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Economic Growth, Liquidity, and Bank Runs

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Huberto M. Ennis
Federal Reserve Bank of Richmond

Todd Keister
Instituto Tecnológico Autónomo de México

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Economic Growth, Liquidity, and Bank Runs*

Huberto M. Ennis
Research Department, Federal Reserve Bank of Richmond
Huberto.Ennis@rich.frb.org

and

Todd Keister
Centro de Investigación Económica, Instituto Tecnológico Autónomo de México (ITAM)
keister@itam.mx

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Abstract

We construct an endogenous growth model in which bank runs occur with positive probability in equilibrium. In this setting, a bank run has a permanent effect on the levels of the capital stock and of output. In addition, the possibility of a run changes the portfolio choices of depositors and of banks, and thereby affects the long-run growth rate. These facts imply that both the occurrence of a run and the mere possibility of runs in a given period have a large impact on all future periods. A bank run in our model is triggered by sunspots, and we consider two different equilibrium selection rules. In the first, a run occurs with a fixed, exogenous probability, while in the second the probability of a run is influenced by banks' portfolio choices. We show that when the choices of an individual bank affect the probability of a run on that bank, the economy both grows faster and experiences fewer runs.

***Journal of Economic Literature Classification Numbers*:** E42, G21, O42
Key Words: Banking panics, endogenous growth, equilibrium selection

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1 Introduction

Bank runs and banking crises in general are an important economic phenomenon, both historically and in recent years. Much has been written about these crises, including analyses of their possible causes and of the magnitude of the economic disruptions that accompany them.[1] We focus on one particular facet: the effect of the possibility of bank runs on capital formation and thereby on economic growth. While there is now a fair amount of empirical evidence on the effects of banking crises on economic growth, not much theoretical work has been done on the subject. In this paper, we present an endogenous growth model where bank runs occur with positive probability in equilibrium. This allows us to examine not only the impact of an actual bank run, but also how the *possibility* of a run changes the decisions made by agents in the economy and how these changes affect long-run economic growth.

Our model of the behavior of banks is in the tradition of Diamond and Dybvig's [7] model, which highlights the role of the banking system in creating liquidity by taking in short-term deposits and making long-term investments. In particular, Diamond and Dybvig [7] show how demand-deposit contracts can easily lead to a situation where there are two pure-strategy equilibria of the game played by a bank's depositors (the "post-deposit" game): one where a bank run occurs and one where there is no run. The optimal contract for the bank to offer in the "pre-deposit" phase therefore depends critically on how an equilibrium of the post-deposit game is selected. One approach is to assume that agents coordinate their actions on a sunspot variable, a publicly observed random variable that is *extrinsic* in the sense that it has no effect on the fundamentals of the economy.[2] Peck and Shell [17] show that if a sunspot-induced run is sufficiently unlikely, depositors can prefer a contract that permits runs, even when a broad set of possible deposit contracts is considered.[3] It is always feasible for a bank to choose a contract that is "run proof" in the sense that it generates a unique, no-run equilibrium of the post-deposit game. However, choosing such a contract is costly *ex ante* because it provides less risk sharing among the bank's depositors. If the probability of a run is small enough (below some critical value), depositors will prefer to have more risk sharing and to live with the possibility of a run.

We keep our model of bank behavior as simple as possible, while retaining the spirit of the Peck-Shell [17] analysis. In particular, we restrict banks to offer simple demand-deposit contracts in order to make the problem tractable even with a large number of depositors. There is a substantial literature on the roles of the

[1] Caprio and Klingebiel [5] provide evidence that the costs of such crises are very large. For example, they estimate the costs of the Chilean and Argentine crises in the early 1980s to be 40% and 55% of GDP, respectively. For a large number of other crises, they report costs in excess of 10% of GDP.

[2] In a recent comprehensive study of modern banking crises, Boyd *et al.* [3] conclude that the available evidence strongly points toward a sunspots-based explanation for the cause of these crises (see also Ennis [8]).

[3] This type of result is shown for a restricted set of deposit contracts in Cooper and Ross [6].

banking system and its microeconomic structure,[4] and it is not our intention to contribute to these issues. Rather, we aim to highlight the basic growth implications of bank runs that are likely to follow from any model where the activities of the banking system matter for real allocations. The only critical aspect of our model of the banking system is that, as in Peck and Shell [17], banks may choose a contract that admits a run equilibrium in the post-deposit game.

Our model has overlapping generations of agents, each of whom lives for two periods. In each period, young agents decide how much of their income to deposit in the banking system, with the remainder being kept in a safe, liquid asset called storage. There is a large number of banks, and competition drives these banks to offer the demand-deposit contract that maximizes the expected utility of depositors. A bank must allocate the deposits it receives between storage and investment in new capital. Investment is illiquid in the sense that much of its value is lost if the project is terminated early.[5] Each bank takes as given an equilibrium selection rule, which determines the probability of a run if the bank offers a contract that is not run proof. Because all banks (and depositors) are identical, all banks will choose the same contract. In addition, all depositors hold the same beliefs, and therefore either all banks will experience a run or none will. We show how the possibility of a run influences the process of capital formation in three distinct ways. The first is obvious: when a run occurs banks liquidate investment, which reduces the amount of new capital created in that period. The second and third are more subtle. The possibility of a run leads agents to keep some of their wealth outside of the banking system in order to self-insure against receiving nothing from the bank during a run. In addition, banks tend to place a higher fraction of deposits into storage. Because there is a large loss from liquidating investment, holding a more liquid portfolio allows a bank to serve a larger number of customers during a run and hence provides depositors with better insurance against a run. However, resources placed in storage (either by agents or by banks) do not produce new capital, and hence both of these effects tend to decrease the level of the capital stock in the following period. Thus the mere possibility of a bank run reduces capital formation, even when a run does not occur.

The long-run impact of the possibility of bank runs depends critically on whether the actions of banks affect the long-run growth rate of the economy. We embed our banking model in an Ak model of growth, [6] which generates the following results. Because the path of real output is history dependent, a bank run necessarily has a permanent effect. This implies that the true cost of a crisis is much larger than a short-run

[4] Freixas and Rochet [13] provide a detailed summary and list of references for this literature.
[5] This modification of the Diamond-Dybvig approach was used by Cooper and Ross [6], who showed that the possiblity of runs could lead the bank to hold excess liquidity.
[6] In this we follow Bencivenga and Smith [2], who studied the role of financial intermediaries in promoting growth. Our model is in many ways similar to theirs, but they do not examine the possibility of bank runs. A key characteristic of the Ak model is that it has no transitional dynamics, which greatly simplifies our computations. The model can be viewed as approximating the balanced growth path of richer models.

estimate (such as those given in the first paragraph above) would indicate.[7] In addition, even small changes in banks' portfolios will change the long-run growth rate and therefore have large effects on the level of real output over time.

An immediate implication of this analysis is that much of the cost of a bank run falls on future generations, whose preferences are not taken into account by banks competing for deposits in the current period. To get a sense of the size of this external effect, we look at the problem of a social planner whose objective is to maximize the discounted sum of the utilities of different generations. We restrict the planner to choose from the same set of deposit contracts available to banks. Because the planner takes future generations into account, the socially optimal contract differs from the equilibrium contract. In particular, the planner offers less risk sharing among agents within a generation and places more resources into investment in order to increase the growth rate of the economy. We calculate the critical (exogenous) probability of a bank run above which the planner would choose a run-proof contract. This critical value decreases rapidly as the weight placed on future generations is increased, and it reaches zero at a fairly low annual discount rate. That is, even a very impatient planner is substantially less tolerant of runs than are banks in the competitive economy, indicating that the intergenerational externalities in our model are large.

We also examine the issue of equilibrium selection in the post-deposit game in more detail, using an approach that we have developed elsewhere (Ennis and Keister [9], [10]). The standard sunspots approach assumes that the probability of a run is a fixed constant (as long as both equilibria exist). In our approach, the probability of a run depends on the strength of the incentive for agents to run as measured by the *risk factor* of the run equilibrium. (A low risk factor corresponds to a strongly risk-dominant equilibrium.) When the risk factor is very low, an agent would choose to run for a wide range of beliefs about the actions of other agents, and therefore we say that a run is relatively likely.[8] The risk factor is determined largely by the deposit contract. We show through examples that when an individual bank's choice of contract affects the probability of a run, banks hold a *less* liquid portfolio. This somewhat counterintuitive result stems from the fact that in order to reduce the probability of a run, the bank must decrease the expected payoff of running (relative to the payoff of waiting). When the bank places more funds in investment and fewer in storage, the payoff of waiting increases if there is no run. The payoff of waiting if there is a run does not change (it is zero). Lower liquidity levels therefore imply a higher expected payoff of waiting, which leads agents to be more likely to wait and thereby lowers the probability of a run. As mentioned above, less liquid bank portfolios lead to more capital formation and therefore to higher growth rates. Hence there is

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[7] Boyd, Kwak, and Smith [4] provide evidence that the effects of modern banking crises are indeed very long lived.

[8] Ennis and Keister [9] show how an equilibrium selection rule of this form is the natural outcome of an adaptive learning process in a stochastic environment.

no tradeoff between growth and stability in this case; less liquid portfolios bring higher growth with fewer bank runs.

The outline of the remainder of the paper is as follows. In the next section we describe our model in detail. In section 3, we describe the equilibrium of the economy and use numerical methods to compute some examples. We trace the growth implications of the possibility of bank runs under the standard sunspots approach, and comment on the (intertemporal) social optimality of eliminating bank runs. In section 4, we study a risk-factor-based equilibrium selection mechanism and its consequences for the decisions of banks and the long-run behavior of the economy. In section 5 we conclude.

2 The Model

The economy consists of an infinite sequence of two-period-lived, overlapping generations, plus an initial old generation. There is a single consumption good in each period, which is produced using capital and labor. Agents in the initial old generation are each endowed with k_1 units of capital, and have preferences that are strictly increasing in consumption during the single period of their life. In each time period t, where $t = 1, 2, ...$, a continuum of agents with unit mass is born. Each of these agents is endowed with one unit of labor when young and nothing when old, and each is either patient or impatient. Preferences are given by

$$v(c_{1,t}, c_{2,t}) = \left\{ \begin{array}{c} b_1 (c_{1,t})^\gamma \\ b_2 (c_{1,t} + c_{2,t})^\gamma \end{array} \right\} \text{ if the consumer is } \left\{ \begin{array}{c} \text{impatient} \\ \text{patient} \end{array} \right\}, \tag{1}$$

where $\gamma < 1$ holds. A fraction ϕ of consumers know at birth that they are patient. We refer to these as type I agents. The remaining (type II) agents will learn their preferences at the end of the first period of their lives, while there is still time to consume that period but after investment decisions have been made.[9] Each type II agent in generation t is impatient with probability u_t and patient with probability $(1 - u_t)$. The realization of types is independent across agents, so that u_t is also the fraction of the population of type II agents in generation t that is impatient. The value of u_t is itself the realization of a random variable that gives the size of the aggregate liquidity shock in each period; high values of u_t correspond to high liquidity demand. We assume that u is independently and identically distributed over time, and that the distribution has a density function f.[10]

[9] We are thus collapsing the first two periods of the Diamond and Dybvig [7] setup into the first period of a young agent's life.
[10] Most of our analysis could be done under the assumption that u is a known constant over time, as in Cooper and Ross [6]. However, as Diamond and Dybvig [7] point out, in this case a simple suspension-of-convertibility policy is a costless way to eliminate the run equilibrium. When u is stochastic, however, the total suspension scheme is no longer optimal and the run equilibrium can exist under the optimal contract, as shown in Peck and Shell [17].

2.1 Production and investment

There is a large number of competitive firms who produce output using capital and labor as inputs according to the production function

$$Y_t = \overline{k}_t^{1-\theta} K_t^{\theta} L_t^{1-\theta},$$

where $\overline{k}_t$ is the average capital-labor ratio in the economy at time t, which is taken as given by each individual firm. Adding the capital externality is one way of preventing the marginal product of capital from falling too low as the economy grows and hence of generating endogenous growth.[11] There are many other, more interesting models with this property, including models of inventive activity. The externality-based approach allows us to keep the model simple and to abstract from transitional dynamics after a crisis, since our economy will always be on a balanced growth path. Nevertheless, our banking model could easily be embedded in a richer model of growth.

Capital is, of course, durable and therefore is one way for young agents to save. There are two other ways of saving, which we refer to as "storage" and "investment." One unit of consumption placed into storage at time t yields one unit of consumption regardless of whether it is liquidated later in period t or held until period $t+1$. One unit of consumption placed into investment in period t yields $R > 1$ units of capital in period $t+1$. This technology is the only way that new capital can be produced. If investment is liquidated early (at the end of period t), it yields $x < 1$ units of consumption per unit invested. Hence investment is an illiquid asset, which yields a higher return than storage if held to maturity but a lower return if liquidated prematurely.

2.2 Timing of events

Period t begins with a stock of capital k_t owned by old agents. This capital is rented out to firms, who also employ young agents and thereby produce output. After production takes place, old agents sell the undepreciated capital. Letting q_t denote the price of capital, an old agent then has $(r_t + (1-\delta)\, q_t)$ units of consumption for each unit of capital she had at the beginning of the period. She consumes all of this and exits the economy.

Type I young agents know that they are patient and therefore will save all of their income in whatever asset yields the highest return. The return to using a unit of consumption to purchase existing capital is

$$\frac{r_{t+1} + (1-\delta)\, q_{t+1}}{q_t}.$$

[11] The assumption that the exponent on the externality term is exactly equal to labor's share of income is, of course, special. Antinolfi, Keister and Shell [1] identify it as a bifurcation point in the parameter space.

The return from investing a unit of consumption in new capital formation is

$$R\left(r_{t+1}+(1-\delta)\,q_{t+1}\right)\equiv\psi_{t+1}.$$

The decision rule of a type I agent is therefore the following:

$$\text{Invest in}\left\{\begin{array}{c}\text{existing capital}\\ \text{either}\\ \text{new capital}\end{array}\right\}\text{ as } q_t\left\{\begin{array}{c}>\\ =\\ <\end{array}\right\}\frac{1}{R}. \tag{2}$$

Since both strategies yield capital in period $t+1$, the agent simply chooses the option that yields more units of capital per unit of consumption invested.

The interesting investment problem is that of type II agents. These agents do not know their preferences until after the opportunity to invest has passed and the market for capital has closed. As a result, they will form coalitions that we call banks. The agents will choose how much of their income to deposit in a bank, and the bank will place some of these resources in storage and the rest in investment in new capital.[12] We assume that income not deposited in a bank must be kept in storage. In other words, investment in capital by type II agents must be intermediated. We also assume that banks offer simple demand-deposit contracts, and that suspension of convertibility is not possible. Each depositor chooses to withdraw her funds from the bank in either period t or period $t+1$. Agents who choose period t arrive at the bank in random order. A bank offers a fixed rate of return on deposits withdrawn in this period, and it must honor this contractual obligation unless it has completely run out of resources.[13] Whatever resources remain in period $t+1$ are divided among the remaining depositors. The deposit contract offered by a bank can therefore be summarized by three numbers: the fraction of an agent's income that is to be deposited (denoted d_t), the fraction of deposits to be placed in storage (denoted η_t), and the return offered to agents who withdraw their deposits in period t (assuming the bank has not run out of resources; we denote this return by $a_{1,t}$). Together with the time t wage rate, these three numbers completely determine the consumption of a depositor under each possible contingency. Competition between banks implies that the equilibrium deposit contract will be the one that maximizes the expected utility of depositors.

After the bank sets the contract and type II agents have made their deposits, we move to what Peck and Shell [17] call the "post-deposit game." Each agent learns whether she is impatient or patient, and then decides whether to go to the bank in period t or in period $t+1$. Following the literature, we focus on

[12] We do not allow banks to purchase existing capital. However, the extreme illiquidity of this asset will make purchasing it a dominated strategy for banks in equilibrium.

[13] Our contract is not fully optimal. Analyzing the optimal contract with a continuum of agents is problematic, in part because increasing the consumption of an individual agent is costless. Peck and Shell [17] have a finite number of agents and show, in an environment similar to ours, that there are bank runs in equilibrium. Our simple contracts lead to equilibrium bank runs in the same spirit, and we believe that the gain from allowing more complex contracts would be small.

symmetric, pure strategy equilibria of the game. There are two possible equilibria of this type: one where all agents go to the bank at the end of period t (a run) and one where only impatient agents go (no run). In choosing the optimal deposit contract to offer, a bank needs to know how likely each of these outcomes is. In order to formulate the bank's problem, therefore, we need to have a theory of how an equilibrium of the post-deposit game is selected. The standard approach in the literature is to assume that agents coordinate their actions based on the realization of a sunspot variable. In particular, suppose that in period t a number s_t is drawn from the uniform distribution on $[0, 1]$. This draw is *extrinsic*, in the sense that it is unrelated to any other variables in the economy and is independent across periods. The realization s_t is publicly observed; we refer to it as the *sunspot signal*. The standard approach is to assume that if both a run and a no-run equilibrium exist in the post-deposit game, then all agents follow the decision rule "run if $s_t \leq \pi$; otherwise do not run" for some number π. Hence the sunspot signal serves to coordinate the actions of agents on one of the equilibria. Peck and Shell [17] call this number π the "propensity to run" and take it to be an exogenous parameter of the economy. If only one equilibrium exists, of course, the sunspot signal is ignored. Our interest is in both this standard approach and a modified version of it in which the probability of a run depends on the parameters of the deposit contract. We begin by defining an equilibrium selection mechanism.[14]

Definition: An *equilibrium selection mechanism* (ESM) is a function that assigns a probability π to the run outcome and $(1 - \pi)$ to the no-run outcome for each possible deposit contract. These probabilities must be feasible, meaning that $\pi = 0$ holds if the run outcome is not an equilibrium for a particular contract, and $\pi = 1$ holds if the no-run outcome is not an equilibrium.

In other words, an ESM assigns a probability distribution over the set of (symmetric, pure-strategy) equilibria of the post-deposit game to each possible deposit contract.[15] We should emphasize that we are still taking a sunspots-based approach. The value of π given by the ESM determines the cutoff point for the sunspot signal s_t, below which agents choose to run. Our definition is more general than the usual sunspots approach in that we allow this cutoff point to vary with the parameters of the contract, instead of being a fixed constant. We should also emphasize that while we speak of "selecting" between the two equilibrium outcomes of the post-deposit game, the actual allocation in each outcome will depend on the (single) contract chosen by banks. Hence the sunspot equilibrium allocation will *not* be a (mere) randomization over the equilibrium allocations of the economy without sunspots. The possibility of a run will affect the contract

[14] See Ennis and Keister [10] for a detailed discussion of the equilibrium selection mechanism approach.

[15] Straightforward calculations show that if the banking contract is run proof, the no-run outcome is indeed an equilibrium of the post-deposit game. In other words, for every banking contract at least one symmetric, pure-strategy equilibrium exists.

chosen by the bank and will therefore affect the growth rate of the economy even in periods when a run does not actually occur.[16]

The feasibility constraints on the ESM are simply a way of including two natural constraints on the bank's problem. As is common in the literature, a bank is able to choose a run-proof contract, in which case the run equilibrium will not exist. For such contracts, the bank recognizes that $\pi = 0$ holds and hence the no-run outcome will obtain. In addition, if a bank is not careful the contract might be such that patient agents always prefer to run; this would usually be referred to as a violation of the incentive compatibility constraint (which requires patient agents to prefer to wait if other patient agents are waiting). In this case the ESM would deliver $\pi = 1$, and the bank would recognize that such a contract could only lead to a run. Hence the incentive compatibility constraint is naturally embedded in the ESM approach. Beyond these two restrictions, the function π reflects the properties of the equilibrium selection process, whatever that may be.

2.3 The bank's problem

We assume that there is free entry into banking, so that competition will drive banks to maximize the expected utility of depositors. Before presenting the problem of a bank formed in period t, we introduce some notation to simplify the statement of the objective function. Define

$$v_{e,t} = \left(a_{1,t}d_t + (1-d_t)\right)^{\gamma}, \quad v_{\ell,t} = \left(a_{2,t}d_t + (1-d_t)\right)^{\gamma}, \quad \text{and} \quad v_{0,t} = (1-d_t)^{\gamma}.$$

These terms are proportional to the utility enjoyed by depositors who: arrive at the bank in the first period and are served, arrive at the bank in the second period and are served, and receive nothing from the bank, respectively. Then the banks' problem can be written as

$$\max_{(d_t,\eta_t,a_{1,t})} \pi\left(d_t,\eta_t,a_{1,t}\right)\int_0^1 \left(ub_1 + (1-u)\,b_2\right)\left(\overline{\overline{u}}_t v_{e,t} + (1-\overline{\overline{u}}_t)v_{0,t}\right) f\left(u\right) du + \tag{3}$$
$$\left(1-\pi\left(d_t,\eta_t,a_{1,t}\right)\right)\left(\begin{array}{c} \int_0^{\overline{\overline{u}}_t}\left[ub_1v_{e,t} + (1-u)\,b_2v_{\ell,t}\right] f\left(u\right) du + \\ \int_{\overline{\overline{u}}_t}^1 \left[\overline{\overline{u}}_t b_1 v_{e,t} + \left((u-\overline{\overline{u}}_t)b_1 + (1-u)b_2\right) v_{0,t}\right] f\left(u\right) du \end{array}\right)$$

[16] Note that this would be true even if we allowed the return offered by banks in the first period to be sunspot-contingent, as in Freeman [11]. Because deposit decisions and bank portfolio choices must be made before the sunspot signal is realized, the growth rate will necessarily depend on the *ex ante* likelihood of a run.

subject to

$$\begin{aligned}
\overline{u}_t &= \min\left[\left(\eta_t / a_{1,t}\right), 1\right] \\
\overline{\overline{u}}_t &= \min\left[\left(\left(\eta_t + (1-\eta_t)\, x\right) / a_{1,t}\right), 1\right] \\
a_{1,t} &\geq 0, \quad 0 \leq d_t, \eta_t \leq 1
\end{aligned}$$

and

$$a_{2,t} = \left\{ \begin{array}{c} \left(\eta_t + (1-\eta_t)\, \psi_{t+1} - u a_{1,t}\right) / (1-u) \\ \left(\eta_t + (1-\eta_t)\, x - u a_{1,t}\right) \left(\psi_{t+1}/x\right) / (1-u) \end{array} \right\} \text{ as } \left\{ \begin{array}{c} u \leq \overline{u}_t \\ \overline{u}_t \leq u \leq \overline{\overline{u}}_t \end{array} \right\}.$$

Notice that the form of the utility function is such that income of a depositor (which will equal the wage) cancels out of the objective and hence does not matter for the solution. This is important because, as the economy grows, wages will grow. We see here that this growth does not affect the decision problem of banks.

Before describing the objective function, we introduce some additional notation and describe the constraints. Let $\alpha_t^s(u)$ denote the fraction of the stored goods that are paid out in period t in the no-run outcome. Similarly, let $\alpha_t^i(u)$ denote the fraction of investment that is liquidated early and paid out in period t in the no-run outcome. Both of these fractions depend on the contract $(d_t, \eta_t, a_{1,t})$. Because $x < 1 < R$ holds, the bank will never choose to pay agents withdrawing in period t with liquidated investment when stored goods are available. In other words, $\alpha_t^i > 0$ implies $\alpha_t^s = 1$. Hence the fourth constraint defines $\overline{u}_t$ as the value of u at which, for the given values of η_t and $a_{1,t}$, all stored goods have been given to withdrawing agents but no investment has been liquidated ($\alpha_t^s(\overline{u}_t) = 1$ and $\alpha_t^i(\overline{u}_t) = 0$). The following constraint defines $\overline{\overline{u}}_t$ as the value of u at which all investment has been liquidated and the bank has just run out of resources ($\alpha_t^s\left(\overline{\overline{u}}_t\right) = \alpha_t^i\left(\overline{\overline{u}}_t\right) = 1$). If the realization of u is greater than $\overline{\overline{u}}_t$, agents arriving at the bank in period $t+1$ will receive nothing. The next two constraints are obvious bounds on the choice variables, and the final constraint simply says that the resources remaining in the bank at period $t+1$ are divided equally among the agents choosing to withdraw in that period.

Turning to the objective function, the first term gives the expected utility of an agent in the event of a bank run (and therefore is multiplied by the probability of a run π). Because the agent's place in line is random and the first $\overline{\overline{u}}_t$ depositors to arrive are served during a run, $\overline{\overline{u}}_t$ also gives the individual probability of being served. With probability u the agent will truly be impatient and hence have preference parameter b_1, while with probability $(1-u)$ she will be patient and will have parameter b_2. An agent who is served receives the rate of return $a_{1,t}$ on her deposits, and every agent has $(1-d_t)$ units of consumption outside of the banking system. The other term in the objective function gives the expected utility of a depositor when there is no run (and therefore is multiplied by $(1-\pi)$). If the fraction of patient agents is less than $\overline{\overline{u}}_t$, the bank will not run out of resources and the agent will receive return $a_{1,t}$ if she is impatient. The return received by a patient agent $(a_{2,t})$ will depend on the number of impatient agents and is given in the final constraint. If the fraction of impatient consumers is above $\overline{\overline{u}}_t$, however, only $\overline{\overline{u}}_t$ of them will receive the return $a_{1,t}$. All other impatient agents, as well as all patient agents, will receive nothing from the bank and have consumption $(1-d_t)$.

Of course, the bank always has the option of choosing a deposit contract such that only the no-run outcome is an equilibrium. In such a case, any feasible ESM will assign $\pi = 0$. It will often be useful to look at the best run-proof contract, which is the solution to

$$\max_{(d_t,\eta_t,a_{1,t})} \int_0^1 \left[ub_1 v_{e,t} + (1-u)\, b_2 v_{\ell,t}\right] f(u)\, du \tag{4}$$

subject to

$$\begin{aligned} \overline{u}_t &= \min\left[(\eta_t / a_{1,t}), 1\right] \\ 0 &\leq a_{1,t} \leq \eta_t + (1-\eta_t)\, x, \quad 0 \leq \eta_t \leq 1 \end{aligned}$$

and

$$a_{2,t} = \left\{ \begin{array}{c} \left(\eta_t + (1-\eta_t)\,\psi_{t+1} - u a_{1,t}\right) / (1-u) \\ (\eta_t + (1-\eta_t)\, x - u a_{1,t})\left(\psi_{t+1}/x\right) / (1-u) \end{array} \right\} \text{ as } \left\{ \begin{array}{c} u \leq \overline{u}_t \\ \overline{u}_t \leq u \leq 1 \end{array} \right\}.$$

The upper bound placed on $a_{1,t}$ in the second constraint guarantees that the bank will not run out of resources in the first period (in other words, that $\overline{\overline{u}}_t = 1$ holds). Therefore a patient agent will not run even if she believes that all other patient agents are running, and hence the run outcome is not an equilibrium. We should emphasize that this problem is contained within problem 3 above, through the feasibility constraint on the selection mechanism $\pi(d_t, \eta_t, a_{1,t})$.

This problem is difficult to address analytically, but an upper bound for $a_{1,t}$ can clearly be chosen large enough to not be binding. Hence the choice set is compact and, as long as the objective function is well-behaved (in both of our approaches below it will be upper semicontinuous), a solution to the problem will exist. We use $\left(d_t^*, \eta_t^*, a_{1,t}^*\right)$ to denote this solution.

3 Equilibrium

We now turn to the analysis of the equilibrium behavior of the economy. We first impose the market-clearing conditions and derive the equilibrium law of motion for the capital stock as a function of the contract offered by banks. We then use numerical methods to compute the solution to the bank's problem and simulate the equilibrium behavior of the economy.

3.1 Market clearing and aggregate investment

Firms are competitive and therefore factors are paid their marginal products

$$
\begin{aligned}
w_t^* &= (1-\theta)\overline{k}_t^{1-\theta} k_t^{\theta} \\
r_t^* &= \theta \overline{k}_t^{1-\theta} k_t^{-(1-\theta)}.
\end{aligned}
$$

In equilibrium all firms will choose the same capital-labor ratio, and hence $\overline{k}_t = k_t$ holds. The marginal product of capital therefore reduces to the constant θ, which is why the economy is always on a balanced growth path.

In the market for existing capital, supply is given by $(1-\delta)\,k_t$. If the price of capital q_t were greater than $(1/R)$, we have from 2 that demand for capital would be zero because new investment would yield a higher return. Therefore the market would not clear. Suppose instead that q_t were below $(1/R)$. Then from 2 we have that type I agents would put all of their income into existing capital so that total demand for existing capital would be equal to

$$\frac{\phi\,(1-\theta)\,k_t}{q_t}.$$

The market-clearing price would then be given by

$$q_t = \phi \frac{(1-\theta)}{1-\delta}.$$

We assume that

$$\phi \geq \frac{1-\delta}{1-\theta}\left(\frac{1}{R}\right) \tag{5}$$

holds, which implies that the candidate price above is at least $(1/R)$, contradicting our original supposition. The role of type I agents in this model is to hold the stock of existing capital (which is completely illiquid) between periods. The assumption in 5 is simply that there are enough type I agents in the economy to prevent existing capital from trading at a discount. (Otherwise, if the discount were large enough, banks might want to invest in existing capital as well.) The only remaining possibility is then $q_t = (1/R)$. Under condition 5 this price clears the market because the demand for capital is perfectly elastic (and large enough). We state this result as our first proposition.

Proposition 1 *Assume condition* 5. *Then the equilibrium price of capital is given by*

$$q_t = \frac{1}{R} \quad \textit{for all } t.$$

Since we have shown both r_t and q_t to be constant over time, the bank's problem is exactly the same in every period and the solution (d^*, η^*, a_1^*) will be independent of time. In other words, banks will offer the same deposit contract in every period.

We use i_t^{I} to denote investment in new capital made by an individual type I agent. This must be equal to the income of the agent less her purchases of existing capital. Using the market-clearing condition for existing capital, we can write the total investment in new capital by type I agents as

$$\begin{aligned} \phi i_t^{\mathrm{I}} &= \phi w_t - q_t (1-\delta) k_t \\ &= (\phi (1-\theta) - (1-\delta)/R) k_t. \end{aligned}$$

Finally, we need to calculate how much new investment is undertaken by banks. This will depend on the fraction of depositors who arrive at the bank in the first period, which we denote μ_t. If there is no run, this is equal to the fraction of depositors who are impatient. If there is a run, however, μ_t is equal to one by definition. Therefore we have

$$\mu_t = \left\{ \begin{array}{c} u_t \\ 1 \end{array} \right\} \text{ in the event } \left\{ \begin{array}{c} \text{no run} \\ \text{run} \end{array} \right\}.$$

We use $\widehat{f}$ to denote the probability measure associated with μ, which is generated by the density function f and the selection mechanism π. Note that since π depends on the contract chosen by the bank, so does $\widehat{f}$. The amount of investment per type II agent is then given by

$$i_t^{\mathrm{II}} (\mu_t) = d (1-\eta) \left(1 - \alpha^i (\mu_t)\right) w_t.$$

In other words, new capital formation depends on the amount an agent deposits in the bank, the fraction of deposits placed in investment, and the fraction of that investment that is not liquidated prematurely. The law of motion for the capital stock is then given by

$$k_{t+1} = (1-\delta) k_t + R \left(\phi i_t^{\mathrm{I}} + (1-\phi) i_t^{\mathrm{II}} (\mu_t)\right).$$

Since both i^{I} and $i^{\mathrm{II}} (\mu_t)$ are linear functions of k_t we have the following result.

Proposition 2 *For any period t and stock of capital k_t, the equilibrium growth rate of capital (k_{t+1}/k_t) is a random variable $g(\mu)$ independent of t. Furthermore, we have*

$$g(\mu) \equiv R (1-\theta) \left[\phi + (1-\phi) \left(1-\eta\right) d (1 - \alpha^i (\mu))\right], \tag{6}$$

where μ is a random variable with probability measure $\widehat{f}$. The function $\alpha^i(\mu)$ is given by

$$\alpha^i(\mu) = \begin{cases} 0 & \text{if} \quad \mu < \overline{u} \\ (\mu a_1 - \eta) / ((1-\eta)x) & \text{if} \quad \overline{u} < \mu < \overline{\overline{u}} \\ 1 & \text{if} \quad \mu > \overline{\overline{u}} \end{cases} .$$

In summary, the difference equation describing the dynamic behavior of k_t is linear and stochastic. Notice that since d, η and the function α^i are the same in every period, the growth rate is a time-invariant, weakly decreasing function of the realization of μ. In equilibrium, aggregate output is given by $Y_t = k_t$ and hence the growth rate of output is the same as the growth rate of the capital stock in this economy.

Because of the complexity of the bank's problem 3, properties of this difference equation are difficult to derive analytically. We now compute solutions numerically and simulate the equilibrium behavior of the economy.

3.2 Implications for growth

In this subsection we investigate the growth implications of the possibility of bank runs by computing a representative example of the model presented above. In particular, consider the utility function 1 with the following parameter values: $\gamma = 0.4$, $b_1 = 2.5$ and $b_2 = 1$. We take the capital share of income θ to be equal to 0.4 and a 20% depreciation rate ($\delta = 0.2$). The return on investment is given by the pair $(R = 3,\ x = 0.3)$. We set the liquidation value of investment x relatively low because for values of x closer to unity the bank finds liquidated investment a not-too-costly instrument for providing consumption to impatient agents. We want to make a clear distinction between storage (which yields consumption goods) and investment (which yields capital), and a low value of x is useful for this purpose. The total return on investment when not liquidated early is then given by

$$\psi = R\,(r + (1-\delta)q) = 2.0$$

We assume that the value of u, the proportion of impatient agents in the population, is drawn from a beta distribution with parameters $(3, 9)$. The mean value of u is then 0.25 and the standard deviation is around 0.12. If the variance of u is set very high, the effects of bank runs will not be substantially different from the effects of regularly occurring high liquidity-demand shocks. Bank runs, however, are extreme events and the way to capture this with our other assumptions is to assume a relatively low variance of u.

Given these parameter values and an ESM, we solve the bank's problem numerically.[17] We first consider the standard sunspots story, where the probability of a run is a fixed number for all deposit contracts under which both equilibria exist. It should be kept in mind that for a sufficiently high value of the probability of

[17] All calculations are done in Fortran. The source code is available from the authors upon request.

a run, banks might choose a run-proof contract (i.e., the contract that solves 4). In the present example, this will occur if the exogenous probability of a bank run is above 11.9 %.[18]

In Table 1 we present the solution of the bank's problem for different levels of the probability of a bank-run. Note that the fraction of income that agents keep outside of the banking system $(1-d)$ is increasing in the probability of a run. The more likely it is that an agent will receive nothing from the bank, the more money she will want to keep "under the mattress." This disintermediation effect is compounded by the fact that the fraction of liquid assets in the bank's portfolio is also increasing. As the probability of a run increases, it becomes more likely that the bank will have to liquidate investment early. But since the liquidation value is relatively low, the bank prefers to hold more of the liquid asset (storage) to deal with a run if it occurs. Together, the movements in d and η lead to substantially less investment in new capital.

Table 1

Prob. of Run (π)	d	η	a_1	Prob[$u \geq \overline{u}$]	Prob[$u \geq \overline{\overline{u}}$]
0.00	0.998	0.449	1.141	0.127	0.0179
0.04	0.983	0.465	1.164	0.119	0.0182
0.06	0.972	0.474	1.177	0.115	0.0184
0.08	0.958	0.483	1.189	0.111	0.0185

The last two columns of the table show the probability that the bank will liquidate some investment early (Prob[$u \geq \overline{u}$]) and the probability that the bank will have to suspend payments in period t after running out of resources (Prob[$u \geq \overline{\overline{u}}$]), both conditional on the bank not suffering a run. We see that as the probability of a run increases and the bank chooses a more liquid portfolio, the probability that it will have to liquidate investment early in the no-run situation falls (from 12.7% of the time to 11.1%). The higher level of liquidity also implies that the bank will have to suspend payments less often when there is no run, although quantitatively this effect is very small. Note, however, that the unconditional probability of suspension clearly increases with the probability of a run.

As a benchmark case, we take the probability of a bank run to be equal to 0.06 (when both equilibria exist). Figure 1 shows the time series of the logarithm of the stock of capital for this economy in a representative 50-period simulation. It shows how a bank run causes an abrupt fall in the level of capital formation in the economy. However, not all of the major downturns in this figure are due to bank runs. In periods where the number of impatient agents u_t is very high, the bank liquidates some investment early to pay these agents, and this liquidation creates some of the observed fluctuations in the stock of capital. There are four bank runs during these fifty periods, at $t = 11$, 29, 36, and 38. There is no bank-run, for example, at

[18] The best run-proof contract has some interesting characteristics. For the parameter values being considered (and for a wide range around them) this contract has a relatively low level of storage in the bank's portfolio and a relatively low return on early withdrawals. This will be important in our discussion of the intertemporal optimality of bank runs in section 3.3.

$t = 17$ and 43.[19]

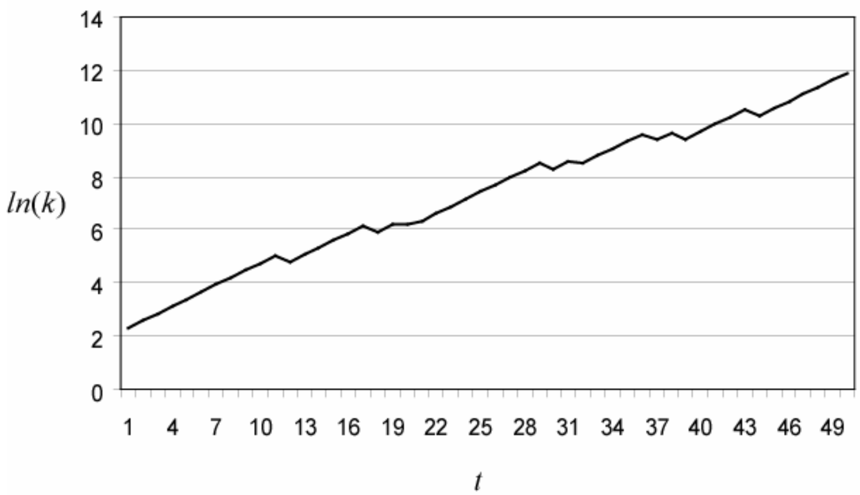

Figure 1: Sample time series for capital

In Table 2 we present the average growth rate of capital conditional on not having a run, as well as the unconditional average growth rate. These numbers are the result of 20 simulations of 50 periods each. We consider a period in this model to represent 5 years and we report the annual growth rates.[20]

Table 2

Prob. of Run	Growth Conditional on No Run	Unconditional Growth
0.00	5.59%	5.59%
0.04	5.03%	4.85%
0.06	4.72%	4.44%
0.08	4.40%	4.03%

Note that the unconditional growth rate of the economy tends to be lower for economies with a higher probability of a bank run. Two effects combine to generate this fact. First, the average growth rate in periods when there is no run is lower because agents keep more resources outside of the banking system and because

[19] During the 50 years of the National Banking Era (1863-1914), there were five major bank panics: 1873, 1884, 1890, 1893, and 1907. The Federal Reserve System was established after that, partly as a response to those regular periods of crisis.

[20] Miron ([15]) studied banking panics in the U.S. during the period 1890-1908. He estimates that the probability of a financial panic in any given year was around 0.30. We are considering much lower probabilities of sunspot-driven runs in our computations. However, two factors make our numbers reasonable. First, the banking system in our model may experience distress due to unusually high levels of the proportion of impatient agents u. In fact, conditional on no run, early liquidation of investment will occur in 11.5% of the periods in our calibration (see Table 1). Some of these events would be included in Miron's definition of a panic. Second, the period studied by Miron seems to be a period with unusually high frequency of runs. Miron also reports the growth rate of output during these 18 years. The growth rate conditional on no run was 6.82% and the unconditional growth rate was 3.75% (see our Table 2 for comparison).

banks place a lower fraction of deposits into investment. This is the effect isolated in the middle column of the table. Second, bank runs induce early liquidation when they occur, and that also reduces capital formation on average. Both of these effects are detrimental for long-run growth.

3.3 On the optimality of eliminating bank runs

For our chosen parameter values, the equilibrium contract has the property that bank runs can occur with positive probability. If the model consisted of only a single generation of agents, we could then say that it is socially optimal to allow bank runs to occur because the equilibrium contract maximizes the expected utility of type II agents (and has no effect on type I agents). However, the above analysis shows that the deposit contract used at time t affects all future generations through its effect on the growth rate of the capital stock. When there are many generations, the intertemporal impact of a bank run can be large, as is evident from Figure 1. A social planner who places sufficient weight on future generations may therefore prefer a deposit contract that eliminates the possibility of runs. In other words, a sufficiently patient planner may prefer a deposit contract that offers (substantially) less risk sharing within each generation, but that generates a higher average growth rate by placing more resources into investment and by making the bank immune to runs. In this subsection, we investigate the conditions under which such a planner would allow bank runs to occur with positive probability.

There is, of course, no clear criterion for aggregating utilities across generations. We take a simple approach which allows us to illustrate our point. Suppose that the planner places equal weight on all members of a given generation, and discounts the expected utility of generation t agents by β^t, for some $\beta \in [0, 1)$. We restrict the planner to choose a simple deposit contract in each period, which implies that the set of feasible allocations for the planner is the same as the set of feasible allocations in the competitive economy. For a given deposit contract, let $v(a_1, \eta, d)$ denote the value of the objective function in 3. Then the expected utility of a generation t agent at birth is given by

$$\begin{aligned} \text{type I} \quad &: \quad b_2 \left(\psi \left(1-\theta\right) k_t\right)^{\gamma}, \\ \text{type II} \quad &: \quad v\left(d_t, \eta_t, a_{1,t}\right) \left(\left(1-\theta\right) k_t\right)^{\gamma}. \end{aligned}$$

To simplify the notation, we define $z \equiv b_2 \psi^{\gamma}$. Then for a given value of β we can write the planner's maximization problem as

$$\max_{\{d_t, \eta_t, a_{1,t}\}} E_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} \left(\phi z + \left(1-\phi\right) v\left(d_t, \eta_t, a_{1,t}\right)\right) \left(1-\theta\right)^{\gamma} k_t^{\gamma} \right] \tag{7}$$

subject to

$$k_{t+1} = g\left(d_t, \eta_t, a_{1,t}; \mu_t\right) k_t,$$

where the function g is given in 6.

In order for social welfare to be well-defined, we need for β to be less than the inverse of the maximum feasible growth rate of per-period utility (so that 7 is finite along every feasible path). Because we are restricting the planner to simple deposit contracts, the maximum growth rate would be obtained if all of young agents' income were placed in investment and there were no liquidation in any period. In this case the growth rate of capital would be $(1-\theta) R$ in every period, and therefore we assume that

$$\beta < \overline{\beta} \equiv \frac{1}{\left[(1-\theta) R\right]^{\gamma}}$$

holds. For the parameter values used in the example above, we have $\overline{\beta} = 0.79$, which corresponds to annual discounting of about 4.6%.

We can replace the capital stock at time t with a product of past growth rates, so that we have

$$k_t^{\gamma} = k_1^{\gamma} g\left(d_1, \eta_1, a_{1,1}; \mu_1\right)^{\gamma} \cdots g\left(d_{t-1}, \eta_{t-1}, a_{1,t-1}; \mu_{t-1}\right)^{\gamma}.$$

Since the random variables μ_t are independent across periods, so are the random variables $g\left(d_t, \eta_t, a_{1,t}; \mu_t\right)^{\gamma}$. Using this and the fact that there is an upper bound on the growth rate of capital, we can rewrite the planner's problem as[21]

$$\max\, (1-\theta)^{\gamma} k_1^{\gamma} \sum_{t=1}^{\infty} \beta^{t-1} \left(\Pi_{s=1}^{t-1} \overline{g}\left(d_s, \eta_s, a_{1,s}\right)^{\gamma}\right) \left(\phi z + (1-\phi)\, v\left(d_t, \eta_t, a_{1,t}\right)\right)$$

where $\overline{g}\left(d_t, \eta_t, a_{1,t}\right)^{\gamma} \equiv E_0\left[g\left(d_t, \eta_t, a_{1,t}; \mu_t\right)^{\gamma}\right]$. Hence, the solution to the planner's problem must satisfy the following Bellman Equation

$$U = \max_{(d,\eta,a_1)} \left\{\phi z + (1-\phi)\, v\left(d, \eta, a_1\right) + \beta \overline{g}\left(d, \eta, a_1\right)^{\gamma} U\right\},$$

which implies that the optimal values of (d, η, a_1) are time invariant (i.e., the planner will choose the same contract in every period). Using this fact, we can further simplify the planner's objective function to

$$\max_{(d,\eta,a_1)} \frac{\left(\phi z + (1-\phi)\, v\left(d, \eta, a_1\right)\right)(1-\theta)^{\gamma} k_1^{\gamma}}{1 - \beta \overline{g}\left(d, \eta, a_1\right)^{\gamma}}. \tag{8}$$

Note that $\overline{g}\left(d, \eta, a_1\right)$ is necessarily less than the maximal feasible growth rate of the economy, and therefore

[21] The upper bound on the growth rate of capital implies that we can find an upper bound for the sequence of partial sums, and therefore this equation follows from the Dominated Convergence Theorem.

the expression in 8 is a finite number.

We want to ask under what conditions a social planner maximizing 8 would choose a contract that allows bank runs to occur. In particular, for a given value of β, we want to find the cutoff value $\widehat{\pi}$ such that the planner would choose a run-proof contract for any $\pi \geq \widehat{\pi}$. We solve 8 numerically, imposing a grid of possible values for (d, η, a_1) and using monte carlo simulation to approximate $\overline{g}$ for each possible contract. The equilibrium described above (see Table 1) corresponds to the planner's allocation when $\beta = 0$, in which case $\widehat{\pi} = 11.9\%$ holds. Using these same parameter values, Figure 2 plots $\widehat{\pi}$ for a grid of values of β between zero and $\overline{\beta}$.

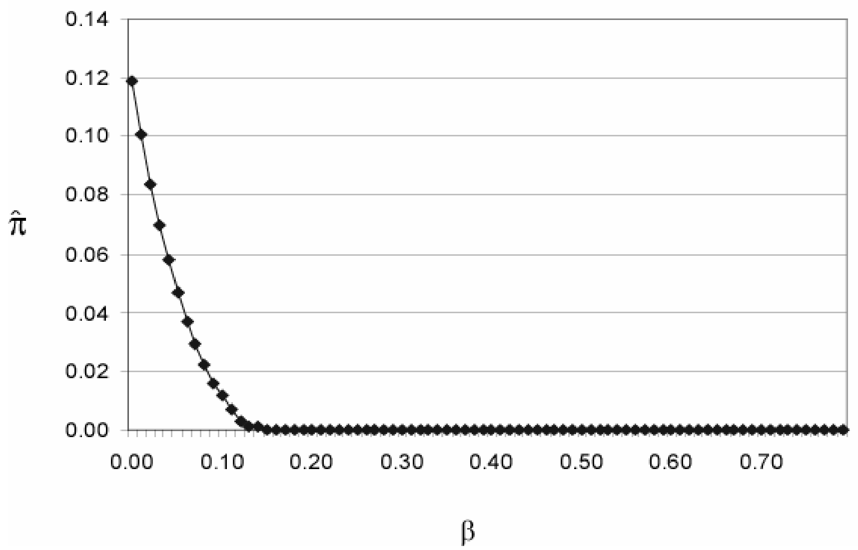

Figure 2: The cutoff value $\widehat{\pi}$ for different social discount rates

The figure shows how the cutoff value falls rapidly as β is increased. In other words, even a very impatient planner is substantially less tolerant of bank runs than are banks in the competitive economy. There are two effects that combine to generate this result. First, a more patient planner will set a higher average growth rate. This is achieved by placing more assets into investment and by lowering the return on early withdrawals. As discussed above (see especially footnote 18), these are precisely the features of the best run-proof contract. Hence the contract chosen by the planner will be closer to being run proof than the equilibrium contract is, which implies that the planner sees a lower cost of choosing a run-proof contract than competitive banks do. Second, a more patient planner also assigns a higher benefit to eliminating runs, since much of the burden of a run falls on generations in the distant future. These two effects combine to imply that a more patient social planner will switch to a run-proof contract for a lower value of π, as the figure demonstrates.[22]

For values of β above 0.15, the optimal contract when $\pi = 0$ holds is already run-proof. That is, even if agents' beliefs are such that bank runs will not occur, a planner who is at least this patient will choose a contract that gives a low enough payment to early withdrawers to eliminate the run equilibrium. Again interpreting a period in the model as representing five years, this value of β corresponds to an annualized discount factor of 0.684, or annual discounting of almost 32%. We interpret this as evidence indicating that, at least in the context of this example, it is very likely that bank runs would not exist in the allocation chosen by a "reasonably" patient social planner.

4 Risk-Factor-Based Equilibrium Selection

So far we have taken the standard sunspots approach to equilibrium selection, where a bank takes the probability of a run as exogenously given and believes that it cannot influence the probability by changing the composition of its portfolio. However, it seems intuitively plausible that the portfolio chosen by banks might actually be a useful predictor of the likelihood of a run. In other words, if banks choose a contract that is "closer" to being run proof, it seems reasonable to think that the probability of having a run might go down. In this section, we examine a more general ESM where the probability of a run can vary continuously with the deposit contract chosen by banks. It seems entirely reasonable to think that the outcome of the post-deposit game can depend on the relative payoffs obtained by an agent in the alternative scenarios, even when both outcomes are Nash equilibria. This is, in fact, precisely the idea that motivates the use of risk dominance as an equilibrium selection mechanism (Harsanyi and Selten [14]).[23] However, risk dominance selects a single equilibrium for each contract that the bank could choose. In other words, whereas the sunspots approach assigns a fixed probability to the run equilibrium (whenever it exists), risk dominance assigns a probability of either zero or one, depending on the contract. We find this unappealing because it implies that the bank can rule out runs entirely by choosing a contract that makes the no-run equilibrium barely risk dominant, while a very similar contract would lead to a run with certainty. In our approach, a run is more likely to occur when the equilibrium is risk dominant, but still can occur when it is not. In other words, we keep the idea that under certain circumstances a bank run is, to some extent, a chance event. However, the likelihood of this chance event now depends on the contract offered by banks. Starting

[22] Paal and Smith [16] use this type of approach to examine optimal monetary policy in an environment similar to ours. They investigate a trade-off between current-period insurance and long-run growth that closely resembles the one studied here.

[23] See Temzelides [18] for an evolutionary justification of using risk dominance as the equilibrium selection mechanism in a model with bank runs.

from any contract that permits a run equilibrium, slightly reducing the relative payoff of running will slightly reduce the probability of a run. Of course, there are still contracts for which a bank run cannot happen, or for which only a run can happen; these are the situations where only one equilibrium exists in the post deposit game. However, we believe that when both equilibria exist, each one can obtain and therefore should be assigned positive probability. Hence our approach retains the probabilistic property of the standard sunspots approach while allowing the portfolio decision of banks to influence the probability of a run.

An important question that arises under this approach is whether the probability of a run on an individual bank is determined by the economy-wide average contract or by the contract offered by that particular bank. We know from the previous section that an individual bank can rule out a run on its assets by choosing a run-proof contract. If the bank does not choose a run-proof contract, however, it is not clear whether the probability that it faces a run should be determined by the "stability" of the banking system as a whole or by the individual bank's actions. We call the former case *systemic* run, and the latter *idiosyncratic* run. In the systemic case, all banks experience a run with the same probability, which is determined by the average deposit contract in the economy. Individual banks behave competitively in that they take this average, and hence the probability of a run, as given. This implies that individual banks behave exactly as in the previous section; they maximize the expected utility of depositors for a given value of π. The difference is that the probability of a run is no longer exogenous; rather, it is determined by the rational-expectations condition that requires the probability of a run that banks take as given to be equal to the probability assigned by the ESM to the contract that they all choose.

In the idiosyncratic case, on the other hand, the probability that an individual bank faces a run is determined by its own deposit contract. As above, depositors in all banks believe that low realizations of the sunspot signal s_t means "run." Bank i's contract determines the cutoff value of π_i such that a run on that bank occurs for $s_t \leq \pi_i$. In equilibrium, all banks will choose the same contract and hence either all banks will experience a run or none will. In this sense the runs will appear to an outside observer to be systemic. The critical difference is that banks in the idiosyncratic case will internalize the effect of the deposit contract they offer on the probability of a run, and this will lead to a lower equilibrium probability of a run.[24] In this section we compare the equilibrium of an economy where runs are systemic with that of an economy where runs are idiosyncratic.

We formalize the dependence of the probability of a run π on the properties of the deposit contract by assuming that π is a decreasing function of the risk factor of the run equilibrium. We begin by defining the

[24] The equilibrium with idiosyncratic runs can also be interpreted as at the within-period-t social optimum when runs are systemic. Because of the exernalities present in the systemic case, all depositors would be made better off if all banks switched to this contract.

risk factor.[25]

Definition: The *risk factor of the run equilibrium* is the smallest probability ρ such that if a patient agent believes that all other agents will run with probability strictly greater than ρ, then running is her unique optimal action.

Roughly speaking, the risk factor measures how "willing" a patient agent is to run to the bank early when she is uncertain about the actions of other patient agents. When she is deciding whether or not to run, a patient agent still does not know the size of the aggregate liquidity shock u_t, which determines the payoff she will receive if she waits. She does, however, have one piece of information: her preferences were drawn from a Bernoulli distribution whose parameter is u_t. Therefore she updates her prior beliefs about u_t (given by the density function f from which u_t was drawn) based on this observation using Bayes' rule. Following Peck and Shell [17], we use f_p to denote the posterior distribution, which represents the belief of every patient agent. The risk factor of the run equilibrium is then determined by the following expression,

$$\rho\left[\overline{\overline{u}}v_e+\left(1-\overline{\overline{u}}\right)v_0\right]+(1-\rho)\left[\int_0^{\overline{\overline{u}}}v_e f_p(u)\,du+\int_{\overline{\overline{u}}}^1\left[\frac{\overline{\overline{u}}}{u}v_e+\left(1-\frac{\overline{\overline{u}}}{u}\right)v_0\right]f_p(u)\,du\right]$$

$$=\rho v_0+(1-\rho)\left[\int_0^{\overline{\overline{u}}}v_\ell f_p(u)\,du+\int_{\overline{\overline{u}}}^1 v_0 f_p(u)\,du\right],$$

where the v_j terms are as defined in the previous section. The left-hand side of this expression is the expected value for a patient agent of running to the bank when she believes that with probability ρ everybody else will run. The right-hand side is the expected value of not running given the same belief. The expression says that if a patient agent assigns probability ρ to the event of a run on the bank, she is indifferent between running and not running. If she assigns a higher probability to a run, she would strictly prefer to run. We assume that the higher the risk factor of the run equilibrium, the lower the probability of a run. Whatever determines the individual agent's prior belief about the possibility of a run, the higher the risk factor ρ, the lower is the likelihood that this belief will be greater than ρ, and hence the lower is the likelihood that the agent would decide to run.

Holding other things constant, the risk factor of the run equilibrium is decreasing in the return offered on period t withdrawals a_1. Higher values of a_1 increase the incentive for agents to withdraw their funds from the bank early. Higher values also make not running less attractive because even if there is no run, the bank will have fewer resources in period $t+1$ and hence a_2 will be lower. The relationship between the risk factor and η, the fraction of the bank's portfolio that is in storage, is not monotonic. If η is very

[25] See Young [20] for an extended discussion of risk factors and risk dominance.

low, then with high probability the realization of u_t will be such that investment is liquidated. In such a situation, increasing η decreases the amount of liquidation and therefore increases the amount of resources available in the second period. This makes waiting a more attractive strategy, and as a result the risk factor is increasing in η. If, on the other hand, η is very high, then with high probability the realization of u_t will be such that no investment is liquidated. In this case *decreasing* η would increase the resources available in the second period, and hence the risk factor is decreasing in η. For moderate values of η, the risk factor is fairly flat and the effects of a change in a_1 will typically dominate the effects of a change in η.

In earlier work, we have shown how an adaptive learning process in a stochastic environment naturally generates an equilibrium selection mechanism in which the probability of an equilibrium is strictly decreasing in a stochastic version of its risk factor (see Ennis and Keister [9]). Because our goal here is to examine the basic implications that follow from this approach, the exact relationship between the risk factor of the run equilibrium and the probability of a run is not very important. Rather than specifying a learning model, we posit a simple linear relationship of the form

$$\pi(\rho) = m - h \cdot \rho,$$

where m and h are constants that allow us to calibrate the equilibrium probability π to a reasonable number. Note that when $h = 0$ holds, this equilibrium selection mechanism reduces to the standard sunspots approach studied earlier. We use this specification to compare the equilibrium of two different economies, one where bank runs are systemic and the other where runs are idiosyncratic. We continue our analysis using the example introduced in the previous section. We assume that $m = 0.1$ and $h = 0.06$.[26] Table 3 shows the equilibrium deposit contracts and growth rates for the two economies.

Table 3

	d^*	η^*	a_1^*	π^*	Growth Conditional on No Run	Unconditional Growth
Idiosyncratic Runs	0.953	0.415	1.037	0.0797	5.33%	4.97%
Systemic Runs	0.952	0.487	1.195	0.0875	4.28%	3.87%

As expected, when runs are idiosyncratic banks choose a contract that implies a higher risk factor for the run equilibrium and thereby lowers the probability of a run π^*. A bank does this by lowering a_1 and η. These adjustments in the contract may seem counterintuitive at first, but they are a direct consequence of

[26] If the slope of the function π is very high, the bank will have an incentive to make ρ high enough to eliminate the runs. We are interested in studying situations where runs are possible and we hence calibrate the value of h to be relatively small. Note that with these parameter values, the ESM is not continuous at the endpoints. Moving from a contract that is run proof to one with a risk factor slightly less than one, for example, will lead to a jump in the value of π from zero to 0.05. By making π a nonlinear function of ρ, we could easily make the ESM continuous without changing the results that we report below. We view the linear function used here as a local approximation that is valid as long as ρ does not change too much.

the equilibrium selection process we are considering. By lowering a_1 and η, the bank lowers the contingent payoff from running to the bank and increases the payoff of waiting to withdraw in the second period of life. The lower value of a_1 allows the bank to put more resources into investment without having to liquidate more often. This leads to a larger return for agents who arrive in the second period. Notice that this is the opposite of the narrow-banking proposal of Friedman [12]. Friedman argued that demand deposits should be backed entirely by safe, short-term assets (such as storage in our model). This would enable a bank to meet all of its obligations during a run, which would in turn prevent a run from happening. The problem with this approach is that preventing the bank from undertaking investment is costly because investment offers a much higher return than storage (see Wallace [19]). Our analysis shows that a better approach for a bank facing the possibility of a run is to structure the contract to reward agents heavily for waiting. This involves holding *fewer* liquid assets and putting more resources into (illiquid) investment.

The last two columns give the implications of these differences in the equilibrium deposit contract for the growth rate of the economy. The economy with idiosyncratic runs has both a higher level of investment and a lower return on early withdrawals. These two facts tend to increase the growth rate of the economy in periods without a run. In such periods, the economy with idiosyncratic runs grows at an average rate that is 105 basis point higher than the economy with systemic runs. Furthermore, the economy with idiosyncratic runs has a lower equilibrium probability of a run. Taking this into account, the economy with idiosyncratic runs grows on average around 110 basis points faster.

In Tables 4 and 5 we further study the economy with idiosyncratic runs. Table 4 shows that when h (the sensitivity of the equilibrium selection function to the risk factor of the run equilibrium) is higher, a bank chooses less liquidity and a lower return on early withdrawals. These two changes have the direct effect of increasing capital formation and hence increasing the growth rate of the economy. In addition, the change in the contract reduces the probability of bank runs, and this further increases the long-run average growth rate. In other words, the more influence an individual bank's portfolio has on the likelihood of runs, the faster the economy will grow.

Table 4

(m,h)	d^*	η^*	a_1^*	π^*	Growth
$(0.1, 0.06)$	0.953	0.415	1.037	0.0797	4.97%
$(0.1, 0.07)$	0.954	0.405	1.019	0.0754	5.15%

In Table 5 we present the equilibrium outcome for different values of the return on the investment technology and the liquidation cost.

Table 5

(R,x)	d^*	η^*	a_1^*	π^*	Growth
$(3.00, 0.3)$	0.953	0.415	1.037	0.0797	4.97%
$(3.25, 0.3)$	0.958	0.397	1.007	0.0773	6.94%
$(3.00, 0.4)$	0.962	0.381	1.056	0.0807	5.37%

The growth rate is higher for higher values of R for several reasons. The first is apparent from 6: when a fixed amount of investment yields more capital, the economy will grow faster. The second reason is also standard: when investment offers a higher return, banks will choose to invest more. However, in our model there is also a third effect: an increase in investment implies that banks are giving a relatively higher payoff to agents in the second period, and hence the risk factor of the run equilibrium is higher. Thus the change in banks' portfolios also decreases the probability of a bank run, which increases the long-run average growth rate even more. In this way the model provides an amplification mechanism for differences in the productivity of investment.

In economies where the liquidation cost of investment is lower (higher values of x), a bank would again choose to place more resources in investment. However, the bank would increase the return for early withdrawals because early liquidation is less costly. This second effect tends to decrease the risk factor of running, and hence in this case the probability of a run increases. Here the two effects are pointing in opposite directions with respect to long-run growth: more investment tends to raise the average growth rate, while a higher frequency of bank runs lowers it. In our example the first effect dominates, because the movement in η is larger than the change in π. As a result, the long-run average growth rate goes up.

5 Conclusion

In this paper, we bring together two major strands of the macroeconomic literature: we study the implications of bank runs for long-run economic growth. We identify three important ways in which the possibility of a bank run affects the growth process: (i) agents tend to lower their participation in the banking system, creating a disintermediation effect, (ii) banks tend to adjust their portfolio of investments towards more liquid, less productive assets, and (iii) when a bank run occurs, early liquidation of investment reduces capital formation. These three effects make both the occurrence and the mere possibility of bank runs detrimental for economic growth. We also show how the cost of a bank run today falls primarily on future generations. As a result, there is a large intergenerational externality and even a fairly impatient social planner would choose to implement arrangements that avoid bank runs altogether. Finally, we examine a model where an individual bank can influence the probability that it experiences a run by changing its portfolio in a way that makes running less attractive to depositors. We show that in this case, banks choose *lower* levels of liquidity, which increases the rate of economic growth.

References

[1] G. Antinolfi, T. Keister, and K. Shell, Growth dynamics and returns to scale: Bifurcation analysis, *J. Econ. Theory* 96 (2001), 70-96.

[2] V. Bencivenga, and B. D. Smith, Financial intermediation and endogenous growth, *Rev. Econ. Stud*. 58 (1991), 195-209.

[3] J. H. Boyd, P. Gomis, S. Kwak, and B. D. Smith, A user's guide to banking crises, mimeo., January 2001.

[4] J. H. Boyd, S. Kwak, and B. D. Smith, The real output losses associated with modern banking crises, or "The good, the bad, and the ugly," mimeo., February 2002.

[5] G. Caprio Jr. and D. Klingebiel, Bank insolvencies: Cross country experience, World Bank Policy Research Working Paper 1620, July 1996.

[6] R. Cooper and T. W. Ross, Bank runs: Liquidity costs and investment decisions, *J. Monet. Econ*. 41 (1998), 27-38.

[7] D. W. Diamond and P. H. Dybvig, Bank runs, deposit insurance, and liquidity, *J. Pol. Economy* 91 (1983), 401-419.

[8] H. M. Ennis, Economic fundamentals and bank runs, Federal Reserve Bank of Richmond *Economic Quarterly*, forthcoming.

[9] H. M. Ennis and T. Keister, Government policy and the probability of coordination failures, mimeo., ITAM and Federal Reserve Bank of Richmond, February 2003.

[10] H. M. Ennis and T. Keister, Aggregate demand management and equilibrium selection, mimeo., ITAM and Federal Reserve Bank of Richmond, June 2002.

[11] S. Freeman, Banking as the provision of liquidity, *J. Bus*. 61 (1988), 45-64.

[12] M. Friedman, *A program for monetary stability*, Fordham University Press, New York, 1959.

[13] X. Freixas and J.-C. Rochet, *Microeconomics of Banking*, MIT Press, Cambridge, MA, 1997.

[14] J. Harsanyi, and R. Selten, *A General Theory of Equilibrium Selection in Games*, MIT Press, Cambridge, MA, 1988.

[15] J. A. Miron, Financial panics, the seasonality of the nominal interest rate, and the founding of the Fed, *Amer. Econ. Rev*. 76 (1986), 125-140.

[16] B. Paal and B.D. Smith, The sub-optimality of the Friedman rule and the optimum quantity of money, mimeo., University of Texas at Austin, October 2000.

[17] J. Peck and K. Shell, Equilibrium bank runs, *J. Pol. Economy* 111 (2003), 103-123.

[18] T. Temzelides, Evolution, coordination, and banking panics, *J. Monet. Econ*. 40 (1997), 163-183.

[19] N. Wallace, Narrow banking meets the Diamond-Dybvig model, *Fed. Reserve Bank Minneapolis Quart. Rev*. 20 (1996), 3-13.

[20] H. P. Young, *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*, Princeton University Press, Princeton, NJ, 1998.