CENTRAL BANKING UNDER THE GOLD STANDARD

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1. INTRODUCTION

This paper is intended as a positive analysis of temporary government policy actions under a gold standard. To understand a gold standard is to understand the private valuation of money and gold as assets, and how their asset values can be influenced by government money and gold policy actions under a fixed money price of gold. An intertemporal, rational expectations, asset-pricing model is employed to address these issues.

The model approaches gold standard analysis by viewing the central bank as choosing policy rules for both money and gold. The procedure is then to see how a fixed money price of gold restricts these rules. Beyond that, it is to ask how an additional central bank objective, such as managing its gold reserve, smoothing interest rates, or smoothing the price level further restricts money and gold policies. The goal is to understand the feasibility and mechanics of various activist policies under a gold standard.

Regardless of the central bank's objective, the short-term interest rate was viewed as the instrument by which the gold standard was managed. A second goal of the analysis is to make use of the explicit determination of rates of return in the asset-pricing framework to understand the effectiveness of a central bank interest rate policy instrument under a gold standard.

A third goal is to use the formal model to discuss issues in gold standard history. Broadly speaking, the paper aims to provide a framework to better interpret historical practices. The model is put to use here to compare a variety of private and public gold standards, to comment on Bank Rate policy, to formally analyze lender of last resort policy, and to explain the emergence of interest rate smoothing in 1914.
The basic asset-pricing model is specified and solved in Section 2. Pricing functions for capital, money, and gold are derived in 2.2. Rate of return functions for the consumption good returns on capital and nominal bonds, and the nominal rate on bonds, are derived in 2.3. The influence of monetary policy on the risk differential between returns on capital and bonds is discussed in 2.4.

Policy analysis under a gold standard is undertaken in Section 3. Restrictions on money and gold policy rules implied by a fixed dollar price of gold are derived in 3.1. Section 3.2 contains an analysis of a classical gold standard in which the central bank follows a money for gold price rule and maintains a fixed gold reserve ratio. Section 3.3 describes the management of a gold standard with an interest rate policy instrument. It illustrates the use of an interest rate instrument to maintain a desired gold reserve ratio, to smooth interest rates, and to smooth the price level.

The formal model is put to use in Section 4 to discuss gold standard history. In 4.1 the model is used to compare a variety of private and public gold standards. The controversy over the feasibility and effectiveness of the Bank of England's interest rate instrument, i.e., the Bank Rate, is addressed in 4.2. In 4.3 lender of last resort policy under a gold standard is interpreted as an irregular interest rate smoothing policy. Finally, the smoothing of interest rates that emerged around 1914 is interpreted and explained in terms of the gold standard model presented in Section 3.
2. THE ASSET-PRICING MODEL

The simple asset-pricing model presented in this section will serve as the basis for discussion of the gold standard in Sections 3 and 4 below. There are four assets. The first is a fixed stock of capital which yields a dividend of nonstorable consumption goods each period. Second, there is a stock of government produced paper money which yields transaction services. Third, the economy has a fixed stock of gold which yields direct satisfaction to its holders in the form of jewelry, ornamentation, safe assets, etc. Fourth, consumers can trade one-period nominal bonds which pay a riskless nominal rate of return. In effect, both government paper money and gold are modelled as consumer durables valued for the direct utility-yielding services they produce. Neither capital or gold depreciate or are producible in the model. Such simplifications will not be misleading if the model is employed as intended, to investigate temporary private and government responses to temporary disturbances.

2.1 MODEL SPECIFICATION AND PRELIMINARY SOLUTION

An infinitely-lived representative consumer maximizes expected utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + v(p_t M_{t+1}) + w(G_{t+1}) \right] \]

subject to a sequence of budget constraints

\[ c_t + p_t M_{t+1} + q_t G_{t+1} + z_t K_{t+1} + p_t B_{t+1} / R_t \leq p_t M_t + q_t G_t + (z_t + d_t) K_{t+1} + p_t B_t \]

where \( \beta = 1/(1+\delta) \)
\( \delta \) = the rate of time preference; \( \delta > 0 \)

\( c_t \) = consumption in period \( t \)

\( M_{t+1} \) = nominal money balances carried from period \( t \) to \( t+1 \), yielding transaction services in period \( t \).

\( G_{t+1} \) = gold carried from period \( t \) to \( t+1 \), yielding utility in period \( t \).

\( K_{t+1} \) = capital goods carried from period \( t \) to \( t+1 \), yielding output in the form of nonstorable consumption goods in period \( t+1 \).

\( d_{t+1} \) = units of nonstorable consumption goods yielded in period \( t+1 \) per unit of capital carried from period \( t \) to \( t+1 \), \( d_{t+1} > 0 \).

\( B_{t+1} \) = one-period nominally-denominated bonds carried from period \( t \) to \( t+1 \), yielding a riskless nominal return in period \( t+1 \).

\( R_t \) = the gross riskless nominal return paid in period \( t+1 \) per dollar of nominal bonds carried from period \( t \) to \( t+1 \).

\( p_t \) = the consumption price of a dollar of nominal money in period \( t \), i.e., the inverse of the price level.

\( q_t \) = the consumption price of gold in period \( t \).

\( z_t \) = the consumption price of capital in period \( t \).

The consumer maximizes (1) subject to (2) by choosing sequences for \( c_t, K_t, M_t, G_t, B_t \), and a Lagrangian multiplier \( \lambda_t \) (associated with the budget constraint) for \( t=0,1,2, \ldots \), given initial conditions \( M_0, G_0, K_0, B_0 \), and prices \( p_t, q_t, z_t, \) and \( R_t \), for \( t=0,1,2, \ldots \).

The first-order conditions necessary for optimality of the consumer's choices include the following

\[
\begin{align*}
(3) & \quad u'(c_t) = \lambda_t \\
(4) & \quad -\lambda_t z_t + \beta E_t[\lambda_{t+1}(z_{t+1} + d_{t+1})] = 0
\end{align*}
\]
(5) \( v'(p_t M_{t+1}) p_t - \lambda_t p_t + \beta E[\lambda_{t+1} p_{t+1}] = 0 \)

(6) \( w'(G_{t+1}) - \lambda_t q_t + \beta E[\lambda_{t+1} q_{t+1}] = 0 \)

(7) \(-\lambda_t p_t / R_t + \beta E[\lambda_{t+1} p_{t+1}] = 0 \)

where \( E[\cdot] \) is an expectation conditional on complete contemporaneous information.

Conditions (3) through (7), together with budget constraint (2), determine current and planned values of \( c_t, M_t, G_t, B_t \) and \( \lambda_t \), for given initial conditions and price paths which are exogenous to the consumer. There is also a relevant transversality condition, but it can be ignored for the issues at hand.

This completes the preliminary specification and solution of the demand side of the model. Before we can solve for asset prices, however, we must specify the disturbance generating processes and the supply side of the model, including government money and gold policy rules.

We begin by specifying the utility functions in (1) as

(8a) \( u(c_t) = u_t \log c_t \quad u_t = u(1+\mu_t) > 0 \)

(8b) \( v(p_t M_t M_{t+1}) = v_t \log(p_t M_{t+1}) \quad v_t = v(1+\nu_t) > 0 \)

(8c) \( w(G_{t+1}) = w_t \log G_{t+1} \quad w_t = w(1+\omega_t) > 0 \)

where \( \mu_t \) = consumption preference shock

\( \nu_t \) = transaction service shock

\( \omega_t \) = gold preference shock

\( \mu_t, \nu_t, \) and \( \omega_t \) are generated by iid white-noise processes.
Capital good productivity shocks are represented by \( d_t \), which is assumed to be a serially uncorrelated nonnegative random variable with mean \( d \). The goods productivity shock \( d_t \) is independent of preference shocks.

Assume that the capital stock is fixed so that the representative consumer has one unit of capital, i.e., \( K_t = 1 \). Likewise the aggregate stock of gold is fixed. The fixed per capita stock of gold is \( G^S \). On a per capita basis, the stock of gold held by the government, i.e., the government stockpile of gold, is \( G^G \). Formally, the gold resource constraint is

\[
G_t = G^S - G^G_t.
\]

From (9), we see that the stock of gold available in utility-yielding form \( G \) varies inversely with the government's gold stockpile. The government may follow a gold stockpiling policy in this model with a rule by which it systematically buys or sells gold as a function of the state of the economy. Gold is assumed to be costlessly transformable between utility-yielding states and a form suitable for the government stockpile. I define pure gold policy as purchases of gold financed entirely by bond sales. I assume that interest on the resulting government bonds is financed with lump-sum taxes. Likewise, funds acquired from pure gold policy actions that reduce the government's gold stockpile are used to retire existing government debt or to make new government loans. In the first case, the reduced interest expense is used to reduce lump-sum taxes; in the second, the increased revenue is paid out in lump-sum transfers.

Pure monetary policy is defined analogously. The government follows a monetary policy by specifying a rule to systematically create or
destroy paper money according to the state of the economy. I define pure monetary policy as the creation or destruction of money financed entirely by purchases or sales of bonds as the case may be. Likewise, I assume the resulting interest flows to be financed by lump-sum taxes or distributed with lump-sum transfers as required.

Since we are interested in analyzing government policy actions in response to temporary disturbances, the shock processes in the model are serially uncorrelated. Therefore, we can also specify the government monetary and gold stockpiling actions as serially uncorrelated. Formally, I write the pure monetary policy and pure gold stockpiling policy rules respectively as

\[
(10a) \quad M_{t+1} = M/(1 - \epsilon_{t+1}) , \quad \epsilon_{t+1} < 1 \\
(10b) \quad G_{t+1} = G/(1 - \eta_{t+1}) , \quad \eta_{t+1} < 1
\]

where \(\epsilon_{t+1}\) and \(\eta_{t+1}\) are generated by white-noise processes reflecting money stock and gold stockpiling actions taken by the government. For convenience the gold stockpiling policy rule is written in terms of its effect on the gold stock in utility-yielding form. The effectiveness of government gold stockpiling policy in this model stems from the fact that gold in the government stockpile yields less utility than the gold in private hands. Gold in the government stockpile has been assumed to yield zero utility for convenience only.

2.2 ASSET PRICING FUNCTIONS

Having described preferences, productive technology, policy rules, and disturbance processes in the previous section, we are now in a position
to solve for asset prices. To do so, we use the first-order conditions for the representative consumer's optimal consumption and asset demands derived above. We impose goods and asset market clearing conditions. And we solve for the goods price of capital \(z\), the goods price of money \(p\), and the goods price of gold \(q\) that achieve aggregate goods and asset market equilibrium.

### 2.2.1 The Price of Capital

Combining (3) and (4) yields the standard Euler equation for consumption

\[ \beta E\left[ \frac{u'(c_{t+1})}{u'(c_t)} \right] r_t = 1 \]  \(\text{(11)}\)

where \(r_t = \frac{z_{t+1} + d_{t+1}}{z_t} \) is the gross consumption return on capital \((K)\) carried from period \(t\) to \(t+1\).

Because \(u(\cdot)\) is strictly increasing, output is nonstorable, and the only source of period \(t\) consumption goods is \(d_t\), goods market equilibrium requires \(c_t = d_t\). Substituting in (11) and rearranging yields

\[ z_t = \beta E \frac{u'(d_{t+1})}{u'(d_t)} (z_{t+1} + d_{t+1}) \]  \(\text{(12)}\)

A solution for (12) may be found by using recursive substitution and the law of iterated expectations as

\[ z_t = \beta E \sum_{j=1}^\infty \beta^j \frac{u'(d_{t+j})}{u'(d_t)} d_{t+j} \]  \(\text{(13)}\)

The price of consumption-good producing capital in (13) is an expected discounted stream of dividends with time-varying and stochastic
discount rates. Expression (13), however, is not a pricing function because it does not express $z_t$ in terms of information available in period $t$. We can derive a simple pricing function for log utility specified in (8a) as

$$z_t = u_d \frac{d}{1 - \beta} u_t$$

The price of capital in terms of consumption goods is stochastic because consumption preferences and goods productivity are stochastic. For example, temporarily low goods productivity lowers the current consumption good price of capital. Why? Below normal output raises the current marginal utility of consumption above normal, encouraging individuals to attempt to sell capital to shift future consumption into the present. In aggregate, however, the economy cannot change the quantity of output available for current consumption. Equilibrium is established by a fall in the current consumption good price of capital sufficient to raise its expected real return enough to bring desired saving back to zero.

2.2.2 The Price of Money

Combining (3) and (5) yields the Euler equation for money

$$P_t = \beta \mathbb{E} \left[ \frac{u'(c_{t+1})}{u'(c_t)} P_{t+1} \right] + \nu (p_t M_{t+1}) \frac{P_t}{u'(c_t)}$$

We can derive a simple pricing function for money in two steps. First, rewrite (15) using the log utility specifications in (8) and the goods market clearing condition $c_t = d_t$ to yield

$$P_t = \beta \mathbb{E} \left[ \frac{u'(c_{t+1})}{u'(c_t)} P_{t+1} \right] + \frac{\nu_r (d)}{M_{t+1} u_t}$$
Second, use recursive substitution and the law of iterated expectations to obtain the solution for (16) as

\[(17) \quad p_t - (d) u_t \sum_{j=0}^{\infty} \beta^j F \left[ \frac{\nu_{t+j}}{M_{t+j+1}} \right] \]

Third, substitute for \( M_{t+j+1} \) in (17) using the money supply rule (10a) to yield

\[(18) \quad p_t = \frac{1}{M} \left[ \nu_t (1-\varepsilon_{t+1}) + \frac{1}{\delta} (\nu-\sigma_{\nu}) \right] (d) u_t \]

Finally, use the specification for transaction service shocks in (8b) to substitute for \( \nu_t \) in (18), and approximate the money pricing function in a way that will be useful below by ignoring \( \nu \varepsilon_{t+1} \) and \( \sigma_{\nu}/\nu \) terms to yield

\[(19) \quad p_t = \frac{\nu_{t+1}}{M} + \nu_t - \varepsilon_{t+1} (d) u_t \]

### 2.2.3 The Price of Gold

Combine (3) and (6), use log utility specifications in (8), and the goods market clearing condition \( c_t = d_t \), to yield an Euler equation for gold

\[(20) \quad q_t = \beta \left[ \left( \frac{d}{d} \right) t+1 (d) u_t c_{t+1} \right] + \omega_t (d) \frac{1}{c_{t+1}} \]

Using recursive substitution and the law of iterated expectations, obtain the solution to (20) as

\[(21) \quad q_t = (d) u_t \sum_{j=0}^{\infty} \beta^j F \left[ \frac{\omega_{t+j}}{c_{t+j+1}} \right] \]
Substitute for $G_{t+j+1}$ in (21) using the gold policy rule (10b) to yield

$$q_t = \frac{1}{G} [w_t (1-\eta_{t+1}) + \frac{1}{\delta} (w - \sigma w_t)] (d_t)^u_t$$

Finally, use the specification for gold preference shocks in (8c) to approximate the gold pricing function as

$$q_t = \frac{w_t (1+t)}{G} + \omega_t - \eta_{t+1} (d_t)^u_t$$

2.3 RATES OF RETURN

Here we use the asset pricing functions derived in 2.2 above to describe some rate of return functions which will be useful when we discuss government policy under a gold standard in Section 3.

2.3.1 The Consumption Rate of Return on Capital

Use (14) to substitute for $z_t$ and $z_{t+1}$ in the definition of the gross consumption rate of return on capital (see definition below (11)) to yield

$$r_t = [(d_t)^u_t + (d_t)^u_{t+1}] (y_t)^d_t$$

The expected gross consumption rate of return on capital is

$$E_t r_t = (1+\delta) \frac{d_t (y_t)}{u_t d_t}$$

It is useful to compare (25) with (14). As discussed below (14), goods market equilibrium is achieved for temporarily low goods productivity by a temporary fall in the goods price of capital that raises the expected consumption rate of return on capital. This is verified in (25). The price
of capital and its expected rate of return move in opposite directions with respect to a temporary consumption preference shock. Finally, we should note that the consumption price and rate of return on capital vary only because of goods preference and goods productivity shocks. They move only in response to goods market shocks because they adjust to equilibrate the goods market. Separability of preferences and production technology across goods, money, and gold markets assures that these markets are independent of each other.

2.3.2 The Nominal Rate of Return on Bonds

Combining (3) and (7), recalling that the gross nominal rate of interest $R_t$ is known in period $t$, yields

$$R_t \beta \left[ \frac{u'(c_{t+1})}{u'(c_t)} \right] \frac{p_{t+1}}{p_t} = 1$$

(26)

Using (8a), (18), and the goods market clearing condition $c_t - d_t$, we can express the nominal interest rate in terms of disturbances and policy actions as

$$R_t = 1 + \delta(\frac{v_t (1-\epsilon_{t+1})}{v - \sigma_{\nu \epsilon}}, v - \sigma_{\nu \epsilon} > 0$$

(27)

Finally, use the specification for transaction service shocks from (8b) to approximate the nominal interest rate pricing function, by ignoring $\nu_{t+1}$ and $\sigma_{\nu \epsilon}/v$ terms, as

$$R_t = 1 + \delta(1 + \nu_t - \epsilon_t + 1)$$

(28)

Expression (28) says that the nominal interest rate is independent of consumption preference shocks $\mu_t$, goods productivity shocks $d_t$, gold preference shocks $\omega_t$, and gold stockpiling policy actions $\eta_{t+1}$. The nominal
rate varies only in response to transaction service shocks \( \nu_t \) and monetary policy actions \( \epsilon_{t+1} \). In particular, the nominal interest rate is independent of the expected real interest rate, i.e., the expected consumption good rate of return on capital. It must be emphasized that this result is by no means general. It is due to the separable log utility specification of preferences and the separable nature of productive technology in this particular model. Nevertheless it serves as a useful benchmark for policy analysis under the gold standard.\(^4\)

2.3.3 The Consumption Rate of Return on Bonds

By definition, the gross consumption rate of return on bonds is \( R_t \cdot \frac{P_{t+1}}{P_t} \). Using (19) and (28) we can express it as

\[
(29) \quad R_t \cdot \frac{P_{t+1}}{P_t} = \left[ 1 + \delta (1 + \nu_{t+1} - \epsilon_{t+2}) \right] (\frac{\delta}{\delta t} \cdot \frac{d}{d t} \cdot \frac{u}{u t+1})
\]

Using (25) and (29) we can express the expected gross consumption rate of return on bonds as

\[
(30) \quad \mathbb{E}[R_t \cdot \frac{P_{t+1}}{P_t}] = \mathbb{E} R_t - (\frac{\delta}{\delta t} \cdot \frac{d}{d t} \cdot \frac{u}{u t+1} \cdot \epsilon_{t+2})
\]

where \( \text{Cov}(\cdot) \) is a covariance conditional on information available in period \( t \).

We see from (30) that the expected consumption good rate of return on bonds equals the expected consumption good return on capital plus a time-varying covariance term representing a risk differential. There is no risk differential as long as monetary policy does not set up a covariance between monetary policy actions \( \epsilon_{t+2} \) and the inverse of the marginal utility of consumption \( \frac{d}{u t+1} \).
2.4 MONETARY POLICY AND THE RISK DIFFERENTIAL BETWEEN CAPITAL AND BONDS

Suppose the government chooses a money supply rule that generates negative policy actions when the marginal utility of consumption is high. From (19) we see that such a monetary rule might be motivated by a desire to smooth the purchasing power of money against temporary goods market disturbances. By (30) such a monetary rule creates an expected return differential in favor of capital, i.e., it lowers the expected consumption yield on bonds below that on capital. Why? By preventing the consumption value of money from falling as much in response to unexpected increases in the marginal utility of consumption, policy reduces the negative covariance between unexpected changes in the marginal utility of consumption and the real return on bonds. For a given expected yield, consumers value a reduction in the negative covariance of an asset return with the marginal utility of consumption, because it transfers resources to states of nature where they are more highly valued from states where they are less highly valued. So price level smoothing makes bonds more desirable than capital.

In equilibrium, however, the bond market must clear, i.e., the net demand for bonds must be zero. Hence, such monetary policy must cause the expected consumption return on bonds to fall relative to capital. Expression (25) shows that the expected real yield on capital is independent of monetary policy. Moreover, expression (27) shows the nominal interest rate to be independent of the covariance between the marginal utility of consumption and the money stock. It is the expected consumption capital gain on money \( \frac{D_{t+1}}{P_t} \) that adjusts to maintain bond market equilibrium for different monetary policy covariances. Specifically, from (19) we see that it is the numerator \( \frac{P_{t+1}}{P_t} \) in the foregoing expression that adjusts. We can see this
formally by taking expectations of (19) to yield

\[
\mathbb{E}_{t}^{p}_{t+1} = \frac{V_{t}(d_{t}(1+\delta)_{t}) - \delta \text{Cov}(d_{t+1}, \epsilon_{t+2})}{M_{t+1}}
\]

In (31) we see that bond market equilibrium is maintained, upon moving from a covariance neutral policy to the one described above, by reducing the expected future purchasing power of money.

The expected future purchasing power of money depends on policy covariance because the goods value of money depends on the product of the monetary policy action and the goods productivity shock. The expected value of the product of the random variables is more positive (negative) the more positive (negative) the covariance between the variables. Hence, by (19) policy that sets up a positive covariance between the money stock and goods productivity shocks drives down the expected future purchasing power of money.

### 3. POLICY ANALYSIS UNDER A GOLD STANDARD

With solutions for prices and rates of return in hand, we are now in a position to study the gold standard. The key to the analysis is the recognition that the fundamental pricing functions derived in Section 2 must also hold under a gold standard. To study a gold standard, we simply use the pricing functions derived in Section 2 together with an additional constraint that the money price of gold is fixed.

In Section 3.1, we derive restrictions on monetary policy and gold stockpiling policy rules, (10a) and (10b), that are implied by a fixed dollar price of gold. Section 3.2 contains an analysis of the classical pre-1914 gold standard. Section 3.3 describes the management of a gold
standard with an interest rate policy instrument. The means by which an interest rate instrument can be used to achieve a gold reserve ratio objective, to smooth interest rates, or to smooth the price level is worked out in detail.

3.1 **RESTRICTIONS ON MONEY AND GOLD POLICY RULES UNDER A FIXED DOLLAR PRICE OF GOLD**

We begin by assuming that the dollar price of gold \( \frac{q_t}{p_t} \) is fixed at \( Q \) so that

\[
q_t/p_t = Q
\]

Substituting for \( p_t \) and \( q_t \) in (32) using (19) and (23) yields two restrictions on the money and gold policy rules that must be satisfied for the dollar price of gold to be fixed at \( Q \). They are

\[
(33a) \quad \frac{M_t}{QG_t} = \frac{\nu_t}{\omega_t}
\]

\[
(33b) \quad \frac{M_{t+1}}{QG_{t+1}} = \frac{\nu_{t+1}}{\omega_{t+1}} \quad \text{or} \quad \epsilon_{t+1} - \eta_{t+1} = \nu_t - \omega_t
\]

Restrictions (33a) and (33b) say that the ratio of the aggregate money stock to the privately held stock of gold (valued at \( Q \)) must equal the ratio of the transaction service to the gold preference parameters. For future use, (33b) is also written in terms of growth rates. In its alternative form (33b) says that the money and gold stock growth rates must differ as the transaction service and gold preference shock growth rates differ.

Importantly, the restrictions on money and gold policy rules come in the form of ratios. This means that the fixed dollar price of gold \( Q \) can be supported with either monetary or gold stockpiling policy. For example, for an arbitrary \( M > 0 \) and \( \epsilon_{t+1} \) white-noise generating process, a gold policy
rule such that

\[ G = \frac{\omega M}{\lambda Q} \quad \text{and} \quad \eta_{t+1} = \epsilon_{t+1} + \omega_t - \lambda_t \]

would support a fixed price of Q, as long as the government were careful to choose M, Q, and \( \epsilon_{t+1} \) so that \( G(1+\eta_{t+1}) \leq G^S \).

Alternatively, for an arbitrary \( G^G \ (G^G < G^S) \) and \( \eta_{t+1} \) white-noise generating process, a monetary policy rule such that

\[ M = (G^S - G^G)Q \frac{\lambda \omega}{\mu} \quad \text{and} \quad \epsilon_{t+1} = \eta_{t+1} + \nu_t - \omega_t \]

would support a fixed price of Q.

Two important points are implied by this analysis. First, the government need not hold a gold stockpile to support a fixed Q, i.e., it can set \( G^G = 0 \). Using conventional terminology, a government need not hold gold "reserves" to maintain a fixed gold value of its paper money. To put it still another way, the government need not "back" its paper money with gold at all to maintain a fixed dollar price of gold. This raises a puzzling question, why if gold reserves are unnecessary to maintain a fixed gold value of government paper money, was there so much discussion of the adequacy of government gold reserves during the gold standard era? We come back to this question in Section 3.3.2.

Second, the government itself need not stand ready to exchange its paper money for gold or vice-versa to fix the dollar price of gold. Instead, the government can provide the money and gold necessary for the desired dollar price of gold to be supported by private competitive financial markets. In short, the government can use quantity rules to maintain indirect convertibility.
In practice, however, the government does not have sufficient information on transaction service and gold preference shocks to guarantee the fixed dollar price of gold by choosing money and gold stocks at the beginning of each period. Therefore, historically commitments to maintain a fixed dollar price of gold have utilized a price rule.

3.2 THE CLASSICAL CASE: A MONEY FOR GOLD PRICE RULE WITH A FIXED GOLD RESERVE RATIO

Analysis of the classical pre-1914 gold standard period, eg., Barro [1979], is carried out in terms of a money for gold price rule plus a gold reserve ratio constraint. Prior to analyzing this classical case as a whole, consider the mechanics of a price rule itself.

Under a money for gold price rule the government agrees to exchange money for gold or vice-versa at a fixed dollar price of gold $Q$. Assume that the government keeps its gold stockpile fixed, so that it makes good its commitment to trade money for gold at the fixed price by using bonds to acquire or dispose of the necessary gold. Now consider a positive gold preference shock. By (23) it puts upward pressure on $q_t$. However, by (19) it leaves the goods price of money $p_t$ unaffected. So by (32) it puts upward pressure on the dollar price of gold. This sets up the following arbitrage opportunity. Borrow money, use it to acquire gold from the government, sell the gold at a higher dollar price in the private market, pay off the loan, and pocket the difference. By (19) the fall in the money stock induced by profitable arbitrage causes $p_t$ to rise, i.e., it causes the price level to fall. The profitable arbitrage opportunity is eliminated by volume sufficient to cause the money stock to fall until $p_t$ rises to match the initial rise in $q_t$. Thereby, arbitrage automatically keeps the market
dollar price of gold equal to \( Q \).

Formally, for an arbitrary \( G^G \) \((G^G < G^S)\) and \( \eta_{t+1} \) white-noise generating process, (35) describes the equilibrium conditions for government money stock behavior with a money for gold price rule. The only difference is that the restrictions in (35) are automatically satisfied by private sector arbitrage under the price rule.

We are now in a position to see how a gold reserve ratio constraint coupled with a money for gold price rule is sustained by appropriate money and gold policies. To begin, define the government's gold reserve ratio \( \alpha \) as the ratio of its gold stockpile valued at \( Q \) to the stock of government paper money such that

\[
\alpha_t = \frac{QG^G_{t+1}}{M_{t+1}}, \quad 0 \leq \alpha_t \leq 1
\]

In Barro, for example, the imposition of a gold reserve ratio constraint takes the form of an implicit assumption that the government chooses its money and gold policy rules to keep \( \alpha \) constant. The popularity of the fixed gold reserve ratio assumption in analysis of the classical gold standard appears to be due to the view that the Bank of England kept its gold reserve ratio within a relatively narrow range. At any rate, the goal here is to investigate the effect of a fixed central bank gold reserve ratio together with a central bank money for gold price rule.

Specifically, we are interested in solving for \( M_{t+1}, G_{t+1}, p_t, \) and \( R_t \) as functions of the fixed dollar price of gold \( Q \), the aggregate stock of gold \( G^S \), and shocks to consumption preferences \( u_t \), goods productivity \( d_t \), transaction service productivity \( v_t \), gold preferences \( \omega_t \), and the government's gold reserve ratio \( \alpha_t \). We solve for variable averages first. Begin
by writing (36) in terms of average values

\[(37) \quad \alpha = QG^g/M\]

Equations (9), (33a), and (37) yield

\[(38a) \quad M = \frac{\nu}{w+av} QG^s\]
\[(38b) \quad G = \frac{\nu}{w+av} G^s\]

From (19) and (38a) we have

\[(39) \quad p = \frac{w+av}{QG^s} \frac{d}{\delta} t+1\]  \(\text{(By (32), q mirrors p)}\)

And from (28) we get

\[(40) \quad R = 1+\delta\]

To solve for the money and gold policy processes, first take logs of (36) and use (38b) together with (10) and a log-linearized version of (9) to obtain

\[(41) \quad \log_{\tau} \alpha_t - \frac{w}{av} \eta_{t+1} - \epsilon_{t+1}\]

Equation (41) together with (35) yields solutions for money and gold policy actions

\[(42a) \quad \epsilon_{t+1} = \frac{w}{w+av} [\nu_t - \omega_t] - \frac{av}{w+av} \log_{\alpha} \alpha_t\]
\[(42b) \quad \eta_{t+1} = \frac{av}{w+av} [\omega_t - \nu_t - \log_{\alpha} \alpha_t]\]

Substituting from (42a) into (19) yields the price level generating process (q_t mirrors p_t from (32))
Finally, substituting from (42a) into (28) yields the nominal interest rate generating process

$$R_t = 1 + \delta[1 + \frac{1}{w+\alpha v}(\omega t + \alpha v t + \alpha v \log \frac{t}{\alpha})]$$

To complete the description of the classical gold standard system, note some of its features with respect to rate of return and policy covariance effects. First, from (14) the goods price of capital $z_t$ generating process is unchanged by the imposition of classical gold standard constraints. Second, expressions (24) and (25) tell us that the processes generating realized and expected consumption good rates of return on capital are also unaffected. These features of the model stem from the separability conditions discussed in Section 2. Basically, the price and rate of return on capital depend only on goods market conditions and are independent of arrangements in money, gold, and bond markets.

Third, (42a) tells us that imposing the classical gold standard constraints leaves the covariance between the marginal utility of consumption and monetary policy actions at zero. So by (30) there is no risk differential between the expected consumption good returns on capital and nominal bonds. Fourth, from (8), (42a), and (42b) we see, however, that the classical gold standard constraints do set up a positive covariance between transaction service shocks and monetary policy actions so that $\sigma_{\epsilon>0}$, and between gold preference shocks and gold policy actions so that $\sigma_{\eta>0}$. That is how policy offsets the effect of such shocks on the dollar price of gold. From (18) and (22) we see that $\sigma_{\epsilon>0}$ and $\sigma_{\eta>0}$ lower the average goods value
of money and gold respectively. We see from (27) that average R is unaffected.

According to (42a) and (42b), a central bank committed to a price rule making money available for gold and vice-versa at a fixed dollar price of gold, and also committed to a fixed gold reserve ratio, completely uses up its degrees of money and gold policy freedom. This accounts for the widespread view that the gold standard completely constrained monetary policy. In particular, we can see from (43) and (44) that if $\alpha_t = \alpha$, the price level and the nominal interest rate generating processes depend on monetary and gold policy only through the fixed gold reserve ratio. 7

3.3 INTEREST RATE AND PRICE LEVEL POLICY

The preceding discussion described how monetary policy can be made to support a fixed dollar price of gold. We saw that if the government also chooses to maintain a fixed gold reserve ratio against its paper money, it cannot pursue other objectives. Here we show how the government can pursue objectives for the price level or the nominal interest rate while simultaneously agreeing to exchange money for gold at a fixed dollar price of gold Q, if it weakens its gold reserve ratio objective, allowing $\alpha_t$ to depart temporarily from its target $\alpha$.

It is easy to see the feasibility of such a policy. By (23) gold policy actions can affect the goods price of gold $q_t$. Arbitrage, as described in Section 3.2, induces money stock innovations $\epsilon_{t+1}$ that support the fixed dollar price of gold Q. Hence, by (32) the goods price of money $p_t$ (the inverse of the price level) must mirror changes in the goods price of gold $q_t$. Since gold stockpiling policy can manipulate the goods price of gold, it can be used to achieve price level and interest rate objectives.
Of course, in practice governments following a gold standard did not explicitly use gold stockpiling policy to achieve price level or interest rate objectives. For example, both the Bank of England and the Federal Reserve discussed their policies in terms of manipulating a nominal interest rate policy instrument, not in terms of gold stockpiling. Yet such policies could be effective only because they had gold stockpiling effects.

Section 3.3.1 describes the mechanism by which an interest rate policy instrument operates under a gold standard. The mechanism is illustrated first by describing the use of an interest rate instrument to maintain a strict \((\alpha_t - \alpha)\) gold reserve ratio objective. The gold reserve ratio objective is dropped in Section 3.3.2 and replaced with an interest rate smoothing objective. Section 3.3.3 analyzes price level smoothing.

### 3.3.1 Management of a Gold Standard with an Interest Rate Policy Instrument

The aim here is to characterize and describe how central bank policy was actually conducted under a gold standard. Discussions of the gold standard, both before and after World War I, focus on use of the nominal interest rate as the policy instrument. So we choose to discuss policy in terms of an interest rate rule here.

From (28) we can write monetary policy actions in terms of the nominal interest rate as

\[
\epsilon_{t+1} = \frac{1+\delta}{\delta} + \nu_t - \frac{1}{\delta} R_t
\]

Using (19) and (45) we can write the price of money as

\[
p_t = \frac{w^{\alpha} v(\delta)}{\delta Q S u^t t_t} R_t
\]
Since we want to interpret central bank policy as being executed by an interest instrument and its effect on monetary policy actions as in (45), we need to rethink the means by which the central bank makes good its commitment to buy or sell money for gold at the fixed price of Q. The discussion of arbitrage in Section 3.2 described gold policy as chosen arbitrarily and monetary policy as dedicated to supporting the fixed dollar price of gold. Here, we must reverse those roles. In practice, the interest rate and monetary policy actions were chosen, as we will show below, to achieve objectives for α, p, or R. Gold policy then supported the fixed dollar price of gold. Arbitrage was allowed to add or drain gold from the government stockpile in order to support a fixed Q. Simultaneously, open market operations in money and bonds accommodated the public's demand for money at the chosen level of the nominal interest rate policy instrument.

In this view, rewrite (35) as

\[ \eta_{t+1} = \epsilon_{t+1} + \omega_t - \nu_t \]  

(47)

Substituting from (45) into (47), express the gold policy action as a function of the interest rate policy instrument

\[ \eta_{t+1} = \frac{1+\delta}{\delta} \omega_t - \frac{1}{\delta} R_t \]  

(48)

Finally, using (41), (45), and (48) express the gold reserve ratio as a function of the interest rate instrument

\[ \log_{\alpha} \alpha_t = \frac{1+\delta}{\delta} \frac{\omega+\alpha}{\alpha} \omega_t - \omega_t - \nu_t + \frac{1}{\delta} \frac{\omega+\alpha}{\alpha} R_t \]  

(49)
Solving (49) for the nominal interest rate under a gold reserve ratio objective \(\alpha = \alpha_t\) yields

\[
R_t = 1 + \delta [1 + \frac{1}{\omega + \alpha \nu} (\omega_t + \alpha \nu_t)]
\]

Equation (50) is the interest rate policy rule for the classical central bank described in Section 3.2. Looking at (48), (49) and (50) we see that the central bank must raise its interest rate instrument to protect its gold reserve and gold reserve ratio against positive gold preference and transaction service shocks. From (46), we see that raising the interest rate instrument to protect gold reserves causes the price level \((1/p_t)\) to fall. That is, a temporary increase in the central bank's interest rate instrument is temporarily deflationary, as was widely recognized at the time.

Before moving on, consider the mechanism by which the central bank interest rate instrument is effective. To begin, the target gold reserve ratio \(\alpha\) anchors the expected future price level by pinning down \(M\) as in (38a) which, together with the policy covariance term pins down \(\epsilon_{t+1}\) according to (31). The interest rate instrument \(R_t\) thereby influences \(p_t\) as described in (30), which we rewrite here as

\[
p_t = R_t \cdot \frac{EP}{\epsilon_{t+1} \cdot \epsilon_t} - \frac{\epsilon_t}{\epsilon_{t+1}} \delta \text{Cov}(\epsilon_{t+1}, \epsilon_{t+2})
\]

By (30) and (51), bond market equilibrium requires a particular relationship between the expected consumption good rates of return on bonds and capital. Although that relationship depends on policy covariance, it is independent of policy actions. Hence, a nominal interest rate policy action must induce a change in expected inflation that preserves the bond market...
clearing expected real return on bonds. Since the expected future price level is fixed, it is the current price level that responds. Raising (lowering) the nominal interest rate instrument causes the current price level to fall (rise).

Price level adjustments, in turn, set up profitable arbitrages between the private gold market and the central bank gold window. Arbitrage adds or drains gold from the government stockpile until the goods price of gold adjusts to fully offset the effect of the change in the price level on the dollar price of gold. Aggregate money stock effects of arbitrage are immediately offset by central bank open market operations in bonds.

Open market operations are also used by the central bank to accommodate the quantity of money demanded at the level of its interest rate instrument. Raising nominal interest rates brings gold into the central bank and raises the gold reserve ratio for two reasons. It lowers the price level, making it profitable to sell gold to the central bank. It lowers nominal money demand both because it reduces the price level and because it raises the opportunity cost of money.

3.3.2 Interest Rate Smoothing

Central bank nominal interest rate smoothing is feasible. It is even possible to peg the nominal interest rate at \( R_t = 1 + \delta \) in this model. To see this, continue to assume that the central bank has a fixed target \( \alpha \) for its gold reserve ratio. But now allow \( \alpha_t \) to depart from \( \alpha \) temporarily.

For a nominal rate peg, we see in (45) that monetary policy actions simply accommodate transaction service shocks. Likewise, by (48) gold policy actions accommodate gold preference shocks. In addition, from (46) we see that moving from a gold reserve objective to a nominal interest rate peg
reduces price level variance. However, in (49) we see that nominal interest rate smoothing can only be achieved by allowing the gold reserve ratio to vary.

The implications of nominal interest rate smoothing for rates of return and policy covariances are qualitatively similar to those of the classical case discussed in Section 3.2. The consumption good price and rate of return on capital are unaffected. Policy still sets up no covariances between the marginal utility of consumption and monetary policy actions, so it generates no risk differential between capital and nominal bonds. However, comparing (45) with (42a), and (48) with (42b) shows that nominal interest rate smoothing raises $\sigma_{\nu_\epsilon}$ and $\sigma_{\nu_\eta}$ relative to the fixed gold reserve ratio case, so it lowers the average goods value of money and lowers the average goods value of gold. Recall, however, that these effects are formally absent here because we are working with approximations.

3.3.3 Price Level Smoothing

It is feasible for the central bank to use its interest rate policy instrument to completely smooth the price level. Specifically, if we continue to assume that the central bank has a target $\alpha$ for its gold reserve ratio, then it can peg the price level at $p$ in (39). From (39) and (46), the interest rate policy rule that completely smooths the price level is

$$ R_t = (1 + \delta) \frac{d(\nu)}{u d t} $$

Substituting from (52) into (45) yields the supporting monetary policy action generating process

$$ \nu_{t+1} = \frac{1+\delta}{\delta} \left[ 1 - \frac{d(\nu)}{u d t} \right] + \nu_t $$
Substituting from (52) into (48) yields the supporting gold stockpiling generating process

\[ \eta_{t+1} = \frac{1+\delta}{\delta} [1 - \frac{d(y)}{u^t d^t}] + \omega_t \]

Finally, substituting from (52) into (49) yields the gold reserve ratio generating process

\[ \log \frac{\alpha}{\alpha} = \frac{1+\delta}{\delta} \frac{w+\alpha v [d(y) - 1]}{\alpha v} - \nu_t \]

Price level smoothing, alone among the objectives we have considered, links the goods market to the money, gold, and bond markets. The consumption good price and rates of return on capital are still determined entirely in the goods market. However, according to (25) and (52) price level smoothing policy sets up a positive covariance between the nominal interest rate and the expected consumption good rate of return on capital. In (53) we see that policy also sets up a negative covariance between monetary policy actions and the marginal utility of consumption. Therefore, as explained in Section 2.4 price level smoothing policy causes the equilibrium expected consumption good return on nominal bonds to fall below that on capital. As for the \( \sigma_{\nu} \) and \( \sigma_{\omega} \) policy covariances, they are identical to those under the nominal interest rate peg. Lastly, gold reserve ratio variability under price level smoothing exceeds that under the nominal interest rate peg.

We are now in a position to answer the puzzling question initially posed in Section 3.1. In terms of the classical gold standard model presented in Section 3.2, it is easy to see that the central bank is fully capable of supporting a fixed dollar price of gold at \( Q \) even if \( \alpha = 0 \), i.e., the government holds no gold. So what accounts for widespread discussion of the
adequacy of gold reserves during the gold standard era? The answer lies in the fact that in order to pursue interest rate or price level smoothing policies, in addition to supporting a fixed dollar price of gold, the government needs to employ activist gold stockpiling policy. For a nominal interest rate peg, the required gold policy action generating process is (48) with $R_{t-1} = 1 + \delta$. For complete price level smoothing, the required gold policy action generating process is (54). Note in particular that the average government gold stockpile ($G^g$) must be sufficiently large to accommodate the relevant shocks to private nonmonetary gold demand.

4. SOME INTERPRETATIONS AND CONTROVERSIES

The formal model is put to use here to discuss some issues in gold standard history. In 4.1 a variety of private and public gold standards are interpreted in terms of the model. The Bank Rate controversy is addressed in 4.2. In 4.3, lender of last resort policy is interpreted in terms of the model. Finally, in 4.4 the widely documented appearance of interest rate smoothing in 1914 is interpreted and explained.

4.1 VARIETIES OF GOLD STANDARD

A brief taxonomy of gold standards is presented in this section. To make the discussion more concrete, an effort is made to associate each with some historical period. Though this should be understood as a heuristic device only. For example, other aspects of policy, such as whether silver also served as a monetary commodity, would also have to be taken into account in a full historical application of the model.

To begin, we can imagine a pure and private gold standard. It is pure in the sense that the medium of exchange is either gold coin or 100%
paper claims to warehoused gold. Whether money circulates as gold embodied directly in coin or as paper claims to gold, we can say that money under this standard has 100% gold reserve backing. This means, in turn, that even though there is no government intervention in the money or gold markets, we can use the fixed gold reserve ratio model in Section 3.2 to study the pure and private gold standard. To do so, we simply set $\alpha = 1$ and relabel $G^g$ to be the sum of gold coin and warehoused gold backing circulating paper claims. We assume that gold coin and privately warehoused gold yield no direct utility. Finally, $e_{t+1}$ and $\eta_{t+1}$ now represent entirely private behavior in the money and gold markets.

Closely related to the pure and private gold standard is the private gold standard with fractional gold reserve backing for paper money. Under the latter standard, gold coin still circulates as money but private banks also issue deposits and notes, whose fixed money price in terms of gold they maintain by a price rule. Importantly, no bank has a monopoly on note issue, so none can pursue interest rate or price level policy as outlined in 3.3. In other words, all banks are price takers. We can get a rough idea of how this standard works too by using the model in 3.2.

To do so, suppose there is no gold coin, so that the money stock consists entirely of private bank deposits and notes. Consider the choice of private gold reserve policy. If it carried no gold reserve a bank would have to issue a bond (or sell a bond it already owned) to obtain funds with which to buy gold to meet any deposit or note conversion. Moreover, whatever gold it received would be sold for funds to buy bonds. Zero gold reserve policy would therefore generate large transaction costs due to repeated trades in the gold and bond markets. While it would be feasible
for a bank to maintain convertibility without holding a gold reserve, at normal rates of return on bonds it would not be efficient. On the other hand, it would not be efficient for a bank to hold 100% gold reserves. To do so would eliminate entirely the transaction costs, but at the cost of its entire interest income. Consequently, one would expect the efficient private gold reserve ratio to lie between zero and one, to depend positively on the magnitude of transaction costs, and to depend negatively on the nominal interest rate.\(^9\)

From the foregoing it is clear that we can use the model in 3.2 to study a private gold standard with fractional gold reserves. One convenient way of doing so is to assume no gold coin, equal optimal gold reserve ratios against deposits and notes, and identical transaction service functions for deposits and notes. Now consider \(\alpha = a < 1\). Interestingly, from (43) we see that lower \(\alpha\) reduces price level \((1/p)\) variability with respect to transaction service shocks and marginal utility of consumption shocks. From (44), lower \(\alpha\) has an ambiguous effect on nominal interest rate variability. From (38b) we see that lower \(\alpha\) also reduces the average stock of gold tied up in gold reserves at private banks.

Freeing gold for direct utility yielding uses is a social benefit. At the level of the individual bank, the substitution of interest-earning bonds for gold raises profits. Competition among banks, however, passes on the excess profit in the form of an increased implicit or explicit yield on deposits. The reduced opportunity cost of holding money causes average real money balances to rise. In effect, the benefits of a lower gold reserve ratio are distributed throughout the population by lower rental prices for gold and money services, supporting greater average stock demands for gold
and money, with accompanying increases in utility. The powerful incentive for banks to substitute interest-earning assets for gold in their portfolios tended to make the pure gold standard unsustainable, evolving instead into a gold standard with fractional gold reserves. The U.S. gold standard before the Civil War was roughly representative of the latter.

Government involvement in the gold standard can take many forms. The simplest is the definition of the value of the unit of account, i.e., the dollar, in terms of ounces of gold, and the regulation or possibly operation of a mint. More significant, however, are government interventions in the money and gold markets themselves. Early U.S. government involvement in the gold standard, in part, took the form of the establishment of the First and Second Banks of the United States. These institutions, though chartered by the federal government, were capitalized privately and run by private owners for a profit. Their size relative to the financial market of the time would appear to have given them some market power. But without a monopoly on note issue, they could not pursue interest rate or price level policy as outlined in Section 3. Because they were profit maximizing, however, they had incentive to manage their fractional gold reserve efficiently as discussed above. Hence, the gold standard with these government chartered banks can also be analyzed by the model in 3.2.

Nationalization of the currency constituted a more significant government involvement in the gold standard. In the United States currency was nationalized under the National Bank Acts during the Civil War, later strengthened by a tax that eliminated state chartered bank notes. However, prior to the establishment of the Federal Reserve in 1914, no agency in the U.S. government could manage the stock of currency, the Treasury possessing only limited powers in that regard.
After resumption of the gold standard in 1879, Treasury gold stockpiling policy was partly dictated by the 100% gold reserve requirement against gold certificates created under its money for gold price rule. Until 1900, however, there was no reserve requirement against greenbacks, issued during the Civil War, or national bank notes. The Gold Standard Act of 1900 required a gold reserve of at least 100 million dollars not to exceed 150 million dollars.

Obviously the most important form of the government currency monopoly has been its management by a central bank. Consider the Bank of England in the second half of the 19th century.11 Except for a fixed fiduciary issue, it was required to hold a 100% gold reserve against its notes. Prior to World War I, the Bank of England was run as a profitmaking concern, so its managers had a powerful incentive to target an efficient gold reserve.12 Because the Bank of England had a considerable monopoly on note issue in England, Section 3.1 makes clear that it could have maintained a fixed pound price of gold without holding any gold at all. Under the circumstances, the best it could do was to meet the required 100% marginal gold reserve requirements against its notes, and hold as small an excess gold reserve as was efficient. This it did. Hence, the Bank of England’s effect on the operation of the gold standard can be studied within the basic models of 3.2 and 3.3.1. The model merely needs to be modified to take account of the marginal gold reserve requirement rule.

During the 19th century private bank deposits had become, along with Bank of England notes, an important source of transaction services. Since deposits and notes are not perfect substitutes in providing transac-
tion services, however, the existence of deposits does not fundamentally alter the operation of the gold standard as depicted in 3.2 and 3.3.1 above.

Under management of the Federal Reserve, the monopoly on currency generated very different money and gold supply rules. More will be said about this in Section 4.4 below. Suffice it to say here that because the Federal Reserve was not run by private citizens for a profit, its management was less constrained to target an efficient gold reserve ratio.\textsuperscript{13} As discussed in Sections 3.3.2 and 3.3.3 above, this was a necessary condition for the Fed to pursue interest rate and price level smoothing objectives.

4.2 \textsc{The Bank Rate Controversy}

There is substantial controversy over how the gold standard worked in its classical period 1880-1913. A number of well-informed commentators and historians of Bank of England practice during the late 19th century describe the Bank as employing the Bank Rate policy instrument to manage its gold reserve ratio.\textsuperscript{14} In fact, J.M. Keynes wrote:

During the latter half of the nineteenth century the influence of London on credit conditions throughout the world was so predominant that the Bank of England could almost have claimed to be the conductor of the international orchestra. By modifying the terms on which she was prepared to lend, aided by her own readiness to vary the volume of her gold reserves and the unreadiness of other central banks to vary the volume of theirs, she could to a large extent determine the credit conditions prevailing elsewhere. [Keynes, 1930, p. 274]

McClosky and Zecher[1976, p. 358] have responded that monetary theory coupled with arbitrage that unifies world goods and asset markets "implies that central bankers did not have control over the variables [e.g. the price level and the nominal interest rate] over which they and their historians have believed they had control."
The model of Section 3.3.1 is directly relevant to this controversy. To see how, interpret it as representing a single world economy where arbitrage unifies world markets for goods, money, gold, capital, and bonds. Assume also that money prices of gold are fixed worldwide. Furthermore, interpret the money and gold policy rules as Bank of England monetary and gold stockpiling actions. It is convenient, though not necessary, to assume that other banks throughout the world, whether public or private, behave as profit-maximizers, targeting efficient gold reserve ratios. Notably, the Bank of England has a national but not a worldwide monopoly on currency issue.

Could the Bank of England still use its Bank Rate instrument to manage its gold reserve ratio as described in Section 3.3.1? Yes, as long as it had an effective local monopoly on currency. Could its actions affect the price level and the nominal interest rate worldwide? Yes, through their effects in the worldwide gold market. Changes in the Bank Rate, by inducing changes in the Bank of England's gold stockpile, could affect the consumption price of gold in world markets which, for a fixed worldwide money price of gold, would affect the world price level and the nominal interest rate.

This point can be made more clearly by contrasting the market power of a central bank having a local currency monopoly with the lack of market power of a bank whose liabilities or assets have no monopoly status. No member in a system made up of the latter banks could exercise direct control over the nominal interest rate. But that is not the nature of the world to which McClosky and Zecher's claim is meant to apply.

The fact that the Bank of England generally did not choose to pursue activist gold policy was a consequence of its incentive to maximize
profit by holding only the minimum gold reserves required by law. This should not obscure the fact that its Bank Rate instrument could be used to attract gold or retain gold in the face of disturbances to the world gold market.

4.3 LENDER OF LAST RESORT POLICY

Walter Bagehot's recommendation that a central bank should behave as a lender of last resort represents another aspect of monetary policy that can be interpreted within the formal model developed in Section 3. Bagehot's policy prescription—summarized as lend freely at a high rate—may be understood as advocating that the central bank's Bank Rate, or simply a rate for buying designated classes of securities in the open market, be kept fixed suitably above the normal range of market rates. Normally the central bank would manage its gold reserve ratio by making quantitative asset and liability choices. Normally, then, the behavior of prices, rates of return, and quantities would resemble that described in 3.2.

The central bank's high lending rate would come into play in the event of a crisis. A sudden loss of confidence in the banking system would be accompanied by a widespread desire on the part of the public to convert bank liabilities into currency. It would also be accompanied by a build up of bank reserves, i.e., currency and central bank deposits. Under fractional reserve banking, a widespread demand for currency could not be satisfied without a departure from the central bank's gold reserve ratio target. To see what would happen short of a banking system collapse, let \( \sigma_t \) and interpret the increased demand for currency as a positive \( \nu_t \). From (43) and (44) we see that this shock deflates the price level and drives the nominal rate up.
To see formally the effects on prices and quantities of last resort lending, set \( R_t \) equal to a ceiling rate \( R \). By (44), with \( \alpha_t \to \alpha \) either a gold preference shock \( \omega_t \) or a transaction service shock \( \nu_t \) could make \( R \) effective. From (49) we see that if \( \nu_t \) becomes large enough for \( R_t \) to equal \( R \), then the central bank must let its gold reserve ratio fall. Any law, such as Peel's Act of 1844 in the case of the Bank of England, that effectively made a central bank meet a minimum gold reserve ratio against its notes would have to be suspended temporarily to allow the bank to act as lender of last resort. Such was in fact done to allow the Bank of England to provide last resort lending in 1847, 1857, and 1866.

From (48) we see that last resort lending in response to a pure transaction service shock \( \nu_t \) does not require any changes in the central bank's gold stockpile. So a banking crisis per se need not threaten the fixed money price of gold. One final point about last resort lending is that it does not even require lending at all, in the sense of advancing funds to particular institutions. Its essence is the provision of currency elastically at a ceiling nominal interest rate. In other words, Bagehot's proposal amounts to a suggestion for irregular use of interest rate smoothing by a central bank in the event that market rates reach a certain height. Such a policy can be executed by buying bonds outright in the open market.

4.4 THE APPEARANCE OF INTEREST RATE SMOOTHING

Barsky et al. provide extensive evidence that the worldwide stochastic behavior of nominal interest rates changed in 1914. Seasonals and other predictable transitory movements, which had previously been a prominent feature short-term nominal rates disappeared. Two institutional changes in the monetary regime also occurred in 1914. Though the United
States remained on a gold standard, the outbreak of World War I was accompanied by widespread suspension of national commitments to maintain the fixed money price of gold. Also, the Federal Reserve began operations in the United States.

Barsky et al. attribute the worldwide change in the behavior of nominal interest rates to the founding of the Federal Reserve rather than to the disruption of the international gold standard. They assume that all countries had a desire to smooth interest rates and to minimize international gold flows. In addition, they assume that, without a U.S. central bank, interest rate smoothing by foreign central banks would necessarily generate undesirable gold flows. But with the Federal Reserve in place, they argue that coordinated central bank monetary policy actions could smooth interest rates without international gold flows. As a result, they claim, interest rates were smoothed.

We can use the interest rate smoothing model in Section 3.3.2 to investigate this question. First of all, it is feasible for a central bank to smooth interest rates under a gold standard, though to do so it must hold a sufficiently large average gold reserve ratio. Hence, interest rate smoothing would be perceived as costly by a central bank, such as the pre-World War I Bank of England, whose management is concerned about maximizing profits. Moreover, comparing $\alpha_t = \alpha$ in (42b) with $R_t = 1 + \delta$ in (48) it is not necessarily the case that interest rate smoothing raises the variability of gold flows in and out of central bank reserves. If the transaction service shock $\nu_t$ variance were sufficiently dominant, then interest rate smoothing could be associated with a reduction in gold reserve flows. Finally, the argument in 4.2 implies that, if it chose to do so, a
single central bank with only a national monopoly on currency issue could support worldwide nominal interest rate smoothing under a worldwide gold standard if it could hold sufficient gold to accommodate worldwide gold shocks itself. Other central banks would need only accommodate their own transaction service shocks.

We are now in a position to comment on Barsky et al.'s proposed explanation of worldwide interest rate smoothing. First, it would not be sufficient for each country to have a central bank to observe interest rate smoothing. If the management of each central bank were concerned about maximizing profits, none would be willing to pursue such policy. Second, it was not necessary for the U.S. to have a central bank for the Bank of England to pursue interest rate smoothing. The reason the Bank did not do so was that it was unwilling to absorb the cost in a higher average gold reserve ratio. Finally, because it is entirely possible that interest rate smoothing might have decreased gold reserve variability, according to Barsky et al. central banks might have absolutely preferred it. For the above reasons, I find Barsky et al.'s explanation of the appearance of worldwide interest rate smoothing unconvincing.

Like Barsky et al., I too associate the appearance of interest rate smoothing with the advent of the Federal Reserve. However, I don't believe the establishment of a U.S. central bank per se to be sufficient to explain it. Rather, I think it crucial that the Fed was deliberately not set up as a private profitmaking institution. The Fed was run in the public interest (with the U.S. Treasury the recipient of net income arising from the Fed's monopoly on currency) so that it could be induced to forego income to eliminate nominal interest rate seasonals.
As pointed out above, although the United States did not leave the gold standard during World War I, there was a widespread suspension of gold convertibility. Hence, in 1914 the emergence of interest rate smoothing overseas was achieved by foreign monetary authorities themselves, independently of gold policy. Nevertheless, when these countries once again fixed their currency values in terms of gold, temporarily in the twenties and thirties, and more permanently under Bretton Woods, interest rate smoothing persisted. It was then that worldwide smoothing had to be supported by costly central bank gold reserve ratio policies, particularly those of the Federal Reserve.

During the 1920s, the Fed also became engaged in smoothing the price level. As discussed in 3.3.3, price level smoothing requires even more gold reserve ratio variability than interest rate smoothing. The Fed held a very high average gold reserve ratio during this period, well above its legal requirement. Such policies were costly to the Fed in terms of lost income. It is inconceivable that they would have been followed had the Fed been set up along private profit-maximizing lines.

5. CONCLUSION

The development of central banking under the gold standard was a crucial step in the evolution of monetary policy. Central banks were key instruments by which societies initially intervened in the monetary system. Moreover, they were the means by which societies eventually asserted their macroeconomic policy will. Hence, to begin to understand the history of monetary policy one needs to appreciate the scope for activist central bank policy under a gold standard.
Though the paper contained considerable detail, the key insights were these. A central bank had two degrees of policy freedom under a gold standard. It could choose policy rules for both money and gold. If, for example, monetary policy actions were used to maintain a fixed money price of gold, then gold policy would be free to pursue a second objective such as maintaining a required gold reserve ratio. Since government gold stockpiling actions could manipulate the relative price of gold in terms of goods, given the fixed money price of gold they could achieve interest rate or price level smoothing objectives as well.

Of course, central banks implemented their policies by standing ready to convert money into gold at a fixed price and by using an interest rate policy instrument to achieve other objectives. Consequently, considerable space in the paper was devoted to illustrating the above principles in terms of such central bank practices. Yet one should remember that interest rate and price level policies could be effective under a gold standard precisely because of gold stockpiling.

The model was meant to be used to address specific issues in gold standard history. The discussion of the evolution of the gold standard was included to put central banking in perspective. The analysis of temporary Bank Rate policy actions showed how the simple rational expectations asset pricing model could be used to study the Bank of England’s operating procedures. Likewise, the model proved useful for exploring the mechanics of last resort lending. Finally, by demonstrating the feasibility and mechanics of interest rate and price level smoothing, the model could usefully contrast the behavior of the Federal Reserve with that of the pre-World War I Bank of England.
ENDNOTES

1. Fischer[1986] and Sargent and Wallace[1983] also use explicit asset-pricing models to study the gold standard. Fischer studies the welfare effects of price level uncertainty. Sargent and Wallace use their model to illustrate Modigliani-Miller propositions by which private agents see through various government policies and undo their effects. Neither paper pursues the set of issues addressed here.

2. See Sargent[1987], p. 96, and p. 143 with respect to (17) and (21) below.

3. Although $\frac{d}{u} < \frac{E(d)}{E'(u)}$, for convenience I use $\frac{d}{u}$ to represent the constant expected future inverse of the marginal utility of consumption.

4. Using (3), (5), (8a), and (8b) we can derive the money demand function of the representative consumer as $p^{\mu}_{t} M_{t+1}^{-v_{t} \left( \frac{R}{t} \right)}$. Note that even though money and consumption are separable in the utility function in (1), consumption still appears as a scale variable in the money demand function. This is due to the fact that high $c_t$ is associated with a low marginal utility of consumption, so when consumption is high there is greater relative value to the service yield from money. Hence, the demand for money is higher. Note too that money demand varies negatively with the nominal interest rate.


6. Automatic adjustment could occur even with sticky prices. In a more complicated model, the price level $1/p_t$ might be predetermined by nominal contracts. And temporary changes in the money stock would affect output instead of the price level. In turn, output disturbances would cause changes in the marginal utility of consumption. By a gold pricing equation like (23) this process could still make $q_t$ adjust to maintain (32).

One might ask how if acquiring gold takes time a central bank could make good its pledge, without holding gold reserves, to keep the dollar price of gold fixed with a price rule? The answer is that instead of providing gold immediately, the central bank could promise to deliver gold in the future at a discount sufficient to keep the current money price of gold unchanged.

7. Barro's[1979] analysis corresponds to the case we have just solved for, except that here we ignore issues of gold production and consumption. Because our models are fundamentally similar at the economic level, though they are very different at the technical level, the impact effects of respective disturbances which appear in the two models are similar. For example, a shock to goods output $d_t$ in my model corresponds to a real income shock in Barro's. Barro's analysis of paper gold is a gold reserve ratio $\alpha$
shock in my model. And his velocity shock is a transaction service shock \( \nu_t \) in my model.

It is useful, at this point, to briefly sketch why the possibility of gold production and consumption imposes an important additional constraint on steady-state equilibrium, i.e., equilibrium with a constant aggregate gold stock. First of all, gold consumption must equal gold production in a steady-state. Gold production depends positively on the goods price of gold \( q \) which governs the portion of economy-wide factors of production producing gold rather than other goods. Suppose gold consumption is a fixed fraction of gold loss per unit of private nonmonetary gold stock per period. Using (3), (6), (8a), and (8c) we can write the gold demand function for the representative consumer under a fixed dollar price of gold as

\[
q_t G_{t+1} = w_t \frac{\sigma R}{1 + \delta}.
\]

By (40), average \( R = 1 + \delta \). The stock demand for gold has unit elasticity with respect to a change in the goods price of gold \( q \). Since the stock demand for gold is negatively related to \( q \), so is gold consumption. Hence, in any steady-state, \( q \) is determined exclusively by gold supply and demand. Since the fixed dollar price of gold is \( q_t / p_t \), the price level \( 1/p_t \) is also ultimately determined exclusively by gold market flow equilibrium conditions. This means that although government gold stockpiling actions can have powerful effects on prices temporarily, their effects on private nonmonetary gold stocks and on prices will eventually be completely offset by cumulative net production or consumption of gold stocks.

Another long-run issue is the viability of price fixing. Townsend[1977] has shown that a government buffer stock program which attempts to fix a relative price over time must eventually fail. Because it is not necessary for the government to hold buffer stocks to fix the dollar price of gold, the viability of a gold standard per se is not subject to Townsend's critique. However, under certain conditions his critique could apply to the case where gold reserves were necessary as part of a program to peg the price level. See Section 3.3.3 below. It would not apply for a fixed gold reserve ratio target or an interest rate peg. See 3.3.2.

8. Nominal interest rate smoothing can be defined in a number of ways. It can mean eliminating deterministic seasonals, as emphasized by Clark[1986] and Miron[1986]. It can mean reducing interest rate surprises as in Goodfriend[1987]. Or it can mean using monetary policy to maintain expected constancy in interest rates as studied by Barro[1987].

In this gold standard model with a fixed total stock of gold and purely temporary shocks, the interest rate generating process must be serially uncorrelated with mean 1+\( \delta \). Interest rate smoothing must obey these restrictions, a peg is only feasible at \( R_t = 1 + \delta \).

9. Frost[1971] and Poole[1968] present theoretical models of the demand for reserves which take these factors into account.
10. Under a gold standard with \( Q = 1 \), the present discounted consumption one-period rental rate for an ounce of gold is \( q \frac{1-R}{R} \). The analogous rental rate for a dollar is \( p \frac{1-R}{R} \). The lower rental rates for gold and money are brought about by a fall in \( q \) and \( p \).

As discussed in note 7, however, temporarily low gold production leads, over time, to a reduction of gold stocks, eventually restoring \( q \) and \( p \) to their original levels. In effect the social benefits of a lower \( a \) come partly in the form of temporarily freeing up of labor for leisure or other productive activities.

11. Until 1826 the Bank of England was the only body of more than six partners which could issue notes in England, although private country bank notes were an important part of the circulation. See Gregory [1929]. pp. xii and xliii. But an act of that year restricted the monopoly to an area within sixty-five miles of London. In practice, the Bank enjoyed a complete monopoly of issue in London, as the London private bankers had all given up issuing notes before the end of the 18th century. After 1826, notes of a growing number of joint-stock banks formed the principal currency of the agricultural districts. Country bank currency issues became less important over time. See also Coppieters[1955].

12. Sayers[1976], volume 1, contains substantial evidence of concern for profits on the part of the Bank of England before World War I. See, for example, page 8. Page 63 contains a discussion of the Bank's holding a larger gold reserve, recognizing the cost. Appendix 2 in the same volume contains the 1914 Memorandum on Reserves, in which is discussed the desirability of increasing the Bank's gold reserve. Again, from the discussion it is clear that the Bank economizes on gold reserves to maximize its profit.

13. Goodhart[1985], pp. 7-8, and Appendix B contains a discussion of the private-public tension in the evolution of central banking throughout the world.

14. See Bagehot[1873], Bloomfield[1959], Clare[1900], Hawtrey[1938], Keynes[1930], Palgrave[1903], Sayers[1936, 1976], Viner[1937], Chapter 5, and Wood[1939].

As explained, however, in Chapter 2 of Sayers[1936], for example, the Bank Rate was not always effective. The Bank of England often let the market rate fall below Bank Rate. Bank Rate was generally employed as an interest rate instrument only when there was upward pressure on market rates which forced banks "into the Bank" to borrow funds.

16. Over the years, the Bank of England came under repeated pressure to hold a greater average gold reserve in order to smooth interest rates and prices. See, for example, Jevons[1884], Palgrave[1903], the 1914 Memorandum on Reserves in Sayers[1976], Appendix 2. Bordo and Schwartz[1984], p. 47 discuss Bagehot's recommendation to this effect. In spite of such pressure, the Bank of England kept only a minimum gold reserve in excess of legal requirements.


18. Cagan[1982], Friedman[1961], and Keynes[1923], pp. 154-59 also comment on the excessive accumulation of gold by the Federal Reserve during this period.

According to Kitchin[1930], Table A, p. 80, as of 1929 47% of the world's stock of gold was held in nonmonetary form. Loveday[1930], Table III, p. 93 reports that $3.9 billion of the world's $10 billion worth of gold reserves were held in North America, essentially at the Federal Reserve. Board of Governors of the Federal Reserve System[1976], Table No. 93 shows the large Federal Reserve gold ratio. Note, in particular, the large quantity of free gold, i.e., reserves held above legal reserve requirements.
REFERENCES


