
<td>0.30</td>
</tr>
<tr>
<td>(2.66)</td>
<td>(3.54)</td>
<td>(3.15)</td>
<td></td>
</tr>
<tr>
<td>Oct. 1979-Dec. 1985</td>
<td>0.73</td>
<td>0.61</td>
<td>0.59</td>
</tr>
<tr>
<td>(7.38)</td>
<td>(7.61)</td>
<td>(7.54)</td>
<td></td>
</tr>
<tr>
<td>Money announcements:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 1977-Oct. 1979</td>
<td>0.072</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>(3.11)</td>
<td>(4.73)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct. 1979-Oct. 1982</td>
<td>0.364</td>
<td>0.338</td>
<td></td>
</tr>
<tr>
<td>(6.58)</td>
<td>(7.59)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct. 1982-Sept. 1984</td>
<td>0.190</td>
<td>0.216</td>
<td></td>
</tr>
<tr>
<td>(5.77)</td>
<td>(5.62)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The funds rate target regression coefficients and the discount rate announcement regression coefficients are from Cook and Hahn [1988, 1989]. The money announcement regression coefficients are from Gavin and Karamouzis [1984].

t-statistics are in parentheses.
this influence diminishes at longer maturities. Hence, if short-term rates are high relative to long-term rates that is an indication that monetary policy is contractionary and a decline in inflation and interest rates is likely in the future.

This explanation for the forecasting power of the yield curve between long- and short-term rates seems especially relevant for the periods from late 1969 to late 1970, mid-1973 to mid-1974, and late 1979 to mid-1982. Near the beginning of each of these periods, the Fed raised the funds rate sharply because of its concern over accelerating inflation, and short-term rates rose well above long-term rates. (See Laurent [1989, Figure 2, p. 26].) In each period the rise in the funds rate and the downward-sloping yield curve were eventually followed by a recession and falling interest rates. As illustrated in Chart 1, the Goldsmith-Nagan survey participants expected large declines in the funds rate throughout these periods. These expectations had considerable accuracy at longer horizons of two to four years, even though they were not very accurate at the three- and six-month horizons.

If the above adjustments to the Mankiw-Miron hypothesis are correct, then one would expect to see the slope of the yield curve out to three months and the slope from one to five years vary more than the slope from three to twelve months in response to policy actions or announcements signaling changes in the funds rate. Numerous studies have provided evidence that the response of interest rates to monetary actions and announcements influencing policy expectations gradually declines at maturities greater than a year. For example, Cook and Hahn [1989] find that the reaction of Treasury rates to funds rate target changes falls from 0.50 at the one-year maturity to 0.29 at the five-year maturity and 0.13 at the ten-year maturity. Likewise, several papers including Hardouvelis [1984] and Gavin and Karamouzis [1984] have reported that the reaction of Treasury rates to money announcements declines at longer maturities.

The evidence at the short-end of the yield curve is more limited and somewhat more ambiguous. The reaction of interest rates to money announcements has been studied by many authors, but only a few have looked at the reaction of rates with shorter maturities than three months. These studies have found that the reaction of the one-month rate to money announcements is smaller than the reaction of longer-term money market rates, which is consistent with the notion that the yield curve out to three months varies more in response to changing policy anticipations than the curve from three months to a year. Husted and Kitchen [1985, p. 460] find that the reaction of Eurodollar rates to announcements of unexpected increases in the money supply—as determined by the coefficients of a regression similar to equation (21) above—rose from 0.28 at the one-month maturity to 0.46 at the three-month maturity and 0.44 at the six-month maturity. Hardouvelis [1984] finds that the reaction of the one-month bill rate (0.24) was smaller than the reaction of the one- to two-month forward rate (0.45), the
two- to three-month forward rate (0.40) and the three- to six-month forward rate (0.35). (The sample period for the studies cited in this and the following paragraph is from late 1979 or early 1980 to late 1982.)

Surprisingly, however, the money announcement literature indicates that the reaction of the one-day funds rate and the one-week bill rate to money announcements is not smaller than the reaction of longer-term money market rates. Hardouvelis finds a coefficient of 0.38 for the one-day funds rate, and Roley and Walsh [1985] find a coefficient of 0.43 for the one-day funds rate, 0.37 for the one-week bill rate, and 0.36 for the three-month bill rate. A possible explanation for the relatively large response of the one-day and one-week rates is that under the lagged reserve accounting system prevailing prior to February 1984 the weekly money announcement provided information on the current statement week's aggregate demand for reserves that influenced the expected average funds rate for the statement week—and, hence, the one-week bill rate— independent of any policy anticipations effect. 28

VI. BEHAVIOR OF THE TERM PREMIUM

The evidence presented in Section III suggests that a variable term premium plays an important role in explaining the negligible forecasting power of the yield curve from three to twelve months. This conclusion raises a final set of questions. First, how does the term premium behave on average and at different maturities? Second, what causes the term premium to change over time? The literature in this area—especially regarding the second question—is voluminous, yet largely inconclusive. Our purpose here is simply to provide a brief review of this literature and the difficulties researchers have faced in trying to measure the term premium.

The Average Term Premium in the Money Market

Researchers have generally estimated the average term premium in the money market by calculating over long periods of time the average excess returns from holding n-month securities for m months versus the return from holding m-month securities. The most common practice is to use the one-month rate for the benchmark (m = 1). The literature in this area has found a positive average term premium in the Treasury bill market at all maturities. The average term premium rises sharply for the first couple of months, increases at a decreasing rate out to around five or six months, and then flattens out. This behavior is illustrated in Chart 2 which shows estimates of the average premium at different maturities using the McCulloch data. 29 McCulloch [1987] shows that the CRSP data provide a similar picture of the relationship between the average premium and maturity in the bill market. 30 Researchers have found no evidence of a significant term premium, on average, in the markets for private money market securities such as commercial paper, CDs, and Eurodollar CDs. Fama [1986, p. 178, Table 11, for example, finds that average term

28 Along these lines, Strongin and Tarhan [1990, pp. 151-152] conclude that "The response of the Fed funds rate [to money announcements] cannot be explained by either [policy anticipations or expected inflation], but instead by the peculiar way money shocks are transmitted to the reserve market under lagged reserves accounting."
premiums for privately issued securities over his whole sample period from January 1967 to January 1985 are close to zero. Fama, however, also divides his sample into months when the yield curve was monotonically upward sloping and months when the yield curve was "humped" (i.e. initially rising and then falling). He finds that in months when the private yield curve was upward sloping, the average term premium was positive and rose with maturity, and in months when the private yield curve was humped, the average term premium initially was positive but then became negative at the longer maturities.

One type of explanation for the positive average premium in the bill market focuses on the preferences of individual investors. Hicks [1946, pp. 144-52] argues that investors have a preference for shorter-term securities because of the greater price volatility of long-term securities when interest rates change. In contrast, he reasons that many borrowers have a preference for long-term borrowing. Hence, there is a "constitutional weakness" on the long side of the market such that in equilibrium investors have to be offered a premium to invest in longer-term securities. In a similar vein, Kessel [1965, p. 45] argues that the market has a preference for shorter-term securities because of their greater liquidity: "The shorter the term to maturity of a security, the smaller is its vulnerability to capital loss, and hence the greater its liquidity and the smaller the yield differential between that security and money."31

More recent papers have analyzed the term premium in the context of individuals who maximize the expected utility of their lifetime consumption.32 An idea that comes out of this literature is that the term premium is likely to be positive if unexpected capital losses (i.e. positive future interest rate surprises) are generally positively correlated with negative consumption surprises. In other words, investors are likely to demand a higher yield on long-term securities if they are likely to experience unexpected capital losses when times are unexpectedly bad and their marginal utility of consumption is relatively high.

A second explanation for the positive average premium in the bill market, suggested by Rowe, Lawler, and Cook [1986, pp. 9-10] and Toevs and Mond [1988], focuses on the unique characteristics of the market. Treasury bills can be used to satisfy numerous institutional and regulatory requirements, such as serving as collateral for tax and loan accounts at commercial banks and satisfying margin requirements on futures contracts. To the extent that the holding period for these purposes tends to be short, investors might prefer to minimize capital risk by holding short-term bills to satisfy them. Moreover, the Treasury is not sensitive to interest rates at different maturities in its supply of bills, so there is no pressure from the supply side to equalize the expected cost of issuing bills at different maturities. In contrast, banks might be expected to issue more three-month CDs if the expected cost of raising funds this way were systematically lower than the cost of raising funds by issuing six-month CDs, and this behavior would raise the three-month rate relative to the six-month rate and reduce the premium.

Measuring the Behavior of the Term Premium over Time

A number of approaches have been taken in the literature to measure the behavior of the term premium over time. The simplest approach is to assume that the forward rate premium is an accurate representation of the term premium. Suppose the expected change in rates at any point in time is negligible so that the forward rate premium is completely dominated by variation in the term premium. Then, as Fama [1986, p. 187] suggests, the forward rate premium can "provide a direct picture of the behavior of the expected term...premium." As discussed earlier, however, the Goldsmith-Nagan survey data suggest that at times market participants have expected large changes in rates. If so, then in these periods the forward rate premium provides an inaccurate picture of the term premium.

A second approach to measuring the term premium is to subtract the expected interest rate level from the Goldsmith-Nagan survey from the comparable forward rate at the time of the survey. Chart 3 shows (a) the difference between the forward rate on three-month bills three months ahead and the expected three-month bill rate three months ahead and (b) the difference between the forward rate on three-month bills six months ahead and the expected three-month bill rate six months ahead.33 The chart shows a clear

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31 There was a huge amount of literature on the expectations theory and the term premium in the 1960s and early 1970s. For a review of this literature see Van Horne [1984, Chapter 4] and Malkiel [1970].

32 For an example of this approach see Sargent [1987, pp. 102-105]. Abken [1990] discusses this literature.

tendency for the survey term premiums to be relatively high in recessions and low in expansions. (This tendency was captured in the regression results reported in Section IV by the significant negative coefficient of the recession dummy variable.)

Chart 4 shows that the survey term premium and the forward premium generally move together, but large differences occasionally occur when the survey indicates large expected declines in rates. The most striking difference in the two estimates of the term premium is in the period from late 1979 through mid-1982 when interest rates were unusually high and were expected to fall by the survey participants. In this situation the survey term premium rose well above the forward rate premium. Chart 5 compares the survey premiums for bills and private money market securities. The private premium generally follows the same pattern as the bill premium—rising in recessions and falling in expansions—although occasionally there are significant differences in the two premiums.

A third estimate of the term premium is the forward deviation, i.e. the difference between the forward rate and the subsequently realized spot rate. As discussed in Section III, the forward deviation can be decomposed into an expected term premium and an interest rate forecast error. Both the Goldsmith-Nagan survey data and futures market data indicate that market participants have had little ability to forecast rates at the three- and six-month horizons. As a result, the forward deviation is an extremely volatile measure dominated by interest rate forecast errors.

A final approach used to estimate the term premium is to employ regression methods to generate “expected” interest rates with data available to market participants at the time of the forecast. These

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estimates can then be used along with contemporaneous forward rates to calculate estimates of the term premium. Since these forecasting equations have little power to predict changes in interest rates, one might expect this approach to provide estimates of the term premium that are similar to the forward rate premium. We are not aware, however, of any studies that have made this comparison.

Determinants of the Variable Term Premium

The estimates of the term premium shown in Charts 3-5 suggest that term premiums in the money market tend to be low in periods of economic expansions and high in periods of weakness. This is consistent with a recent conclusion by Fama and French [1989, p. 43] that term premiums “move opposite to business conditions.” This is not a universally accepted description of the behavior of term premiums, however. Numerous variables are correlated with economic conditions, and the charts might be capturing a correlation of the premium with some other variable such as the level of interest rates. Moreover, even if one accepts the description that term premiums move opposite to business conditions as accurate, there is still no generally accepted explanation for why term premiums rise around recessions and fall in expansions.

Numerous papers have attempted to make judgments about the determinants of the term premium by regressing one of the measures of the premium described above on various possible explanatory variables. Two explanatory variables often included in these regressions are the volatility of interest rates and the level of interest rates. Hicks [1946] reasons that the term premium is compensation for the capital risk resulting from interest rate movements and, therefore, increases in interest rate volatility should increase the premium demanded by investors. The argument that the level of rates should be a determinant of the term premium is generally associated with Kessel [1965, pp. 25-26]. He argues that short-term bills are better money substitutes than long-term bills, and since an increase in interest rates increases the cost of holding money, the yield on short-term bills should fall relative to the yield on long-term bills when rates rise.

Papers that find the volatility of interest rates to be a significant determinant of the premium include Fama [1976], Heuson [1988], and Lauterbach [1989]. Papers that find the level of rates to be a determinant of the premium include Kessel [1965], Pesando [1975], and Friedman [1979]. Other explanatory variables that have been used in studies of the premium include the relative supplies of securities at different maturities, the unemployment rate, industrial production, and the spread between yields on high- and low-risk securities. As Shiller [1987, pp. 56-57] concludes in his survey article on the term structure of interest rates: “It is difficult to produce a useful summary of [the] conflicting results” from the empirical studies of the term premium. The main conclusion is that no consensus has emerged in the literature on what macroeconomic variable the term premium is most closely related to or on why the term premium varies so much.

VII. CONCLUSION

The studies surveyed in this paper find that over long periods of time the yield curve from three to twelve months has had negligible power to forecast interest rates three and six months in the future. The

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For example, on the basis of the Goldsmith-Nagan survey data and a chart similar to Chart 3, Froot [1989, p. 299, Figure 1] concludes that the surveys suggest that term premia rose substantially during periods of high interest rate volatility [p. 300]. He also concludes that the survey premia are highly positively correlated with nominal interest rates and inflation [p. 303]. Friedman [1979, p. 972] on the basis of regressions using the Goldsmith-Nagan data from September 1969 through March 1977 concludes that the results make clear that the basic relation is between the term premium and interest rate levels, not economic activity.

For example Friedman [1979] uses the premium calculated from the survey data as the dependent variable in his regressions, Kessel [1965] uses the forward deviation in his regressions, and Pesando [1975] estimates interest rate forecasting equations to calculate an estimate of the premium to use as the dependent variable in his regressions.
yield curve out to three months has had forecasting power for the one-month ahead rate, however, and the yield curve from one to five years has had forecasting power for the one-year rate over the following three or four years.

The research in this area has suggested two broad reasons for the poor forecasting power of the yield curve from three to twelve months. The first is that the variation in the term premium at the three- and six-month horizons has been substantial relative to the variation in the expected change in rates. The second is that even when market participants have forecasted significant changes in interest rates at the three- and six-month horizons, their forecasts have been poor at these horizons.

An understanding of how market participants form monetary policy expectations may provide insight into some of the results in this literature. A monetary policy explanation for the poor forecasting power of the yield curve from three to twelve months is that market participants expect changes in the Fed’s federal funds rate target to be persistent. According to this explanation, three-, six-, and twelve-month rates tend to move the same amount in reaction to changes in the funds rate target and, therefore, changes in the slope of the yield curve over this range are dominated by movement in the term premium. The forecasting power of the yield curve out to three months may reflect the ability of market participants to forecast over short horizons the reaction of the Fed to new information influencing its policy decisions. And the forecasting power of the yield curve from one to five years may partially reflect the belief of market participants that over longer periods of time changes in the funds rate target are likely to be reversed, especially after the Fed has raised the funds rate sharply in reaction to rising inflation.

The evidence cited in this paper in favor of a monetary policy explanation for the regression results is limited, however, and the explanation has not been universally, or even widely, accepted. There is also no general agreement on why the term premium varies so much, although the Goldsmith-Nagan survey data strongly suggest that the premium rises when economic conditions deteriorate. A brief assessment of the literature surveyed in this paper is that it has done a good job of documenting the forecasting power of various parts of the yield curve, and it has suggested some plausible and interesting answers to some of the major questions in this area. A comprehensive explanation for these questions, however, awaits further research.

APPENDIX I
INTEREST RATE CONVERSIONS

All interest rates in the paper are continuously compounded annual rates. No conversion is necessary for the McCulloch Treasury bill rate data, which come in this form. Three-month Treasury bill rate forecasts from the Goldsmith-Nagan survey are on a 360-day discount basis, however, as are all commercial paper rates used in the paper. Eurodollar, CD, and federal funds rates are quoted on a simple interest 360-day basis. Prices per $1 of return are calculated from the quoted yields, Q, using the formulas:

\[ P = 1 - \frac{(Q/100)}{(t/360)} \]  

(1)

for bills and commercial paper and

\[ P = \frac{1}{\left[\frac{(Q/100)}{(t/360)} + 1\right]} \]  

(2)

for Eurodollars, CDs, and federal funds rates. “t” is the days from settlement to maturity: 30, 90, and 180 days for commercial paper, CDs, and Eurodollars; 91 days for Treasury bills; and 1 day for federal funds. Prices are converted to continuously compounded yields using the formula:

\[ r = -(365/t)\ln P \]  

(3)

where \(\ln P\) is the natural logarithm of \(P\).
Appendix II
The Coefficient of the Forward Rate Premium
in the Standard Regression

The standard regression equation is:

\[ r(3: t + 3) - r_3 = a + b[(6, 3) - r_3] + u:t + 3 \]  \hspace{1cm} (1)

To simplify the notation rewrite this as:

\[ \Delta r = a + b(FP) + u:t + 3 \]  \hspace{1cm} (2)

where \( \Delta r \) is the rate change and \( FP \) is the forward rate premium. Recall also that the forward rate premium can be decomposed into the expected rate change and the expected term premium, \( \theta \), and the actual change in the interest rate can be decomposed into the expected change and a forecast error, \( e \):

\[ FP = E(\Delta r) + \theta \]  \hspace{1cm} (3)

\[ \Delta r = E(\Delta r) + e \]  \hspace{1cm} (4)

The probability limit (abbreviated as plt) of the ordinary least squares estimate of \( b \) in equation (2) is:

\[ \text{plt } b = \frac{\text{cov}(FP, \Delta r)}{\text{var}(FP)} \]  \hspace{1cm} (5)

Substituting (3) and (4) into (5) yields:

\[ \text{plt } b = \frac{\text{cov}[E(\Delta r) + \theta, E(\Delta r) + e]}{\text{var}[E(\Delta r) + \theta]} \]

\[ = \frac{\text{cov}[E(\Delta r) + \theta, E(\Delta r)]}{\text{var}[E(\Delta r) + \theta]} \]

\[ + \frac{\text{cov}[E(\Delta r) + \theta, e]}{\text{var}[E(\Delta r) + \theta]} \]  \hspace{1cm} (6)

Suppose the rational expectations assumption is valid. Then the forecast error, \( e \), is not correlated with information available at the time of the forecast, which includes the expected change in rates and the expected premium. Then the second term on the right-hand side of equation (6) equals 0 and we get the expression:

\[ \text{plt } b = \frac{\text{cov}[E(\Delta r) + \theta, E(\Delta r)]}{\text{var}[E(\Delta r) + \theta]} \]  \hspace{1cm} (7)

Denote the variance of \( x \) as \( \sigma^2(x) \), the standard deviation as \( \sigma(x) \), and the correlation coefficient between \( x \) and \( y \) as \( \rho \). Recall that \( \text{cov}(x, y) = \rho \sigma(x) \sigma(y) \). Then equation (7) can be written:

\[ \text{plt } b = \frac{\sigma^2[E(\Delta r)] + \text{cov}[\theta, E(\Delta r)]}{\sigma^2[E(\Delta r)] + \sigma^2(\theta) + 2\text{cov}[E(\Delta r), \theta]} \]

\[ = \frac{\sigma^2[E(\Delta r)] + \rho \sigma(\theta) \sigma[E(\Delta r)]}{\sigma^2[E(\Delta r)] + \sigma^2(\theta) + 2\rho \sigma[E(\Delta r)] \sigma(\theta)} \]  \hspace{1cm} (8)

This is the expression in Hardouvelis [1988, p. 342]. It is also similar to the expression in Mankiw and Miron [1986, p. 219], except that the term premium in their framework is equal to one-half the premium above. Note that the probability limit of \( b \) is one if the premium is a constant and one-half if the standard deviation of the term premium equals the standard deviation of the expected change in rates.

Now substitute equation (4) into (5) to get:

\[ \text{plt } b = \frac{\text{cov}(FP, E(\Delta r) + e)}{\text{var}(FP)} \]

\[ = \frac{\text{cov}(FP, E(\Delta r)) + \text{cov}(FP, e)}{\text{var}(FP)} \]  \hspace{1cm} (9)

Substituting (3) into (9) yields:

\[ \text{plt } b = \frac{\text{cov}(FP, FP - \theta) + \text{cov}(FP, e)}{\text{var}(FP)} \]

\[ = \frac{\text{var}(FP) - \text{cov}(FP, \theta) + \text{cov}(FP, e)}{\text{var}(FP)} \]

\[ = 1 - \frac{\text{cov}(FP, \theta)}{\text{var}(FP)} + \frac{\text{cov}(FP, e)}{\text{var}(FP)} \]  \hspace{1cm} (10)

Equation (10) says that a positive correlation of the term premium with the forward rate premium or a negative correlation of forecast errors with the forward rate premium will reduce the coefficient of the forward rate premium below the value of one predicted by the expectations theory.
References


