Working Paper Series

A Road Map for Efficiently Taxing Heterogeneous Agents

WP 13-13R

Marios Karabarbounis
Federal Reserve Bank of Richmond

THE FEDERAL RESERVE BANK OF RICHMOND
RICHMOND ▪ BALTIMORE ▪ CHARLOTTE

A Road Map for Efficiently Taxing Heterogeneous Agents*

Marios Karabarbounis
Federal Reserve Bank of Richmond

June 2, 2015
Working Paper No. 13-13R

Abstract

This paper characterizes optimal labor-income taxes that depend on age, household assets, and filing status (one or two earners) within a life-cycle model with heterogeneous, two-member households and endogenous human capital. The key innovation is a labor supply elasticity that varies endogenously among households. I find that tax distortions should be hump shaped in age, decrease in household assets, and be lower for joint relative to single filers. Age and assets act as complements within the optimal tax policy. In contrast, filing status neither complements nor crowds out the age and asset tag. Overall, a tax system using all three tags can increase consumption up to 6.4% and welfare up to 1.5%.

JEL Codes: E2; H21; H31.
Keywords: Heterogeneous Agents; Labor Supply Elasticity; Life Cycle; Optimal Taxation.

*Contact information: Federal Reserve Bank of Richmond, Research Department, 701 Byrd St., Richmond, VA, 23219; email: marios.karabarbounis@rich.frb.org. I thank Richard Rogerson and three anonymous referees for extremely valuable advice. I would also like to thank Yongsung Chang, Jay Hong, Yan Bai, Rudi Bachmann, Mark Bils, Nezih Guner, Ellen McGrattan, Jose-Victor Rios-Rull, Juan M. Sanchez, Gustavo Ventura, and seminar participants at ASU, Universitat Autonoma de Barcelona, Ecole Polytechnique, Federal Reserve Bank of Minneapolis, Federal Reserve Bank of Richmond, Federal Reserve Bank of St. Louis, SED Cyprus, and Vanderbilt. Earlier versions of this paper circulated under the title "Heterogeneity in Labor Supply Elasticity and Optimal Taxation." The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System. All errors are my own.

1 Introduction

Standard public finance principles imply that the government should decrease tax distortions on workers with a larger value of labor supply elasticity. To distinguish between workers of low and high labor supply elasticity, the government can use information on their observable characteristics. For example, a worker closer to retirement is more likely to quit her job if her taxes increase, as is a worker who is not the only financial provider in the household. This paper examines the potential of a tax system that depends on such personal characteristics: age, household assets, and household composition (one or two earners).

This is not the first paper to highlight the potential of tagging as part of an optimal tax policy. One set of studies focuses solely on the insurance and redistributive aspects of these policies (for example, Weinzierl (2011), Farhi and Werning (2013), and Golosov, Troshkin, and Tsyvinski (2015), who analyze optimal age- and asset-dependent taxation). In contrast, this paper explores an additional, perhaps more natural, motivation for tax tagging: the heterogeneity in labor supply elasticity. On the other hand, papers that do underline differences in labor supply elasticity usually focus on a single tag (for example, Guner, Kaygusuz, and Ventura (2012b) on taxing based on gender). This paper analyzes jointly multiple tags (age, household assets, and filing status) allowing one to assess if these tags act as substitutes or complements within the optimal policy.

The key modeling assumption that generates differences in labor supply elasticity is an extensive margin of labor supply coupled with rich heterogeneity. Heterogeneity is introduced through i) a life-cycle dimension, ii) permanent and temporary uninsurable labor productivity shocks, iii) endogenous human capital accumulation (learning-by-doing), and iv) households with two (potential) earners, a male and a female.[1] Changes in tax rates will affect only those workers whose reservation wage is sufficiently close to the market wage, the marginal

[1]This framework is related to heterogeneous-agent life-cycle models of labor supply with a single earner (Rogerson and Wallenius, 2009; and Erosa, Fuster, and Kambourov, 2013) or two earners (Guner, Kaygusuz, and Ventura, 2012a).

workers. Hence, heterogeneity in labor supply elasticity arises endogenously in the model from differences in reservation wages. These results stem from the important insights of Hansen (1985), Rogerson (1988), and Chang and Kim (2006).

The main experiment replaces the current U.S. labor-income tax code with a revenue-neutral labor-income tax system that depends separately or jointly on age, household assets, and filing status (single or joint). The objective is to maximize the welfare of the newborn household at the new steady state.

The first finding is an optimal tax schedule that is hump shaped in age. Young households receive tax cuts since they have little experience (and thus receive lower wages). Middle-aged households are strongly attached to their jobs, so larger distortions have a small efficiency loss. In contrast, older households are very sensitive to tax changes. Households older than age 60 have a Frisch elasticity of labor supply (at the extensive margin) around 1.4, much larger than the average of 0.7. By decreasing their tax rates, the new system encourages these households to delay retirement.

This is a new finding in the literature of age-dependent taxation. Weinzierl (2011), Farhi and Werning (2013), and Golosov, Troshkin, and Tsyvinski (2015) find an increasing labor wedge to be optimal, i.e., to decrease distortions on younger and increase tax rates on older workers. In my model, older households are more likely to retire early if their taxes increase. In this sense, this paper is closer to the findings of Conesa, Kitao, and Krueger (2009), who argue in favor of high capital taxation to implicitly tax very elastic old workers less.[2]

The second finding is an optimal labor-income tax that decreases (linearly) in household assets. This negative dependence has also been analyzed in Kocherlakota (2005), Albanesi and Sleet (2006), and Kitao (2010). The novelty in my case is the strong complementarity

[2]Their intuition is based on Erosa and Gervais (2002), who make an argument for tax rates that should follow the life-cycle labor supply profile. Both Erosa and Gervais (2002) and Conesa, Kitao, and Krueger (2009) choose a utility specification that allows the labor supply elasticity to vary inversely with working hours. In contrast, in my model, endogeneity in labor supply elasticity arises naturally through the presence of an extensive margin of labor supply and uninsurable idiosyncratic labor income shocks.

between age- and asset-dependent taxes. Although a single tag on age increases welfare by 0.4% and a single tag on household assets by 0.1%, a joint tag on age and household assets increases welfare by 1.0%. When age tagging is also used, tax rates decrease not for all wealthier households but only for those closer to retirement.

The third finding is a large tax subsidy toward households with two earners. Given the new incentives, most of the single-earner households switch to a two-earner household, while only a small fraction switch to nonemployment. These effects reflect the large labor supply elasticity for the secondary earner and the relatively lower elasticity for the primary earner. When age (or household assets) is used together with filing status, the gains are almost equal to the additive sum of the separate policies. Hence the policies do not interact but do not crowd out each other either.

The gains associated with these reforms turn out to be large. Compared to the current U.S. economy, capital increases up to 19.7%, and total supply of labor, measured in efficiency units, increases up to 2.7%. Consumption can increase up to 6.4% and welfare up to 1.5%.

This paper also contributes to the growing literature analyzing the relation between human capital accumulation and optimal taxation. Peterman (2012) analyzes labor- and capital-income taxes in the presence of human capital, while Kapicka and Neira (2015) study optimal tax policies with an emphasis on risky human capital accumulation. Krueger and Ludwig (2013) analyze progressive taxation with endogenous education decisions. da Costa and Santos (2013) study age-dependent taxes with both learning-by-doing and learning-or-doing human capital accumulation. Finally, Stantcheva (2015) derives optimal tax policies considering insurance, efficiency, and human capital investment motives.

In contrast to these papers, my model emphasizes the effect of endogenous human capital accumulation on labor supply elasticity. Endogenous human capital accumulation decreases the sensitivity of very young and very old households to tax changes. Hence, without endogenous human capital, the hump shape of age-dependent taxes becomes more pronounced.

The paper is organized as follows. Section 2 develops a static example to highlight the main idea of the paper. Section 3 sets up the model. Section 4 describes the quantitative specification of the model. Section 5 describes the main tax experiment and Section 6 different model specifications. Finally, Section 7 concludes.

2 Static Model

This section builds a simple static model of labor supply to explain how a simple policy reform can increase participation in the labor market. Each household has only one agent i who is endowed with asset holdings a and has preferences over consumption c and hours worked h:

$$U = \max_{c,h} \left\{ \log c + \psi \frac{(1-h)^{1-\theta}}{1-\theta} \right\} \tag{1}$$

subject to

$$c = w(1-\tau)h + (1+r)a \tag{2}$$

where w is the wage rate per effective unit of labor, τ is the proportional tax rate, r is the real interest rate, and a_i is i's initial asset holdings. The parameter ψ defines the preference toward leisure and θ the intertemporal substitution of labor supply.

Intensive Margin Adjustments Intensive margin adjustment is how much existing workers change their labor supply supply in response to a wage variation. Worker i equates the marginal rate of substitution between consumption and leisure to the real wage rate.

$$\psi(1-h)^{-\theta} = \frac{w(1-\tau)}{c} \tag{3}$$

The optimal supply of hours h^* will depend on initial asset holdings. If worker i has a lot of assets she will buy more leisure and work less (income effect). The (intensive) Frisch elasticity of labor supply for i is given by:

$$\frac{1}{\theta}\frac{(1-h^*)}{h^*}. \tag{4}$$

The preference specification makes the intensive margin labor supply elasticity endogenous to working hours. Agents working many hours will respond more inelastically than those working a few hours. Hence the amount of heterogeneity in the intensive margin elasticity of labor supply will depend on the distribution of hours across workers.

Extensive Margin Adjustments The extensive margin of labor supply is defined by how many people enter or exit the labor market in response to wage variations. To make the extensive margin active, I assume that workers have to pay a fixed cost FC every working period. This cost will not affect the optimal choice of hours but will affect the decision to be employed in the first place. Worker i with initial asset holdings a will participate if the value of employment $V^E(a)$ is at least as large as the value of being unemployed $V^U(a)$. These two are given by:

$$V^E(a) = \log(w(1-\tau)h^* + (1+r)a) + \psi \frac{(1-h^*)^{1-\theta}}{1-\theta} - FC \tag{5}$$

$$V^U(a) = \log((1+r)a) + \psi \frac{1^{1-\theta}}{1-\theta}. \tag{6}$$

The reservation wage is the wage net of taxes that makes the agent indifferent about working or not. It is given by:

$$w^R(a) = \frac{(1+r)a}{h(a)} \left[\exp\left\{ -\psi \frac{(1-h^*)^{1-\theta}}{1-\theta} + const \right\} - 1 \right] \tag{7}$$

where $const = \psi \frac{1^{1-\theta}}{1-\theta} + FC$. Participation amounts to $w(1-\tau) > w^R$. Ceteris paribus, a rich agent will demand a higher wage to enter the labor market. The participation schedule is a step function and consists of three parts. If $w(1-\tau) < w^R$, the worker is not participating. If $w(1-\tau) = w^R$, the worker is indifferent about working or not. If $w(1-\tau) > w^R$, the worker enters the labor market. Worker i's extensive margin elasticity depends on the distance between her reservation wage and the market net wage. If her reservation wage is much lower or much higher than the market net wage, small variations in the market wage will leave the worker unaffected. If her reservation wage is sufficiently close to the market wage, she is very elastic to wage variations. Taking into account both the intensive and the extensive margin, we can construct the labor supply decision

$$l_i^s(w^R(a), w) = \begin{cases} h^* & \text{if } w(1-\tau) \geq w^R(a) \\ 0 & \text{if } w(1-\tau) < w^R(a) \end{cases} \tag{8}$$

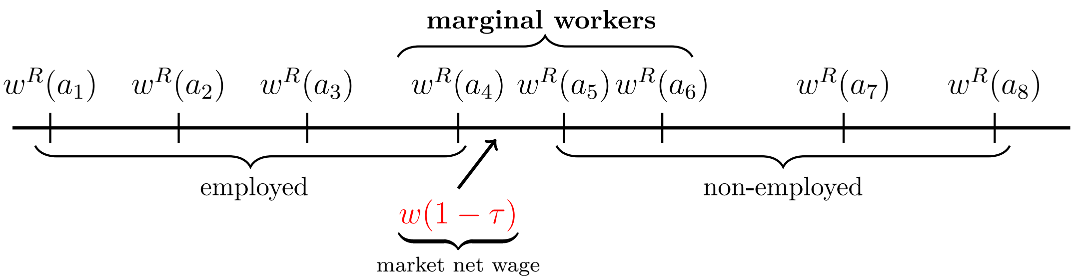
Aggregate Response of Labor Supply Let the distribution of reservation wages be denoted as $\phi(w^R)$. The aggregate labor supply at the market wage w equals total amount of hours supplied: $L^s(w) = \int_0^w l^s(w^R)d\phi(w^R)$. Then, differentiating with respect to the market wage and using the Leibnitz rule, we can decompose the aggregate labor supply elasticity to its intensive margin and extensive margin components.

$$\underbrace{\frac{L'^s(w)w}{L^s(w)}}_{\text{Total Elasticity}} = \underbrace{\frac{\int_0^w l'(w^R)d\phi(w^R)w}{L^s(w)}}_{\text{Intensive Margin Elasticity}} + \underbrace{l^s(w)w\frac{\phi(w)}{L^s(w)}}_{\text{Extensive Margin Elasticity}} \tag{9}$$

The first term at the right-hand side of equation (9) is the aggregate intensive margin elasticity. The magnitude of the response depends on the curvature of the labor supply function l'. The second term at the right-hand side of equation (9) is the aggregate extensive margin elasticity. Its value depends mostly on the distribution of the reservation wages around the market wage $\phi(w)$. If the reservation wage distribution is very concentrated, the ratio $\frac{\phi(w)}{L^s(w)}$ increases and hence the labor supply elasticity increases. The Hansen-Rogerson limit of infinite elasticity is reached if the reservation wage distribution is degenerate. On the other hand a dispersed reservation wage distribution will imply a small aggregate labor supply elasticity.

Figure 1 displays how the model economy works. In this simple example there are eight agents. Each is endowed with initial asset holdings a_i where $a_i < a_j$ with $i < j$. The initial asset holdings distribution will imply a distribution of reservation wages $\phi(w^R(a))$. Low number agents are employed. Their reservation wage is lower than the net market wage. High

Figure 1: Reservation wages and marginal workers



number, wealthy agents are non-employed since the net market wage is not high enough. In this example the employment rate is equal to 50%.

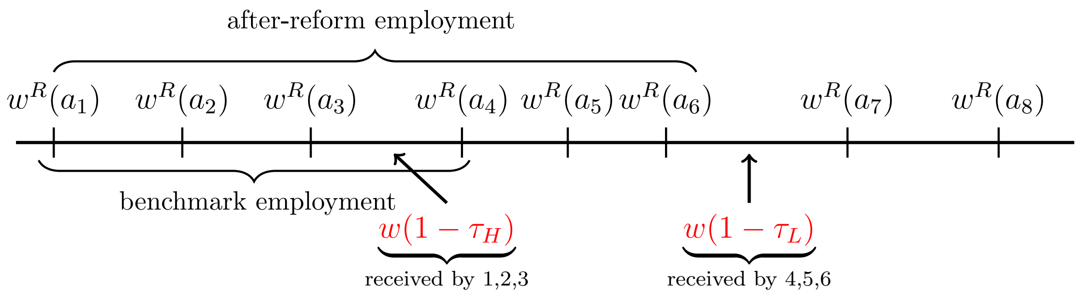
A wage variation will affect mostly agents 4, 5, and 6 whose reservation wages are sufficiently close to the net market wage. These marginal workers have very high labor extensive margin elasticities. The larger the density of workers around the market wage, the larger the aggregate response of the economy to a wage change. Agents 1, 2, and 3 will respond only at the intensive margin. Finally, agents 7 and 8 have very large assets so they cannot be affected by small variations in the market wage. Hence, differences in reservation wages generate heterogeneity in labor supply elasticity.

Tax Reform To distinguish between agents of low and high labor supply elasticity, the government can use information on their asset holdings. An example of such a (revenue-neutral) tax code is the following:

$$\tau(a) = \begin{cases} \tau_H & \text{if } a \leq a_3 \\ \tau_L & \text{if } a > a_3 \end{cases}$$

with $\tau_H > \tau_L$. Under this tax system, workers with low assets who also have a low labor supply elasticity pay higher labor income taxes. Figure 2 describes the outcome. Agents 1, 2, and 3 receive a lower net wage $w(1-\tau_H)$. However, their reservation wages are low enough

Figure 2: Effects of tax reform on employment



to keep them employed. Adjustment will take place only at the intensive margin. Marginal worker 4 continues to work and pays lower taxes. Marginal workers 5 and 6 enter the labor market in response to the tax cuts. Under the new system they receive a higher net wage $w(1-\tau_L)$. Agents 7 and 8 are indifferent to this policy. The new policy increases employment.

3 Fully Specified Dynamic Model

The model is an overlapping generations economy with endogenous savings and labor supply decisions. I focus on a steady state equilibrium and thus abstract from time subscripts.

Demographics The economy is populated by a continuum of households. Each household consists of two members, a male (m) and a female (f). I will use the notation $i = \{m, f\}$. Both household members are assumed to be of the same age j. There are a total of J overlapping generations in the economy, with generation j being of measure μ_j. In each period a continuum of new households is born whose mass is $(1+n)$ times larger than the previous generation. The probability of surviving at year j is s_j. Households that reach the age of J^R have to retire and receive Social Security benefits ss financed by proportional labor taxes τ_{ss}. Household members have the option to exit the labor market earlier, but if they do so, they will not receive Social Security benefits before the age of J^R.[3]

Preferences Households derive utility from consumption (c) and leisure. Both members are endowed with one unit of productive time, which they split between work (h^m and h^f) and leisure. Households' decisions depend on preferences representable by a time separable utility function of the form

$$U = \mathbf{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \prod_{j=1}^{J} s_j \quad \left\{ \log c_j + \psi_j^m \frac{(1-h_j^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h_j^f)^{1-\theta}}{1-\theta} \right\} \right] \tag{10}$$

where β is the discount factor and θ affects the Frisch elasticity of labor supply. Households make a decision of how much to consume/save and how much time each member will allocate to the labor market. In addition, I make the assumption that leisure is valued differently by households at different ages (parameter ψ is age-dependent).

Human Capital Accumulation The model allows for endogenous human capital accumulation for both males and females. A higher level of working experience (denoted κ) implies a higher productivity component ϵ. Similar to Blundell, Costa-Dias, Meghir, and Shaw (2013), I assume that experience and productivity are related through the following function

$$\log \epsilon^i(\kappa) = \chi_1 \log(1 + \chi_0^i + \gamma(\kappa - 1)) \tag{11}$$

where $\kappa = \kappa_{-1} + \mathbf{I}_{\{\text{employed at } -1\}}$. $\mathbf{I}$ is an indicator function that takes the value of 1 if a household member was employed in the previous period and zero otherwise. Parameter χ_0^i depends on gender to reflect the average wage differences between males and females of

[3]If such a case was allowed, early retirees would start retirement with a lower amount of money in their retirement funds than late retirees. This is exactly what happens in this model when early retirees start eating their assets earlier and hence have a lower amount of money throughout retirement than late retirees. Since both modeling techniques have the same implications about retirees' wealth, I choose the simpler modeling assumption.

similar labor market experience. Parameter χ_1 affects the returns to labor market experience. An employed household member will move to a higher experience level with a probability $p = \frac{1}{\gamma}$. γ is a scale parameter equal to $\frac{J-J^R}{N_{hc}}$, where N_{hc} is the number of grid points set for κ and $J - J^R$ is the maximum amount of working years.

Productivity Every period, household members receive a wage $\hat{w}$ that depends on the prevailing market wage w, their skill z, their experience κ, and a persistent idiosyncratic shock x. Skills are distributed across households as $\log(z) \sim N(0, \sigma_z^2)$. I assume that household members share the same level of skill.[4] Moreover, each household member draws an idiosyncratic shock that follows an AR(1) process in logs:

$$\log x_j = \rho \log x_{j-1} + \eta_j, \qquad \text{with} \quad \eta_j \sim \text{ iid } N(0, \sigma_\eta^2). \tag{12}$$

Following Attanasio, Low, and Sanchez-Marcos (2008), I assume that both household members draw from the same process. However, the specific realization of x may very well differ between members. As usual, the autoregressive process is approximated using the method developed by Tauchen (1986). The transition matrix, which describes the autoregressive process, is given by $\Gamma_{xx'}$. Summing, the natural logarithm of wage for member i living in a household of skill type z is given by

$$\log \hat{w}^i = \log w + \log z + \log \epsilon^i + \log x^i. \tag{13}$$

Asset Market and Borrowing Constraints The asset market is incomplete. From an empirical standpoint, incomplete markets support the evidence that consumption responds to income changes. At the same time, in the absence of state-contingent assets agents use labor effort to insure against negative labor income shocks. This mechanism lowers the correlation

[4]There is ample evidence that schooling decisions of husband and wife are positively correlated. Pencavel (1998) reports that the odds of being married to someone with the same schooling level are 1.03 and the odds of being married to someone with almost the same years of schooling are 8.62.

between hours and wages, a pattern well documented in the data (Low, 2005, and Pijoan-Mas, 2006). With this in mind, I restrict the set of financial instruments to a risk-free asset. In particular, households buy physical claims to capital in the form of an asset a, which costs 1 consumption unit at time t and pays $(1 + r)$ consumption units at time $t + 1$. r is the real interest rate and will be determined endogenously in the model. Moreover, households are allowed to borrow up to a limit $\underline{a}$. This assumption can greatly affect labor supply responses.[5] In the model, saving takes place for three reasons. Households wish to smooth consumption across time (intertemporal savings motive), to insure against labor market risk (precautionary savings motive), and to insure against retirement (life-cycle savings motive).

Production There is a representative firm operating a Cobb-Douglas production function. The firm rents labor efficiency units and capital from households at rate w (the wage rate per effective unit of labor) and r (the rental rate of capital), respectively. Capital depreciates at rate $\delta \in (0, 1)$. The aggregate resource constraint is given by

$$C + (n + \delta)K + G = f(K, L) \tag{14}$$

where C is aggregate consumption, K is aggregate capital, and L is aggregate labor measured in efficiency units. G represents government expenditures.

Government The government operates a balanced pay-as-you-go Social Security system. Households receive Social Security benefits ss that are independent of the members' contributions and are financed by proportional labor taxes τ_{ss}. This payroll tax is taken as exogenous. Moreover, the government collects the accidental bequests. The government pays any existing debts (due to unexpected mortality) using part of the bequests and uniformly distributes the rest to living households. These transfers are denoted Tr.

[5] According to Domeij and Floden (2006) borrowing constrained individuals can smooth their consumption only by increasing their labor supply. Hence, in the presence of borrowing constraints the labor supply elasticity is downward biased.

The government needs to collect revenues in order to finance a given level of government expenditures G. To do so it taxes consumption, capital, and labor. Consumption and capital income taxes (τ_c and τ_k, respectively) are proportional and exogenous. Tax rates are computed based on a household's total pre-tax labor earnings $W = \hat{w}^m h^m + \hat{w}^f h^f$ with $\hat{w} = wz\epsilon x$. Tax rates also depend on the filing status F. Households file a single (S) or a joint (J) tax return based on the number of earners.[6] Following Heathcote, Storesletten, and Violante (2014), I use a nonlinear tax schedule of the form:

$$T_L(W; \mathrm{F}) = W - (1 - \tau_0) W^{1-\tau_1(\mathrm{F})}. \tag{15}$$

If $\tau_1 = 0$, the tax function becomes a proportional tax schedule. For $\tau_1 > 0$, the system becomes progressive. τ_1 depends on filing status F to reflect different marginal tax rates paid by single and joint filers in the U.S. tax system. A high value of τ_0 implies that working agents face both higher average and higher marginal tax rates.

Exogenous Separations Every period each household member faces an exogenous probability of losing their job. These probabilities are denoted λ^m, λ^f for the male and female, respectively.

Fixed Cost and Search Cost Every working period, employed males and females pay a fixed cost. The fixed cost FC_j is expressed in utility terms and depends on age. In addition, non-employed household members at age $j - 1$ have to pay an extra cost in order to work at age j. This is rationalized as a search cost sc_j. Hence, we have to track previous employment status $E_{-1} = \{u, e\}$ for each household member. Search cost also depends on age. In summary,

[6]In reality the U.S. tax system is much more flexible. For example, two-earner households can choose between filing jointly or separately. In addition, households can file jointly even if one spouse has no income. In this paper, for simplicity, I associate single and joint filing status with the number of working members in the household.

the total cost of working is

$$\zeta_j^i(E_{-1}) = \begin{cases} FC_j^i + sc_j^i & \text{if } E_{-1}^i = u \\ FC_j^i & \text{if } E_{-1}^i = e \end{cases}. \tag{16}$$

Household's problem Households are indexed by their skill type and their age (z, j). The state variables are the household's asset holdings a, the stochastic productivity components of its members $\mathbf{x}^i = \{x^m, x^f\}$, the labor market experience of its members $\boldsymbol{\kappa} = \{\kappa^m, \kappa^f\}$, and the previous employment status for each member $\mathbf{E}_{-1} = \{E_{-1}^m, E_{-1}^f\}$.

The household's decision is constrained by the borrowing constraint $a' \geq \underline{a}$ and the nonnegative consumption constraint $c \geq 0$. In the following, I take these constraints as given. In every period, each household member can be employed or non-employed. For each case, the value functions will be denoted $V^{\{E,E\}}, V^{\{E,NE\}}, V^{\{NE,E\}}$, and $V^{\{NE,NE\}}$. Since household members can exogenously lose their jobs, a set of continuation values needs to be defined. In particular, the continuation value if no member loses their job is V^0, if the female does it is V^1, and if the male does it is V^2. These are given by

$$V_{zj}^0 = \max\ \{V_{zj}^{\{E,E\}}, V_{zj}^{\{E,NE\}}, V_{zj}^{\{NE,E\}}, V_{zj}^{\{NE,NE\}}\} \tag{17}$$

$$V_{zj}^1 = \max\ \{V_{zj}^{\{E,NE\}}, V_{zj}^{\{NE,NE\}}\} \tag{18}$$

$$V_{zj}^2 = \max\ \{V_{zj}^{\{NE,E\}}, V_{zj}^{\{NE,NE\}}\} \tag{19}$$

Moreover, the experience levels $\boldsymbol{\kappa}$ evolve stochastically. I denote $\boldsymbol{\kappa}^0 = \{\kappa^m+1, \kappa^f+1\}$, $\boldsymbol{\kappa}^1 = \{\kappa^m+1, \kappa^f\}$, $\boldsymbol{\kappa}^2 = \{\kappa^m, \kappa^f+1\}$, and $\boldsymbol{\kappa}^3 = \{\kappa^m, \kappa^f\}$. The probability of these events taking place are $\boldsymbol{p}^0 = p^2$, $\boldsymbol{p}^1 = \boldsymbol{p}^2 = p(1-p)$, and $\boldsymbol{p}^3 = (1-p)^2$. In the following, I write the value functions for a household employing both members $V^{\{E,E\}}$ and for a household employing only the male member $V^{\{E,NE\}}$. For convenience, the value functions for a household employing the female member $V^{\{NE,E\}}$ and a household with no earners $V^{\{NE,NE\}}$ can be found in Section E in the Appendix.

The value function for a household that employs both members is:

$$
\begin{aligned}
V_{zj}^{\{E,E\}}(a,\mathbf{x},\boldsymbol{\kappa},\mathbf{E}_{-1}) = \max_{c,a',h^m,h^f} & \Bigg\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta} - \sum_{i=\{m,f\}} \zeta(E_{-1}^i) \\
& + \ \beta s_{j+1} \sum_{x_m{}'} \sum_{x_f{}'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \ * \\
& [\ (1-\lambda^m)(1-\lambda^f) \sum_{s=0}^{3} \boldsymbol{p}^s V_{z(j+1)}^0(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E}) \ + \ (1-\lambda^m)\lambda^f \sum_{s=0}^{3} \boldsymbol{p}^s V_{z(j+1)}^1(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E}) \\
& + \ \lambda^m(1-\lambda^f) \sum_{s=0}^{3} \boldsymbol{p}^s V_{z(j+1)}^2(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E}) \ + \ \lambda^m\lambda^f \sum_{s=0}^{3} \boldsymbol{p}^s V_{z(j+1)}^{\{NE,NE\}}(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E}) \ ] \Bigg\}
\end{aligned} \tag{20}
$$

$$
\textbf{s.t.} \qquad (1+\tau_c)c + a' = (1-\tau_{ss})W - T_L(W;\mathrm{J}) + (1+r(1-\tau_k))(a+Tr) \tag{21}
$$

$$
\mathbf{E} = \{e,e\} \tag{22}
$$

The value function for a household that employs only the male member is:

$$
\begin{aligned}
V_{zj}^{\{E,NE\}}(a,\mathbf{x},\boldsymbol{\kappa},\mathbf{E}_{-1}) = \max_{c,a',h^m} & \Bigg\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta} - \zeta(E_{-1}^m) \\
& + \beta s_{j+1} \sum_{x_m{}'} \sum_{x_f{}'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \ * \\
& [\frac{(1-\lambda^m)}{1-p} \sum_{s=\{1,3\}} \boldsymbol{p}^s V_{z(j+1)}^1(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E}) + \frac{\lambda^m}{1-p} \sum_{s=\{1,3\}} \boldsymbol{p}^s V_{z(j+1)}^{\{NE,NE\}}(a',\mathbf{x}',\boldsymbol{\kappa}^s,\mathbf{E})] \Bigg\}
\end{aligned} \tag{23}
$$

$$
\textbf{s.t.} \qquad h^f = 0 \tag{24}
$$

$$
(1+\tau_c)c + a' = (1-\tau_{ss})W - T_L(W;\mathrm{S}) + (1+r(1-\tau_k))(a+Tr) \tag{25}
$$

$$
\mathbf{E} = \{e,u\} \tag{26}
$$

Equilibrium The policy functions for saving, consumption, and hours are denoted g^a_{zj}, g^c_{zj} and g^{hm}_{zj}, g^{hf}_{zj}, respectively. Let $\Phi_{zj}(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1})$ denote the cumulative probability distribution of states $(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1}) \in \Omega$ across households of type (zj).[7] The marginal density is denoted by ϕ.

Given a tax structure $\{\tau_c, T_L, \tau_k, \tau_{ss}\}$ and an initial distribution $\Phi_{z1}(a = 0, \mathbf{x} = \bar{x}, \boldsymbol{\kappa} = \{1,1\}, \mathbf{E}_{-1} = \{u,u\})$ a stationary competitive equilibrium consists of functions $\{V^{\{E,E\}}_{zj}, V^{\{E,NE\}}_{zj}, V^{\{NE,E\}}_{zj}, V^{\{NE,NE\}}_{zj}, g^a_{zj}, g^c_{zj}, g^{hm}_{zj}, g^{hf}_{zj}\}^J_{j=1}$, prices $\{w, r\}$, inputs $\{K, L\}$, benefits $\{ss\}$, transfers $\{Tr\}$ and distributions $\{\Phi_{zj}(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1})\}^J_{j=2}$ s.t.

1. Given prices $\{w, r\}$, benefits $\{ss\}$, and transfers $\{Tr\}$, the functions solve the household's problem.

2. The prices satisfy the firm's optimal decisions, $r = F_K(K, L) - \delta$ and $w = F_L(K, L)$.

3. Capital and labor markets clear

$$K = \sum_{j=1}^{J-1} \mu_{j+1} \int_{\Omega} g^a_{zj}\phi_{zj} \quad \text{and} \quad L = \sum_{j=1}^{J} \mu_j \int_{\Omega} (zx^m \epsilon^m_j g^{hm}_{zj} + zx^f \epsilon^f_j g^{hf}_{zj})\phi_{zj}.$$

4. The Social Security system clears: $\tau_{ss} wL = ss \sum_{j=j^R}^{J} \mu_j$ and the transfers are given by: $Tr = \int_{\Omega} \mu_j(1 - s_j) g^a_j$.

5. The government balances its budget: $G = \tau_c C + \tau_k rK + \sum_{\mathrm{F}=\{\mathrm{S,J}\}} \int_{\Omega} T_L(.;\mathrm{F}) d\phi$.

4 Quantitative Analysis

4.1 Stylized Facts on Life-Cycle Labor Supply

Information on employment rates, average hours of work, and households' assets are based on data from the Panel Study of Income Dynamics (PSID). Moreover, I collect information on

[7] Ω is the state space, defined as $\Omega = A \times \mathbf{X} \times Z \times \Sigma$. $A = [\underline{a}, \overline{a}]$ is the asset space. The lower bound is

reservation wages from the Survey of Income and Program Participation (SIPP). Section A in the Appendix includes a full description of the data. The main findings on life-cycle labor supply are the following:[8]

1. For both males and females, employment rates and hours of work are roughly hump shaped over the life cycle. Changes in hours of work are mainly driven from the extensive margin of labor supply.

2. The probability of being employed at age $j+1$ is very high if a household member is employed at age j. The probability of switching to employment at age $j+1$ for non-employed members at age j is decreasing along the life cycle. This implies that nonemployment becomes an absorbing state.

3. Conditional on age, wealthier households are more likely to employ the male but less likely to employ the female member. Hence, income effects are stronger for the secondary earner.

4. Reservation wages increase in both age and household assets. Females have lower reservation wages than males.

4.2 Calibration

This section describes the calibration of the model. A group of parameters is set based on values commonly used in the literature. The remaining parameters target a set of statistics.

based on our assumption. Since the agents cannot save more than what they earn over their lifetime, I can safely assume an upper bound $\overline{a}$. $\mathbf{X} = R$ is the productivity space for the primary and the secondary earner, and $Z = R$ is the space for the household's skill level. $\Sigma = \{ee, eu, ue, uu\}$ is the set of possible values for the previous employment status of the household's members.

[8] The patterns are consistent with other studies focusing on the life-cycle labor supply of males (Prescott, Rogerson, and Wallenius, 2009; and Erosa, Fuster, and Kambourov, 2013) and females (Attanasio, Low, and Sanchez-Marcos, 2008).

Section B in the Appendix summarizes all parameter values and moment statistics in the data and the model (Table 7).

The model period is set to one year. The agents are born at the real life age of 21 (model period 1) and live up to a maximum real life age of 101 (model period 81). Agents become exogenously unproductive and hence retire at the real life age of 65 (model period 46). The survival probabilities are taken from the life table (Table 4.C6) in Social Security Administration (2005). I use an average of the survival probabilities reported for males and females.

The population growth rate is set to $n = 1.1\%$, the long-run average population growth in the United States. The production function is Cobb-Douglas, $f(K, L) = K^{\alpha}L^{1-\alpha}$, where $\alpha = 0.36$ is chosen to match the capital share. As already noted, preferences are separable in consumption and leisure. Parameter θ, which determines the Frisch labor supply elasticity, is set to 2, based on Erosa, Fuster, and Kambourov (2013).

Parameters χ_0^m and χ_1 are chosen to match the position and slope, respectively, of the age-profile of male real-hourly log wages as estimated from the PSID. I then choose χ_0^f to capture an average female to male hourly wage ratio equal to 0.72.[9]

The tax rates are set based on Imrohoroglu and Kitao (2012) and Kitao (2010). The consumption tax is $\tau_c = 5\%$ and the capital tax rate $\tau_k = 30\%$. The Social Security tax is set at $\tau_{ss} = 10.6\%$. This gives a replacement ratio around 45%. Finally, I estimate the parameters τ_1(S) and τ_1(J) using data from the Current Population Survey (CPS) for single and joint filers respectively, for the time period 1992-2007. The values found are $\tau_1(\text{S}) = 0.073$ and $\tau_1(\text{J}) = 0.065$.

The remaining parameters are chosen to match specific moments. For convenience, I associate each parameter with the moment it affects the most. The discount factor (β) targets a capital-output ratio equal to 3.2. The depreciation rate (δ) targets an investment to output ratio of 0.25. The tax parameter τ_0 is set to guarantee a government spending to output ratio

[9]Blau and Kahn (2000) report the same male-to-female ratio for the periods 1978-1998.

equal to 0.22.

Both household members face an age-dependent fixed cost of working. I use a parametric function $FC_j^i = a_0^i + a_1^i j + a_2^i j^2$, for $i = \{m, f\}$. The larger the fixed cost, the smaller the incentive to be employed. Hence, I choose a's to match the average employment rate at three stages of the life cycle: early working years (ages 21-35), middle ages (35-50), and the rest of working life cycle (ages 51-65). Similarly, to calibrate the utility parameter ψ, I use the average hours of the employed but allow a more flexible parametric form $\psi_j^i = \nu_0^i + \nu_1^i j + \nu_2^i j^2 + \nu_3^i j^3$, $i = \{m, f\}$.

The separation rates (λ^i for $i = \{m, f\}$) target the probability of staying employed. Moreover, the search cost targets the average transition from nonemployment to employment. Intuitively, a large search cost makes re-employment harder. I assume that $sc_j^i = \eta_0^i + \eta_1^i j$ for $i = \{m, f\}$ and use the average transition probability between ages 21-42 and 43-65 as targets.

The borrowing limit $\underline{a}$ targets the fraction of households with negative assets. In the PSID, 8.7% of households have negative net worth. Finally, to pin down the productivity parameters $(\sigma_z, \rho, \sigma_\eta)$, I follow the identification strategy of Storesletten, Telmer, and Yaron (2004). My main target is the life-cycle profile of the variance of log-labor earnings. Using information from the PSID, I find that the variance evolves in a linear manner. The profile starts from 0.27 at age 21 and increases linearly to 0.75 by the age of 65. In this model all agents start off their lives having the same transitory shock x. As a result, any dispersion in labor earnings is caused by the dispersion in the fixed effect z, i.e., by the parameter σ_z. As the cohort ages, the distribution of transitory shocks converges towards its invariant distribution. The variance of log-labor earnings at the stationary distribution is pinned down by the variance of the transitory shock, σ_η. Lastly, the persistence of the transitory shock determines how fast the economy converges to the invariant distribution. The slower the rate, the flatter the slope of the life-cycle variance. This helps pin down ρ.

4.3 Results

This section describes the labor supply patterns generated by the model and compares them to the data. I am interested in the relationships between labor supply and age and labor supply and household assets.

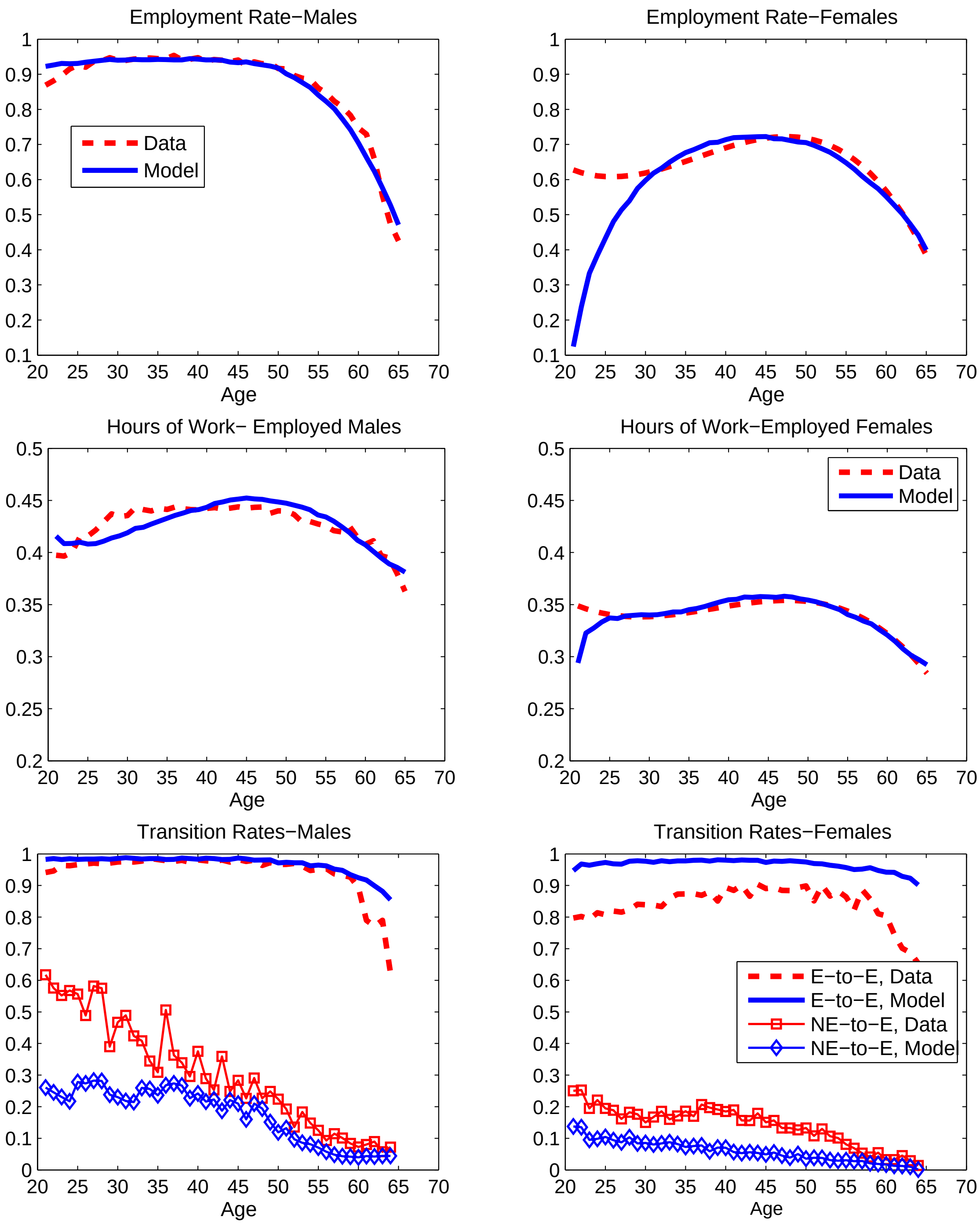
Labor Supply by Age The upper two panels of Figure 3 plot the life-cycle profile of employment rates for males and females, respectively. Similar to the data, the employment rate is very high for males, younger than the age of 50. A long time-horizon makes working (and thus building up human capital) very attractive. As the time horizon becomes smaller and as households accumulate more assets, males gradually move to nonemployment. As a result, the employment rate of males begins to decline after the age of 50.

The model also replicates the lower average female employment rate. In the data, the employment rate for males is 0.87 while the employment rate for females is 0.63. In the model, these numbers are 0.87 and 0.59, respectively. Males enjoy a wage premium relative to females. Hence, males are usually the household's primary earner. Similar to the data, females postpone labor market entry for a longer time relative to males. The fixed cost of working is calibrated at a relatively high value in the first period of the life cycle. As a result, and given consumption insurance within the household, females stay out of the market for a longer time.

The middle panels of Figure 3 plot the average working hours for employed males and females, respectively. Both employed males and employed females do not vary their hours much along the life cycle. A relatively high value of θ helps to generate a small response of hours of work to wage increases during the life cycle.

Finally, the lower panels of Figure 3 plot transition rates: employment to employment and nonemployment to employment. The model matches the high and constant probability of staying employed for males. But the model generates a higher probability of staying employed for females than what we observe in the data. The model also replicates the

Figure 3: Labor Supply by Age



Note: **Upper Left Panel**: Life-cycle employment rate of males, data and model. **Upper Right Panel**: Life-cycle employment rate of females, data and model. **Middle Left Panel**: Life-cycle average hours of work for employed males, data and model. **Middle Right Panel**: Life-cycle average hours of work for employed females, data and model. **Lower Left Panel**: Transition rates for males by age a) Employment-to-Employment and b) Nonemployment-to-Employment, data and model. **Lower Right Panel**: Transition rates for females by age a) Employment-to-Employment and b) Nonemployment-to-Employment, data and model.

decreasing life-cycle probability of switching from nonemployment to employment for both males and females. In the presence of the search cost, household members prefer to spread their working years as little as possible and (most of them) retire once and for all once they have accumulated enough assets. This explains the very small probability of moving to employment for non-employed household members close to retirement.

Labor Supply by Household Assets The relationship between labor supply and household assets in the PSID follows the non-monotonic relationship between labor supply and age. Wealth-poor (relatively young) and wealth-rich (relatively old) households have a lower employment rate relative to households with assets around the mean (the middle-aged). To separate the two effects, I regress employment rates to the education level, age, age squared, and household assets. The regression is run separately for males and females. The results are reported in Table 1. Consistent with Figure 3, the age coefficients reveal a concave decreasing profile for both males and females. Conditional on age and education, males living in wealthy households are more likely to be employed. In contrast, females living in wealthier households are less likely to be employed.

The differential response of males and females to household assets is successfully captured by the model. For households with a low amount of assets (and thus consumption), the returns from having both members employed are high. Hence, conditional on age and skill, households with a low amount of assets are more likely to have two earners. In contrast, having a secondary earner is less beneficial for households with a large amount of assets. Since males enjoy a premium in the labor market, single-earner households will choose to employ the male member. This explains the negative response of females to household assets.

Reservation Wages In Table 2, I compare the model-generated reservation wages to direct evidence on self-reported reservation wages from the Survey of Income and Program Participation (SIPP). To my knowledge, this is the first paper to use empirical evidence on reservation wages to test the predictions of a heterogeneous-agent model. Reservation wage

Table 1: Regression Results: Employment Rates

	Data (PSID)		Model	
	Males	Females	Males	Females
Employment				
Constant	0.2426***	-0.0198	0.3917***	-0.9210***
Education	0.0559***	0.1123***	0.0570***	0.1723***
Age	0.0373***	0.0371***	0.0332***	0.0741***
Age^2	-0.0005***	-0.0004***	-0.0004***	-0.0008***
HH Assets	0.0038	-0.0083**	0.0036***	-0.0043***

Note: The table reports regression coefficients of employment on education, age and household assets. Coefficients with one, two, or three stars denote significance at 10%, 5%, and 1% level, respectively. Education is a dummy variable taking the value of 1 if education is equal or larger than 12 years of education. In the model education equals 1 if the household's skill is larger than the median. Employment takes the value of 1 if the person is employed and 0 otherwise. I normalize household assets to their mean.

is regressed on age, age squared, household assets, sex, and wage on the previous job. Each regressor is also run separately. The main findings are the following.

First, in the data, reservation wages are higher for older individuals. In some specifications the life-cycle profile of reservation wages is increasing and convex, and in others it is increasing and concave. However, in none of the specifications are the coefficients statistically significant. The model is in general consistent with an increasing age-profile of reservation wages. The returns to accumulating human capital are small for older household members. Hence, they ask for a larger wage in order to participate in the labor market.

Second, in the data, household assets are also positively related to reservation wages (Alexopoulos and Gladden, 2002). The coefficients are all statistically significant. In the model, males and females living in wealthier households can more easily substitute between employment and nonemployment. Hence, a regression of reservation wages on household assets delivers a positive coefficient. When controlling for age (specification (6)) the model coef-

Table 2: Regression Results: Reservation Wages

				Data (SIPP)			
Reservation Wage							
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.3045***	0.4031***	0.2957***	0.3183***	0.3972***	0.3568***	0.3249***
Age	0.0001	-0.0052	–	–	–	-0.0030	0.0021
Age^2	–	0.00006	–	–	–	0.00003	-0.00001
HH Assets	–	–	0.0277***	–	–	0.0275***	0.0269***
Sex (Male=1)	–	–	–	-0.0139	–	–	0.0375
Wage_{t-1}	–	–	–	–	−0.0122***	–	−0.0143***

				Model			
Reservation Wage							
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.3911***	4.2099***	0.9657***	1.4334***	1.4742***	4.1764***	4.8809***
Age	0.0141***	−0.1812***	–	–	–	−0.1801***	−0.1700***
Age^2	–	0.0022***	–	–	–	0.0022***	0.0020***
HH Assets	–	–	0.0091***	–	–	−0.0093***	0.0744***
Sex (Male=1)	–	–	–	−0.8662***	–	–	−0.6229***
Wage_{t-1}	–	–	–	–	−0.4675***	–	−0.6891***

Note: The table reports regression coefficients of reservation wages on age, household assets, sex, and wage on the previous job. Coefficients with one, two, or three stars denote significance at 10%, 5%, and 1% level, respectively. Reservation wages are based on declared reservation wages from the SIPP and are reported in 1984 hourly wage rates. I normalize the reservation wages to their mean both in the data and the model. I also normalize household assets to their mean.

ficient becomes negative. However, when I also include the previous wage as a control (a measure of ability) the relationship again becomes positive. High-skill household members can afford a lower market wage since their effective wages are high. Without a measure of ability, household assets will capture this negative relationship.

Third, in the model, males have lower reservation wages than females. This is true in specification (4), in the data. However, when I control for age, household assets, and sex (specification (7)) the coefficient becomes positive.

Overall, the model seems consistent with the positive relationship linking reservation

Table 3: Labor Supply Elasticity

	Intensive Margin		Extensive Margin	
	Males	Females	Males	Females
Age				
21-30	0.57	0.75	0.42	3.94
31-40	0.51	0.77	0.20	0.38
41-50	0.48	0.70	0.19	0.37
51-60	0.51	0.73	0.14	0.18
61-65	0.58	0.74	1.68	0.81
Wealth Quartile				
1^{st}	0.56	0.70	0.55	4.24
2^{nd}	0.51	0.75	0.22	0.49
3^{rd}	0.50	0.73	0.24	0.30
4^{th}	0.49	0.74	0.30	0.32
Aggregate	0.53	0.74	0.34	1.12

Note: The table reports the Frisch elasticity of labor supply across age and wealth groups. The intensive margin labor supply elasticity is the percentage change in labor supply in response to a one-percent change in the wage for previously employed workers. The extensive margin labor supply elasticity is the percentage change in labor supply in response to a one-percent change in the wage due to changes in the employment rate.

wages to age and household assets, as documented in the SIPP.

Labor Supply Elasticity In this section, I present how the labor supply elasticity varies across population groups. To compute the labor supply elasticity, I simulate the effects of a one-time unanticipated increase in the wage, holding the steady-state wealth distribution constant. Hence, one can interpret the elasticity as a Frisch elasticity of labor supply (Blundell, Costa-Dias, Meghir, and Shaw, 2013). Table 3 presents our results.

The intensive margin labor supply elasticity is the percentage change in labor supply in response to a one-percent change in the wage for previously employed household members.

The intensive margin labor supply elasticity is 0.53 for males and 0.74 for females. This value depends on parameter θ, which is calibrated at the value of 2. Intensive elasticities across age groups range from 0.51 to 0.58 for males and from 0.70 to 0.74 for females. In general, the dispersion of intensive margin elasticities is small because, conditional on employment, households work more or less the same amount of hours.

The extensive margin labor supply elasticity is the percentage change in labor supply in response to a one-percent change in the wage due to changes in the employment rate. The elasticity depends on the relative density of marginal workers around the market wage. The extensive margin elasticity (including both males and females) is equal to 0.73 and accounts for about 53% of the total elasticity of labor supply. The labor supply elasticity is 0.34 for males and 1.12 for females. Females can move easily between employment and nonemployment as they are usually not the only financial provider in the household.

There is the significant variation in the value of labor supply elasticity across age groups. Middle-aged males have small extensive margin elasticities relative to males closer to retirement. Periods of nonemployment are very costly for middle-ages males. Being employed allows household members to consume more and, more importantly, to build up human capital and financial savings. Hence, only a small fraction will be indifferent between employment and nonemployment. In contrast, due to a short time horizon, older males find it less costly to give up their job. At the same time, they can afford to do so since they have a larger amount of assets.

Females follow the same life-cycle pattern but feature much larger labor supply elasticity at very young ages. This means that many young (wealth-poor) households are willing to employ the secondary earner for a small wage increase. Indeed, females living in households whose assets are in the first quartile of the wealth distribution have very large elasticities. As with males, the elasticity increases in ages 61-65, but the increase is much less pronounced.

Most of the literature that focuses on the intensive margin points to a relatively small labor supply elasticity, especially for males. For example, Pistaferri (2003) reports a value of 0.70 which is a little higher than my value of 0.53. Erosa, Fuster, and Kambourov (2013) calculate the labor supply elasticity within a model with incomplete markets and nonlinear wages. They focus only on male workers and abstract from endogenous human capital. Their intensive margin elasticity is 0.67, relatively close to the value found in this paper. However, I find an extensive labor supply elasticity equal to 0.34 which is lower than their value of 1.08. In both papers, the age profile of the elasticity is U-shaped. Rogerson and Wallenius (2009) also find employment responses to a wage change that are concentrated among young and old workers. My values are very close to French (2005) who finds that at age 40 the labor supply elasticity is around 0.25 while at age 60 it is around 1.15. Finally, Jaimovich and Siu (2009) report that young and old cohorts experience much greater cyclical volatility in hours than the prime-aged. My findings are in general consistent with the age profiles reported in these papers.

4.4 Discussion

Wealth Heterogeneity It is important to check if the model generates a realistic amount of wealth heterogeneity. Table 4 reports the mean household assets for specific age groups (normalized by the average household assets). In both model and data, households gradually build up their assets to prepare for retirement. In the data, age group 31-40 own 0.51 of the average assets while age group 51-60 own 1.53 times the average. In the model, these numbers are 0.46 and 1.80, respectively.

Table 4 also reports wealth Gini coefficients. In the PSID, the coefficients are highest for households in their 20s, decrease between the ages 30-50, and rise again for ages 51-65. The model replicates this U-shaped profile. However, the average wealth Gini is 0.68 lower than 0.83 found in the data. The difficulty generating a concentrated wealth distribution is common in models with incomplete markets and idiosyncratic risk. Nevertheless, our model manages to generate a wealth Gini relatively close to the data without resorting to unrealistic amounts of wage dispersion.

Table 4: Household Assets

Age	Mean Household Assets: Data (PSID)	Mean Household Assets: Model	Wealth Gini: Data (PSID)	Wealth Gini: Model
21 – 30	0.12	0.07	1.01	0.83
31 – 40	0.51	0.46	0.83	0.61
41 – 50	1.01	1.11	0.82	0.54
51 – 60	1.53	1.80	0.87	0.52
61 – 65	1.80	2.10	0.96	0.54
Average	1.00	1.00	0.83	0.68

Note: The table reports the mean household assets and the wealth Gini for both model and data. Mean household assets are normalized by the average household assets for all age groups between 21 and 65.

Age-Dependent Parameters Our calibration employs several age-dependent parameters. To evaluate their importance, I re-calibrate the model under the restriction that parameters $FC^m, FC^f, \psi^m, \psi^f, sc^m, sc^f$ are not age-dependent. Figure 9 (Section B in the Appendix) shows the results. In spite of the minimal structure, the model can still replicate most of the basic features of the labor market. Hence, primarily, the model can match life-cycle behavior based on its internal mechanics.

5 Optimal Tax System

This section sets up the main quantitative experiment. I examine the properties of a tax system that depends separately or jointly on age, household assets, and filing status (single or joint). The objective is to maximize the welfare of the newborn household at the new steady state. I report the properties of the optimal tax system(s) and show the aggregate effects of each reform.

Ramsey Problem The welfare function is the ex-ante expected life-time utility of the

Table 5: Policy Instruments

Tag	$\tau_0(.)$
Age	$\tau_{00} + \tau_{01}j + \tau_{02}j^2 + \tau_{03}j^3$
HH Assets	$\tau_{00} + \tau_{01}a + \tau_{02}a^2$
Filing Status	τ_{00}^i for $i = \{S, J\}$
Age and HH Assets	$\tau_{00} + \tau_{01}j + \tau_{02}j^2 + \tau_{03}j^3 + \tau_{04}a + \tau_{05}aj$
Age and Filing Status	$\tau_{00}^i + \tau_{01}^i j + \tau_{02}^i j^2 + \tau_{03}^i$ for $i = \{S, J\}$
HH Assets and Filing Status	$\tau_{00}^i + \tau_{01}^i a + \tau_{02}^i a^2$ for $i = \{S, J\}$
Age, HH Assets, and Filing Status	$\tau_{00}^i + \tau_{01}^i j + \tau_{02}^i j^2 + \tau_{03}^i j^3 + \tau_{04}^i a + \tau_{05}^i aj$ for $i = \{S, J\}$

Note: The table describes the parametrization of tax parameter τ_0 for different tax functions that depend separately or jointly on age, household assets, and filing status.

newborn household.[10] Formally, it can be written as

$$SWF = \int V_{z1}^0(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1}) d\phi_{z1}(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1}) \tag{27}$$

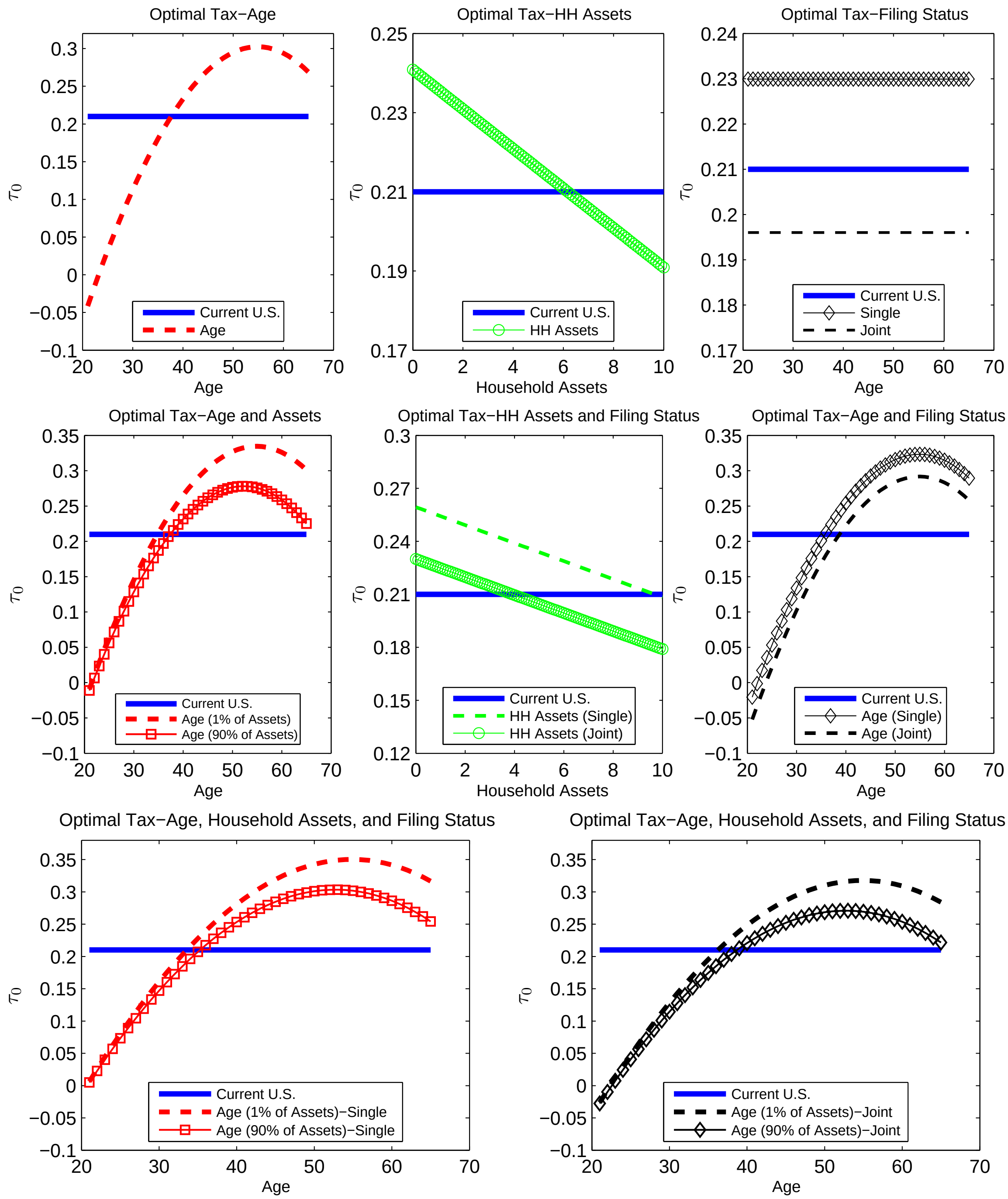
where a is the initial asset holdings, $\mathbf{x}$ is equal to the mean productivity, $\boldsymbol{\kappa} = \{1, 1\}$, and $\mathbf{E}_{-1} = \{u, u\}$. The integral is taken over possible types z.

The set of policy instruments includes a wide range of tax functions that depend separately or jointly on multiple tags such as age, households assets, and filing status (single or joint filers). This takes place using the parameter τ_0. I describe our parametrization in Table 5. Our specification is flexible enough to capture nonlinear effects as well as interaction between the tags. The problem is solved in two stages. For a given set of tax instruments, I find the competitive equilibrium allocations that also satisfy the government budget constraint. I then iterate over the tax parameters to find the set that maximizes the social welfare function.

Properties of the Optimal Tax Function Figure 4 describes the optimal labor-income

[10] The social welfare function corresponds to a Utilitarian view of tax policy. Although a commonly used welfare criterion, it is possible that this objective does not represent the true preferences of the society (Weinzierl, 2014).

Figure 4: Optimal Tax Properties



Note: **Upper Left Panel**: Optimal tax rates by age: age-dependent taxes. **Upper Middle Panel**: Optimal tax rates by household assets: asset-dependent taxes. **Upper Right Panel**: Optimal tax rates by filing status: household-dependent taxes. **Middle Left Panel**: Optimal tax rates by age and household assets (1^{st} percentile and 90^{th} percentile of wealth distribution): age- and asset-dependent taxes. **Central Panel**: Optimal tax rates by household assets for single and joint filers: asset- and household-dependent taxes. **Middle Right Panel**: Optimal tax rates by age for single and joint filers: age- and household-dependent taxes. **Lower Left Panel**: Optimal tax rates by age and household assets (1^{st} percentile and 90^{th} percentile of wealth distribution) for single filers: age-, asset- and household-dependent taxes. **Lower Right Panel**: Optimal tax rates by age and household assets (1^{st} percentile and 90^{th} percentile of wealth distribution) for joint filers: age-, asset- and household-dependent taxes.

tax code for every possible set of tax instruments (Table 5). The main findings are the following:

1. Average tax rates are hump shaped in age. In the benchmark economy, a two-earner household with average income ($86,509 in the PSID) pays 19% of its income in taxes. In an economy using all tax tags, the same household pays 2% at age 25, 26% at age 45, and 24% at age 65.[11]

2. Average tax rates linearly decrease in household assets. If the government tags both age and households assets, tax rates decrease only for wealthier households closer to retirement. Hence, age and household assets interact within the optimal policy.

3. Two-earner households face a smaller tax rate relative to single-earner households with the same income. Under the current U.S. system a one-earner household pays 19.4% of its income compared to 19.6% for a household filing jointly.[12] In the optimal economy, the single filers pay (on average) 23.8% in taxes while the joint filers pay (on average) 18.6%. If the government can differentiate between single and joint filers, the negative interaction between age and household assets decreases, but only by a small amount. Hence, filing status neither substitutes nor complements the other tags.

Aggregate Effects of Reforms The proposed reforms are associated with large gains. Table 6 reports the percentage change in aggregate variables between the benchmark and the optimal economy for each tax system. Both age- and asset-dependent taxes can increase capital by 9.5%. Consumption also increases by 0.9% and 2.6%, respectively. However, a tax system tagging age can distribute the consumption gains more evenly (the consumption Gini decreases by 1.7% compared to the benchmark). As a result, welfare increases by 0.4%, compared to just 0.1% in the case of asset-dependent taxes.[13]

[11]These calculations are for a two-earner household with assets equal to the mean assets ($229,008 in the PSID).

[12]This is due to parameter τ_1, which is calibrated at different values between single and joint filers.

[13]Welfare is measured in terms of consumption equivalent variation. The welfare gains are computed using

Table 6: Aggregate Effects of Tax Reforms

Tag	K	L	C	w	r	Cons. Gini	CEV
Age	+9.5%	-0.8%	+0.9%	+4.1%	-0.6%	-1.7%	+0.4%
HH Assets	+9.5%	+0.6%	+2.6%	+3.0%	-0.5%	+1.7%	+0.1%
Filing Status	+2.8%	+1.9%	+3.5%	+0.2%	0.0%	-0.2%	+0.5%
Age and HH Assets	+17.7%	+0.6%	+3.7%	+5.8%	-1.1%	+0.3%	+1.0%
Age and Filing Status	+11.7%	+1.1%	+3.8%	+3.6%	-0.7%	-2.1%	+0.9%
HH Assets and Filing Status	+12.3%	+2.9%	+5.7%	+3.2%	-0.6%	+1.5%	+0.8%
Age, HH Assets, and Filing Status	+19.7%	+2.7%	+6.4%	+5.6%	-1.0%	-0.2%	+1.5%

Note: The table reports the percentage change in macroeconomic variables for every possible tax reform. For the interest rate and consumption Gini I report the change in percentage points between the optimal and the benchmark economy.

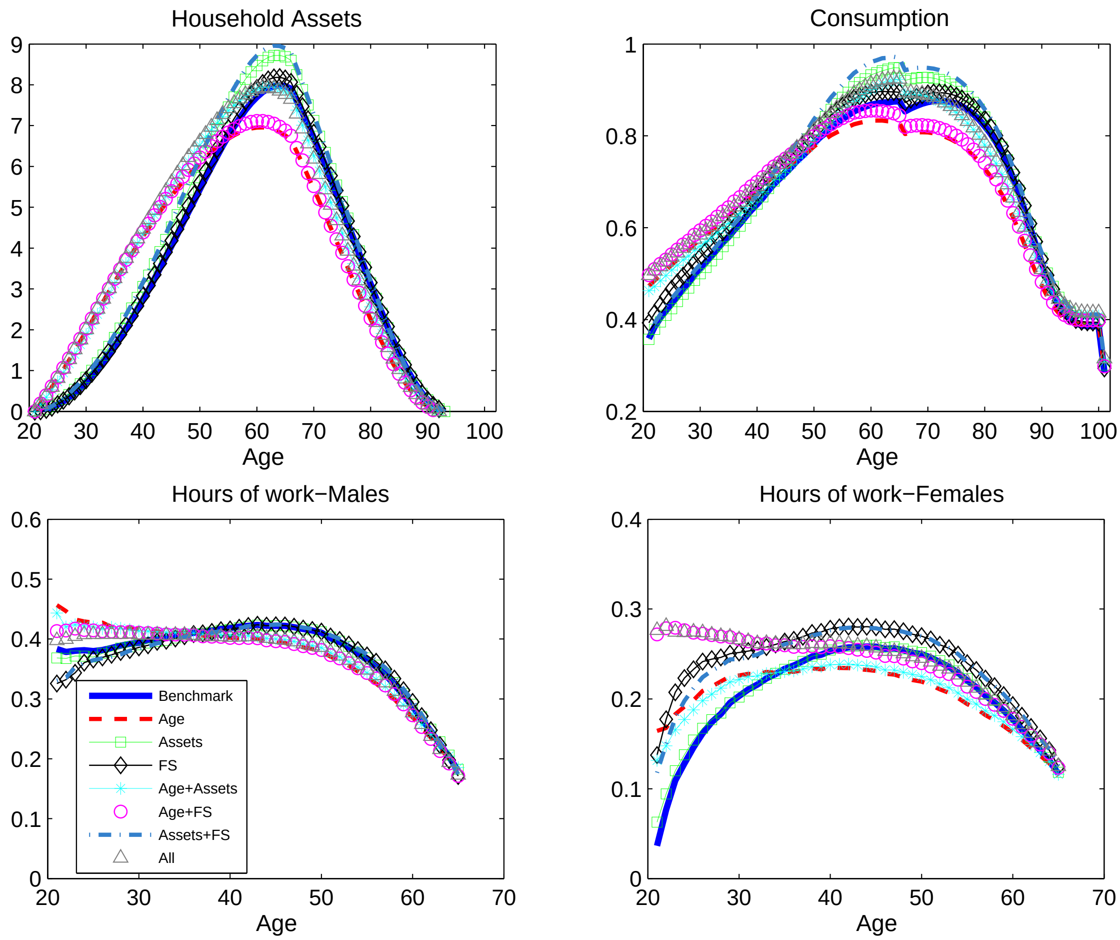
Age and household assets acts as complements within the optimal labor-income tax policy. In the case of age- and asset-dependent taxes, capital increases by a large 17.7%. This happens because each tag affects a different part of the life cycle. Age-dependent taxes encourage savings at younger ages while asset-dependent taxes increase savings during older ages (see Figure 5). Moreover, in spite of the much larger capital stock, labor supply (measured in efficiency units) increases by 0.6%. As a result, welfare increases by 1.0%, almost twice the sum of the separate policies.

While age and household assets can greatly affect savings incentives, they have a small impact on labor supply. The opposite is true for filing status. In this case, labor supply increases by 1.9%. The number of two-earner households increase significantly in the new steady state. More importantly, in spite of the heavier tax burden, very few single-earner households choose nonemployment. This policy can also increase aggregate consumption (by 3.5%) and decrease consumption inequality (by 0.2%). Welfare increases by 0.5%.

Filing status seems orthogonal to age and household assets tags. When age (or household assets) is used together with filing status, the gains are almost equal to the additive sum of

the formula $CEV = e^{\frac{1-\beta}{1-\beta^{J+1}}(W_2 - W_1)} - 1$ where W_1 and W_2 are the exante welfare of the newborn at the old and the new steady state, respectively.

Figure 5: Life-cycle Profiles



Note: The figure shows the effects of each reform on average life-cycle profiles: household assets, consumption, and hours of work (males and females).

the separate policies. Hence, the policies do not interact significantly, but they do not crowd out each other either. Filing status can encourage labor supply (especially by females) while age and household assets can encourage saving. If filing status depends on age the welfare gains are 0.9%; if it depends on household assets, they are 0.8%.

Overall, a policy that uses all tags can increase capital by a large 19.7%, labor supply by 2.7%, consumption by 6.4%, and welfare by 1.5%.

Figure 5 shows the effects of each reform on life-cycle profiles: household assets, consumption, and hours of work (males and females). Age-dependent taxes tilt hours of work (and thus consumption and savings) toward younger ages. When combined with asset-dependent taxes they can encourage savings throughout the life cycle. The relative decrease in tax rates allows older households to work almost as much as they do in the benchmark economy. Finally, filing status encourages average female hours without decreasing significantly the average male hours. When combined with age and household assets, the total effect is nearly the sum of the individual policies.

6 Alternative Models

The benchmark model features a) endogenous human capital accumulation and b) heterogeneity in labor supply elasticity. I consider two alternative specifications. The first shuts down endogenous human capital accumulation and is titled "Exogenous Human Capital." The second shuts down endogenous human capital and considers an environment with a constant elasticity of labor supply. This specification is titled "Constant Elasticity Model." For both I calculate the optimal labor-income tax system that depends on household characteristics using a parametric form similar to the benchmark. Table 9 in Section C in the Appendix shows the aggregate effects of the reforms under the new specifications.

Exogenous Human Capital In this case, life-cycle productivity evolves deterministically. Equation 11 is replaced with:

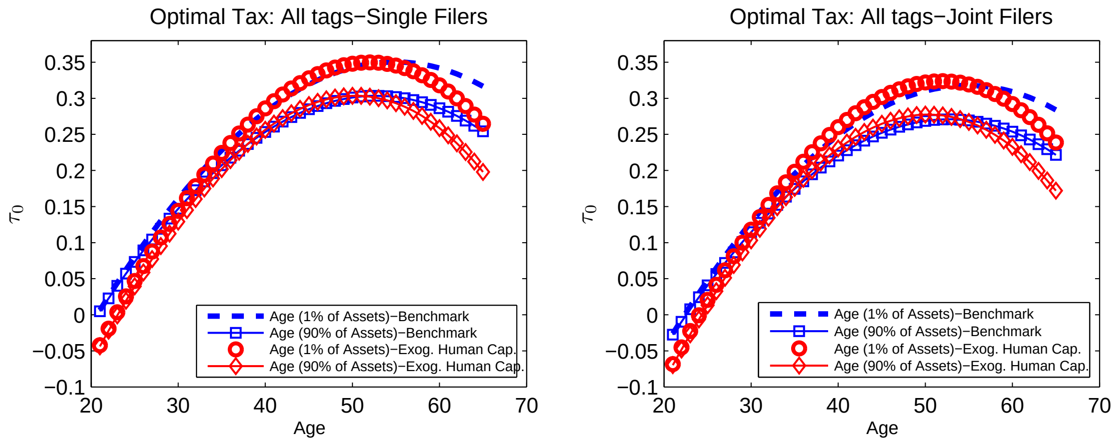
$$\log \epsilon_j^i = \chi_1 \log(\chi_0^i + j), \tag{28}$$

which substitutes experience κ with the exogenous variable age j.

Figure 6 plots the optimal labor-income tax that depends on all tags. Without endogenous human capital, the hump shape of life-cycle taxes becomes more pronounced. Young and old households pay (relatively) smaller taxes compared to the benchmark case. The opposite is true for middle-aged households.

In the "Exogenous Human Capital" model, nonemployment spells carry no wage penalty. As a result, young households can relatively easily move between employment and nonem-

Figure 6: Optimal Tax Properties-Benchmark vs. Exogenous Human Capital



Note: The figure compares the optimal tax rates in the benchmark and the "Exogenous Human Capital" economy. **Left Panel**: Optimal tax rates by age and household assets for single filers. **Right Panel**: Optimal tax rates by age and household assets for joint filers.

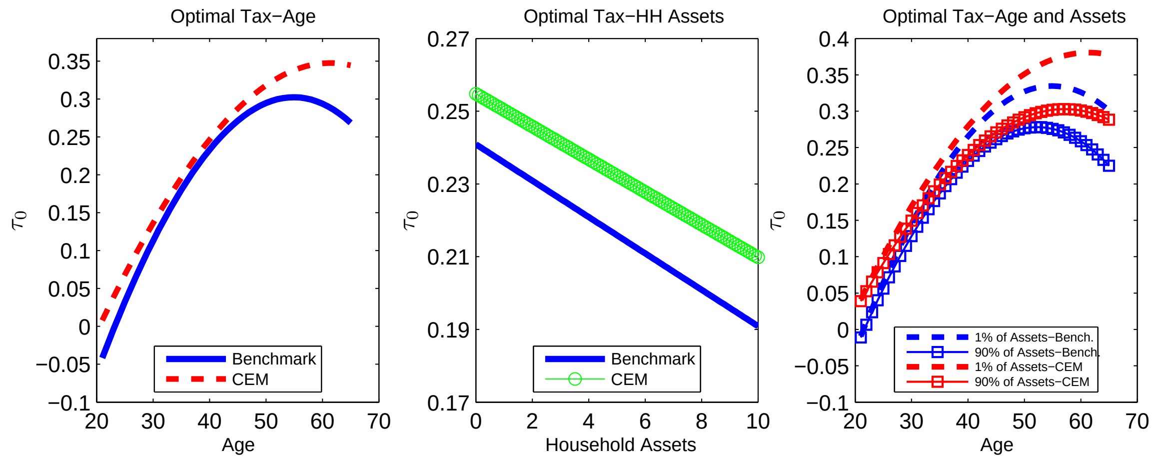
ployment (i.e. they have a larger value of labor supply elasticity). Hence, tax cuts for young households have a larger effect compared to the benchmark model. The second difference is the larger tax cut for older households. Without endogenous human capital, households expect a high wage at older ages independently of their labor market history. As a result, they have a smaller incentive to accumulate assets when young and a larger incentive to accumulate assets when old. A higher stock of assets at older ages gives households the option to retire early. The optimal labor-income tax decreases tax rates to prevent this outcome.

Constant Elasticity Model In the second specification, I explore the implications of heterogeneity in labor supply elasticity on optimal labor-income taxation. In particular, I consider a single-earner household with a divisible labor supply decision. Preferences are given by a Frisch utility function

$$U = \log c_j + \psi \frac{h_j^{1+\frac{1}{\theta_c}}}{1+\frac{1}{\theta_c}}. \tag{29}$$

Given the utility specification, and absent of an extensive margin of labor supply, the labor

Figure 7: Optimal Tax Properties-Benchmark vs. CEM



Note: The figure compares the optimal tax rates in the benchmark and the "Constant Elasticity Model." **Left Panel**: Optimal tax rates by age: age-dependent taxes. **Middle Panel**: Optimal tax rates by household assets: asset-dependent taxes. **Right Panel**: Optimal tax rates by age and household assets (1^{st} percentile and 90^{th} percentile of wealth distribution): age- and asset-dependent taxes.

supply elasticity is the same across agents (and equal to θ_c). I use a value of $\theta_c = 0.73$ equal to the average labor supply elasticity found in the benchmark economy.[14]

Since this is a single-earner economy, I characterize the optimal tax system that depends on age and household assets (Figure 7). In the benchmark economy, the optimal tax schedule is hump shaped in age. In contrast, in the CEM, the tax schedule is monotonically increasing in age, albeit at a decreasing rate. In the benchmark economy, it is optimal to provide tax incentives for older workers with a large value of labor supply elasticity. There is no such consideration in the CEM. These findings are in line with Weinzierl (2011), Farhi and Werning (2013), and Golosov, Troshkin, and Tsyvinski (2015) who find an increasing life-cycle tax distortion to be optimal.

7 Conclusion

In this paper I analyze optimal labor-income taxation that depends on age, household assets, and filing status (single or joint). The optimal labor-income tax rates are hump shaped

[14] In Section C in the Appendix, I show the aggregate effects of an age- and asset-dependent reform for a range of labor supply elasticities.

in age, decrease in household assets, and are lower for joint filers compared to single filers. The motivation for such a reform is the significant heterogeneity in the elasticity of labor supply among households. In the absence of such heterogeneity, tax rates should be monotonically increasing in age.

The paper also evaluates how tags interact within the optimal labor-income tax policy. There is a strong complementarity between age and assets. In particular, when combined, age and asset increase welfare twice as much as the sum of the individual policies. Filing status neither crowds out nor complements the other tags. A policy using all tags can increase capital by 19.7%, labor by 2.7%, consumption by 6.4%, and welfare by 1.5%.

References

Albanesi, S., and Sleet, C. (2006). "Dynamic optimal taxation with private information". *Review of Economic Studies*, *47*(1), 1-27.

Alexopoulos, M., and Gladden, T. (2002). "Wealth, reservation wages and labor market transitions in the U.S.: Evidence from the SIPP". *Working Paper, University of Toronto*.

Attanasio, O., Low, H., and Sanchez-Marcos, V. (2008). "Explaining changes in female labor supply in a life cycle model". *American Economic Review*, *98*(4), 1517-1552.

Blau, F. D., and Kahn, J. (2000). "Gender differences in pay". *The Journal of Economic Perspectives*, *14*(4), 75-99.

Blundell, R., Costa-Dias, M., Meghir, C., and Shaw, J. (2013). "Female labour supply, human capital and welfare reform". *NBER Working Paper, No. 19007*.

Chang, Y., and Kim, S. (2006). "From individual to aggregate labor supply: a quantitative analysis based on a heterogeneous agent macroeconomy". *International Economic Review*, *47*(1), 1-27.

Conesa, J. C., Kitao, S., and Krueger, D. (2009). "Taxing capital? Not a bad idea after all!". *American Economic Review*, *99*(1), 25-48.

da Costa, C., and Santos, M. (2013). "Age-dependent taxes with endogenous human capital formation". *Working Paper*.

Domeij, D., and Floden, M. (2006). "The labor supply elasticity and borrowing constraints: Why estimates are biased". *Review of Economic Dynamics*, *9*(2), 242-262.

Erosa, A., Fuster, L., and Kambourov, G. (2013). "Towards a micro-founded theory of aggregate labor supply". *Working Paper, University of Toronto*.

Erosa, A., and Gervais, M. (2002). "Optimal taxation in life cycle economies". *Journal of Economic Theory*, *105*(2), 338-369.

Farhi, E., and Werning, I. (2013). "Insurance and taxation over the life cycle". *Review of Economic Studies*, *80*(2), 596-635.

French, E. (2005). "The effects of health, wealth, and wages on labor supply and retirement behavior". *Review of Economic Studies*, *72*(2), 395-427.

Golosov, M., Troshkin, M., and Tsyvinski, A. (2015). "Redistribution and social insurance". *Working Paper, Cornell University*.

Guner, N., Kaygusuz, R., and Ventura, G. (2012a). "Taxation and household labour supply". *Review of Economic Studies*, *79*(3), 987-1020.

Guner, N., Kaygusuz, R., and Ventura, G. (2012b). "Taxing women: A macroeconomic analysis". *Journal of Monetary Economics*, *59*(1), 111-128.

Hansen, G. (1985). "Indivisible labor and the business cycle". *Journal of Monetary Economics*, *16*(3), 309-327.

Heathcote, J., Storesletten, K., and Violante, G. (2014). "Optimal tax progressivity: An analytical framework". *NBER Working Paper, No 19899*.

Imrohoroglu, S., and Kitao, S. (2012). "Social security reforms, benefit claiming, labor force participation and long run sustainability". *American Economic Journal: Macroeconomics*, *4*(3), 96-127.

Jaimovich, N., and Siu, H. E. (2009). "The young, the old, and the restless: Demographics and business cycle volatility". *American Economic Review*, *99*(3), 804-826.

Kapicka, M., and Neira, J. (2015). "Optimal taxation with risky human capital". *Working Paper*.

Kitao, S. (2010). "Labor-dependent capital income taxation. *Journal of Monetary Economics*, *57*(8), 959-974.

Kocherlakota, N. (2005). “Zero expected wealth taxes: A mirrlees approach to dynamic optimal taxation”. *Econometrica*, *73*(5), 1587-1621.

Krueger, D., and Ludwig, A. (2013). “Optimal progressive labor income taxation and education subsidies when education decisions and intertemporal transfers are endogenous”. *American Economic Review*, *103*(3), 496-501.

Low, W. H. (2005). “Self insurance in a life-cycle model of labour supply and savings”. *Review of Economic Dynamics*, *8*(4), 945-975.

Pencavel, J. (1998). “Assortative matching by schooling and the work behavior of wives and husbands”. *American Economic Review*, *88*(2), 326-329.

Peterman, W. (2012). “The effects of endogenous human capital accumulation on optimal taxation”. *Working Paper*.

Pijoan-Mas, J. (2006). “Precautionary savings or working longer hours?”. *Review of Economic Dynamics*, *9*(2), 326-352.

Pistaferri, L. (2003). “Anticipated and unanticipated wage changes, wage risk, and intertemporal labor supply”. *Journal of Labor Economics*, *21*(3), 729-754.

Prescott, E., Rogerson, R., and Wallenius, J. (2009). “Lifetime aggregate labor supply with endogenous workweek length”. *Review of Economic Dynamics*, *12*(1), 23-36.

Rogerson, R. (1988). “Indivisible labor, lotteries and equilibrium”. *Journal of Monetary Economics*, *21*(1), 3-16.

Rogerson, R., and Wallenius, J. (2009). “Micro and macro elasticities in a life cycle model with taxes”. *Journal of Economic Theory*, *144*(6), 2277-2292.

Stantcheva, S. (2015). “Optimal taxation and human capital policies over the life cycle”. *Working Paper*.

Storesletten, K., Telmer, I. C., and Yaron, A. (2004). "Consumption and risk sharing over the life cycle". *Journal of Monetary Economics*, *51*(3), 609-633.

Tauchen, G. (1986). "Finite state markov-chain approximations to univariate and vector autoregressions". *Economics Letters*, *20*, 177-181.

Weinzierl, M. (2011). "The surprising power of age-dependent taxes". *Review Economic Studies*, *78*(4), 1490-1518.

Weinzierl, M. (2014). "The promise of positive optimal taxation". *Forthcoming, Journal of Public Economics*.

Appendix

A Data Sources

Data sources include the Panel Study of Income Dynamics (PSID), the Survey of Income and Program Participation (SIPP), and the Current Population Survey (CPS). Data from PSID include information on employment rates, hours of work, wages, and household assets. I use waves from 1970 to 2005. The survey was conducted annually up to 1997 and biannually from 1999 to 2005. For each year, data are collected for both the head of the household and the "wife" of the household. These are the total amount of hours supplied, their annual labor income, and their sex. For hours, I use the variables "Head Annual Hours of Work" and "Wife Annual Hours of Work." These variables represent the total annual work hours on all jobs including overtime. Information on labor income is collected using variables "Head Wage" and "Wife Wage," which include wages and salaries. Households with a single female primary earner are excluded from the analysis. The measure of wealth is the variable WEALTH2 as found in specific waves of PSID. This variable is constructed as sum of values of several asset types (family farm business, family accounts, assets, stocks, houses, and other real estate etc.) net of debt value.

Information about reservation wages is collected using the SIPP. This is available in the topical module of Wave 5 for 1984. The SIPP sample design consists of 21,000 household units. Each household was interviewed at four-month intervals and was asked questions about the four months before the interview day. The data offers information on household residents like education, age, gender, race, marital status, etc., as well as employment history for the past four months. Individuals who also experienced at least one spell of unemployment in between the interviews are also asked about the minimum wage for which they would be willing to work. In addition, the SIPP makes available information on the total net worth of the household assets. Information about assets is included in the topical module of Wave 4. Alexopoulos and Gladden (2002) state that collective evidence supports that wealth information from the SIPP is comparable to the wealth information from the PSID.

To estimate the progressivity of the U.S. tax schedule, I use data from the CPS for the period 1992-2005. In particular, I gather information for the individual's annual working hours (usual weekly hours $\times$ weeks worked), family income, marital status, type of filing (nonfiler, single, joint), and marginal tax rates. The sample is restricted to individuals who work between 800 and 5200 hours, who report positive family income, and who are between the age, of 21 and 70. I describe the estimation of parameters in the following section.

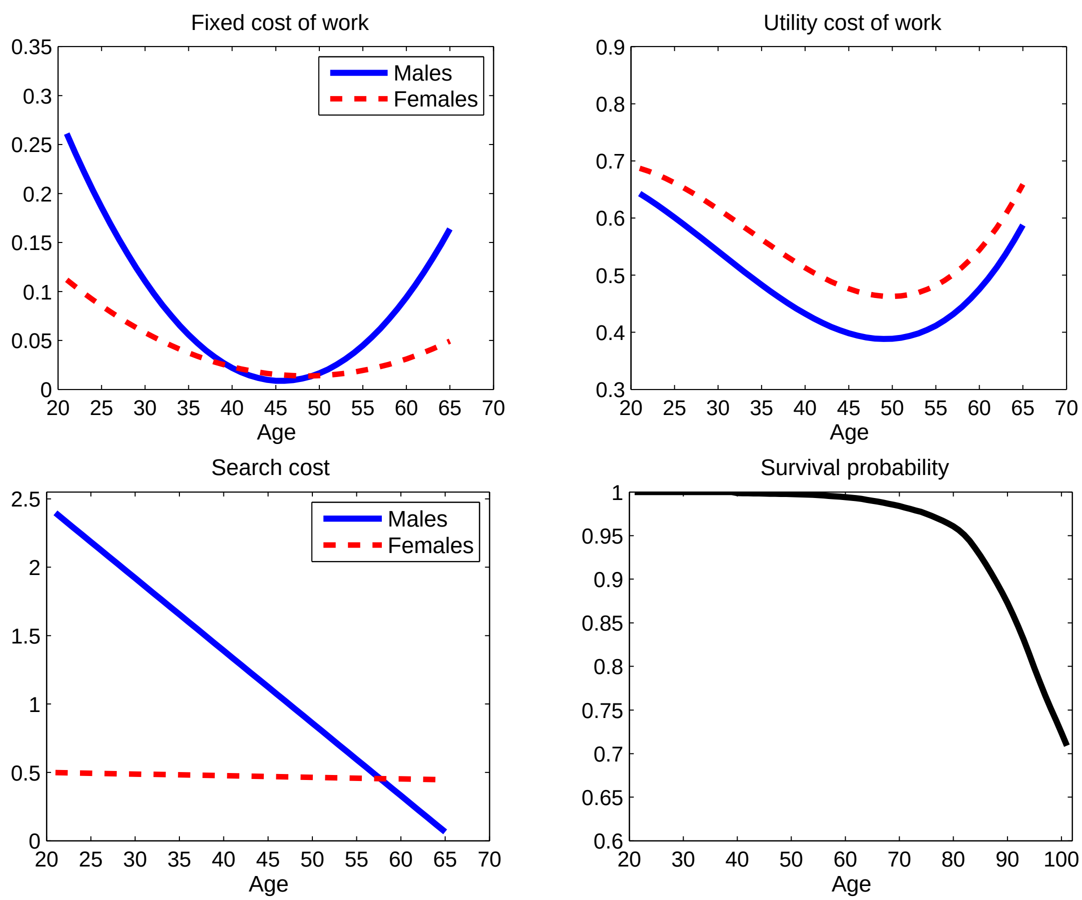
B Calibration

Table 7 reports the parameter values used in the benchmark model. I also report the accuracy of the calibration.

B.1 Tax Parameters

Parameters τ_1(S) and τ_1(J) are estimated using data from the CPS. Differentiating and taking the logarithm of (15), I obtain a linear relationship between the logarithm of (one minus) the marginal tax rate and the logarithm of household labor earnings. The elasticity is our estimate of τ_1. I estimate two separate equations, one for married individuals who file jointly and one for single filers regardless of whether they are married or not. The values found are $\tau_1(\text{S}) = 0.073$ and $\tau_1(\text{J}) = 0.065$.

Figure 8: Parameter Values



Note: **Upper Left Panel**: Fixed cost of work (FC) for males and female. **Upper Right Panel**: Utility cost (ψ) for males and females. **Lower Left Panel**: Search cost (sc) for males and females. **Lower Right Panel**: Survival probability.

Table 7: Parameter Values

Parameter	Description	Value	Reference	
J	Length of lifetime	81	Standard	
J^R	Retirement age	45	Standard	
n	Population growth	1.1%	US long-run average	
α	Technology parameter	0.36	Capital share	
θ	Preference parameter	2	Erosa, Fuster, and Kambourov (2013)	
τ_{ss}	Social security tax	0.106	Kitao (2010)	
τ_c	Consumption tax	0.05	Imrohoroglu and Kitao (2012)	
τ_k	Capital tax	0.30	Imrohoroglu and Kitao (2012)	
τ_1(S)	Tax parameter	0.073	CPS	
τ_1(J)	Tax parameter	0.065	CPS	
γ	Scale parameter	11.25	$\gamma = \frac{J-J^R}{N_{hc}}$	
χ_0^m	Human capital parameter	1	PSID	
χ_0^f	Human capital parameter	0.7	PSID	
χ_1	Human capital parameter	0.07	PSID	
$\{s_j\}$	Survival probabilities	Figure 8	Social security admin. (2005)	
Parameter	**Description**	**Value**	**Target (Data)**	**Model**
β	Discount factor	0.996	Capital-output ratio=3.20	3.16
δ	Depreciation rate	0.0816	Investment-output ratio=0.25	0.25
$\{\alpha_i^m\}_{i=0}^3$	Fixed cost males	Figure 8	Employment rate males	Figure 3
$\{\alpha_i^f\}_{i=0}^3$	Fixed cost females	Figure 8	Employment rate females	Figure 3
$\{\nu_i^m\}_{i=0}^4$	Utility cost males	Figure 8	Average hours males	Figure 3
$\{\nu_i^f\}_{i=0}^4$	Utility cost females	Figure 8	Average hours females	Figure 3
λ^m	Prob. of separation males	0.015	Probability males: $E \rightarrow NE$= 0.055	0.052
λ^f	Prob. of separation females	0.015	Probability females: $E \rightarrow NE$= 0.158	0.055
η_0^m	Search cost param. males	2.45	Probability males (21-42): $NE \rightarrow E$= 0.44	0.24
η_1^m	Search cost param. males	-0.053	Probability males (43-65): $NE \rightarrow E$= 0.16	0.11
η_0^f	Search cost param. females	0.5	Probability females (21-42): $NE \rightarrow E$= 0.18	0.08
η_1^f	Search cost param. females	-0.0012	Probability females (43-65): $NE \rightarrow E$= 0.09	0.05
$\underline{a}$	Borrowing limit	-0.10	Fraction of HHs with net debt=0.087	0.090
τ_0	Tax parameter	0.21	Government spending-output ratio=0.20	0.20
σ_z^2	Variance in skill distribution	0.27	Log-wage variance among 21-years old=0.27	0.27
ρ	Persistence of AR(1)	0.965	Average increase in log-points=0.013	0.011
σ_η^2	Variance of AR(1)	0.029	Log-wage variance among 60-years old=0.75	0.76

B.2 Female Labor Supply

Female labor force participation has been steadily increasing over the last decades. The average employment rate is 38% for females born between 1920-1930, 54% for females born between 1940-1950, and 66% for females born between 1960-1970. Since I use data for the period 1970-2005, there is no information on all age groups for every cohort. For example, for females born between 1920-1930, there is no information on age groups younger than 50. Similarly, for cohorts born after the year 1960 there is no information on age groups older than 50.

To calculate the age-profile of female employment rate, one would have to use information from different cohorts which might bias the estimates. Hence, I separate the age from the cohort effect using the following strategy. The employment rate for age groups between 21-42 is calculated based on cohorts born after 1950. For age groups 43 and onwards, I run an age-cohort dummy regression using cohorts before the year 1950. The age dummies are used to estimate the employment rate between 43-65 while the cohort effect of the last cohort is used to estimate the position of the schedule. The average employment rate for females in the middle-right panel of Figure 3 is a combination of these two profiles smoothed using a polynomial of the third degree.

B.3 Age-Dependent Parameters

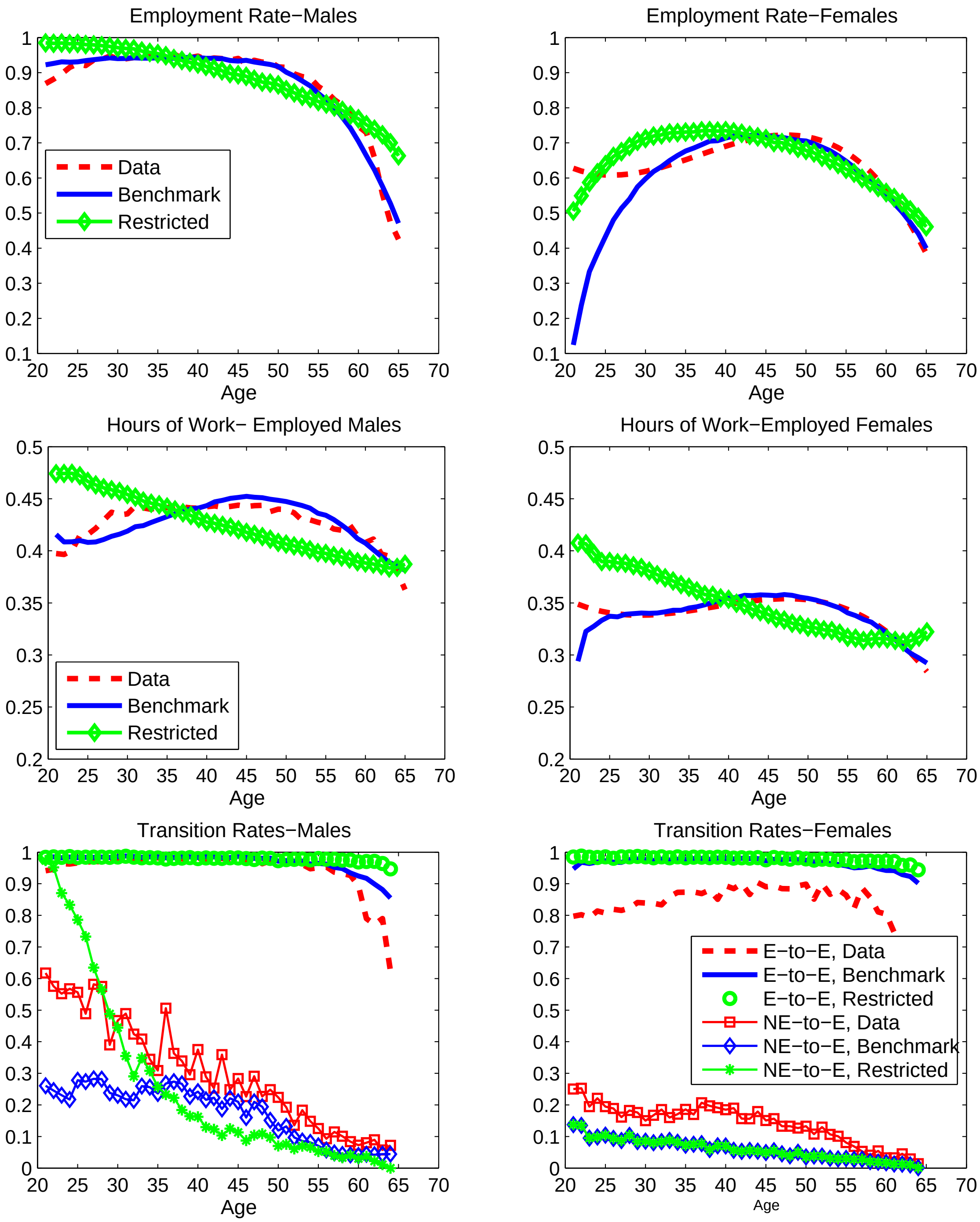
As can be seen in Table 7, some of our parameters are age dependent. It is possible that age dependence is crucial for the model to replicate the labor market statistics presented in Figure 3. Hence, one may wonder how well the model can do in the absence of age-dependent parameters. This part re-calibrates the model under the restriction that parameters $FC^m, FC^f, \psi^m, \psi^f, sc^m, sc^f$ are not age dependent but set to a constant value. The life-cycle profiles for this model (named "Restricted" model) are presented in Figure 9. In spite of the minimal structure, the model can still replicate most of the basic features of the labor market. The employment rate for males and females is still hump shaped and the transition rates are similar to the benchmark. The most notable difference is the hours of work, which is monotonically decreasing along the life cycle. Hence, primarily, the model has enough internal mechanics to generate reasonable life-cycle behavior even in the absence of age-dependent parameters.

C Alternative models-Aggregate Effects

This section reports the aggregate effects of the optimal labor-income policy in the "Exogenous Human Capital" model and the "Constant Elasticity Model."

Table 8 reports the effects and welfare gains for the model with exogenous human capital. Overall, the aggregate effects are in the same range. In addition, the presence of endogenous human capital seems to add 0.1% to our welfare gains. Table 9 compares the effects of the the age- and assets- dependent tax reform for both models. Changes in macro variables and welfare gains are close to the ones found in the benchmark economy. Table 9 also reports the effects in macro aggregates and welfare gains for different values of labor

Figure 9: Results without Age-dependent Parameters



Note: **Upper Left Panel**: Life-cycle employment rate of males, data, benchmark, and restricted model. **Upper Right Panel**: Life-cycle employment rate of females, data, benchmark, and restricted model. **Middle Left Panel**: Life-cycle average hours of work for employed males, data, benchmark, and restricted model . **Middle Right Panel**: Life-cycle average hours of work for employed females, data, benchmark, and restricted model. **Lower Left Panel**: Transition rates for males by age a) Employment-to-Employment and b) Nonemployment-to-Employment, data, benchmark, and restricted model. **Lower Right Panel**: Transition rates for females by age a) Employment-to-Employment and b) Nonemployment-to-Employment, data, benchmark, and restricted model.

Table 8: Aggregate Effects of Tax Reform: Benchmark and Exogenous Human Capital

Model	Benchmark	Exogenous Human Capital
K	+19.7%	+20.3%
L	+2.7%	+2.8%
C	+6.4%	+6.0%
w	+5.6%	+5.7%
r	-1.0%	-1.0%
Cons. Gini	-0.2%	-0.5%
CEV	+1.5%	+1.4%

Note: The Table reports the percentage change in aggregates due to the tax reform. The tax reform includes age, household assets, and filing status as tax tags. Results are presented for the benchmark case and two alternative specifications. A model with exogenous human capital accumulation and a model with constant elasticity of labor supply.

Table 9: Aggregate Effects of Tax Reform: Benchmark and CEM

Model	Benchmark	CEM		
		$\theta_c = 0.4$	$\theta_c = 0.73$	$\theta_c = 1.5$
K	+17.7%	+18.9%	+18.4%	+22.2%
L	+0.6%	+0.6%	+0.9%	+1.7%
C	+3.7%	+3.0%	+3.1%	+4.2%
w	+5.8%	+6.1%	+5.9%	+6.8%
r	-1.1%	-0.3%	-0.3%	-0.4%
Cons. Gini	+0.3%	+0.2%	+0.2%	+0.2%
CEV	+1.0%	+1.1%	+1.1%	+1.0%

Note: The Table reports the percentage change in aggregates due to the tax reform. The tax reform includes age and household assets as tax tags. Results are presented for the benchmark case and a model with constant elasticity of labor supply. For the latter case, I present the effects of the optimal tax system for a variety of labor supply elasticities: $\theta_c = 0.4$,$\theta_c = 0.73$, $\theta_c = 1.5$.

supply elasticity.[15] A larger value of labor supply elasticity can increase labor supply (and capital) by a larger

[15] In the benchmark model the (heterogeneity in) labor supply elasticity depends mostly on the distribution of reservation wages and not on the value θ (see Hansen (1985), Rogerson (1988), and Chang and Kim (2006)). In contrast, varying parameter θ_c in the CEM will affect uniformly all agents.

amount. However, welfare gains seem robust across specifications.

D Simpler Policies

In this section, I restrict the age-dependent tax system to simpler forms. In particular, I analyze a linear form, $\tau_{00} + \tau_{01}j$, a second-degree polynomial, $\tau_{00} + \tau_{01}j + \tau_{02}j^2$, and compare them to our benchmark specification: $\tau_{00}+\tau_{01}j+\tau_{02}j^2+\tau_{03}j^3$. The aggregate effects and optimal tax properties are shown in Table 10 and Figure 10, respectively.

Table 10: Age-dependent Taxes: Simpler Functional Forms

	Linear	Second-Degree	Benchmark
Functional Form	$\tau_{00} + \tau_{01}j$	$\tau_{00} + \tau_{01}j + \tau_{02}j^2$	$\tau_{00} + \tau_{01}j + \tau_{02}j^2 + \tau_{03}j^3$
K	+5.3%	+5.0%	+9.5%
L	-1.0%	-0.7%	+0.8%
C	+0.1%	+0.4%	+0.9%
w	+2.2%	+2.0%	+4.1%
r	-0.4%	-0.4%	+0.6%
Cons. Gini	-1.1%	-1.3%	-1.7%
CEV	+0.2%	+0.3%	+0.4%

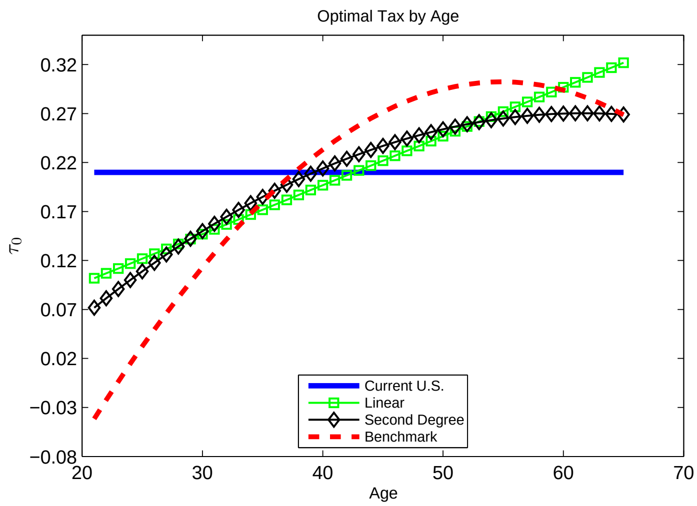
Note: The table reports the percentage change in aggregates due to an age-dependent tax reform. Results are presented for a linear tax function, a second-degree polynomial and the benchmark specification.

A linear increasing function increases capital by a smaller amount compared to the benchmark. Moreover, labor supply decreases as the tax distortion induces older households to retire earlier. If the tax function is a second-degree polynomial then the tax distortion also increases but a smaller rate for older households. This makes the employment reduction smaller. Using the benchmark specification (third-degree polynomial), we can increase capital by a larger amount but also increase employment. The flexibility allows to increase tax distortions steeply up to age 50 and decrease taxes after that age. Although relatively small, welfare gains double compared to a linear tax function.

E Household's Problem: Value Functions

In this section, I write the value function for a household employing the female worker $V^{\{NE,E\}}$ and a household whose members are not employed $V^{\{NE,NE\}}$. The value function for a household employing the

Figure 10: Optimal Age-dependent Taxes: Simpler Functional Forms



Note: The figure plots the properties of optimal age-dependent taxes for different functional forms: linear, second-degree polynomial and benchmark case (third-degree polynomial).

female worker is:

$$V_{zj}^{\{NE,E\}}(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1}) = \max_{c,a',h^f} \Bigg\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta} - \zeta(E_{-1}^f)$$

$$+\beta s_{j+1} \sum_{x_m'} \sum_{x_f'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \ *$$

$$[\frac{(1-\lambda^m)}{1-p} \sum_{s=\{2,3\}} \boldsymbol{p}^s V_{z(j+1)}^1(a', \mathbf{x}', \boldsymbol{\kappa}^s, \mathbf{E}) + \frac{\lambda^m}{1-p} \sum_{s=\{2,3\}} \boldsymbol{p}^s V_{z(j+1)}^{\{NE,NE\}}(a', \mathbf{x}', \boldsymbol{\kappa}^s, \mathbf{E})] \Bigg\} \quad (30)$$

$$\textbf{s.t.} \qquad h^f = 0 \quad (31)$$

$$(1+\tau_c)c + a' = (1-\tau_{ss})W - T_L(W; \mathrm{S}) + (1 + r(1-\tau_k))(a + Tr) \quad (32)$$

$$\mathbf{E} = \{u, e\} \quad (33)$$

The value function for a household with no earners is:

$$V_{zj}^{\{NE,NE\}}(a,\mathbf{x},\boldsymbol{\kappa},\mathbf{E}_{-1}) = \max_{c,a'} \Bigg\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta}$$

$$+\beta s_{j+1} \sum_{x_m{}'} \sum_{x_f{}'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \;\; V_{z(j+1)}^2(a',\mathbf{x}',\boldsymbol{\kappa}^0,\mathbf{E})] \Bigg\} \quad (34)$$

$$\textbf{s.t.} \qquad h^m = 0, \quad h^f = 0 \quad (35)$$

$$(1+\tau_c)c + a' = (1 + r(1-\tau_k))(a + Tr) \quad (36)$$

$$\mathbf{E} = \{u, u\} \quad (37)$$